**Analog Electronic Circuits**
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**Lecture – 96**
**Effect of Feedback on Frequency Response (Part-B)**

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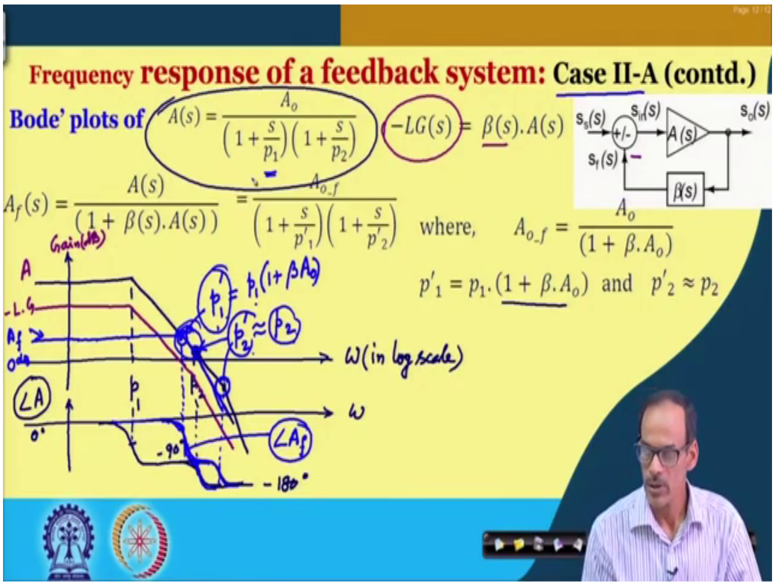


Welcome back after the short break. So before the break, we are talking about the location of the pole of the feedback system. And as you can see here, the condition and in fact, I should have taken this p 2 even beyond this location of p 1 dash for a clearer picture. So, if I consider say, p 2 it is even beyond this point, something like this and then if it is having p 2 ok.

So, if I consider this is the p 2m then the corresponding phase it could have been like this. For a and then p 2 dash on the other hand, it is instead of here, this is p 2 dash and which is same as p 2 then this blue line, the phase here for A f we could we can say that the phase it is having very clean step like this one.

Now if this p 2 it is in the near vicinity of p 1 dash, then what will happen? That is what we need to investigate in our next discussion. And we consider that case, it is case 2 b there also we consider 2 poles for A, but the conditions the condition is different. So let me clear the board and let me go to that case yeah.

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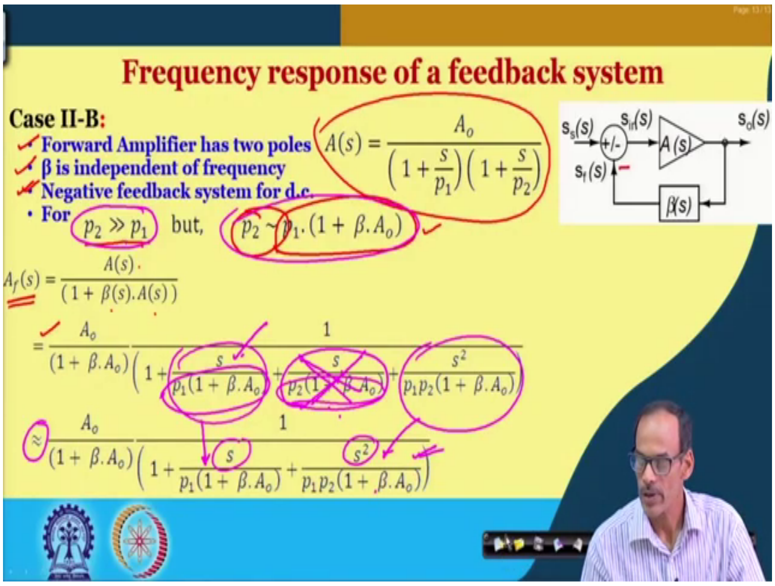


So here we do have the that situation. So first of all, forward amplifier it is having 2 poles; p 1 and p 2, beta is independent of frequency and the system of course, it is negative feedback system. And we consider p 1; it is lower frequency than p 2 which means that p 1 is referred as dominant pole.

But the anticipated shifted version of this p 1 namely p 1 dash, if it is comparable will with p 2, then what happens? So if I again if I come back to the feedback system gain A f which is having an expression of a divided by 1 plus beta into A and then we obtain this expression. Now while we will be doing the approximation of this equation, we can consider this case.

And we may say that instead of really considering this part, let we completely ignore this part to have a meaningful conclusion. So what we are doing is, we are retaining this part and we are keeping this part here, but this part since it is very small compared to this one, we may drop this part.

So with this approximation, what we are getting here it is we do have only these 3 terms and this is of course, valid because as I say that p 2 it is much higher than p 1. So if I multiply boths this pole by 1 plus beta into A naught, then this condition remain same which means that this part it is very very much very very large compared to this part.

And the reciprocal wise, if I see then I can say this is dominant term compared to this one. And hence, we can do this approximation, we can eliminate this part or we can drop this part. Now after dropping this part, what do we have? We do have of course, the second order equation and from this second order equation, if I say that these 2 are comparable, then a factorization of this pa equation may or may not be having meaningful factorization.

Or rather, I should say that we can do factorization, but while we will be doing the factorization the co-efficient need not be real. Forget about the integer, but the co-efficient what we will what we will be getting it may not be real, rather it may be even complex. So depending on the coefficient of s square, here and s and their relative value, we may get expression of the poles it will be complex. So in the next slide, we can do the you know, derivation of the location of the pole.

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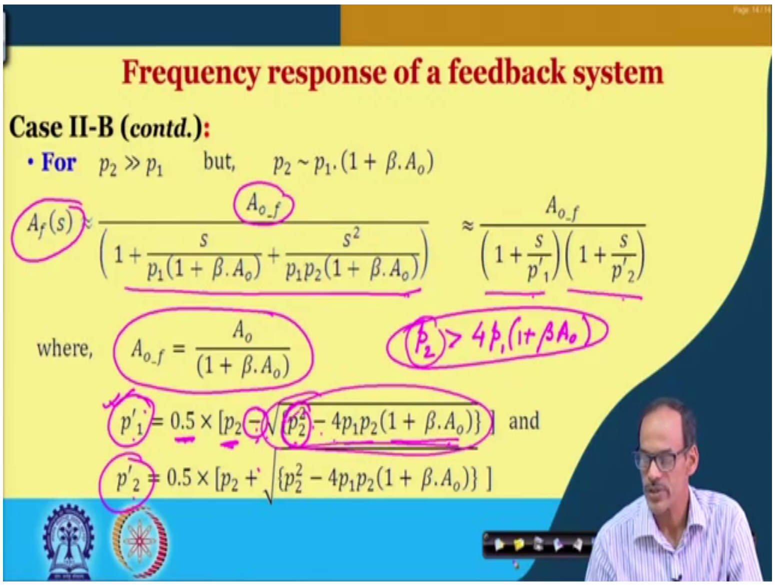


So if you yeah, so here we do have the expression of A f it is having in the numerator, we do have a naught A f and A naught f it is defined here A naught divided by 1 plus beta into A naught. And then this part if you see based on this co-efficient, we can write in this form.

However, this p 1 dashed and p 2 dashed, they are they need not be real number. In fact, if you consider the second order equation and then if you write the corresponding routes, what will be finding is that the routes are or other location of the pole p 1 dashed and p 2 dash, it is 1 by 2 multiplied by p 2 minus square root of p 2 squared minus 4 p 1 into p 2 multiplied by 1 plus beta into A naught.

So this is one possible route. The other one it is same except this sign instead of minus, it will be plus. So we can say that now based on the relative value of this p 2, p 2 square and 4 p 1 p 2 multiplied by 1 plus beta into A. So based on their relative value, you may get this part either real or it may be imaginary depending on the relative value of the positive part and negative part.

So, if I say that if p 2, it is higher than 4 times p 1 1 plus beta into A naught, then we can get this part it is real. On the other hand if it is less, then this part it will be imaginary. And then what we will be getting is that both p 1 and p 2, they are complex and incidentally, they are complex conjugate also.

So the real part it will be p 2 by 2, in that case and then imaginary part it is plus minus. In fact for p 1 and p 2, we do have plus and minus. And then the imaginary part, then we have to find what will be the value here. So this condition it is finally, telling us that whether these 2 poles they are really real poles remaining as real pole or they are becoming complex conjugate.

So depending on this value, as I said that the location of the pole of A f it will be different. So the corresponding Bode plot if you try to sketch what you will be getting here it is yeah.

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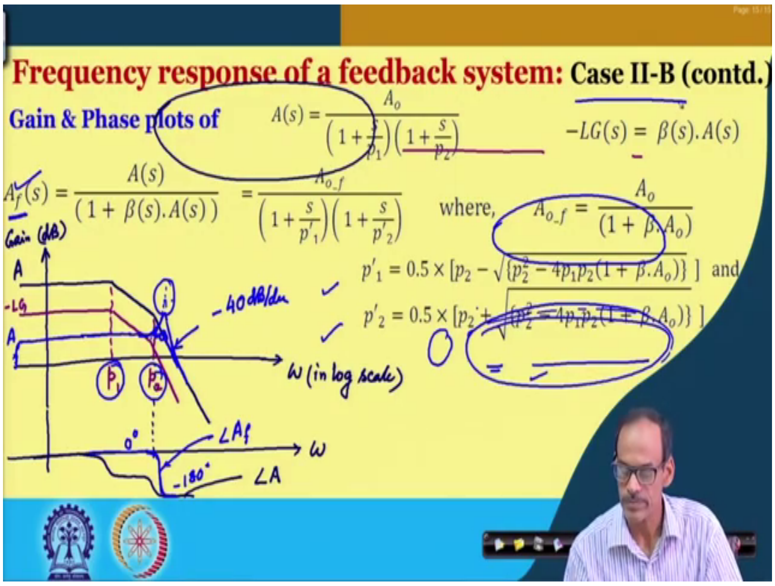


So I, let me try to sketch the Bode plot of A, A f and maybe the loop gain also.

So let me start with A. So if I consider this A in dB and then if we plot against omega omega in log scale again. So what we are expecting here it is we do have a pole here, p 1 and then we do have the second pole here maybe, in the near vicinity of the anticipated pole p 1 dashed which is p 1 multiplied by 1 plus beta into A naught.

So, this is A this is in dB scale and then beta we do have I should not say beta, rather loop gain minus of loop gain. So we do have loop gain, it is having the same poles namely, p 1 and p 2 same as A. So this is gain plot of the loop gain and then we do have the A f. So A f it is above 0 d B.

So what we are expecting here, it is remaining there to the value here or rather if I convert this into dB so that will be the level here, for A f in the low frequency region. And then it is it is expected to be heating this A and we are expecting that there will be sharp corner. Why is it sharp? Because it is from the flat line it is getting converted into minus 40 dB per decade.

So as if it is having 2 combined poles are there. In fact this pole they are not really they are real and in fact, they are imaginary. As a result in the Bode plot, because of the imaginary rather, I should not say imaginary, it is complex pole rather complex conjugate pole. And because of the complex conjugate poles in the gain what we can see here it is it will be having a kind of overshoot kink kind of things.

So strictly speaking, this A f it will be having a kink now. So the overshoot of this kink it is directly proportional or rather directly depends on the magnitude of the imaginary part here and the real part here. So if the imaginary part it is getting higher and higher so then we are expecting, this will be having higher kink.

On the other hand, if these 2 poles are distinct pole of course, then or rather if these 2 poles are real poles; namely, whatever we see here if it is also a real entity or if p 2 square it is higher than the remaining part here. then of course, this p 1 dashed and p 2 dashed, they will be distinct real poles and in that case, this overshoot it will not be there.

Now if I consider the corresponding phase, probably we can try to go to the phase plot also yeah. So we can try to make a phase plot here, I am just retaining the gain plot. So that I do not have to redraw it and so the phase plot for A; so, phase plot of A it is having a step here and then it is having another step here like this.

But if I consider so, this is the phase plot of A and now if I consider phase plot of A f, it is remaining here and then all of a sudden, it will get a short change of the phase starting from 0 degree to minus 180 degree. So this is the phase of the feedback system transfer function. And higher this overshoot, this phase roll-off it will be faster or rather rapid.

So based on the again relative position of this p 1 and p 2, this the pole of A f it can be distinct pole or it can be the 2 poles can be distinct real poles or they can be complex conjugate pair. Now if I try to compare the location of poles for these 2 cases, case 2 A and case 2 B, you can probably try to correlate the location of poles particularly for the forward amplifier poles location and then probably the pole location of the feedback system.

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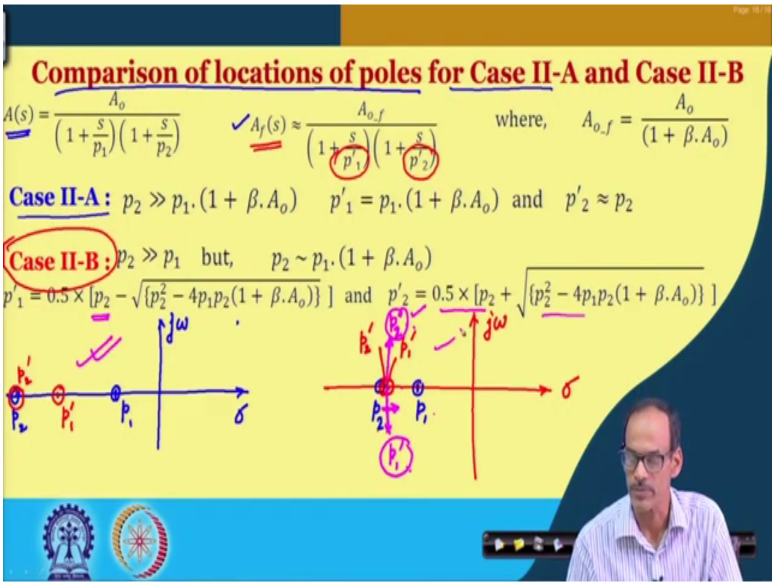


So, in the next slide we are going to compare the location of poles. So we like to compare location of poles for these two cases. So let me try to see here this A s, the forward amplifier circuit it is having 2 poles p 1 and p 2. So it is let me consider this case first. So this is the A s plane, this is sigma and this is j omega and both p 1 and p 2, they are representing left half plane poles.

So, this is a p 1 and this is a p 2, it is quite far apart. So let you consider p 2 it is quite far apart. Now if I consider the corresponding A f assuming that these 2 poles are really wide apart and if I try to see their location of poles particularly p 1 dashed and p two dash.

So, p 1 dashed what we are expecting here it is it got shifted to higher frequency, p 1 dashed and p 2 dashed on the other hand, it is almost remaining to this place, ok. So the blue colour indicates the location of the pole of a and red colour indicates the location of the pole of A f. Now and this is the situation for case 2 A. Now let us consider case 2 B.

So, we do have situation for case 2 B and here again, this is the real part of the s and this is the imaginary part of s and in this case, what we said is p 1 it is say let me again use the blue colour for A s. So we do have p 1 here, but then p 2 it is quite close. So we do have p 2 here. In this case, what we are expecting is that this p 1 dash, it may be in the near vicinity. So this is p 1 dashed and since it is very close to this p 2, it will interfere with that and p 2 will also be changing and the corresponding p 2 it will be coming in the near vicinity.

And if I further try to push this pole, this pole closer. So then what you are expecting is that this p 1 dashed and p 2 dashed, they will collide and then they will create complex conjugate pole pair. So we can say that this is maybe p 1 dashed, it is having minus sign here. So, I am considering this is p 1 dashed and this is p two dashed. So, if we take this p 1 and p 2 closer and closer, then bifurcation of this complex conjugate pole pair, it will be more. Note that p 1 and p 2, they are the real part it is half of p 2 alright half of p 2.

While they are getting bifurcated as complex pole, the real part it is half of p 2. So if I take this second pole closer towards p 1, then real part it is also getting changed and the imaginary part on the other hand, it is overshooting. So however, of course, both p 1 dashed and p 2 dashed both are remaining on the left half plane. As a result, these 2 poles since they are remaining on the left half plane, the system A f or the feedback system remains stable.

But then since these poles are complex conjugate pole, they will be having very strange behaviour in step response. So that part we will be talking later, but this is just to tell you that yes the position of p 1 and p 2, the relative position of p 1 and p 2. They are very important for a well-behaved feedback system. And in fact, we prefer this condition rather than this condition, ok.. Now suppose, we do have a case where A is having only one pole and suppose, the feedback network beta is having another pole and then what it may happen ?

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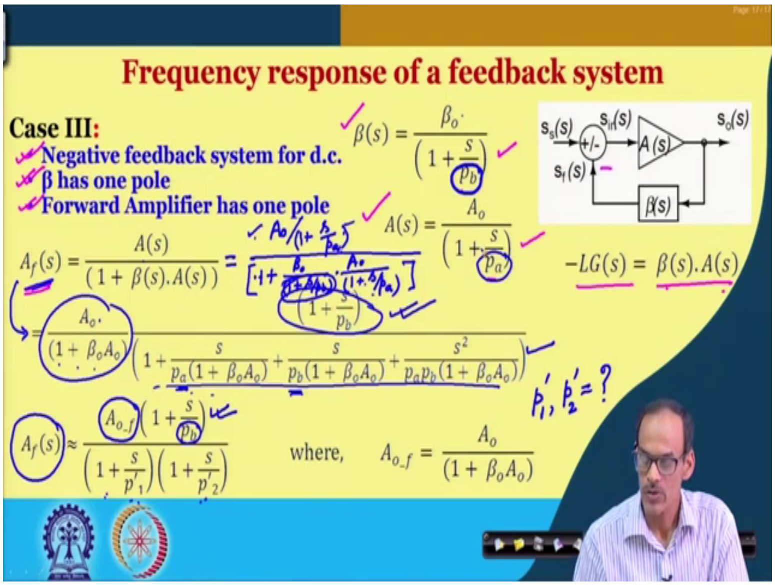


So in the next slide, we are going to discuss this situation. Here what we have it is of course, again we are considering it is negative feedback system in DC condition. Then we are considering beta is having 1 pole called p b and forward amplifier it is having a pole called p a.

So the beta and forward amplifier gain or transfer function both are having 1, 1 pole each. So if I consider of course, the loop gain then this part beta into A it is having both these poles appearing, right. But then if I consider A f, the feedback system of course, it will be having 2 poles that is what we are anticipating, but also it will be having a 0. In fact, if you if you write this here, the expression of A in the in the expression of A f. So what we do have in the numerator it is A naught divided by 1 plus s divided by p a, and in the denominator we do have 1 plus beta which is beta naught divided by 1 plus s by p b into a naught divided by 1 plus s divided by p a right.

So, here we do have only one factor 1 plus s divided by p a in that de numerator part, but then in the denominator, we do have these 2 parts. So if I take these 2 here we will be getting almost the similar situation as we have worked in before, but this factor this factor it is remaining it is remaining there. As a result, what we can get this A f, the constant part it is of course, A naught divided by one plus beta naught into A naught and in the denominator we do have the second order polynomial.

In addition to that, we do have 1 factor in the numerator that represents a 0. So these 2 depending on of course, the location of p a and p b what we have discussed for case 2 A and 2 B they may create real poles or may be complex conjugate poles. In addition to that, we do have a we do have a 0 here. So I can say that A f, it is having frequency independent part, it is having this factor representing a 0 at p b and then it is having 2 poles called p 1 dashed and p 2 dashed.

And p 1 dashed and p 2 dashed it is as I said that it is it can be obtained by the same method what we got for case 2 A and 2 B. So I am not going to repeat that. So that part we can see here. Now if I consider the Bode plot of A f and A and also the beta you will be finding very some similarities are there, but of course, it is having some differences also compared to the previous case.

So let me try to sketch that for you. You know what? It may be the situation for a situation where in case if this pole this pole it is dominant pole over this p a. So in the next slide let me try to sketch that, yeah.

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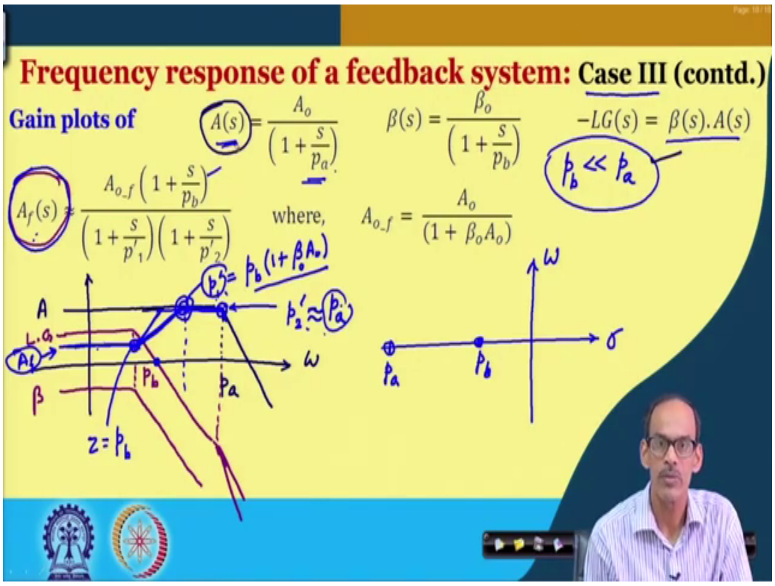


So here I do assume here I do assume that p b it is much smaller than p a and under this condition, if I try to sketch the Bode plot of say, A since it is A has only 1 pole since A has only 1 pole, so it is having say A naught gain and then it is having pole p a, but because of this condition, we are assuming that p a it is quite far and then we do have a this pole p a.

So this is p a on the other hand, if I consider beta, beta is having low frequency gain of beta naught. Say like this and then it is having a pole called p b. So it is having a so this is beta this is p b.

So if I combine this 2 to get the loop gain, what we can get here it is loop gain it is having a this pole p b and then after that it is continuing and then it is hitting this p a. Sorry, this this is having steeper slope here. So this is the loop gain. And now if I try to plot say A f you will be finding very interesting information that A f it is approximately if I consider in the low frequency region, it is approximately 1 by beta naught.

So it is continuing here and then it is having a 0 here. So because of this 0, it will increase. So this is A f and then it is having a pole and then it is having the next pole. So we can say that this pole it is p 1 dashed and that is equal to p b multiplied by 1 plus beta naught into A naught and then it is having a 0 here which is equal to p b. And then of course, it is having the other pole p 2 dashed which is we can say that it is well approximated by this p a.

So for A f what I said is that low frequency gain here it is approximately 1 by beta naught, then it is having a 0 here, and then it is having a pole and then you do have another pole and so and so. And of course beyond this point, beyond this point it is in fact, so in the sketch I do have it is not very neat. In fact, this point and this point supposed to be almost close to each other. In fact, it should have gone like this and then like this.

But anyway, so what I like to say that this A f it is having a additional 0. So depending on the location of the pole of the feedback network, it may be having in the situation like this. And as this p 1 dashed and its expression is given here, if it is approaching to this p 2 dashed or p a, then they these 2 poles may get combined and it may creates the complex conjugate pole p a.

Probably you can try to sketch the location of the poles for different cases; particularly, when you consider both A and beta they do have the pole. And if they are approaching each other, what may be the situation? What may be the location of the 0 poles of this one, you can try yourself and convince yourself that yes; A f it is having one 0 and 2 poles.

Now this is the case when we do have loop gain it is having 2 poles. It may have a situation where loop gain may have 3 poles or even more than that. So suppose, if I consider the next case, case 4 where A is having 3 poles, beta may not be having a pole, ok.

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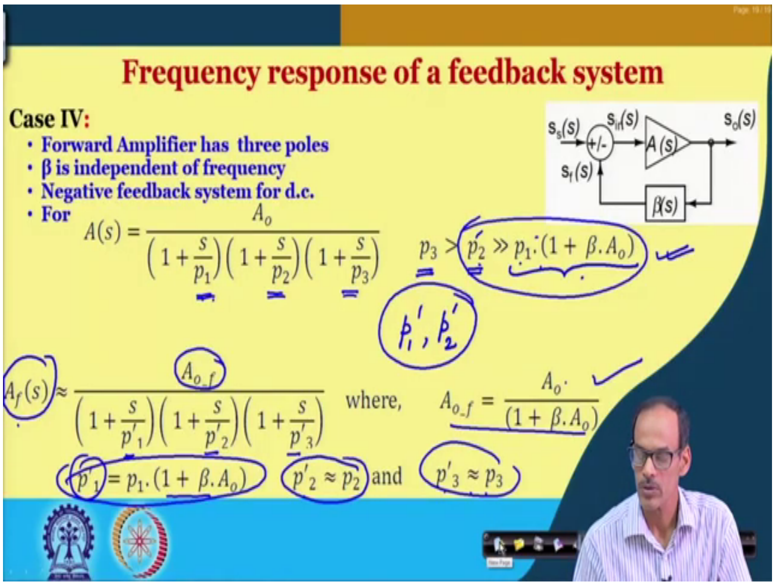


So in the next slide, we do have this situation where this A is having is having 3 poles and if I consider p 1 it is dominant pole. Not only it is dominant pole, even after if I consider it is getting shifted by a factor of 1 plus beta into A naught even in this case, if I assume that this is less than p 2 and p 3, then what we can say that A f we can we can extrapolate our previous analysis. That A f it is having low frequency gain defined by this equation A naught divided by one plus beta into A naught.

And then it is having 3 poles namely p 1 dashed, p 2 dashed and p 3 dashed. p 1 dashed it is shifted version of p 1 by a factor of one plus beta into A naught. p 2 dashed on the other hand, yeah so, p 2 dashed and p 3 they are approximately remaining to the same position of p 2 and p 3 only p 1 dashed, it is getting shifted by this factor ok.

So that is how we can you can analyse a circuit. In fact if you consider its Bode plot probably you yourself can find what will happen. And of course, if this condition it is getting violated namely, if these 2 are becoming comparable, then this 2 may be getting mixed up and as a result then p 1 and p 1 dashed and p 2 dashed they may be appearing as complex conjugate pole.

And then also we do have the third pole. If the third pole it is also coming into a picture then of course, then we will be having still 2 complex conjugate pole pair and one real pole. And then all the 3 poles will be having the contribution to define the location of the of the poles of the A f. So if I try to plot the gain for A f and A for this condition, probably you yourself can find out, but let me help you to elaborate on that, ok.

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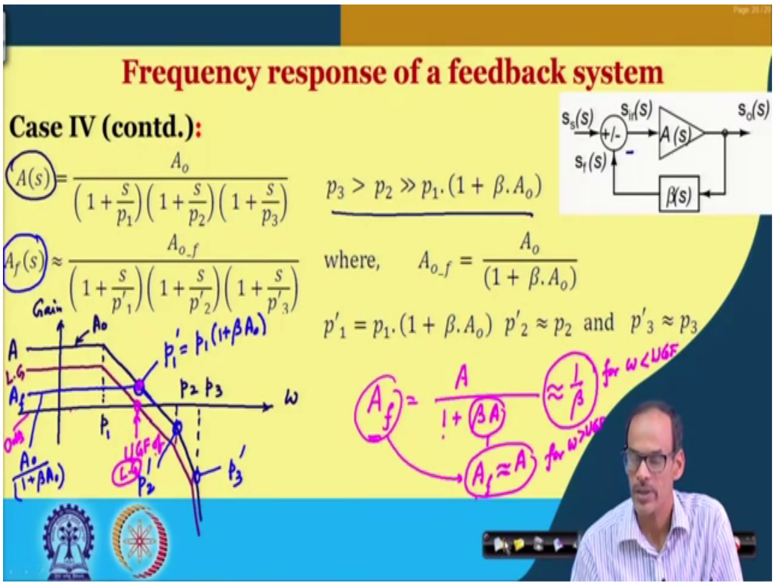

So if I try to make a sketch of it namely, if I try to make the gain and particularly gain plot let me restrict the discussion here, we will be covering the phase plot later.

So what we said is that if I consider this is the condition. Namely, p 1 it is dominating and then p 2 it is there, but p 2 it is really far then maybe p 3 also alright. So we do have p 3 here you do have p 2 here and then we do have p 1 here. So this is the plot of A and this is low frequency gain on this A and we consider beta it is independent of independent of frequency, but then it is less than 1.

So the corresponding beta into A f I plot or rather, if I plot the loop gain. So this is the loop gain and it is having the first pole and then it is having the second pole and then third pole and then if I plot the feedback system transfer function namely, A f. So what we are expecting here it is, it is having low frequency gain which is A naught divided by 1 plus beta into A naught and it is continuing till it is hitting A.

And this is the point where it is hitting A and beyond this point, it will it is following A. So we can say that it creates a pole here which is p 1 dashed. This is p 1, this is p 1 dashed which is equal to p 1 multiplied by 1 plus beta into A naught. And then of course, the corresponding pole here and here they are remaining same. So we can say that p 2 dashed and here we do have p 3 dashed.

One important point, you we have simply missed it if you see this corner point and the frequency at which the loop gain it is becoming crossing 0 dB level. So this is 0 dB level, they do have good you know coincidence. They are very close to each other which indicates that if I consider A f which is A divided by 1 plus beta into A.

In fact, if you see here in the low frequency situation where this part it is dominating. So in low frequency, we can approximate this by 1 by beta. But if you go to higher frequency, particularly beyond this frequency where this part it is less than 1, beyond that frequency then this is this A f need to be approximated by A. So this frequency which is the unity gain frequency of loop gain it is very vital point.

So later, we will say that this A f its behaviour or rather we can make a we can make 2 segments of A f over the frequency range, below this unity gain frequency and beyond that. So below the unity gain frequency the feedback system gain, it is 1 by beta. On the other hand, if it is beyond this unity gain frequency, A f is equal to approximately A. For omega more than unity gain frequency and this is for omega less than unity gain frequency.

In fact, importance of this unity gain frequency it will be discussed later in our in our stability analysis system. Probably, that will be discussed in the next week. So let me wind up whatever the discussion, we do have today. Primarily, change of frequency response of the amplifier due to feedback connection.

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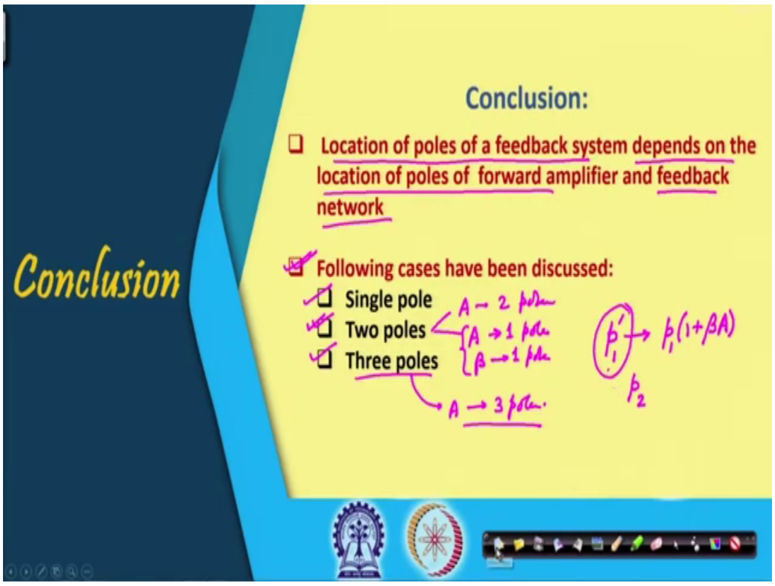

So it is what we have discussed in today's class in 2 half, we have discussed about the location of poles of a feedback system. And it is their dependencies on the location of pole of the forward amplifier and also the location of the pole of the feedback network.

And what we have done is that we have considered 4 cases in fact where the loop gain it is having only 1 pole, then loop gain it is having 2 poles, it is having 3 possible deviation where A is having 2 poles and then A is having 1 pole and beta is having 1 pole.

And then we have consider a situation where A is having 3 poles, ok. So this gives you fair idea that how the poles locations are getting changed. And what we have seen is that the dominant pole p 1, it is getting shifted. Dominant pole of the loop gain, it is getting shifted to p 1 in the form of p 1 dashed which is p 1 multiplied by 1 plus beta into A.

And relative position of this p 1 dashed and the second pole and third pole they are creating a situation whether the shifted version poles are having clear radial value or are they complex conjugate. I think that is all. In the next class, we will be talking about some numerical examples.

Thank you.