

Analog Electronic Circuits
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Lecture - 42
Frequency Response Of CE/CS Amplifiers Considering High Frequency Models of BJT and MOSFET (Part C)

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Numerical example:
Frequency response of equivalent circuit of CE / CS amplifier

- $R_1 = 1.3\text{k}\Omega$, $R_2 = 3.3\text{k}\Omega$, $R_s = 650\Omega$
- $C_1 = 100\text{pF}$, $C_2 = 10\mu\text{F}$, $C_3 = 10\text{pF}$, $C_4 = 5\text{pF}$
- $A_v = -240$
- Find the frequency response of the circuit.
 - Mid-frequency gain
 - lower-cutoff frequency and
 - upper-cutoff frequencies

$$f_{p1} = \frac{1}{2\pi(R_s + R_1)C_1} = \frac{1}{2\pi \times 1950 \times 10^{-5}} = 8.14\text{ kHz}$$

$$f_{p2} = \frac{1}{2\pi R_2 \| R_L C_2} = \frac{1}{2\pi \times 433 \times 1215 \times 10^{-12}} = 302.4\text{ kHz}$$

$$f_{p3} = \frac{1}{2\pi \times 3300 \times 105 \times 10^{-12}} = 459\text{ kHz}$$

$$C_{in} = C_4(1 - (-240)) + C_3 = 5 \times 241 + 10 = 1215\text{ pF}$$

$$\frac{V_i}{V_{in}} = \frac{R_1}{R_s + R_1} = \frac{2}{3} \Rightarrow A_{v, \text{overall}} = \frac{2}{3} \times 240 = 160$$

Welcome back after the short break. So, we are going to discuss about numerical example, and the circuit it is still that equivalent circuit we do have and what we are. So, we do have the generalized equivalent circuit, but then also we do have additional information namely the value of different components, R_1 this input resistance is 1.3 k, then R_2 output resistance it is a 3.3 k and then let you consider source resistance 650 ohm that is also a typical value one possible value of typical signal source.

And then the load capacitance C_L 100 picofarad, C_1 it is given here it is say 10 microfarad and then C_3 which is one of the element contributing to input capacitance it is say 10 picofarad, C_4 the Miller effected capacitance the capacitor which is breezing the input and output terminal of the circuit is 5 picofarad. And let you consider this voltage gain A_v or A_{naught} in this case we are denoting this by A_{naught} which is a 240 with a minus sign.

So, that means, actually this is minus and this is plus. So, anyway with this information let we try to get the frequency response and particularly containing the mid frequency gain and then lower cutoff frequency and then upper cutoff frequency. So, how do we proceed? First of all this resistance directly given there, but then need to calculate the input capacitance.

So, to start with let we calculate C_{in} and C_{in} equals to C_4 multiplied by $1 - A_{naught}$ plus C_3 . So, we do have 5 here and then multiplied by 241 plus 10. So, that gives us how much; we do have 1215, 1215 picofarad, yes. So, we do have the C_{in} is given here, with this C_{in} we can calculate the location of the second pole and let we calculate the first pole first. So, p_1 which is defined by if I express this in the unit of Hertz then we have to consider 2π . So, 2π multiplied by R_s plus R_1 into C_1 , so this is equal to $1 / 44$ by 7.

C_s it is 650 and then R_1 , R_1 it is 1.3 k, so that is 1950 this resistance and then C_1 it is 10 microfarad which means 10^{-5} . And if you calculate it, I have done this calculation for you. So, what you will get it is 8.16 Hertz, right. So, you got the lower cutoff frequency is this one and then we can get the second pole p_2 which is coming from again R_s and R_1 in parallel and since it is in Hertz.

So, we have to consider 2π , so $2\pi R_s$ in parallel with R_1 into C_{in} and the value of the C_{in} it is given here. So, this becomes $1 / 44$ divided by 7 into this resistance, these two resistances in parallel, so we do have 650 and 1.3 k it is coming close to 400. In fact, to be more precise it is 433.3 ohm and then C_{in} it is 1215 into 10^{-5} . And if you do this calculation what you will be getting it is close to 300 and to be more precise 302.4 kilo Hertz.

So, we do have the second pole it is given here 302 kilo Hertz. Now, you can also calculate the third pole p_3 which is coming from R_2 and then output resistance. So, p_3 it is $\frac{1}{2\pi R_2 C_4}$ it is 3.3 k, so you have 3300 ohms and then output resistance it is when you have C_L equals to 100 picofarad and then the C_4 it is almost coming as is, so we can see roughly 105 picofarad 10^2 to the power minus 12. So, if you do this calculation what you will be getting it is 459, in my calculation that is what I obtain 459 kilo Hertz.

And as I said that since this is higher. So, the upper cutoff frequency it will be decided by p_2 and lower cutoff frequency it will be coming from this one. And also to get the mid frequency gain, so mid frequency again it is of course, this multiplied by whatever the attenuation coming from these two elements. So, what is that attenuation? So, I should say V_1 divided by V in mid frequency range equals to R_1 divided by R_s plus R_1 and R_1 it is 2.3 this is 0.65 kilo, so that is equal to $\frac{2}{3}$ and that gives us the overall gain, let me use different color. So, overall gain, A_v overall, so that is equal to two-third multiplied by 240 with a minus sign and this is equal to 160, right.

So, we should say that the overall frequency response of this circuit, it is something like this. So, I do have some space here. So, let me utilize this space. So, mid frequency gain it is given here, maybe we can convert that into dB and then we do have the lower cutoff frequency here and then upper cutoff frequency. So, the lower cutoff frequency it is 8.16 and then the upper cutoff frequency it is this one and this is because this is lower than the p_3 part.

Now, let we go to a one axial circuit. So, as I say that this is equivalent circuit and of course, the value we have picked up here it is very close to whatever the equivalent circuit we get out of actual circuit, but for you to get a feel of that let me consider one practical circuit.

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Numerical Example: CE amplifier – Fixed-bias

- $V_{CC} = 12V$; $\beta = 100$; $V_{BE(on)} \approx 0.6V$; $R_B = 570k\Omega$; $R_C = 3.3k\Omega$, f_U
- Find Operating point, small signal parameters and voltage gain
- $C_1 = C_2 = 10 \mu F$; $C_L = 100 pF$; $R_s = 650\Omega$ f_L
- Find Lower and upper cutoff frequencies considering $C_{\pi} = 10 pF$ and $C_{\mu} = 5 pF$

$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12V - 0.6V}{570k\Omega} = 20 \mu A$
 $I_C = \beta I_B = 100 \times 20 \mu A = 2 mA$
 $g_m = \frac{I_C}{V_T} = \frac{2 mA}{26 mV} = \frac{1}{13} S$
 $r_{\pi} = \frac{\beta}{g_m} = \frac{100}{1/13} = 1.3 k\Omega$
 $R_{in} = R_B \parallel r_{\pi} = 570k \parallel 1.3k \approx 1.3 k\Omega$
 $A_v = -g_m R_C = -\frac{1}{13} \times 3300 = -254.7$
 $A_{v, overall} = \frac{A_v}{1 + \frac{R_s}{R_{in}}} = \frac{-254.7}{1 + \frac{650}{1300}} = -169$
 $C_{in} = C_{\pi} + C_{\mu}(1 + A_v) = 10 pF + 5 pF(1 + 254) = 1285 pF$
 $f_L = \frac{1}{2\pi(R_s + R_{in})C_1} = 8.16 Hz$
 $f_U = \frac{1}{2\pi(R_{in} \parallel R_C)C_{\pi}} = 286 kHz$

So, in the next slide we do have CE amplifier having fixed bias arrangement and different components of, so different components of these bias as well as the other value the capacitors and all it is given here. So, R B, R B it is 570 kilo ohm, supply voltage it is 12 volt and then V BE on it is approximately 0.6. So, that gives the base current equals to 20 microampere.

Earlier we have done this calculation and with multiplying with beta we do get the collector current it is 2 milli ampere, question collector current is 2 milli ampere. So, that gives us the g_m equals to 2 milli divided by 26 millivolt. So, this is equal to 1 by 13 mho. And in r_{π} , r_{π} of the transistor which is beta divided by g_m , so that is 100 divided by g_m is 1 by 13 that means, 13 into 100, so that is equal to 1.3 kilo ohm.

And then we have C 1, C 1 it is given here. So, this is C 1, C 2 it is also equals to same 10 microfarad. And then we can connect a C L here, and this C L it is given equals to 100

picofarad. In addition to that we are assuming that we do have a source resistance R_s to see the effect of this C_{pi} and C_{in} . So, C_{pi} it is given in 10 picofarad and then we do have the C_{mu} it is given us 5 picofarad.

So, from that let we and of course, the R_C it is given here. So, from this parameter, so we can probably we can try to find the voltage gain, and then input capacitance, input resistance and so and so. So, first of all input resistance R_{in} which is equal to R_B in parallel with the input resistance coming out of the device, so that is r_{pi} . And this is equal to 570 k in parallel with only 1.3 kilo ohm. So, you can approximate this by considering 1.3 kilo ohm, it is dominating. So, that gives us the input resistance.

Now, before we calculate the input capacitance, so we need to know what will be the gain from this point to this point and that gain if I call say A_{naught} equals to g_m into R_c , and so this is equal to 1 by 13 into 3.3 k 3300. So, that is becoming equal to close to whatever earlier we obtain or earlier we have taken, but to be more precise I think it is 254, of course, with a minus sign. So, the voltage gain here it is minus 254, input resistance it is 1.3 k and then we can calculate the input capacitance, C_{in} equals to C_{pi} as is plus 5 picofarad C_{mu} into whatever 1 plus 254.

So, if I put the value here it is 10 and then value here it is 5, so that gives us 10 plus 5 into 255. So, that is giving us 1285 picofarad capacitance, ok. So, now, we can calculate by considering this R_s , we can calculate the pole location of the pole. So, let you consider first pole p_1 which is equal to 1 by 2 pi into R_s which is 650 plus R_{in} equals to 1300 multiplied by this C_1 . In fact, this calculation we already have done for the previous example and that gives us 8.16 Hertz.

On the other hand, we can calculate p_2 , p_2 also it will be very similar. So, let me use this space here to calculate p_2 . So, the expression of p_2 it is 1 by 2 pi 5 sorry 650 in parallel with 1300 multiplied by C_{in} , so that is 1285 picofarad 10 to the power minus 12. So, with this, so this is again this part it is coming whatever 433 and with that what we get of course, this capacitance it is slightly different and this gives us 286 kilo Hertz.

Now, if I calculate the third pole namely the pole coming from the output resistance which is R_C and then the C_L , and R_C it is given here. So, the third pole, let me use this space. To calculate the third pole which is $1 / (2\pi \times \text{output resistance} \times \text{output capacitance})$ which is 100 and then also C_{μ} coming there, so that is 105 picofarad. In fact, if we calculate this what we can get by putting the value of this R_C equals to 459 kilo Hertz.

Now, if I compare the p_2 and p_3 since p_2 it is lower, so p_2 defines the upper cutoff frequency f_U . And then of course, p_1 it is defining the lower cutoff frequency f_L . And then the gain overall gain, of course it will be this multiplied by; this multiplied by whatever the attenuation coming out of R_s and R_{in} . So, the overall gain A_v overall equals to, so this is equal to $1.3 / (1.3 + 0.65)$, in fact, that is two-third multiplied by this 254 of course, with a minus sign and that is becoming close to minus 169.

So, in summary the mid frequency gain it is 169, lower cutoff frequency it is 8.16 Hertz and then upper cutoff frequency it is 286. So, that is the overall frequency response or the common emitter amplifier given here. Now, we can similar kind of exercise you can try for common source amplifier.

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Numerical Exercise (CS amplifier)

- Given: $(K.W/L) = 1\text{mA/V}^2$; $V_{th} = 1\text{V}$, $V_{dd} = 12\text{V}$, $R_1 = 9\text{k}\Omega$, $R_2 = 3\text{k}\Omega$, $R_D = 3\text{k}\Omega$
- Find small signal parameter, A_v , R_{in} and R_o
- $C_1 = C_2 = 10\ \mu\text{F}$; $C_L = 100\ \text{pF}$, $R_s = 750\ \Omega$.
- Find Lower and upper cutoff frequencies considering $C_{gs} = 10\ \text{pF}$ and $C_{gd} = 5\ \text{pF}$

$A_{v, \text{small}} = -6 \times \frac{2.25}{(2.25 + 0.75)} = -6 \times \frac{2.25}{3} = -4.5$
 $I_{D5} = \frac{1\text{mA/V}^2 (3-1)^2}{2} = 2\text{mA}$, $g_m = 2\text{mA/V}$, $r_{ds} \rightarrow \infty$

$A_0 = -g_m R_D = -6$

$R_{in} = R_1 \parallel R_2 = 2.25\text{k}\Omega$

$f_{L1} = \frac{1}{2\pi(R_s + R_{in})C_1} = \frac{1}{4.7 \times 3000 \times 10^{-6}} = 5.3\text{Hz}$

$f_{L2} = \frac{1}{2\pi(R_s \parallel R_{in})C_{in}} = \frac{1}{1.9 \times 650 \times 10^{-6}} = 629\text{Hz}$

$f_{H3} = \frac{1}{2\pi R_D C_{out}} = \frac{1}{4.7 \times 3000 \times 10^{-6}} = 505\text{kHz}$

$C_{in} = 10 + 5(1-(-6)) = 45\text{pF}$

$C_{out} \approx C_L + C_{gd} = 105\text{pF}$

So, again this circuit is very similar, but we need to be careful here that g_m of this transistor MOS transistor, since it is much lower than g_m of b j t, so we are expecting the gain here it will be much lower and then we can see what is its consequence in the frequency response.

So, here we do have the value of different bias elements namely R_1 and R_2 are given here, 9 k and 3 k respectively. We do have the supply voltage 12 volt. And then for this device parameters are given here namely transconductance factor it is 1 milli ampere per volt square, threshold voltage it is 1 volt and then R_D the passive load it is 3 k.

Now, if I analyze say this part to find the voltage here, since it is 9 and since it is 3 the voltage coming here it is 3 volt from this 12 volt, right. And once we have the gate voltage 3 volt and

threshold voltage is 1 then we can say that this $I_{D,D}$ equals to this 1 milli ampere per volt square by $2 \times 3 \text{ minus } 1 \text{ V}_{gs} \text{ minus } V_{th} \text{ square}$, so that is equal to 2 milli ampere.

In fact, with this we can also get g_m of the transistor it is 2 milli ampere per volt. Earlier we have done this calculation, and we can assume that the λ is very small. So, we can say that r_{naught} or r_{ds} is very high, we may consider this as very high. So, the voltage gain from gate to drain of this amplifier it is $g_m \text{ into } R_D$ with a minus sign and that is equal to 6. So, we got voltage gain from here to here it is only 6.

Now, we can calculate the input capacitance using this information. So, we do have voltage gain it is only 6, so the C_{in} , so the C_{in} part it is we do have C_{gs} which is 10 picofarad and then $C_{gd} \times 5$ multiplied by $1 \text{ minus } 6$. So, that gives us 45 picofarad only. And on the other hand, if we connect the C_L here which is 100 picofarad given here and if this is 10 micro farad, so effectively these two together it is giving us 100 picofarad and of course, we have to consider effect of C_{gd} and this is C_{gs} , so effect of C_{gd} it is coming here almost as is.

So, we can say that C_{out} it is equal to $C_L \text{ plus } C_{gd}$ approximately, right and so that is we can see it is equal to 105 picofarad. Now, based on this information of course, we need to know the input resistance here R_{in} and gate to source resistance of coming out of the device it is very high. So, input resistance it is coming from the bias circuit only and also we consider the source resistance to get the effect of C_{in} and the source resistance it is given here it is 750.

So, R_{in} now it is R_1 in parallel with R_2 , so that is equal to 2.5 kilo ohm. So, we do have R_{in} is equal to 2.25 kilo ohm, then R_s is 750. So, the first pole, let me use different color here. First pole p_1 coming due to series connection of R_s and R_{in} and C_1 , C_1 it is here C_2 it is here So, this is 2π , then R_s in series with R_{in} multiplied by C_1 . So, that is equal to $44 \text{ by } 7$ into we do have together series connection of R_s and R_{in} this is actually 3 k, 3 k and then C_1 it is 10 to the power minus 5.

So, that gives us the lower cutoff frequency of this circuit it is even lower than the previous case. So, this is equal to 5.3 Hertz only. So, we got the lower cutoff frequency it is coming here. Then p_2 , if you consider p_2 which is $\frac{1}{2\pi}$ then R_s in parallel with R_{in} and then multiplied by C_{in} and C_{in} it is only 45 picofarad, so this is equal to $\frac{1}{44 \times 7}$ into. So, these two coming in parallel, so that gives us let me check in my calculation.

So, this is R_s is equal to point or let us say 750 in parallel with 2250 and then 45 picofarad. In fact, this is becoming quite high particularly the capacitance, it is low this is equal to 6.28 mega Hertz. So, note that the value of this capacitance it is higher, sorry the lower that makes this p_2 it is higher.

Now, let me also calculate p_3 which is coming from R_D the output resistance here let me use this space here, ok. So, we do have 2π here and then R_D and then C_{out} . In fact, if you consider the corresponding numerical value here, so this is $\frac{1}{44 \times 7}$ into R_D it is 3 k and then C_{out} it is, so this is 3 k sorry. C_{out} it is 105 picofarad and in fact, this is becoming 505 kilo Hertz.

Now, this is the case where what we are getting is that this frequency it is lower than this one. So, naturally the upper cutoff frequency it will be decided by p_3 of course, the lower cutoff frequency it will be decided by p_1 , ok. So, this is the; this is the example where I was talking about that this p_2 it is not defining the upper cutoff frequency, it is rather p_3 the output node pole it is still defining the upper cutoff frequency.

And then what is the mid frequency gain? We do have the gain of 6, that need to be multiplied by the attenuation coming from R_s and R_{in} and the attenuation there it is of course, that is 0.75 and we do have the input resistance it is 2.25. So, overall gain A_v overall equals to we do have minus 6 multiplied by 2.25 divided by 2.25 plus 0.75. So, this part it is becoming 3 and. So, this is minus 6 into 2.25 divided by 3, so that gives us minus 4.5. So, the mid frequency gain it is only 4.5 and the lower cutoff frequency it is 5.3 Hertz, upper cutoff frequency it is 505 kilo Hertz, ok.

So, anyway we know that this circuit will be having low gain, but the exercise here what you have seen here it is because of the low gain the input capacitance, it is not so high. As a result the upper cutoff frequencies it is still getting decided by the pole coming from R D and C L or output capacitance. Now, we can do probably one more exercise which is cell biased, but I suggest that probably you can try it out to solve this problem.

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Numerical Exercise: CE amplifier –Self-bias

- $V_{CC} = 12V$; $\beta = 200$; $V_{BE(on)} \approx 0.6V$; $R_1 = 9.9k\Omega$; $R_2 = 3.3k\Omega$; $R_C = 2.7k\Omega$; $R_E = 1.2k\Omega$
- Find Operating point, small signal parameters and voltage gain
- $C_1 = C_2 = 10\mu F$; $C_E = 100\mu F$; $C_L = 100pF$, $R_s = 650\Omega$
- Find Lower and upper cutoff frequencies considering $C_\pi = 10pF$ and $C_\mu = 5pF$

$$V_B \approx 3V, I_E = 2mA \Rightarrow I_C \approx 2mA \Rightarrow g_m = \frac{1}{13} S$$

$$r_{\pi} = \frac{200}{g_m} = 2.6k\Omega \Rightarrow R_{in} = R_1 || R_2 || r_{\pi} = 1.268k\Omega$$

$$A_o = -g_m R_C = \frac{-2.7 \times 10^3}{13} = -207.7$$

$$C_{in} = C_\pi + C_\mu(1 + 207.7) = 1053pF$$

$$p_1 = \frac{1}{2\pi(R_s + R_{in})C_1} = ? \quad p_2 = ?$$

$$p_3 = ?$$

So, I will be giving little hint to solve this problem. First of all here again we shall try to find the operating point of the transistor. So, if you consider R 1 of 9.9 k and R 2 it is 3.3 k. Again, the voltage coming here it is 3 volt. So, we do have the value of this resistance it is 9.9 k and we do have 3.3 k, so that gives us 3 volt here. And with this 3 volt if you consider that we do have the base to emitter voltage drop of 0.6, in fact, that gives us this voltage equals to 2.4 volt.

Now, of course, we are assuming that while the base node it is connected here and then the base current it is flowing here we are assuming that this voltage the base voltage it is not changing. And in this case it is valid because the internal current here through this register R_1 and R_2 before we connect this base terminal it was much higher than the anticipated base current. So, even after connecting this base terminal we can see that the V_B still it is very close to 3 volt, right.

So, once we have 3 volt here then if we reduce this drop of point 6 then we do have 2.4 volt coming at the emitter. And then if we consider R_E which is 1.2 k that gives us the emitter current I_E equals to 2.4 divided by 1.2 which means that the I_E is equal to 2 milli ampere. And we can also approximate that this I_C is also equal to same, so that is also 2 milli ampere. And then from that you can calculate g_m which is 1 by 13 mho, then we can calculate r_{π} which is 200β is 200 divided by g_m . So, that is 1 by 13, so that is equal to 2.6 kilo ohm.

Now, the input, so that gives us the input resistance R_{in} equals to R_1 in parallel with R_2 in parallel with r_{π} . Probably, you can check I think in my calculation it was coming 1.268 kilo ohm and the voltage gain of course, the internal circuit voltage gain from base to collector A_{naught} which is g_m into R_C assuming that this emitter register it is successfully bypassed by this capacitor C_E . So, this it becomes R_C we do have 2.7 and then g_m it is 1 by 13, so that gives us a gain. So, this is k of course, multiplied by 1000. So, this gain it is coming close to 200, in fact, 2.7.7 with a minus sign.

And using this you can calculate what is the input capacitance, namely C_{π} plus C_{μ} into 1 plus 207.7. I think in my calculation I was getting close to 1 nanofarad, in fact, 1053 picofarad, right. So, using this information of say C_{in} and then R_{in} you can calculate what will be the p_1 by considering the source resistance R_s which is given here. So, p_1 again you can calculate this is equal to $2 \pi R_s$ plus R_{in} multiplied by C_1 and C_1 it is given here it is 10 microfarad.

So, you can find the value of this one. Likewise you can calculate p 2 and then you can calculate p 3. So, I will not be doing this one, probably, you can simply try it out and get the corresponding solution.

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Conclusion:

- Frequency response of CE and CS amplifier considering high frequency model of transistors particularly with R_s
 - Lower cutoff frequency
 - Upper cutoff frequency
 - Mid-frequency gain
- Equivalent circuit of CE and CS has been analyzed Using
 - Miller's theorem and $\Rightarrow C_{in}$
 - Frequency response of R-C-R||C circuit
- Numerical examples considering high frequency models
 - CE with fixed bias and self bias
 - CS

So, in summary what we have so far we have covered it is. So, in this module what we have covered here it is basically we have considered high frequency model of transistor and particularly in presence of source resistance R_s , what is its impact on the frequency response of common emitter and common source amplifiers. And what you have seen primarily, it is the change of the lower cutoff frequency and upper cutoff frequency and also the mid frequency gain.

So, this is the main thing we have done and that has been done by properly calculating the input capacitance for which we have considered Miller's theorem. So, we have touched upon

the basic Miller's theorem and then we have seen that that theorem it was helping us to calculate the input capacitance C in which primarily it was defining the upper cutoff frequency. And then, also to get the overall frequency response we also have analyzed $R C$ followed by $R C$ in parallel circuit and we obtain the corresponding frequency response.

So, these two underlying theory it was helping us to get the frequency response of the CE and CS amplifier. So, we started with analysis and then the expression of the poles and all. And later in the third part of it we have considered numerical examples and particularly for common emitter amplifier with fixed bias and common source amplifier in detail, and we have given a hint of how to do similar kind of you know analysis for common emitter amplifier having self-bias arrangement. I think that is all I do have.

Thank you for listening.