

Analog Electronic Circuits
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Lecture - 41
Frequency Response of CE/CS Amplifiers Considering High Frequency Models of BJT and MOSFET (Part B)

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Effective input/output capacitances of CE / CS amplifiers using Miller's theorem

• C_4 (C_{μ} / C_{gd}) can be "converted" into input port cap and output port cap

$$C_{4-in} = C_4 (1 - A_o)$$

$$C_{4-out} = C_4 (1 - \frac{1}{A_o})$$

$$C_{in} = C_3 + C_4 (1 - A_o)$$

$$C_{out} = C_L + C_4 (1 - \frac{1}{A_o})$$

The slide also shows a simplified equivalent circuit at the bottom left with V_{in} , R_s , C_1 , R_{in} , and C_{in} in parallel, leading to voltage V_1 .

Yeah. So, welcome after the break. So, we are talking about the, in fact, what we got it is the generalized model of CE and CS amplifier here. What it is having here it is the input signal source, having the source resistance of R_s , and then signal coupling capacitor C_1 , and then if I consider this is the main amplifier where we do have the input resistance represented by this R_1 .

And then we do have voltage dependent voltage source, which means that this is the core of the amplifier, then we do have the output resistance R_2 . And then C_3 and C_4 , they are representing you know either C_{π} or C_{gs} and C_{gd} based on whether the circuit it is CE amplifier or CS amplifier.

So, this particular this capacitor it can be converted into two equivalent capacitance; one is for the input port, the other one is for the output port. And then, the input port part coming out of the C_4 it is what we said is that C_4 multiplied by $1 - A_v$ or in this case A_v is equal to V_o . In fact, if you see here we are putting a minus sign here assuming that the polarity of the voltage dependent voltage source, here it is positive.

And on the other hand, so this is the contribution coming to the input port earlier we used to call C_1 . Now, let me put a different name C_{4in} ; that means, the input port capacitance coming due to C_4 . So, likewise the output port capacitance coming due to C_4 , let you call this is C_{4out} . So, this is equal to C_4 into $1 - 1/A_{naught}$. So, if I consider this gain it is very high and in case if it is having minus sign here or the polarity here it is opposite then we can consider that C_{4in} it is just C_4 multiplied by $1 +$ magnitude of the voltage.

So, this capacitance it is coming in addition with C_3 as a result we are getting the net input capacitance C_{in} equals to C_3 plus C_{4in} and then multiplied by $1 - A_{naught}$. On the other hand the output capacitance net output capacitance of course, we do have C_L . So, the C_L is coming as is plus this part namely C_{4out} multiplied by $1 - 1/A_{naught}$.

Yeah, I like to mention one thing here it is in the actual circuit C amplifier or CS amplifier, typically we do have one DC decoupling capacitor or a C coupling capacitor and typically used to name as C_2 . And then the C_2 and C_L if I consider their typical magnitude, this may be in the order of say 10 microfarad whereas, the C_L may be in the range of say 100 picofarad.

So, as a result the load coming at this node due to the series connection of C_2 and C_L practically it is dominated by C_L . So, that is why at this node or at this node whatever the

effective load capacitance we do have coming out of say these two parts it is getting to be equivalent to C_L , ok. So, that is why we are not considering this C_2 in this analysis.

So, in summary what we have it is at this node we do have the C_{in} and then at this node we do have the net C_{out} . Now, to get the frequency response of this circuit namely starting from this point till the primary output what we have it is we do have one network here and then we do have of course, the main amplifier starting from this point to this point and then of course, at this point we do have the C_{out} . The board looks like clumsy, so, let me clear and then summarize it.

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**Effective input/output capacitances of CE / CS amplifiers
using Miller's theorem**

- C_4 (C_{μ}/C_{gd}) can be "converted" into input port cap and output port cap

$V_1(s) = ??$
 $V_{in}(s)$
 $R-C - R||C$

What we have what I said is that from here to here the gain it is defined by A_{naught} , in addition to that we do have this resistance coming in series with or getting loaded by net output capacitance called C_{out} . On the other hand, from this point to this point if we see the

circuit we do have one R, we do have one C and then we do have one R here and then the effective input capacitance coming from C 3 and C 4 together. So, that gives us a network something like this.

So, once to get the overall frequency response we need to get the frequency response from this point to this point and then of course, here to here. So, just to start with let we see the first part, namely this part and the corresponding circuit it is given here, in this diagram. We do have the input signal, we do have the source resistance, we do have the C 1 in series and then we do have the effective input resistance, in this case you may call it is R 1, but whatever the resistance we may name as R in. And then C 3 and C 4 together it is contributing to the net input capacitance with respect to ground that is called C in.

So, our first task is to find the frequency response from this point to this point, namely V 1 by V in may be in Laplace domain we can see, and then we can find what is the corresponding transfer function we are getting. So, in the next step, next slide what we are going to do? We are going to analyze this R in series with C, in series with R in parallel with C.

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Frequency response of R-C-R||C circuit

$$\frac{V_1(s)}{V_{in}(s)} = \left\{ R_s + \frac{1}{sC_1} + R_{in} \parallel \frac{1}{sC_{in}} \right\}^{-1} \cdot \frac{R_{in}}{1 + sR_{in}C_{in}}$$

$$= \frac{R_{in}}{\left(R_s + \frac{1}{sC_1} + \frac{R_{in}}{1 + sR_{in}C_{in}} \right)}$$

$$A = \frac{V_1}{V_{in}} = \frac{R_{in}}{R_{in} + R_s} \cdot \frac{1}{1 + s(R_{in} + R_s)C_1} \cdot \frac{1}{1 + sR_{in}C_{in}}$$

$$Z = 0 \Rightarrow \frac{R_{in}R_sC_{in}}{R_{in} + R_s} = -\frac{1}{s} \Rightarrow s = -\frac{R_{in} + R_s}{R_{in}R_sC_{in}}$$

$$p_1 = -\frac{1}{(R_{in} + R_s)C_1}; \quad p_2 = -\frac{1}{\frac{R_{in}R_sC_{in}}{R_{in} + R_s}}; \quad Z = 0$$

$$\approx \frac{sR_{in}C_1}{1 + s(R_{in} + R_s)C_1} \cdot \frac{1}{1 + sR_{in}C_{in}}$$

So, we do have this circuit, we do have R, and then C, and then R and the C coming in parallel. Now, if you see here to get the frequency response of this circuit namely in Laplace domain V_1/s divided by V_{in}/s we can get the transfer function by considering this impedance which is R_{in} in parallel with $1/sC_{in}$ and then R_s in series with $1/sC_1$ plus R_{in} in parallel with $1/sC_{in}$.

Now, the numerator part, you can further simplify, and you can see that this is R_{in} divided by $1 + sR_{in}C_{in}$. And in the denominator, we do have R_s plus $1/sC_1$ plus, this part it will be the same as this R_{in} divided by $1 + sR_{in}C_{in}$. Now, if I further simplify, so this factor, once it is coming in the numerator it is getting cancelled with this factor, but then we do have s into C_1 , so that gives us in the numerator we do have $sR_{in}C_1$.

And in the denominator we do have all these things getting multiplied. So, what we have? We do have, s this R s multiplied by s $C - 1$ multiplied by $1 + s$ R in C in, so that is the first term. And then we do have the second term where we do have $1 + s$ R in into C in and then we do have the third term which is s R in into $C - 1$.

So, in the denominator we do have force one and then we do have the s term; s term we do have here we do have here and also we do have here, in addition to that we do have a square term as well. So, in the denominator if I further expand it, what we are getting is that numerator is remaining s R in into $C - 1$ divided by $1 + s$. We do have R in into C in plus, we do have R in into $C - 1$, and then also we do have a s into R s into $C - 1$, and then we do have a square into $C - 1$ C in R in and R s , right. So, the transfer function here which is written here you can see that it is having a 0 here, due to this s term, and also since it is second order polynomial in the denominator, so we are expecting two poles.

Now, let we consider a typical numerical value and based on that we make some assumption here. So, what we have here it is C in which is a typically much smaller than $C - 1$. So, probably we can ignore this part and then we can consider this term and then we can take $C - 1$ outside. So, in the denominator if we do this approximation we do have s R in and then $C - 1$ that is coming in the numerator and in the denominator we do have $1 + s$, then R in plus R s into $C - 1$ and then we do have the s square term, coefficient of s square it is given here which is R in into R s into C in into $C - 1$.

In fact, this can be well approximated by considering two factors and if I consider a typical value again we can do some assumption valid assumption to simplify the denominator and what we can get that we can get two factors one factor is coming from here, namely $1 + s$ into R in plus R s into $C - 1$. So, this is the first factor.

And then the second factor, it should be such that the coefficient of s square should match. In fact, that can be shown that the second part; second factor it will be $1 + s$. So, then we do have R in then R s divided by R in plus R s into C in, ok. So, in fact, if you if you multiply of course, there is an assumption that here this term of course along with this term we do have

one more term which is one multiplied by this part, but we assume that term it is very small and then product of this s term and this s term it is matching with this.

So, here we may say that the first term it is giving a pole called p_1 . So, we can say that p_1 it is coming from the first one and the location of the pole it is $\frac{-1}{R} + Cs$. And then the second pole p_2 , it is coming from the second factor and it is given by $\frac{-1}{R} + Cs$. In addition to that, this since we do have the s term here in the numerator. We also have a 0 at 0 frequency.

Also if you see that the in the mid frequency range, what may be the attenuation? When I say mid frequency, what does it mean is that if you consider the coefficient of s is dominating over this one and s^2 term, and if I say that if this is dominating; that means, if I drop this two part then what we can get in the numerator we do have C and in the denominator we have $1 + Cs$, C is getting canceled, s is getting canceled. So, we can say that mid frequency gain, if I call say whatever it is, A defined by V_1 divided by V_2 in mid frequency that is coming $\frac{R_1}{R_1 + R_2}$.

Let me explain the intuitive way of analyzing this circuit. What we have it is the frequency response let me use this space. The frequency response if you see probably I can use this space no; that means, this space. So, if I see that the gain of this circuit it is, so this is the frequency in log scale and this is $20 \log \frac{V_1}{V_2}$ in dB. So, in very low frequency we do have a 0 and then we do have pole, and after that the gain it is getting stabilized; that means, this is the attenuation and that attenuation is given by this $\frac{R_1}{R_1 + R_2}$ and then we do have the second pole.

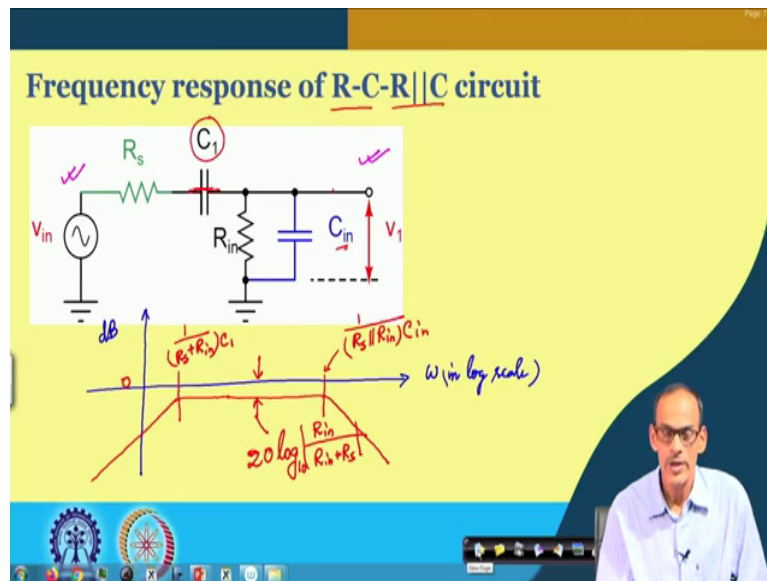
So, we do have the first pole here p_1 given by that expression and then we do have the p_2 given by this expression and it starts with a 20 dB per decade slope at the beginning which means that it is having a 0 at 0 frequency. And this attenuation it is essentially $\frac{R_1}{R_1 + R_2}$ and this need to be converted in dB.

So, now coming to the intuition in very low frequency this capacitor it is blocking this signal to propagate here. So, that is why you do have lot of attenuation here, but then with progress of frequency this capacitor it starts allowing the signal to come to this node and that is what the indication that as we are increasing the frequency the signal it is propagating here. And then we do have a situation where this capacitor almost it is working as a short and leaving behind we do have only this R_s and R_{in} .

This capacitor yet to show its effect. So, in the mid frequency range what we have it is R_s and R_{in} series input we are applying at this node and then the output you are observing here. So, if you see this circuit of course, the attenuation from this node to this node it is defined by R_{in} divided by R_s plus R_E . So, that is what the attenuation we are getting here in the mid frequency range.

Now, once we have this circuit. So, entire portion it is working as a Thevenin equivalent signal source, and its Thevenin equivalent resistance now it is R_s in parallel with R_E , right. And then we do have then C_{in} connected here, right. And this signal it is not just V_{in} it is attenuated version of V_{in} . So, I should say this is V_{in} multiplied by R_{in} divided by R_{in} plus R_s . So, from this frequency onwards this is the model. And then based on this resistance R_s in parallel with R_{in} and then C_{in} we do have a pole. So, that is why the expression of this pole if you see it is coming from this resistance which is R_{in} in an R_s in parallel and then we do have the C_{in} . So, that is why the pole we do have here. So, that is the interpretation of this frequency response.

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So, in summary what we like to say here it is the frequency response of this circuit starting from this point to this point is given by; so, this is the frequency maybe we call it is radian per second and then we do have the gain in dB. So, this is in log scale, typical bode plot, right, and very low frequency we do have we do have a 0 and then we do have a pole where this capacitor it is just sorting the signal beyond that that frequency, and then after that this capacitor it starts sorting the signal creating a pole here. And location of this pole and this pole as I said this is 1 by R_s in series with R in multiplied by C_1 and location of the other pole it is 1 by R_s in parallel with R in multiplied by C_{in} .

So, that is the frequency response of $R C$ in series with parallel connection of another $R C$. And this attenuation it is we should see this value, so this is the 0 dB level. So, this value it is minus of course, it will be coming minus, but they to write the expression $20 \log_{10} \frac{R_{in}}{R_{in} + R_s}$. Now, we shall use this frequency response for our frequency to get

the frequency response of the CE and CS amplifier considering the high frequency model of the transistor in the next slide, ok.

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**Frequency response of generalized equivalent ckt of CE / CS amps
(considering high frequency models)**

• Find the expressions of lower- and upper-cutoff frequencies

$\omega_L = p_1$
 $\omega_U = \min(p_2, p_3)$
 $p_1 = \frac{1}{(R_s + R_1)C_1}$
 $p_2 = \frac{1}{R_1 C_2}$
 $p_3 = \frac{1}{R_2(C_L + C_4(1 - \beta))} \approx \frac{1}{R_2(C_L + C_4)}$

$\frac{V_1}{V_{in}}$
 $\frac{V_2}{V_1}$
 $\frac{V_o}{V_2}$
 $\frac{V_o}{V_{in}}$

So, this is, so we are coming back to the generalized equivalent circuit; generalized equivalent circuit of the C and C is amplifier which is given here and from this point to this point we got the frequency response and then from here to here will be getting the another part of the network. And hence, we can get the frequency response starting from the primary input till the primary output.

So, to start with we already have discussed that if you consider say V_1 by V_{in} and we obtain the corresponding frequency response like this, so this is V_1 by V_{in} and we already have discussed the location of the pole and then we do have from this point to this point or say this point. So, we can say V_2 by V_1 and that is defined by this A_{naught} . So, this A_{naught} it is

pretty high. So, we can say that A_{naught} it is constant. So, this is V_2 divided by V_1 . So, if I combine this two; if I combine this two, so what we can get it is that V_2 by V_{in} . So, V_2 by V_{in} if I plot, V_2 by V_{in} it will be similar to this one, but it got lifted up. So, this is the this is V_2 by V_{in} .

And then V_2 to V_{out} we do have another RC circuit and that RC circuit of course, we do have the resistance here, we do have the capacitance maybe primarily coming from the load capacitance and maybe some part of the C_4 it is also contributing to the C. So, if I consider on the other hand V_2 to V_o . So, there what we get it is of course, here to here it is passive network, so it is starting with almost 0 dB and probably it is having a pole like this, right. So, this is V_o divide V_2 .

Now, if I combine say V_o by V_2 and this curve together. So, then we can get the overall frequency response now let me use a different color here maybe black color yeah. So, to get V_o divided by V_{in} that is the complete overall frequency response to do that I need to combine this curve and this curve, so that V_2 and V_2 are getting canceled we do have V_o in the numerator and then V_{in} in the denominator.

Since, it is having this curve it is having 0 dB here, so I should say this is not disturbing. So, this characteristic curve it is supposed to be same as like this and then it will be coinciding like this and then probably we do have the pole here, second pole, so this pole.

So, now, we like to say that the location of this pole, so the net frequency response now we obtain it is like this, and so this is the overall frequency response blue color. And location of this pole p_1 it is coming from this network, initial network you may recall its expression is 1 by R_s in series with R_1 and then we do have the C in. And C in, we have C in that C_3 and C_4 they are contributing, this pole we say that p_2 which is R_s in parallel with this R_1 , earlier who is to say that this is R_{in} that is multiplied by sorry, this is not C in this is rather C_1 and here we do have the C in.

And then also we do have the other pole coming from this network and this pole it is we may call this is p_3 and that is defined by this R_2 , and the net capacitance at this output node

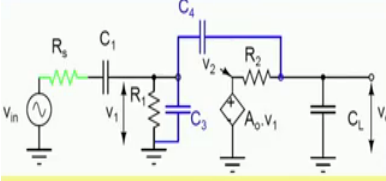
probably this may be $C L$ and then contribution coming from the $C 4$ and that may be $C 4$ multiplied by $1 - 1/A$ and all practical purposes you may consider this is $1/R^2, C L + C 4$, ok.

So, this is typically what the frequency response we do expect. But of course, it depends on the numerical value of the different components. Say for example, in case if this pole third pole if it is appearing before the second pole which means that if this violet colour instead of; instead of having a pole here if it is having a pole somewhere here, then obviously, if I call say this is a possible situation maybe say this is $p 3$ dash. And then if I combine this pole or rather this frequency response; this frequency response with this one and if the location of the third pole it is here which is a before this one obviously, that will define the overall frequency response having a pole here.

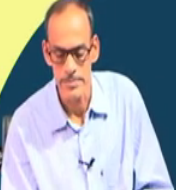
In other words, the lower cutoff frequency typically it is coming from this one, so we may say that ω lower cutoff frequency it is defined by $p 1$. On the other hand, the upper cutoff frequency it is defined by minimum of this $p 2$ and $p 3$ whichever is minimum, so that defines the, so that defines the upper cutoff frequency. So, in our numerical example, so we will see that two cases where for one case maybe this $p 2$ is lower, in the other case maybe the $p 3$ is lower. So, probably with that numerical example things it will be even clearer.

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Numerical example:
Frequency response of equivalent circuit of CE / CS amplifier



- $R_1 = 1.3\text{k}\Omega$, $R_2 = 3.3\text{k}\Omega$, $R_s = 650\Omega$
- $C_L = 100\text{pF}$, $C_1 = 10\mu\text{F}$, $C_3 = 10\text{pF}$, $C_4 = 5\text{pF}$
- $A_v = -240$
- Find the frequency response of the circuit
 - Mid-frequency gain
 - lower-cutoff frequency and
 - upper-cutoff frequencies



So, we are going to talk about numerical examples, but before that let me take a short break and then will come back.