

Analog Electronic Circuits
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Lecture – 40
Frequency Response Of CE/ CS Amplifiers Considering High Frequency Models Of
BJT And MOSFET (Part A)

So, dear students so, we will come back to our NPTEL online certification course on Analog Electronic Circuits, myself Pradip Mandal from E and EC Department of IIT Kharagpur. Today's topic of discussion it is Frequency Response of CE CS Amplifiers Common Emitter and Common Source Amplifiers Considering High Frequency Model of BJT and MOSFET.

In fact, we already have started about this frequency response of CE amplifier and CS amplifiers, but there we did not consider capacitances associated with the MOS transistor itself. So, today's discussion it is a we will see what will be the impact of the capacitances associated with the devices the transistors on its frequency response particularly for common emitter and common source amplifier.

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Flow of Discussion (Bottom-up) – Building blocks

- **System/ Sub-systems** (*for specific application*)
 - **Modules** (*performing specific tasks*)
 - **Building blocks** (*having specific characteristics*)
 - *Components (devices/circuit elements)*

• **Week 4:**

- ✓ Frequency response of CE and CS amplifiers,
- ✓ High frequency models of BJT and MOSFET, and their usages.
- Limitations of CE/CS amplifiers and hence the need of buffers.

Compared to our overall flow and overall plan where we stand today it is we are in module 4 we have done quite an extent about the frequency response of common emitter and common source amplifier. Today we will be extending that for frequency response considering high frequency model of BJT and MOSFET transistor.

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The slide features a dark blue background on the left with the text 'CONCEPTS COVERED' in yellow. The right side has a yellow background with the title 'Concepts Covered:' in blue. Below the title is a list of four items, each with a red square icon:

- Impact of high frequency models of transistors on Frequency response of CE and CS amplifiers
- Use of Miller's theorem
- Frequency response of R-C-R||C circuit
- Numerical examples
 - CE amp. with fixed-bias
 - CS amp
 - CE amp with self bias

To the right of the list is a hand-drawn circuit diagram showing a resistor, a capacitor, and a transistor symbol. At the bottom of the slide, there is a Windows taskbar with various icons and a system tray.

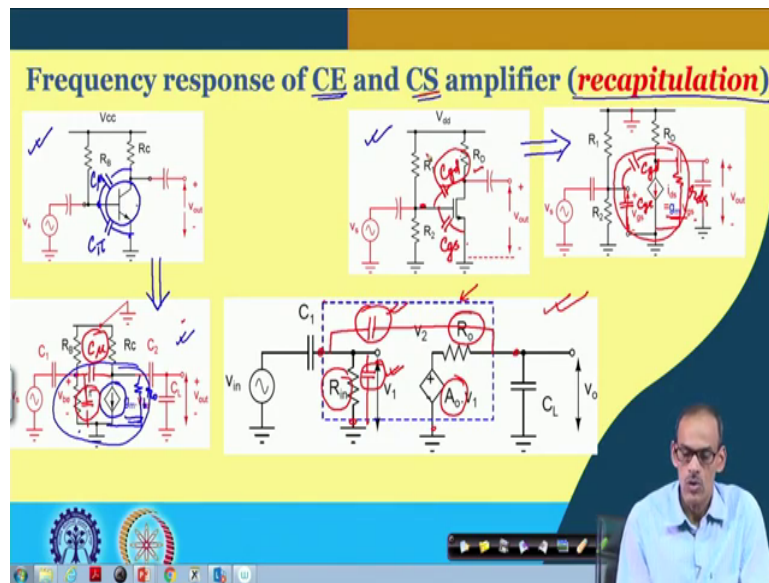
So, the concepts we are planning to cover today is the following. First of all we like to; we like to highlight the points that the impact of the high frequency response on the frequency response of CE and CS amplifiers and then we will see that there is a need of some theory, proposed by Miller called Miller's theorem.

And then we shall use that Miller's theorem to calculate effective capacitance associated with the transistor in the frequency response and then we shall see the need of a frequency response analysis for a special kind of circuit namely R-C followed by R and C in parallel. So, this frequency response of this kind of circuit we have not discussed.

We have discussed only R-C and CR circuit, but here whenever we will be talking about the equivalent circuit of common emitter and common source amplifier, containing the capacitance is coming from the transistor, then we will see that there is a need of a R-C

circuit followed by CR circuit. So, that is what we have to consider. So, we will be having R-C and then R and C in parallel. So, this kind of circuit we have to consider and then after that we will be talking about some numerical examples. So, this is what the overall plan.

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So, let us look back what is the frequency response we have discussed for common source and a common emitter amplifier. So, this is a recapitulation as I say recapitulation of whatever we have discussed so far. So, here we do have the CE amplifier and here is the corresponding small signal equivalent circuit for that.

And so, likewise here we do have the common source amplifier and its corresponding small signal equivalent circuit is given here. So, note that in this model in this small signal equivalent circuit, for this transistor we do have r_{π} from base to emitter terminal and then we do have g_m into V_{be} . In fact, this will be V_{be} .

So, this current source it is basically voltage dependent current source in addition to that we may consider r_o , but whatever the model we have considered so far it does not include the inherent capacitances associated with this transistor namely base to emitter terminal capacitance called C_{π} and then base to collector terminal capacitance called C_{μ} . So, if you consider these two capacitances in this equivalent circuit, we are expecting we will be having a C_{μ} part here and then C_{π} part here.

So, likewise if you see the common source amplifier, here in the model of the MOS transistor we do have the voltage dependent current source g_m into V_{gs} and then we may consider from here r_{ds} , but so far we have not considered inherent capacitances of the MOS transistor namely gate to source capacitance C_{gs} and then C_{gd} . So, if I consider that C_{gs} and C_{gd} what we are expecting that we will be having C_{gd} here and then C_{gs} here.

Now, if I consider generalized model and here we do have the generalized model of the two amplifiers namely common emitter amplifier and common source amplifier. So, what you can see that the dotted portion is the macro model or the voltage source or other voltage amplifier where we do have this is the input port and then this is the output port within that we do have output resistance, input resistance and then also the voltage gain.

Now, if I consider the C_{π} C_{μ} what we are expecting that from say input to output node there will be one capacitance. So, likewise from input to ground there will be another capacitance. Now, this capacitance input port capacitance for C_e amplifier it represents primarily the C_{π} part and if it is a common source amplifier, then this capacitor represents C_{gs} part on the other hand the input to output port bridging capacitance this one it is representing C_{μ} for common emitter amplifier and then C_{gd} for common source amplifier ok.

So, in summary so far we have discussed these two small signal equivalent circuit and they are we have seen the frequency response now in our present discussion, we need to consider two more capacitances in our discussion and that will lead to a frequency response particularly for high frequency behavior.

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Frequency response of CE and CS amplifier
(considering high frequency models of transistors)

- C_{π} / C_{gs} at input port and
- C_{μ} / C_{gd} between input and output terminals
- Presence of source resistance r_s

So, what we have to consider now, it is given here as I said just now we need to consider C_{π} and C_{μ} for common emitter amplifier. Likewise, here we need to consider C_{gs} and C_{gd} for common source amplifier. So, this C_{π} or C_{gd} depending on whether we are talking about common emitter or common source amplifier, that increases the input capacitance or that contributes to input port capacitance and then we do have C_{μ} or C_{gd} it is the bridging capacity capacitance between input and output terminal.

Now, we also consider since so, this C_{π} and C_{μ} effectively they are providing input capacitance, input parallel capacitance called say C_{in} to see its effect it is also we consider the source resistance R_s . Of course, if I do not consider source resistance even if you consider say input capacitance, in the frequency response it will not be having any effect

because the signal applied at the primary input, it will be directly coming to the input port of the amplifier.

On the other hand if I am having this r_s if I consider the source resistance r_s and then in presence of C_{in} , I will be expecting one RC circuit and naturally that will affect that will change the frequency response of the amplifier. So, while we will be talking about the frequency response considering high frequency model, we do consider this non zero value of this r_s , r_s here and r_s are here so we will consider the source is having r_s here and this source is also having source resistance r_s .

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**Frequency response of CE / CS amplifier
considering high frequency models of transistors (contd.)**

The slide contains four circuit diagrams illustrating the frequency response of CE/CS amplifiers. The top row shows a common-emitter (CE) amplifier with V_{cc} , R_b , R_c , and V_{out} . The middle row shows a common-source (CS) amplifier with V_{dd} , R_1 , R_2 , R_0 , and V_{out} . The bottom row shows a more detailed model with source resistance r_s , input capacitance C_{in} , and output capacitance C_{out} . A red circle highlights a specific model with nodes v_1 and v_2 , and capacitors C_3 and C_4 . A presenter is visible in the bottom right corner.

So, if I consider this model and this model together we do have generalized model which is given here. So, we do have C_3 . C_3 representing either C_{μ} or C_{gs} likewise, C_4 it is representing C_{μ} or C_{gd} depending on C_e or C_s amplifier. So, I should say this is the

generalized model of the amplifier whether it is common emitter or common source that can be analyzed by considering this circuit.

So, here while we will be talking about the frequency response, what we can see that impact of these two capacitances we have to consider. And in this frequency response of course, I have committed a small mistake in this circuit we should be having C 2 here. So, this is the C 2 and this additional capacitance. So, whatever the capacitance you are putting here that is basically the C L instead of C 2 it is CL ok.

So, now our task is to find the frequency response of this equivalent amplifier representing both common emitter and common source amplifier. So, to analyze this circuit let we try to see that, what are the additional things we have to do.

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Frequency response of CE / CS amplifier
considering high frequency models of transistors (contd..)

$C_1, C_2 \gg \mu F$
 $C_3, C_4 \sim 10 pF$

- Effect of C_3 (C_{π}/C_{gs}) and R_s : Additional R-C effect at the input port
- Effect of C_4 (C_{μ}/C_{gd}) connecting input and output terminals

Miller's theorem

So, if you see carefully in this circuit as I said again I have repeated this mistake, we should be having C_2 here and then this should be C_L . Now, as I said that this C_3 , C_3 it is the input port capacitance and this input port capacitance, it is forming RC circuit with this R_s and whatever the equivalent parallel capacitance. So, do have. So, it is introducing additional RC effect at the input port earlier in absence of say R_s and R_3 we used to consider only C_1 and R_1 to get the frequency response.

Now, we do have this resistive element and this capacitive element. In fact, we also have this capacitive element, but before we translate this capacitive element we need to be careful that C_4 it is connected between the input port of the amplifier and the output port of the amplifier.

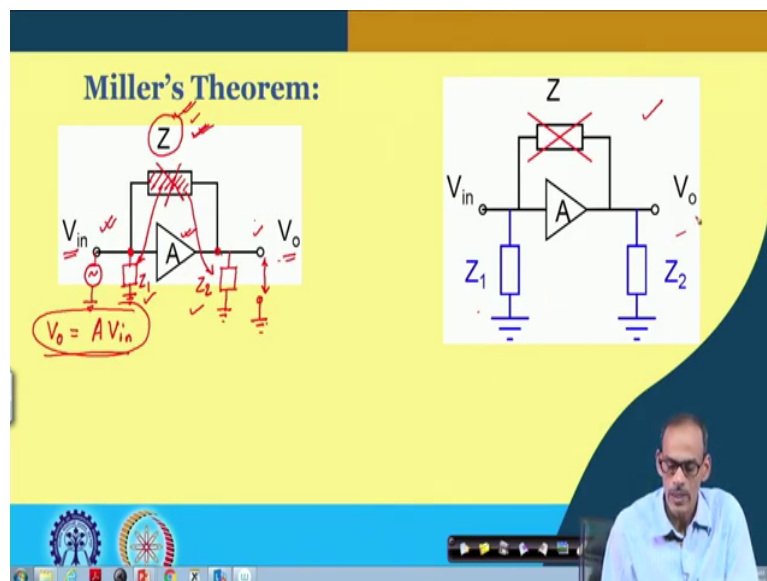
So, we need a special treatment to translate this C_4 in terms of or into two equivalent element one is at the input port and another one it is at the output port either you can consider here or here. Now, I must say that this splitting of this C_4 capacitance a one for input port another is for output port, it is normally done by a theory proposed by Miller which is commonly known as Miller's theorem.

So, before we go into the detail analysis let me go through this Miller's theorem first and then we split this C_4 into these two equivalent component and then we will be going for the frequency response of the amplifier right. So, and the other point I like to mention is that whenever we will be talking about the frequency response considering say r_{pi} sorry C_{pi} and C_{mu} or C_{gs} and C_{gd} we may consider that whenever these capacitors are prominent the C_1 and then C_2 they may be successfully allowing the signal to go through this.

So, for simplicity whenever we will be talking about the effect of C_3 and C_4 we may short this one. So, that you have to keep in mind mainly because the typical value of say C_1 and C_2 if you see, this these capacitors maybe having higher than microfarad. On the other hand typical value of say C_3 and C_4 they may be in the range of 10 picofarad ok.

So, naturally the frequency range over which we will be considering the effect of C 3 and C 4 there you may consider that C 1 and C 2 effectively they are simply shorting their two terminals. So, now, we like to go into this Miller's theorem and try to see what is it and how we split the input to output bridging capacitance into two parts; one is for input port another is for the output port.

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Now in general here suppose we do have one amplifier and let me consider the signal at the input it is voltage. So, likewise we do have output signal also in the form of voltage and then we do have that mean amplifier A , which means that V_o equals to A times V right. So, the signal it is going from left to right. So, we do have input port and then we do have the output.

So, you may say whenever you are talking about V_o we are considering the voltage at this terminal with respect to common terminal called ground likewise at the input we are feeding

the signal V in with respect to the common terminal. Now, we do have this bridging element. So, in general it may be resistive, it may be inductive or it may be capacitive or it may be combination of that but whatever it is, this element it is bridging the input port and output port.

Now this input port to output port relationship signal relationship can be represented by this one excluding this Z element the bridging element. Now, in presence of this Z that makes the analysis a little complicative. So, a better approach and efficient approach is to split this capacitor sorry I should say this Z element into two equivalent element; one is for the input port, which is connected with respect to common node ground and let me call this is Z_1 and since the other terminal it is connected to the output port we also need to consider its effect before we remove it and let we call this is Z_2 .

So, we can say that effect of Z it is getting captured by the Z_1 and Z_2 . So, we can see once it is getting translated into the its equivalent parts then you can remove this one. So, that is what your this diagram it is representing that we are splitting the bridging element into two equivalent parts called Z_1 and Z_2 one for input port another is for the output port.

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Miller's Theorem:

$$I_1 = \frac{V_{in} - V_o}{Z} = \frac{V_{in}(1-A)}{Z}$$

$$Z_1 = \frac{Z}{(1-A)}$$

$$I_1' = \frac{V_{in}}{Z_1}$$

Now, when do I say that they are equivalent? So, first of all if we feed a signal V_{in} , then if I call that these two circuits are equivalent and then if you feed the same signal V_{in} , then the condition for this circuit and this circuit should be same which means that if I am feeding a voltage here and then whatever the current supposed to be flowing through this element, that should be seen for this circuit also.

Now, for this circuit the current or the if I call say this is I_1 . So, for this case the current expression I_1 it is equal to $V_{in} - V_o$ divided by the impedance Z . Now, we also know that V_o equals to A times V_{in} . So, this can be written as V_{in} multiplied by $1 - A$. So, by replacing this V_o by expression we are getting V_{in} divided by Z .

Now, on the other hand if you see in this case and if I call this current it is say I_1' . So, I_1' dashed we can write this is equal to V_{in} divided by Z_1 . Now, to claim that these two circuits

are equivalent we have to equate these two currents or these two currents should be equal and so, we can say if I equate these two, what we are getting here it is Z_1 equals to Z divided by $1 - A$ right.

So, likewise if I say that these two circuits are equivalent for the output port, then similar kind of things we can do for the output port. So, please try to remember this relationship Z_1 is equal to Z divided by $1 - A$.

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Miller's Theorem:

$$I_2 = \frac{V_o - V_{in}}{Z}$$

$$= \frac{V_o}{Z} \left\{ 1 - \frac{1}{A} \right\} = \frac{V_o}{Z_2}$$

$$\Rightarrow Z_2 = \frac{Z}{1 - \frac{1}{A}} = \frac{AZ}{(1-A)}$$

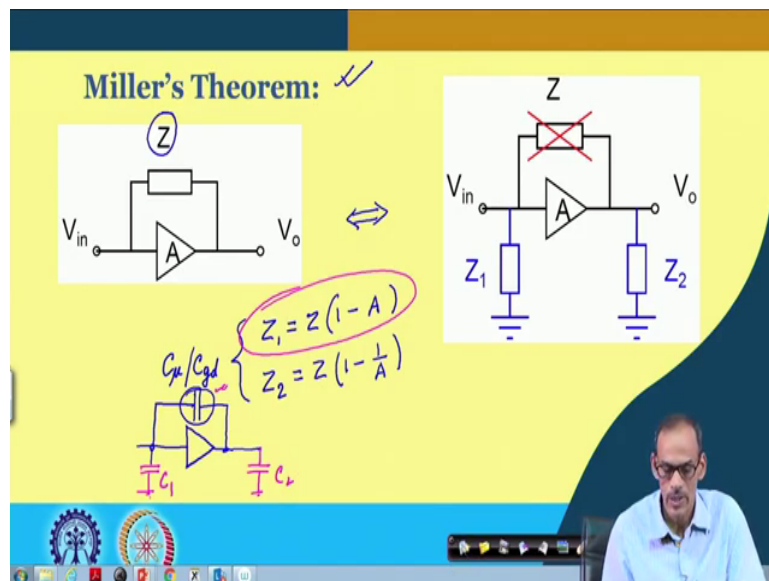
$$I_2' = \frac{V_o}{Z_2}$$

So, then if I consider a output port and if I stimulate this circuit by say V_x or instead of calling say V_x say this V_o . So, if I say that this V_o if it is coming here. So, let us not stimulate sorry. Let us not stimulate suppose we are applying V_{in} and then we do have V_o here and if the voltage here it is V_o the current run through this circuit, it is say if I call I_2 dashed.

So, I_2 dashed it is equal to V_o divided by Z_2 . On the other hand the current flowing through this circuit, if I call it is I_2 and in this case I_2 equals to V_o minus V_{in} divided by Z and we know that V_o equals to A times V_{in} . In other words we can write V_{in} equals to V_o divided by A . So, this V_{in} you can replace by this expression. So, what we are getting here it is V_o multiplied by 1 minus 1 by A divided by Z .

Again if I say that these two circuits are equivalent for this output port, then we can say that this current and this current expression they should be equal. So, if I equate this with V_o divided by Z_2 . So, that gives us the expression of Z_2 equals to Z divided by 1 minus 1 by A or you can say that this is equal to A divided by 1 minus A into Z .

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So, in summary these two circuits it will be claimed as equivalent provided the Z_1 equals to Z multiplied by 1 minus A and then Z_2 equals to Z into 1 minus 1 by A right. So, and so, this

is what the whatever it is called Miller's theorem and as I said that this Z it may be capacitive, it may be the inductive or it may be resistive element or it may be combination.

Now, in our application in our today's discussion we do have one amplifier and then we do have input to output bridging capacitance either it may be C mu or it may be Cgd depending on CE or CS amplifier and then we can try to see what will be this capacitance it can be splitted into two parts one for the input port another is for the output port ok.

So, the in the next slide we consider a special case of this Z and we consider this C and then we will try to find what will be the corresponding equivalent capacitance, we do have a here and here.

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Miller's Theorem (contd.):

$C = C_{\mu}/C_{gd}$
 $V_o = AV_{in}$
 $Z_1 = \frac{1}{sC_{in}} = \frac{1}{sC} \{1 - A\} \Rightarrow C_{in} = C \{1 - A\}$
 $Z_2 = \frac{1}{sC_{out}} = \frac{1}{sC} \left(1 - \frac{1}{A}\right) \Rightarrow C_{out} = C \frac{A-1}{A}$

$V_o \approx V_{in} A \Rightarrow C_{in} = C \{1 - A\} \quad A' \approx A$
 $C_{out} = C \left\{ \frac{A'-1}{A'} \right\}$

So, here we do have the circuit of our discussion namely we do have this amplifier and its gain it is A and we do have the C it may be as I said it may be C_{in} or a C_{gd} that you call it a C and this capacitor it can be replaced by its equivalent two parts. One is for input port another is for the output port and how do we say that this is the equivalent? So, the impedance of this capacitance which is $1/sC$ in should be equal to $1/sC$ multiplied by $1 - A$.

Sorry, this is divided by; this is divided by. Now if I rearrange this equation what we are getting here it is C_{in} equals to C multiplied by $1 - A$. Note that here this whenever we are defining this A we consider V_o is equal to A times V_{in} . Now, in case if A is having minus sign so; that means, this part it will be getting plus. So, that you have to keep in mind whenever we will be talking about the actual circuit. On the other hand the $1/sC_{out}$ so, that is the you see Z_2 .

So, this was Z_1 this is Z_2 . So, this is equal to $1/sC_{in} - A$ divided by or I should say this is multiplied by $1 - A$ right ok. So that gives us C_{out} equals to C multiplied by $A - 1$ divided by A . In fact, ok. So, these two equation it will be used for our analysis and this is valid even if you consider the finite output resistance. So, in this case of course, the bridging element it is not connected here instead it is connected after the resistors, because this may be part of the model of the amplifier.

So, we can say this is the terminal equivalent resistance of the amplifier output port right. So, even for this circuit if I consider the signal coming here and the signal coming here they are almost equal. So, if I say that even though we do have this resistance, if I say that this V_o it is practically V_{in} multiplied by A . So, then also we can use this relationship namely C_{in} equals to C multiplied by $1 - A$ and C_{out} equals to C into $A - 1$ divided by A .

Now, in case if you want to really consider this R_o then this A part. So, this A part should be replaced by whatever the gain you get from this point to this point. So, in case the corresponding gain from here to here it is A_{dashed} , then this A should be replaced by A_{dashed} this A should also be replaced by this A_{dashed} right. So, depending on the situation

we may consider this A dashed or we may say that A dashed it is approximately equal to A .
So, this is the Miller's theorem it will be used in our frequency response analysis ok.

So, let me take a break and then we will come back for continuing this topic.