

Analog Electronic Circuits
Prof. Pradip Mandal
Department of Electronics and Electrical Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture – 36
Frequency Response of CE and CS Amplifiers (Part B)

So, welcome back to this R-C circuit frequency response after the short break.

(Refer Slide Time: 00:31)

The slide, titled "Frequency response of R-C circuit (recapitulation)", features a circuit diagram on the left. It shows an AC voltage source V_{in} in series with a resistor R , connected to a parallel combination of a capacitor C and an output terminal. The output voltage is V_o . Handwritten notes in red ink include $V_{in}(s)$ and $V_o \leftrightarrow V_o(s)$. To the right of the circuit, the following equations are written:

$$V_o(s) = \frac{V_{in}(s) \times \frac{1}{sC}}{R + \frac{1}{sC}}$$
$$\frac{V_o(s)}{V_{in}(s)} = \frac{1}{1 + sRC}$$

Below these, the Fourier domain transfer function is given as:

$$\frac{V_o(j\omega)}{V_{in}(j\omega)} = \frac{1}{(1 + j\omega RC)}$$

Additional handwritten notes include $s = j\omega + \cancel{\lambda}$. At the bottom of the slide, there are two bullet points:

- Analyses in Laplace Domain and then in Fourier Domain
- Gain plot and Phase plot of Transfer function

The slide also includes a small video inset of a man in the bottom right corner and a Windows taskbar at the very bottom.

So, similar to the C-R circuit again here what you are doing is that we are taking the circuit into Laplace domain, namely the impedance of both the elements we are going in Laplace domain for C it is $1/s$ and this is directly same as this R. And then input it is V_{in} in Laplace domain s and then the corresponding output also we are considering in Laplace domain which is V_o .

And, if we analyze this circuit what you are getting here it is $V_o(s)$ equals to V_{in} divided by R plus 1 by sC into 1 by sC . So, from that what your yeah what you are getting is $V_o(s)$ divided by $V_{in}(s)$ equals to 1 by $1 + sRC$. It is similar to the previous circuit except of course, we do not have the sRC part rather in the numerator we do have simply 1 .

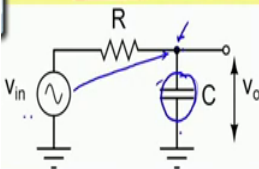
Now, to get the frequency response as I said for the previous case what you have to do, we have to take this replace this s by $j\omega$ or rather I was having say sigma part in s which I am making it 0 . So, we are dropping this part and that gives us the transfer function in Fourier domain and it becomes 1 by $1 + j\omega RC$.


Now, this transfer function in Fourier domain again you can make the gain plot and the phase plot to get the frequency response.

(Refer Slide Time: 02:47)

Page 13/17

Frequency response of R-C circuit (recapitulation)

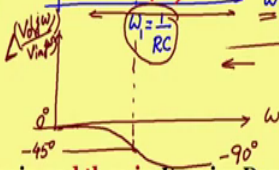





$$\frac{V_o(s)}{V_{in}(s)} = \frac{1}{1 + sRC}$$

$$\frac{V_o(j\omega)}{V_{in}(j\omega)} = \frac{1}{1 + j\omega RC}$$

- Analyses in Laplace Domain and then in Fourier Domain
- Gain plot and Phase plot of Transfer function





So, again here what you can do? We can plot the magnitude. So, if we consider the magnitude of V_o / V_i with respect to ω . So, if you see here at low frequency; if we ignore this part with respect to 1, then the corresponding magnitude it is 1. But then if you go to higher and higher frequency and if this part it is dominating over this 1, then the magnitude wise what we will be seeing here it is $1/\omega$ nature. So, I should say it is more like a hyperbolic curve if we consider this part, it is dominating.

So, beyond some frequency it is going through this hyperbolic nature. So, initially it may be it was approximated like this and beyond this point it is approximated by this hyperbolic part. And, the changeover it is happening again at the same frequency where ωRC magnitude wise ωRC it becomes equal to 1. So, I should say if I call this is ω_1 at $\omega_1 RC = 1$ what you are getting is that $\omega_1 RC = 1$ which means $\omega_1 = 1/RC$.

So, if I consider the actual curve instead of considering this two asymptotic or approximated curve will be getting the actual curve going like this. So, here again, it is having complementary behavior as I said with respect to the previous one. Before this frequency the circuit it is passing the signal to the output and that is very obvious from the intuition of this R-C circuit because if the signal frequency it is low enough, then the capacitor it may not be shorting this output signal as a result at the output we can get almost this input signal.

On the other hand, if you go to higher and higher frequency, this capacitor it is impedance it may drop and then the signal at this output it will be dropping. So, if you consider frequency beyond this ω_1 , then you can say that behavior of the circuit it is working like a the signal is getting attenuated. So, beyond this frequency the signal it is getting attenuated. So, I can say that the circuit it is again it is having two range of frequency – one is pass band, another is the stop band. And, in this case it is we can say low pass low lower frequency it is getting allowed through this circuit.

Now, the similar to the previous case here again we can make the phase plot and then the if you consider the phase plot at very low frequency this part it is almost 0. So, we do have 0

phase shift. So, you can see that the phase shift here it is 0 degree. So, this y-axis is the phase shift offered by this network.

So, V_o divided by V_{in} at $j\omega$ if you consider it is phase then at low frequency it will be 0 degree and then as we are approaching towards this $\omega = 1$, then this part it will be more and more prominent and then at $\omega = 1$ that will be equal to it is magnitude wise it will be 1 and so, it will be $j\omega$. So, in this part it will be simply ω .

So, the corresponding phase here it will be minus 45 degree that is because in the denominator at this point I do have $1 + j$ all right and if you go to a higher frequency beyond this $\omega = 1$, this part it will be dominating and as a result then will be going towards minus 90 degree. So, this phase plot and this gain plot they are giving the total frequency response of this R-C circuit.

Now, similar to the previous case again since we like to see the wide range of frequency and the behavior of the frequency response over wide range. So, we need to change this scale into logarithmic form and magnitude also we like to change in logarithmic form. So, whatever we get is the Bode plot. So, here again in the Bode plot what will be seen it is similar to the previous case that if you consider ω in log scale then of course, 0 frequency it will be pushed to infinite distance and then this one part it will be coinciding with 0 dB.

And, then this hyperbolic part on the other hand, it will be having a linear behavior and then at the corner frequency, there will be there will be rule of started and exactly at corner frequency at ω is equal to $\omega = 1$, we will be having minus 3 dB. So, again here we can say that this corner frequency whether it is in; so, this is gain in gain or magnitude in dB. So, these this corner frequency and whether it is in this frequency response or in Bode plot, it is related to this location of the pole.

So, here again we are getting a direct relationship of this pole and this corner frequency. So, just by seeing the transfer function; if we know that the location of the pole probably from that we can tell what is the location of this corner frequency. And, in this case if we consider lower frequency with respect to this corner frequency, then the gain it is remaining constant.

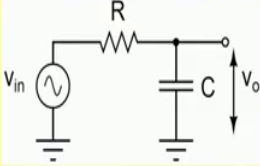
And, beyond this point the role of it is linear and in fact, mathematically it can be shown that or you may be already knowing from other subject that the slope of this gain variation with ω it is minus 20 dB per decade.

So, whenever you are talking about frequency response in fact, what we are looking for is that in some range gain it is remaining constant and then we are hitting say something called corner frequency and beyond that it is having a roll off like this minus 20 dB per decade. In case if we consider lower side for the other circuit, then we have seen that beyond some other corner frequency or below of this corner frequency, the slope here it is 20 dB per decade.


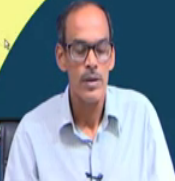
So, I should say that the whenever you are talking about this frequency response what we are trying to capture it is the location of the corner frequency and that can be directly obtained from the pole of the transfer function in Laplace domain. So, the natural curiosity is there why this pole and this frequency response cutoff frequency why they are related. So, let us see that part yeah.

(Refer Slide Time: 11:27)

Frequency response of R-C circuit (contd.)

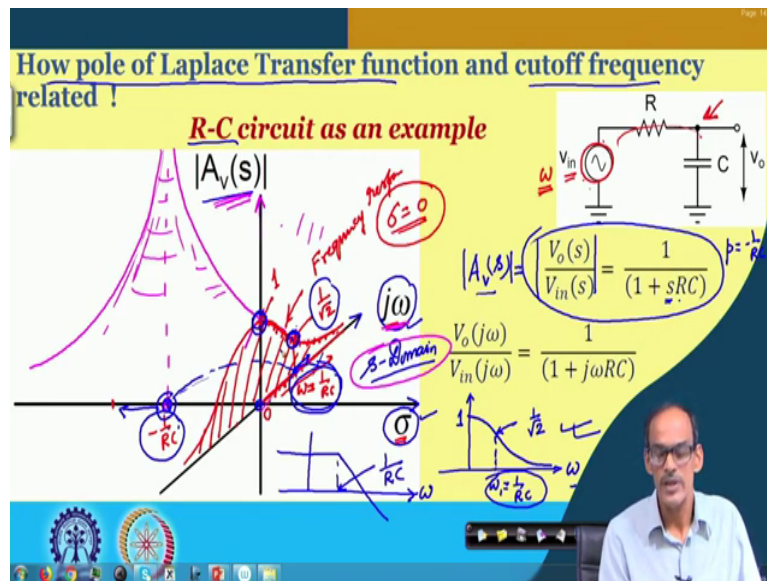

$$\frac{V_o(j\omega)}{V_{in}(j\omega)} = \frac{1}{(1 + j\omega RC)}$$

• **Bode' plots: Gain plot and Phase plot**



Yeah, this part we already have covered; Bode plot we already have covered yeah.

(Refer Slide Time: 11:31)



So, this is what we are about to discuss that the transfer function pole of transfer function in Laplace domain and the cutoff frequency they are related. And, let us consider this simple R-C circuit which is having one pole and the location of the pole, it is if you see here the location of this pole it is p equals to minus 1 by RC which means that s is equal to minus 1 by RC the transfer function the magnitude here it will be going to be infinite.

So, if I say that this gain or transfer function if we denote by say $A_v s$ since it is we are dealing with voltage and if you plot this magnitude as function of s and s is note that s is having two components; one is the sigma part the real part and the imaginary part. So, if we plot this magnitude over this s domain; so, this horizontal part, the horizontal part consists of this sigma axis and j omega axis basically this is the axis. So, we may call this is the s plane s domain.

So, over this domain s domain if we plot this magnitude of this transfer function what we are seeing here it is since it is having a pole it is having a pole at this frequency or say let me consider this one. So, at $\frac{1}{RC}$, so, if we sketch this plot obviously, or the surface if you see this will be going to infinite at this point. So, if you see this surface it is more like a tent and at this frequency the tent it is going to infinite.

So, this is imagine that this is 3-dimensional plot and the location of the pole it is here. So, if you see this surface and then if you consider the vertical plane constructed by this $j\omega$ axis and then this A/V axis; that means, this plane consists of this vertical plane and this plane it is cutting the surface whatever the surface just now we are plotting here which is basically representing the variation of the gain over this s domain.

Now, if I consider this cut set of this surface which is getting cut by this vertical plane constructed by this A/V and $j\omega$. What we are what we can see here it is, it is a cut set having a profile like this. So, the cut here will be seeing a profile like this and if you see this cut what is the significance or importance of this one, it is that along this axis this is also this axis is also representing some value of s , but along this axis of course, the sigma part equal to 0.

So, which means that whenever we are talking about sigma is equal to 0; we are restricting our s domain only along this line. So, if we move along this line each of this point is representing one frequency component corresponding to whatever the omega we do have. And whatever the surface profile we are seeing here, it is basically representing how much the gain or the attenuation of the input signal it will be experiencing through this circuit based on its frequency this variation it will be obtained.

In other words, if I say that in this experiment if we change this omega frequency starting from say 0 towards higher and higher frequency and keeping the magnitude input magnitude of the input signal say constant and if you observe the magnitude of the output signal what we will be seeing that the with omega how the magnitude its changing that profile we can get by this line.

So, this line as I said this line it is nothing, but the response of this circuit for say stimulus whose magnitude is not changing with time. Why magnitude is not changing with time because we consider the corresponding stimulus it is having a sigma is equal to 0. So, whatever the signal we are feeding here if we consider it is a steady sinusoidal signal that represents two omega component - one is plus omega and minus omega part. And if you consider this part plus part, then if you see the corresponding variation of the magnitude with the omega nothing that is nothing, but this line this curve.

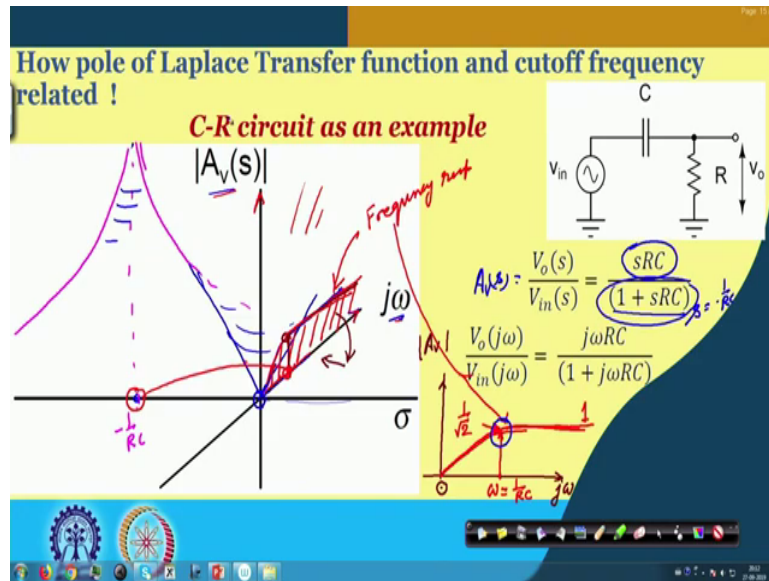
And, incidentally this curve it is the frequency response right and the interesting thing is that in this case for R-C circuit, the value here it is equal to 1 and if you go to a frequency say omega equals to $1/RC$, the corresponding magnitude here you will see it is $1/\sqrt{2}$. So, if I compare the value of this frequency response at this point and at this point, then where it is $1/\sqrt{2}$. And if you see that omega is equal to $1/RC$ they are what they are revealing that the location of the pole at $1/RC$. It is having an impression at the frequency response.

So, I should say if I draw one say circle centering the origin having a radius of $1/RC$, then that circle it will be cutting this $j\omega$ axis at this point and at this point the magnitude here, it is $1/\sqrt{2}$. So, you may recall that whenever we plot this frequency response of this circuit, we have seen that magnitude it was diminishing with omega. Note that this is in of course, it is in linear scale and this is the magnitude and here it was 1 and at omega $1/RC$ which is equal to $1/RC$ the magnitude here it is $1/\sqrt{2}$.

So, this point the cutoff point or cutoff frequency point which is basically the image of the location of the pole. In fact, if this location of the pole it is going far away then the point at which the frequency response it becomes $1/\sqrt{2}$ of this point that will also be moving away. So, location of this pole in the Laplace domain and the location of the cutoff frequency they are related. So, in fact, if you see this circuit this is of course, RC circuit. So, based on the location of the pole, we can directly as I said that we can find a cutoff frequency whether it is whether it is the frequency response in linear scale or whether it is in Bode plot the cutoff frequency in fact, this is also $1/RC$.

So, now let us see the; so, here we have considered R-C circuit which it was having a pole and we have seen the relationship to cutoff frequency. Let us look into the other circuit namely the C-R circuit which is having slightly different transfer function.

(Refer Slide Time: 20:55)



So, yeah so the C-R circuit the denominator part it is same which means that this is also having a pole at s is equal to minus 1 by RC in addition to that it is also having a 0 which means that numerator polynomial of this transfer function $A V s$. It becomes 0 at s is equal to 0 which means that here also it is having a pole it is having a pole at minus 1 by R C. So, if I am having only one pole, the function supposed to be going to be the surface supposed to be going to be infinite this point.

But, then unlike the previous case here it is having a 0 which means that. So, it is also having a 0 at this point. So, if it is having a 0 then of course, this surface it will be turned down to 0

here. And, if you see the impact of this the pole and 0 combination that gives us a nice surface here so, this part it is going to infinite on the other hand this part it is going to be 0.

And, if you see here if we travel along this sigma axis we can see this infinite and 0. But, if you consider see in other direction maybe along the $j\omega$ also so, what kind of profile we will be seeing here it is it will be going up here and then it will get saturated like this. So, earlier whatever the cut set we have drawn here along the along the $j\omega$ axis or rather if I consider this surface whatever the surface, we are trying to explain here. If that surface it is cut by this vertical plane constructed by $A V$ and $j\omega$ axis, then whatever the cut set will be seeing here it will be having a cut like this.

So, whatever the cut set we are getting here again this is representing the frequency response. And, again depending on the location of this pole this frequency response of course, it will be going to steady and it will be having a corner frequency where the yeah. So, let me use a different color here yeah. So, we do have a corner frequency where the magnitude of this frequency response it will be $1/\sqrt{2}$ times the saturated level.

If I turn this curve and if I see the in this direction what we can see here it is this is $j\omega$ axis and this is the sigma axis which is coming out of the surface and this is $A V$ magnitude. What we are trying to explain here it is having a plot like this. So, this is what the frequency response. In fact, so, this line it is basically the same line here and again the location of this pole it is basically deciding the point where it is going to a point which is having a magnitude which is $1/\sqrt{2}$ times of this saturated level and the saturated level of course, it is 1.

So, this point it will be whatever you call $1/\sqrt{2}$ points or in Bode plot it is referred as minus 3 dB point and this frequency again which is $\omega = 1/RC$. So, what is the summary here we like to say here it is that based on the location of the pole based on the location of the pole of this transfer function in Laplace domain we can directly say that what is the corresponding cutoff frequency.

Now, so, we have so far discussed about simple circuit R-C circuit and C-R circuit and if we combine this R-C circuit and C-R circuit together of course, then whatever the transfer

function will be getting it will be having multiple poles and zeros those things we will see that. So, let us see the combination of C-R and R-C circuit and while you are combining we need to be careful that loading effect if we cascade the two circuits of course, one circuit it may load the other one.

(Refer Slide Time: 26:32)

Frequency response of (C-R)-Amplifier-(R-C) circuit

• **Transfer function:**

$$\frac{V_o(s)}{V_{in}(s)} = \frac{V_1(s)}{V_{in}(s)} \times \frac{V_2(s)}{V_1(s)} \times \frac{V_o(s)}{V_2(s)}$$

$$= \left\{ \frac{sC_1R_1}{1 + sC_1R_1} \right\} \times A_o \times \frac{1}{(1 + sC_2R_2)}$$

$p_1 = -\frac{1}{R_1C_1}$
 $p_2 = -\frac{1}{R_2C_2}$

To avoid the loading effect what we have done here it is we do have the C-R circuit we do have the C-R circuit and also we do have the RC circuit. And, in between we are putting some ideal kind of block saying that this block it is what it is doing is that whatever the voltage it is getting developed called say V 1 it produces a voltage here which is a V 2 which is equal to constant times this V 1. So, I should say that this model this ideal model of this voltage dependent voltage source I should say this voltage dependent voltage source ensures that loading effect of this R-C circuit is not falling on this C-R circuit.

So, if this A is having some magnitude which is higher than 1, so, we can say that this voltage dependent voltage source is an ideal voltage amplifier. So, the whole circuit if you see you can call it is combination or cascaded of C-R circuit followed by amplifier and then followed by R-C circuit and if we stimulate this circuit from this point and if we like to observe the corresponding output here, what we can see here it is we can get the transfer function of the whole system by considering each of this part.

So, from primary input to primary output transfer function if we try to see we can split this transfer function into three components one is coming from this C-R circuit, namely V_1 divided by V_{in} . So, this is the first part this is coming from the C-R circuit and then the second part which is coming from the amplifier. So, we do have this amplifier here. So, this part it is basically representing the amplifier. On the other hand, the third part it is the R-C circuit and its transfer function it is basically the V_o divided by V_2 here.

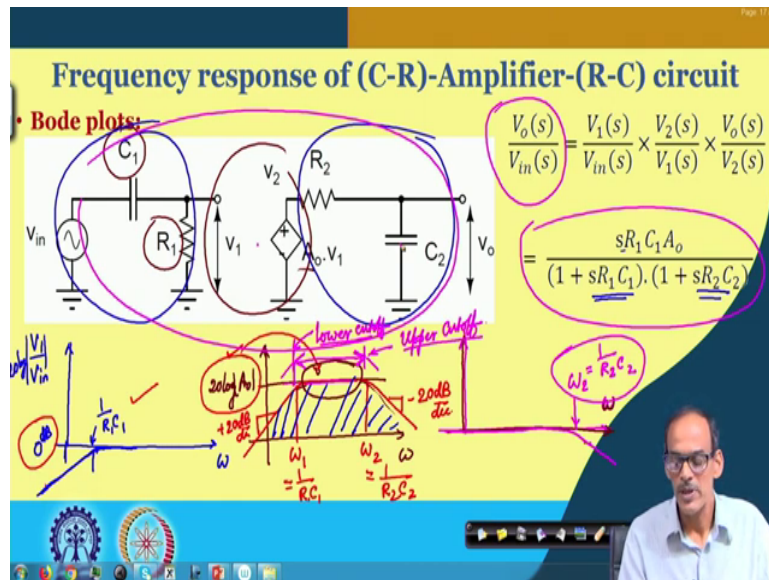
So, now, if we recall that transfer function of this C-R circuit let me use different color here. So, this part it is having a transfer function of s corresponding C and corresponding R divided by $1 + s$ corresponding C and R . So, this is the first part and then the second part is from here to here. So, the corresponding transfer function it is simply this A . And, then the third part which is R-C circuit and we have seen before that its transfer function it is 1 by s multiplied by the corresponding C and the corresponding R .

So, overall the input to output transfer function what we are getting here it is this one and what you can see here again what we are looking for is the frequency response and hence the pole of the transfer function or whatever the factors we do have in the denominator polynomial one is here another is here.

So, the if I consider this factor it is having a pole called say p_1 equals to 1 by $R_1 C_1$ with a minus sign. Likewise if I consider this part this part we do have the second pole say p_2 which is equal to 1 by $R_2 C_2$. And, this is C-R circuit and we have seen that it is a frequency response it is high pass in nature this is low pass in nature. And, for our

convenience let us consider that the high pass frequency cutoff frequency it is lower than this one and then try to see what kind of frequency response intuitively we can get out of that.

(Refer Slide Time: 31:20)



So, so, let us yeah. So, as we have discussed that the transfer function of the whole system it is given here and it is having as I said it is having two poles one is coming from here another it is here and then also it is having a 0. And, if you see the frequency response and let you consider the first one C-R circuit and the C-R circuit frequency response it is.

So, if we plot the omega in log scale and V_1 divided by V in dB; that means, if we take 20 log base 10 what will be getting here, it is basically the high pass behavior. So, we will be getting a frequency response like this. If I consider this is 0 dB level and then it is corner frequency it is here which is 1 by $R_1 C_1$. So, the frequency response of the first part it is given here.

Second part it is if I consider it is remaining constant. So, I should say that this part. So, it is remaining constant and this level it is $20 \log$ whatever A_{naught} we do have. So, it is remaining constant. On the other hand, the third part which is R-C circuit it is having, it is low pass kind of behavior and let me use different color here for axis. Let me use this color and the Bode plot of the R-C circuit it is going like this and then it is having a role of like this. And, the role of it is happening here which is say the second pole let me call this is ω_2 which is equals to $\frac{1}{R^2 C^2}$.

Now, if I combined this pink characteristic curve and then the middle one and then the blue one what we are expecting that it will be just if we overlay the three parts what will be getting here it is. So, this part it is coming from the C-R circuit and the amplifier again this remains high pass, but then instead of having 0 dB here I do have some positive gain here. And then if I combine the pink part then what will be getting here it is at some frequency beyond ω_2 it will be having a roll off.

So, the overall frequency response what we are getting here which is shown here by this red color which is having a cutoff frequency ω_1 and ω_2 in the middle range we do have very good gain which is defined by this one. And, this is coming from $\frac{1}{R_1 C_1}$ and this is ω_2 it is coming from $\frac{1}{R_2 C_2}$. So, I think this intuition will be helping you or and of course, this role of since it is single pole roll off later. We will discuss about what will happen in case if you have say double pole roll off.

Namely if you have say two poles exactly at the same frequency what will happen but, for the time being they are single pole roll off. So, this is minus 20 dB per decade slope and here we do have the slope it is positive plus 20 dB per decade and the middle gain, it is this one. So, so far whenever we are talking about say CE amplifier or common source amplifier we were dealing the amplifier in the mid frequency range, to be more precise we are talking about this frequency range and then we talked about the voltage gain of CE amplifier and other things.

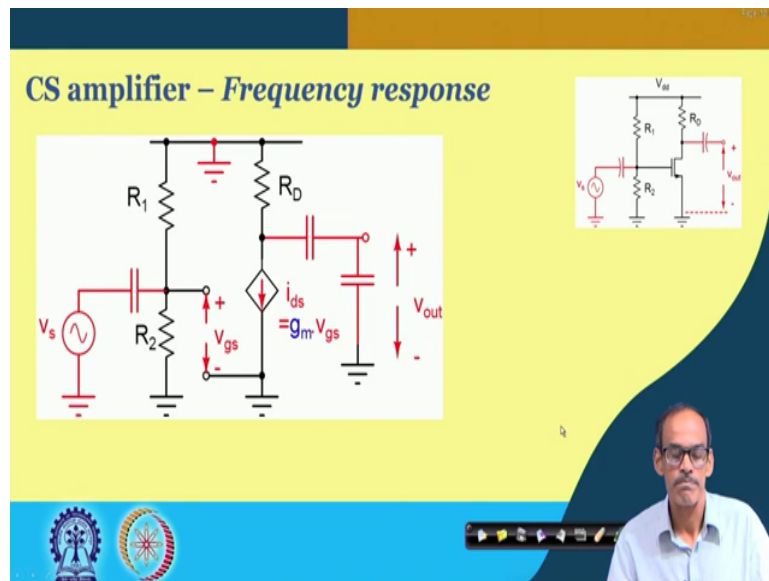
But, if you go to lower and lower frequency depending on the value of this capacitor and then this R the corresponding gain definitely it will drop. And, on the other hand depending on this

R and C then again beyond some point it will be dropping. So, the actual frequency response or the circuit behavior it is given by this combined Bode plot and as I said that it is coming from the three elements. So, whenever we will be dealing with the actual circuit it is better to identify which is the R-C circuit and which is the C-R circuit, they are defining the corresponding that the cutoff frequency or corner frequency.

So, since our main purpose here it is to use this circuit as an amplifier. So, we can say that this is the suitable range of the amplifier and if you go beyond this point then the circuit it is it is gain is dropping so, we are not interested to that. So, likewise if you go beyond this point then again gain is dropping. So, we are not interested to that. So, we can say that our suitable range it is only this one and. So, this two are the limit of the suitable range or we can say cutoff range or cutoff frequency one is called lower cutoff frequency and this one it is called the upper cutoff frequency.

So, whenever we will be talking about say amplifier along with the mid frequency range gain, these two informations basically these two frequency cutoff frequencies are also very important to define the frequency response of the amplifier. So, presently whatever the circuit we will be dealing with this model or this C R followed by amplifier followed by R-C circuit. This model it is pretty handy and pretty sufficient to explain the behavior of simple frequency response of the simple voltage amplifier.

(Refer Slide Time: 38:24)



So, we will be talking about this C-R circuit sorry, common source amplifier and common emitter amplifier circuit, but let me take a short break and then we will come back to the frequency response of the amplifiers the actual amplifier.