

Analog Electronic Circuits
Prof. Pradip Mandal
Department of Electronics and Electrical Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture - 35
Frequency Response of CE and CS Amplifiers (Part A)

Yeah, dear students welcome back to this NPTEL online program on Analog Electronic Circuits, myself Pradeep Mandal from the E and EC E Department of IIT Kharagpur. We are going through different modules and presently we are in the 4th module of this course. And, today's topic of discussion it is Frequency Response of CE and CS Amplifier; Common Emitter and Common Source Amplifier.

In the previous module we have seen how to find the gain of common emitter and common source amplifier. And, today what we will see that if we change the frequency of the input signal how the gain of the circuit whether it is common emitter or common source amplifier, it is it changes with frequency. So, the overall situation if you see the flow wise as I say that we are in the 4th module.

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Flow of Discussion (Bottom-up) – Building blocks

- **System/ Sub-systems** (for specific application)
 - **Modules** (performing specific tasks)
 - **Building blocks** (having specific characteristics)
 - Components (devices/circuit elements)
- **Week 4:**
 - ✓ Frequency response of CE and CS amplifiers,
 - High frequency models of BJT and MOSFET, and their usages.
 - Limitations of CE/CS amplifiers and hence the need of buffers.

And, today we are going to discuss in the 4th module we are going to discuss about frequency response of common emitter and common source amplifier. And, subsequently we will be covering the frequency response considering high frequency model. So, today primarily we will be ignoring high frequency model of the BJT and MOSFET and whatever the frequency response we will be seeing it is due to the coupling capacitor and then bias circuits and so and so.

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CONCEPTS COVERED

Concepts Covered:

- ☐ Frequency response of R-C and C-R circuits ✓
- ☐ Frequency response of CS amplifier ✓
 - Circuit analysis
 - Numerical example
- ☐ Frequency response of CE amplifier (with fixed bias) ✓
 - Circuit analysis
 - Numerical example
- ☐ Frequency response of CE amplifier (with self bias) ✓
 - Circuit analysis
 - Numerical example
- ☐ Design guidelines ✓

Now, what do we have in this plan, in this module; it is the or rather today and the next class are the following. So, we are planning to cover as I said that we are going to cover frequency response of common source and common emitter amplifier. To understand that first what we will do that we will revisit the frequency response of R-C circuit and C-R circuit. And, then based on the R-C and C-R circuit we will be talking about the transfer function of a typical system R-C or C-R combination.

And, then we will also discuss about what is the relationship between transfer function and then frequency response and then location of the pole zeroes in Bode in Laplace domain transfer function. So, and then what is the relationship between the pole and zeroes and the cut off frequency in the frequency response. So, those things we will be discussing in detail with simple R R-C and C-R circuit as an example. And, then using that knowledge you will be discussing about the frequency response of common source amplifier; particularly the

analysis part we will be covering today, the numerical part we will be covering on the next day.

So, today's plan is only up to the analysis and then also we will be discussing about the frequency response of common emitter amplifier having fixed bias. And, there also we will be going only up to the circuit analysis, numerical examples who we will be covering in the next class. And of course, subsequently we will be discussing about the CE amplifier having self bias and its frequency response it is slightly tricky compared to the fixed bias. So, we like to consider this frequency response analysis in a separate lecture. Then also we do have plan to cover design guidelines, once we once we recover the frequency response.

I like to mention one thing in the previous module we have covered numerical examples of common source amplifier, but we did not discuss about the design guidelines of common source amplifier. If time permits we will be covering that after covering the frequency response. So, the design guidelines we will be covering both CE and CS amplifier considering frequency response of the amplifiers. So, to start with let we go for the R-C and C-R circuit; I must say that this is more like recapitulation of whatever you know.

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Frequency response of C-R circuit (recapitulation)

$$\frac{V_o(s)}{V_{in}(s)} = \frac{sRC}{1 + sRC}$$

$$\frac{V_o(j\omega)}{V_{in}(j\omega)} = \frac{j\omega RC}{1 + j\omega RC}$$

$$V_o(t) = \frac{R \cdot V_{in}(t)}{R + \frac{1}{sC}}$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{sRC}{1 + sRC}$$

- Analyses in Laplace Domain and then in Fourier Domain
- Gain plot and Phase plot of Transfer function

And, to start with let us go with C-R circuit. So, the C R circuit is given here, the input we are applying across the series connection of C and R. And, then the output we are observing across the resistance. And the how do we find the frequency response? First we go to the Laplace domain then we analyze the circuit. Namely, let us draw the equivalent circuit in Laplace domain where the C part its impedance it is $1/sC$ and for the resistor on the other hand it is directly it is same as R.

And, then the input we are feeding a signal which is V in time domain whereas, in Laplace domain we call this is capital V in s. So, this is in time domain whereas, in the Laplace domain the signal it is in Laplace domain. So, we are using different notation here capital V in s. And, the output on the other hand we are observing across this resistance and the corresponding output, it is capital V o s. So, if you simply analyze this circuit what we are

getting is that $V_{out} / V_{in}(s)$ equals to R into the current flow which is $V_{in}(s)$ divided by the series connection of R and the capacitor; so, R plus $1/sC$.

So, if we simplify this equation what we are getting is that $V_{out}(s)$ divided by $V_{in}(s)$ so, this becomes sC into R in the numerator and in the denominator we do have $1 + sC$ into r . So, this is what the input to output transfer function in Laplace domain. So, this is what we are we have summarized here. So, V_{out} by V_{in} in Laplace domain it is sRC divided by $1 + s$ into RC . Now, this in the Laplace domain of course, the s is basically I should say it is having two part; two parts one is the real part and then the imaginary part.

And, corresponding to each of this s the basis function it is e to the power s into t which is essentially e to the power σt into e to the power $j\omega t$. Now, this σ it is significance of this σ it is in case the signal amplitude it is changing with time, then that may be captured by this σ part. And, in case if the signal frequency is changing or rather if we have sinusoidal component and its frequency it is represented by this ω .

So, on the transformation of the transfer function from Laplace domain to the frequency domain it can be obtained simply by considering rather dropping this σ is equal to 0. In other words special case of e to the power $s t$ it becomes e to the power $j\omega t$. Or, we can say that in the transfer function in Laplace domain whatever the s we do have, if we simply replace this s by $j\omega$, then whatever the transfer function we will we will get that that basically gives the frequency response of the circuit.

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The slide, titled "Frequency response of C-R circuit (recapitulation)", illustrates the analysis of an RC circuit. It features a circuit diagram with an input voltage V_{in} , a capacitor C , and a resistor R in parallel, with output voltage V_o . The Laplace domain transfer function is given as $\frac{V_o(s)}{V_{in}(s)} = \frac{sRC}{1+sRC}$. The Fourier domain transfer function is $\frac{V_o(j\omega)}{V_{in}(j\omega)} = \frac{j\omega RC}{1+j\omega RC}$. The magnitude plot shows $|V_o/V_{in}| = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}}$, with a corner frequency $\omega_c = 1/RC$ where the magnitude is 0.707. The phase plot shows a phase shift $\angle V_o/V_{in}$ that approaches 90° at high frequencies. Handwritten notes include "Analysis in Laplace Domain and then in Fourier Domain" and "Gain plot and Phase plot of Transfer function".

So, what we said here it is if we have say Laplace domain transfer function from that we can get the frequency domain transfer function or rather frequency response can be obtained by this one. Now, if we say that this is input to output transfer function in frequency domain or Fourier domain, then if we take the magnitude of it; so, it is basically it is a complex number.

And, if we consider its magnitude and phase then we can see how the system behavior or this network behavior changes with so, the frequency of the stimulus. In other words the whenever we are talking about frequency response of a circuit say in this case this C R circuit, what does it mean is that how the behavior of this block it is changing with the frequency of the input stimulus. And, whenever in this case in this study whenever we are talking about behavior is basically the output to input ratio, we may call it is voltage gain or transfer function or whatever it is.

So, this transfer function it is changing with frequency and that has been captured by this equation. So, if we simply make a plot of this function with ω , then we can get how the individual signal it is getting transformed before it is arriving to the output. So, if we plot in fact, the frequency response it is having as I said that since it is a complex number it is having two parts; one is the magnitude part and then the phase part.

So, if I consider that the magnitude part namely $V_o(j\omega)$ divided by $V_i(j\omega)$ and then if you take mode of it and then if we plot with respect to ω . In this case what will be seeing here it is due to the presence of ω in the numerator at very low frequency or if I consider ω is equal to 0, due to this ω in the numerator it will be 0. And, then as you go higher and higher so, probably at then this part it may be prominent compared to 1.

And, then beyond some frequency it becomes this part it becomes dominate over this one. And, whenever this the $j\omega RC$ part it is dominant over 1, then it becomes the whole ratio it becomes 1. So, in other words we can say that beyond some frequency it becomes 1. So, it is going to be the value it is going to be 1. And, the change over from this linear part to the whatever you say the constant part it is happening at a frequency, where ω into RC this part it becomes 1 or if I say that this is a particular ω called say ω_1 .

So, the corresponding ω_1 ; so, this gives us this ω_1 equals to 1 by R into C . In fact, what is the significance of this part? It is before this frequency ω_1 we are assuming that 1 was dominating over ωRC magnitude wise and then beyond this point we started saying that you know this part it is now it is going to be dominant. And, exactly at this point at ω_1 equals to 1 by RC both this part and this part they are equal. So, if I ignore one of them I may be getting the approximated 1 say in this part I may be getting the approximated 1 which is linear function of ω .

On the other hand, if I say that this part it is dominating namely the $j\omega RC$ part it is dominating over 1, then it becomes constant. So, again if I consider the second part it is completely dominating then I can get another approximation. This two approximation curve they are intersecting precisely at the same point, where ωRC it becomes 1. In fact, yeah

so we can many a times instead of going through this blue line which is the actual 1 for simplicity we consider these two linear parts. And, this is what we said the magnitude part.

In fact, if you see here from this equation if I consider the magnitude it is ω into R into C in the numerator and in the denominator it is a square root of 1 plus ω RC square. So, that is the gain plot of the transfer function. Now, since it is a complex number, it may happen that if we are giving a sinusoidal signal here at the output while it is going through this system, not only its magnitude may be getting changed because of this magnitude variation ah, but also its phase might get shifted.

So, that phase shift can be captured by considering whatever the phase we can see here. So, the next to the gain variation with respect to ω we can also observe the corresponding phase variation. So, if you see in this circuit or this transfer function if we plot the phase of this V_o divided by V_{in} ; what we have in the numerator? We do have the $j\omega$ part and in the denominator we do have 1 and $j\omega$ part.

So, at low frequency before this $\omega = 1$ frequency what you call it is cut off frequency we may say that so, the numerator part it is primarily defining the phase. And, the corresponding phase here it is you may say it is 90 degree plus. And, as we are approaching to this $\omega = 1$ frequency the corresponding phase it is becoming 45 degree. So, exactly at this point it is 45 degree. And, beyond that once you start increasing this ω further, then ω into RC it may be dominating and then this ω and numerator ω and rather numerator j part and denominator j part they are getting cancelled.

And, it becomes basically 1 real part so, which means that phase shift it will be 0 degree. So, in other words we can say that suppose we stimulate this circuit by one frequency maybe at this point. And, then if I say that this is ω say x , the output will be obtaining that you can see that compared to 1 this is much lower. So, at the output we can say that the amplitude it will be diminishing, in addition to that it will also be having the phase shift almost 90 degree.

On the other hand, if I change this frequency to exactly at $\omega = 1$, then the magnitude here it will be it is magnitude change it can be obtained from this one. And, if you see that at ω

1 this part it becomes 1 and this is 1 so, we can say that this part is of course 1. So, at $\omega = 1$ the precisely this value it is $1/\sqrt{2}$. Approximated equation it is saying that it will be 1, but actual value if you see the blue line and if you consider the equation there in the denominator you will be getting $\sqrt{2}$.

Which means that if we stimulate this the circuit with a frequency exactly equal to $\omega = 1$, the output at the output whatever the signal will be getting it is $1/\sqrt{2}$ times of the input signal amplitude. And, also there will be 45 degree phase shift in fact, this is phase lead I should say. And, then if we if we further change this stimulus to even higher frequency maybe somewhere here.

And, then if we see the gain plot it is showing that the its magnitude it will be; it will be almost the same at the output with respect to its corresponding input and the phase shift is also going to be almost 0. Which means that this meaning of the gain plot and phase plot in fact, intuitively if you see that if we feed a signal here sufficiently high frequency, where in case if the capacitor is almost behaving like a short circuit then at the output we will get the same input signal coming there. So, that is corresponding to say maybe a frequency at this point where now the gain it is 1 and then phase is 0.

On the other hand, if you consider now very low frequency then the signal it may fail to propagate here and at least the magnitude wise that you can say that the signal did not arrive there. So, if I based on for a given value of C and R or CR or RC time constant depending on the value of this stimulus frequency the behavior of this circuit you may say that either it is passing the signal to the output or it may be blocking the signal. In fact, if you see here the high frequency region the circuit it is working like a pass circuit.

And, on the other hand in the low frequency region, if you see in the low frequency region they the circuit it is attenuating the signal which means that the signal is the circuit it is working like a filter. And, this behavior of the filter it is like say a high pass kind of nature. So, whenever we are talking about as I said the frequency response is basically we want to see

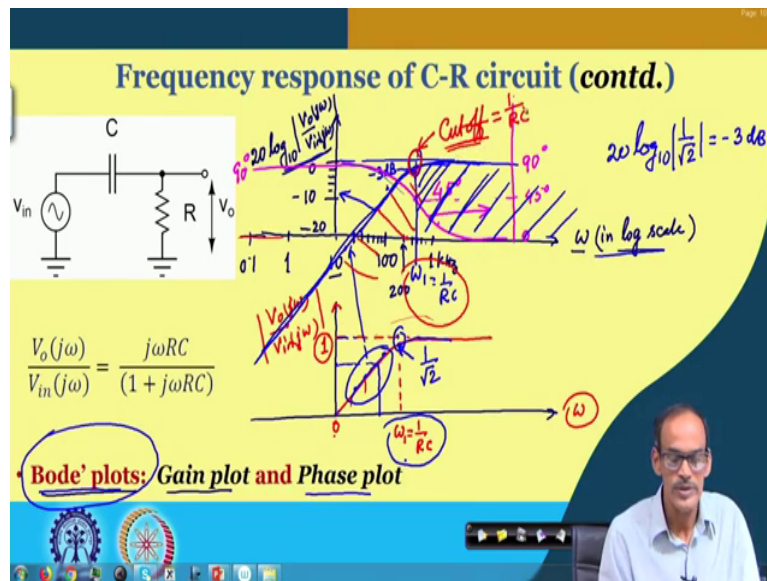
how the circuit behavior changes with the frequency of the stimulus. Now, these two plots; these two plots are as I said that they are storing the information of the behavior.

Typically, instead of considering these two plots the commonly used plot is something called Bode plot, where this ω the frequency it is in log scale. And, and the corresponding magnitude here instead of considering magnitude, the corresponding data is converted into decibel form. So, instead of taking only the magnitude, it is log transformed with a base 10 multiplied by 20. It is having some reason why we multiply with 20, but at least to make you understand that why we consider log and why do you consider log scale is basically capturing wide range of the variation of the gain; particularly in this portion and in this portion.

This is where the gain it is going to be 0 and also along the frequency axis we like to cover wide range of frequency all the way from almost 0 frequency, 0 Hertz to maybe in the range of maybe 100s of mega Hertz or it may be even giga Hertz. So, in case if you want to keep this ω in linear scale of course, it will be difficult to cover that entire scale. And, getting the inside of a the variation of the behavior over the frequency, wide frequency range.

So, the natural tendency is to use this log scale. So, that within a same graph we can see the behavior of the circuit over a wide range of frequency. So, whenever we are going to change this log scale as I said the corresponding plot name it is different and it is referred as Bode plot.

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So, what is this Bode plot? Is as I said basically this is a gain plot and phase plot, but the instead of gain or you can say magnitude of the transfer function it is $20 \log$ base 10. And, multiplied the log base 10 of V_o divided by V_{in} and of course, they are in frequency domain and this is of course, the omega. Note that in this is in log scale.

So, once say this is in log scale; obviously, we cannot reach to the 0 frequency, we may be starting with some positive frequency. Say for example, this may be 1 Hertz, this may be 10 Hertz, next point it is say 100 Hertz and so and so, this may be 1 kilo Hertz and so and so. And, then previous one it is 0.1 Hertz and so and so. On the other hand if you see the data since we are converting the data in log scale, the y axis scale on the other hand it is linear.

So, if it is say this is maybe say 0 dB or maybe minus this 1 is maybe say 20 dB minus 20 dB and this may be say minus 10 dB, this may be 0 dB and so and so. And, the scale here of

course, it is linear whereas, the x axis which is basically the omega since it is in log scale so, its ah spacing of this minor lines are getting compressed here. And, then again it starts with so, this is say 10 Hertz, then this point it is 100 Hertz and the next one it is 200 Hertz and then again the scale is getting compressed.

So, this graph it is referred as semi log graph paper where x axis is in log scale, y axis it is in linear scale ah, but then y axis data it is getting converted into logarithmic form. So, what is the change you will see compared to; so, in this Bode plot with respect to the frequency response in linear scale. If it is a linear scale then let me also try to sketch that linear scale whatever we have done just now. So, if I consider the linear scale and only the magnitude, we have seen for this RC circuit the frequency response it was like this.

So, the y axis is V_o divided by V_i in and its magnitude only and what we said is this corner frequency or we call cut off frequency, where omega equals to what we call omega $\frac{1}{RC}$. Now, this is 0 frequency of course, this 0 frequency we cannot see in log scale, it has gone to infinite so minus infinite distance or rather towards the left extreme beyond our from this graph. On the other hand depending on whatever the value we do have and in case if we are focusing somewhere here, probably we can we can map all the data points into this Bode plot.

So, how will you plot? First of all this magnitude since it is simple CR circuit and we have seen that as it is saturating towards a value of 1 and if you take log of 1 we will get 0. So, which means that this level it is coinciding with 0 dB ok. So, this is coinciding with 0 dB and see this frequency suppose it is somewhere here, maybe it is say here. So, we call this is omega 1 which is equal to $\frac{1}{RC}$ and at this frequency the corresponding difference here if you see it is 1 by root 2 times.

And, if we if we take log of 1 by root 2; so, $20 \log$ of log base 10 or from 1 by root 2 what we do get here it is minus 3 unit and its unit is disabled dB. So, with respect to 0 at this frequency what we are seeing here it is that it reached to minus 3 dB. And, then if you go to some other lower frequency of course, depending on the value here it may be point something or so, we will be getting some other point here and so and so on. So, if I consider different data points

here and if we try to plot here what we will be seeing here it is yeah actually we will be getting a linear almost linear here.

And, then there will be a bend like this and then it is getting saturated. And, if you consider if the data is going to be the magnitude it is going to be 0 which means that the corresponding log it is going to be infinite. In fact, this will be linearly dropping. So, I should say this part it is getting extended here, since it is a log scale in the x axis. And, the linear behavior it remains here also it is linear and this saturation part it remain saturation.

And, whatever the point we do have here where the magnitude it was 1 by root 2 times of this one, the steady level and that gives us a point here which is minus 3 dB. So, I should say this corner point here it is corresponding to minus 3 dB point which is which is commonly referred as minus 3 dB point. So, this plot this gain plot in log scale it is referred as the gain plot of the Bode plot. So, Bode plots it is also having two component: one is the gain part and the phase part.

So, similar to the gain if you consider the phase with omega in log scale, what will be seeing there it is; let me use the same graph here and same axis here. So, at this frequency, the phase it was 45 degree and then at lower frequency it was going 90 degree. So, maybe this was 90 degree and then it was going to 0 degree. So, of course, I do have different scale for this phase. So, this part the phase part it is I should say this is 0 degree, this is 90 degree and this is 45 degree and so and so.

So, the phase plot it is having the scale here which is again linear and the corresponding the scale corresponding to the gain plot in dB it is in this it which is shown here. So, this gain, part and the phase part together it is giving us the Bode plot. So, typically when we are talking about frequency response of any circuit, say a CR circuit what we refer it is basically the this gain plot and the corresponding phase plot.

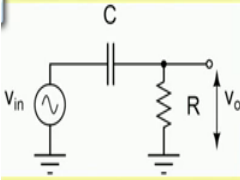
Now, here this frequency as I said that this is behaving like a high pass and this frequency it is referred as the corner frequency or cut off frequency beyond which the signal it is passing. So, I should say that this frequency range, this frequency range it is the pass band. And, on the

other hand this portion; this portion or the lower frequency or frequency lower than this corner frequency it is referred as the stop band. And, this corner point it is referred as the cutoff of this behavior or the bandpass.

I should not say bandpass, this is a basically high pass filter; so, this point is the cutoff frequency. So, for C R circuit we can say that cutoff frequency it is 1 by RC , in case if the frequency it is expressed in radian per second. If it is Hertz then of course, it will be 1 by $2\pi RC$.

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Frequency response of C-R circuit (recapitulation)



$$\frac{V_o(s)}{V_{in}(s)} = \frac{sRC}{(1 + sRC)}$$

at $s = \frac{1}{RC} \rightarrow \infty$

$$\omega_c = \frac{1}{RC}$$

$$\frac{V_o(j\omega)}{V_{in}(j\omega)} = \frac{j\omega RC}{(1 + j\omega RC)}$$

- Analyses in Laplace Domain and then in Fourier Domain
- Gain plot and Phase plot of Transfer function

So, one interesting thing is that this if you see here the cutoff frequency expression of cutoff frequency. And, if you see the previous slide where you are talking about the we are talking about the yeah the transfer function in Laplace domain. There is one important information

that this denominator polynomial of this transfer function it becomes 0 at s is equal to 1 by RC .

So, this s is equal to 1 by RC so, at this point this transfer function it becomes infinite which means that this transfer function it is having a pole at s is equal to 1 by RC . So, this pole and whatever just now we said that the cutoff frequency of the frequency response which is $\omega_c = 1/RC$ they are related. So, we will see that the pole in the transfer function in Laplace domain and the cutoff frequency of the frequency response they are having a direct co-relationship.

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The slide is titled "Frequency response of C-R circuit (contd.)". It features a circuit diagram of an RC network where an input voltage V_{in} is applied to a series combination of a capacitor C and a resistor R . The output voltage V_o is measured across the resistor R . Below the diagram, the transfer function is given as
$$\frac{V_o(j\omega)}{V_{in}(j\omega)} = \frac{j\omega RC}{1 + j\omega RC}$$
. At the bottom of the slide, there is a bullet point: "• Bode' plots: Gain plot and Phase plot". The slide also includes logos of institutions and a small video inset of a presenter.

We will discuss that in detail ah, but let we cover one more circuit which is complementary of this circuit, namely that is RC circuit and its behavior it is more like a low pass kind of behavior. So, let me take a short break and then we will come back to this frequency response

of this RC circuit. And, we will then further we will correlate the frequency response on the pole of the transfer function in Laplace domain.