

Analog Electronic Circuits
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Lecture - 27
Common Emitter Amplifier (Contd.) (Part B)

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CE amplifier – Self-bias (contd...)

• Small signal equivalent circuit and Mapping on V. Amp.

$i_c = g_m v_{be}$

$v_{out} = -g_m R_C v_{be} = -\frac{g_m R_C}{1 + g_m R_E} v_s$

$A_v = \frac{v_{out}}{v_s} = -\frac{g_m R_C}{1 + g_m R_E}$

$v_{be} = \frac{v_s}{1 + g_m R_E}$

$v_s = v_{be} + R_E (1 + \beta) i_b = v_{be} + R_E (1 + \beta) \frac{v_{be}}{r_{\pi}} = v_{be} + R_E g_m v_{be}$

So, welcome back after the short break. So, where we are discussing? We are talking about the small signal equivalent circuit and then we are trying to find the corresponding gain of the circuit.

So, the output voltage as I said that output voltage, it is this one. So, v_{out} equals to minus g_m into R_C into v_{be} . Now, this v_{be} of course, it is function of V_s , but we need to find what

is the exact expression of that. So, let me erase whatever the scribbling I have done and start afresh again drop across this R_E which is R_E times $1 + \beta$ times i_b .

On the other hand, this i_b it is; i_b it is it can be expressed in terms of v_{be} and r_{π} alright. So, we can write this as again v_{be} plus R_E into $1 + \beta$ times v_{be} by r_{π} . In fact, if you see here β divided by r_{π} it is nothing, but g_m . So, if we drop this 1 and then if we consider this is equal to β approximately divided by r_{π} so, this part it becomes g_m . So, we can further simplify I am saying that this is v_{be} plus R_E into g_m into v_{be} . In other words, we may say that v_{be} equals to divided by $1 + g_m$ into R_E .

So, interestingly depending on the value of this R_E , we can see that v_{be} it is rather a small fraction of v_s . Whatever it is this output voltage we said equals to minus $g_m R_C$ into v_{be} . So, that becomes minus $g_m R_C$ into v_s divided by $1 + g_m$ into R_E .

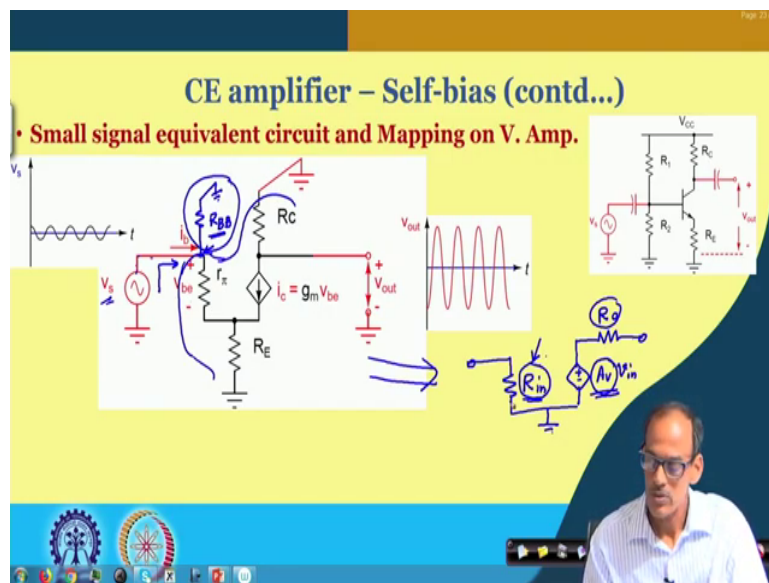
So, as a result if I say that what is the gain of this circuit starting from primary source to the primary output, we can say the voltage gain A_v equals to minus g_m into R_C divided by $1 + g_m$ into R_E . So, this is one important parameter of the voltage amplifier out of this circuit.

Now of course, we will be talking about its numerical value and all. But if we; if we recall the if it is fixed by a circuit the expression of the voltage gain, it was only this much. Now, we do have additional factor here which is in fact, degrading the gain of the circuit. We will be talking about that, the circuit of this self-bias circuit because we do have this R_E present at the emitter, it is degrading the gain.

In fact, the a purpose more main motivation of putting this R_E , it is to stabilize the operating point of the circuit in case if β is changing. So, you can think of that R_E , it is desensitizing the circuit or rather its operating point it is getting desensitized against the variation of this β . However unfortunately, this is also desensitizing this circuit against input signal and as a result it is making the gain much smaller than whatever the original gain of the CE amplifier potentially can provide.

So, we have to see what we can do for this part, but before that at least to you obtain the expression of the voltage gain and the whenever we are mapping this small signal equivalent circuit on a voltage amplifier model. So, apart from the voltage gain open loop voltage gain, we do have two more important parameters namely input resistance and output resistance of the model.

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So, you may recall that whenever we are going to map this circuit into voltage amplifier at the input, we do have the equivalent resistance call R_{in} and then also we do have voltage controlled voltage source, namely $A V$ times whatever the v_{in} and then we do have the Thevenin equivalent resistance alright. And so, we obtain this parameter now and we like to get the other two parameters of this model.

By the way, we have ignored this part; this part, if you want you can keep that as well. Many a times we do ignore, but is for practical purposes we may consider this R BB and since this R BB this node the voltage source it is directly coming here and it is a short. So, the voltage at this point of course, it will be same as v s. However, whenever we are talking about input resistance, whatever the input resistance we can see at this input port, it is parallel connection of this R BB and whatever the input resistance coming from the rest of the circuit ok.

So, while we will be talking about input resistance, we may consider this R BB, but while here we are deriving this voltage gain AV, we have ignore because this voltage source it is predominantly defining the voltage at the base node. So, let us find the expression of this input resistance of this circuit. So, I do have another slide for that, yes.

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CE amplifier – Self-bias (contd...)

• Finding parameters in Voltage Amplifier

$R_{in} = ? = \frac{V_x}{i_x}$
 $R_{in} = R_{BB} \parallel R_{in}'$
 $R_{in}' = R_{BB} \parallel \{ r_{\pi} + R_E(1+\beta) \}$
 $V_x = i_x r_{\pi} + i_x(1+\beta) R_E$
 $R_{in} = \frac{V_x}{i_x} = r_{\pi} + (1+\beta) R_E$

$i_c = \beta i_b = \beta i_x$
 $i_x + \beta i_x$

V_{be}
 r_{π}
 R_E
 R_c
 V_{cc}
 R_1
 R_2
 V_{out}

So, what we have? This is the main circuit and whenever we are going to find a small signal parameter. In this case may be R_{in} in expression of R_{in} in what we will be doing it is as a generalized methodology at the input we will be stimulating the circuit by a known signal source and then we will be monitoring or observing the corresponding current say i_x . So, we call this is v_x and then we are observing the i_x and then ratio of this v_x and i_x that is giving us the resistance. So, v_x by i_x is the resistance.

So, one of this v_x and i_x is the cause and the other one is the effect. So, you may consider say i_x is the stimulus and then you can observe the voltage at the base node with respect to ac ground or you may say that we are giving a stimulus called v_x and then you are observing the i_x . Either way you will be finding the correct expression of the input impedance.

While we are doing this exercise we can keep rest of the circuits in DC operating condition and we will be considering this is the only stimulus. And if I say that this is the v_x , we may say that this current it is flowing through this circuit as the base current. So, i_b equals to i_x .

And the voltage v_x it is again it is having two components; one is the voltage across this r_{π} which is $V_{bb} - V_{be}$ and also the drop across this R_E . To simplify what you can do, we may say that if I do have i_b which is i_x flowing from base to emitter terminal. The current flow on the other hand in the collector terminal it is i_c which is β times the i_b which is incidentally, this is β times i_x . So, the total current flowing through this R_E it is i_x plus β times i_x .

So, we can say that the voltage drop v_x equals to v_{be} ; v_{be} it is i_x times r_{π} plus i_x times 1 plus β multiplied by R_E . So, from that we can say that v_x by i_x equals to r_{π} plus 1 plus β times R_E . So, this is what we are defining the input resistance. So, this circuit is its input resistance is this r_{π} in series with R_E , but then R_E multiplied by 1 plus β times.

So, likewise you can find the output impedance of this circuit or output resistance of this circuit. Note that this input resistance we obtained only coming from this part in addition to that we do have the R_{BB} . So, I should say this is only one part of it. So, let me call this is R_{in}

in dash; R in dash. So, R in dash it is this one and then total input resistance of the voltage amplifier, it will be R BB coming in parallel with in dash which is of course, this is R BB in parallel with r_{π} plus R_E times $1 + \beta$.

So, similarly let us analyze the output port and let me see that what is the corresponding expression of the output resistance. I think I do have another slide let me see, no let me do it here itself.

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CE amplifier – Self-bias (contd...)

• Finding parameters in Voltage Amplifier

The slide displays a small-signal equivalent circuit for a CE amplifier. The circuit includes a dependent current source $g_m v_{be}$, a base-emitter resistance r_{π} , a collector resistor R_c , and an emitter resistor R_E . Handwritten red annotations show the derivation of output resistance $R_o = R_c$. A test voltage source v_x and current source i_y are connected to the output terminals. A small inset circuit shows the full self-biasing network with V_{cc} , R_1 , R_2 , and R_E .

While we will be doing this similar kind of exercise, we need to find as I said that we need to find what will be the output resistance R_O of this voltage amplifier, while we are mapping this small signal equivalent circuit into a voltage amplifier. What you have to do? Again we will be stimulating this circuit from this port by say a signal source called v_x or say v_y and then we can observe the corresponding current let we mark it as say i_y .

And while we are doing this exercise we have to keep this signal 0 since it is voltage signal so, we are making this is ground. So, what we are and then if we take the ratio of this v_x and i_x so, that gives us the output resistance, so v_y by i_y right. And here whatever you do here in fact, this r_{pi} it is coming in parallel with R_E . But since we do have ideal current source here so, these two elements it will be; it will be blocked by this ideal current source because its resistance looking into this circuit it is infinite.

So, at the output port what we have it is only R_C remaining. So, if I am applying v_y here, the current flow through this circuit it is mainly this is the current. Now, the current of course, that will be v_y so, that is v_y divided by R_C so, that is the i_y . So, from that we can say directly that R_O equals to R_C . Now, this is of course, we are assuming that the conductance here it is 0. Now, if I consider the additional conductance of course, will be having the influence of this part also.

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CE amplifier – Self-bias (contd...)

• Finding parameters in Voltage Amplifier

The slide displays a small-signal model of a CE self-bias amplifier. The model consists of a dependent current source $g_m V_{be}$ in parallel with r_{π} and R_E . The output is taken across R_C . Handwritten notes in blue and red ink provide the following expressions:

- Input resistance: $R_{in} = R_{BB} \parallel \{r_{\pi} + (1+\beta)R_E\}$
- Output resistance: $R_o = R_C \parallel \left\{ \frac{g_m R_C}{(1+g_m R_E)} \right\}$
- Voltage gain: $A_v = \frac{g_m R_C}{(1+g_m R_E)}$

A video inset in the bottom right corner shows a person speaking, likely the instructor.

We will see that but so far what we have discussed, it is this amplifier it can be mapped into mapped into the small signal model into a voltage amplifier. And expression of this RO, it is R_C this is A_v time's v_{in} in where A_v is g_m into R_C divided by $1 + g_m$ into R_E with a minus sign and then R_{in} ; R_{in} equals to.

So, if I consider R_{BB} also, so r_{in} equals to R_{BB} in parallel with r_{π} plus $1 + \beta$ times R_E . Now, yeah, so this is the voltage amplifier model. If we consider as I said that if you consider the resistance here due to early voltage called R_O or finite conductance here, then; obviously, I need to consider the resistance of this part coming in parallel with whatever the resistance we can see in lower side.

So, let me discuss about that and that will of course, change the expression of this output resistance. In fact, that will make the output resistance is R_C in parallel with some other

component. So, let us see what is that component coming here. I think I do have next slide to discuss that, yes.

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CE amplifier – Self-bias (contd...)

• Finding parameters in Voltage Amplifier considering r_o

$$v_e = i_y \cdot (R_E \parallel R_L)$$

$$\frac{v_y - v_e}{r_o} = g_m v_e + i_y$$

$$\frac{v_y}{r_o} = \left[\left(g_m + \frac{1}{r_o} \right) \times (R_E \parallel R_L) + 1 \right] i_y$$

Here again, what we will be doing to find the output resistance in presence of r_o , we will be stimulating this circuit by say v_y . So, this is we are stimulating and then we are observing this i_y and while we are doing this exercise, we have to keep in mind that we have to make this voltage input signal to be 0. And in fact, once you do that this r_{π} , it is coming in parallel with this. So, you may say that this is coming in parallel with r_{π} .

So, R_E and r_{π} they are coming in parallel, so that is the representation. And on the other hand since we are doing the exercise to find the output resistance which is of course, parallel connection of this R_C and whatever the resistance you are seeing in the lower side. So, for simplicity we may for the time being, we drop this part and let me analyze only this part.

So, if you see this circuit, we may frequently come to this circuit again and again and so, let me draw this circuit as a general one. So, we do have current source here which is g_m times whatever you say v_{be} and then we do have the r_{π} here and then we do have the resistance here.

And incidentally this is v_{be} , so this is minus and this is plus. So, we may call since the base is connected to ground and emitter voltage whatever nonzero voltage, it is having. So, we may say that this v_{be} , you may write this is g_m into v_e with a minus sign. So, we can probably you can write this v_e with a minus sign or we can put the current direction in this other way and then we can say that whatever the current is flowing will; let we call this is i_y and the corresponding stimulus here it is v_y .

So, since this current is flowing in this direction and of course, since we have changed this direction of the current, we are we need to put the plus sign here. And what is v_e ? v_e is the voltage at this point and this is the resistance R_E coming in parallel with r_{π} .

So, now, if you see this circuit that since we have we are considering only lower part and we are not considering this R_C at the moment. So, this i_y it is actually flowing through maybe bifurcating and then coming back to this ground. So, the current flow here it is same as this i_y .

As a result v_e voltage drop voltage at this point, it is i_y multiplied by R_E in parallel with r_{π} right. So, that is the voltage here. So, the current flowing on the other hand, it is function of this v_e and then current flow through this part of course, it is v_y minus this voltage divided by r_{π} .

So, at this node if you see since this current is coming here and then the current flow here, it is basically summation of this two that must be equal to whatever the current is flowing. So, current flow through this r_{π} it is having one expression is v_y minus v_e divided by r_{π} . And the other expression it is g_m into v_e plus i_y and v_e , it is having this expression right.

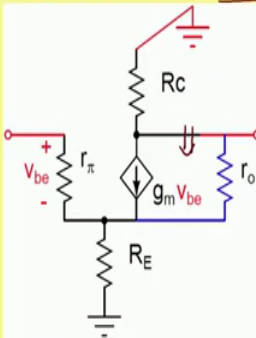
So, what we can say that v_e it since it is function of i_y . So, we can take this v right side. So, what we are getting it is v_y divided by r_{naught} equals to g_m plus 1 by r_{naught} into v_e and v_e , it is having this expression which is R_E in parallel with r_{pi} into i_y and also we do have this i_y . So, we can say this plus 1 into i_y alright.

So, that gives us the v_y divided by i_y expression and it can be shown that from this one, it can be shown that v_y ; v_y divided by i_y equals to r_{naught} plus R_E in parallel with r_{pi} plus r_{naught} into R_E in parallel with r_{pi} multiplied by g_m . So, this resistance whatever the equivalent resistance, it is quite large primarily because of this term. But whatever it is the through this analysis, what we can say that this is the resistance of the lower part then total resistance of course, this will be R_C in parallel with that.

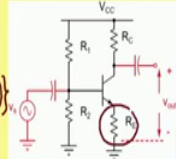

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CE amplifier – Self-bias (contd...)

• Finding parameters in Voltage Amplifier considering r_o



$$R_o = R_C \parallel \left\{ r_o + (R_E \parallel r_{\pi}) + g_m r_o (R_E \parallel r_{\pi}) \right\}$$

$$|A_v| = \frac{g_m R_C}{1 + g_m R_E} \approx \frac{R_C}{R_E}$$



So, let me rewrite whatever I just now have said that output resistance R_O which is R_C in parallel with whatever the resistance it is coming from this circuit and that resistance it is r_o plus R_E in parallel with r_{π} plus $g_m r_{\pi}$ naught R_E in parallel with r_{π} and then whole thing it is coming in parallel with R_C .

So, that is the this mapping of the CE amplifier with self-bias things onto a voltage amplifier and the corresponding different parameters. Now, what we have said that we are placing this emitter resistor to make the circuit's operating point desensitized against beta variation, but it is making the gain also dropping to a smaller value namely what we say it is that voltage gain A_V , it is magnitude wise g_m into R_C divided by 1 plus g_m into R_E .

So; obviously, this is not acceptable particularly if R_E it is significant and this multiplying this two, it will be quite large and if this may give us a value which is close to R_C by R_E that is in the order of maybe sometimes it may be less than 10 numerically. So, that is not acceptable. So, we need to unless we address this issue this circuit definitely, we cannot use it. So, let us see how this problem can be addressed.

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CE amplifier – Self-bias (contd...)

• Getting back high gain

$R_{BB} \ll (1+\beta) R_E$

$R_{BB} \leq \frac{(1+\beta) R_E}{10}$

So, what we say that how do you get back gain of the circuit. So, as you can as you have discussed before in that whenever we are feeding the signal here say, v_s significant part of that voltage it is getting dropped across this one. And as a result we do have only a small fraction as v_{be} and making this corresponding output voltage very small.

So, if we make this voltage whatever the emitter voltage; if we make 0, then we can then force this v_b to be equal to v_s and then we can get back the gain. So, we like to make this voltage 0, but then moment we make this is 0 just by hard connection. Again we will be having the issue of the operating point getting sensitive to the beta variation. So, the clever thing is that for dc, we do not want this circuit to be working; but for ac, we want the circuit to be working.

So, as a result we can put a capacitor here and so the what you are looking for is basically this capacitor, it will not be interfering the dc operating point, but then for ac signal this will be making this ground. So, this is the solution of getting the voltage gain back and this is; this is not disturbing the circuit for dc operating point ensuring that the operating point of the circuit it will remain insensitive to beta variation. But for small signal or small signal or high frequency signal, this is working as a short making the emitter node connected to ground and making the this emitter node it is going to be shorted to ground, as a result we can simply shunt it.

So, if you draw the small signal equivalent circuit of this one, it becomes similar to whatever we already have discussed for the fixed bias circuit. By the way we also have this R_1 and R_2 coming in parallel and practically, we need to be careful that while we are keeping it similar to the fixed bias. But in this case what we have said is that R_{BB} need to be very small compared to $1 + \beta R_E$. So, that is the basic difference remains there even after cunning connecting this CE that R_{BB} should be small compared to very small compared to this one.

Typically to satisfy this condition what we said is that R_{BB} , it will be less than or equal to one-tenth one-tenth of this one into $1 + \beta R_E$ to get this approximation is getting valid ok. So, that is the sorry this is not two this is ten. So, this is what the practical design guidelines, we follow for this circuit.

So, you may say that smaller this resistance are better. So, can I make this resistance really small or is there any trade off. Of course, if I make this if I want to make this resistance smaller maintaining dc volt same; that means, both of these registers I need to make it smaller and smaller.

One consequence is that of course, there will be a dc current flow here so, that practically increases the power dissipation. But then even more serious problem is that if this resistance is getting smaller and smaller, then this capacitor will be having a difficult time to feed the signal at this node.

In technical terms, you may call that this capacitor and then these two resistors coming in parallel call R_{BB} , they do define the lower cutoff frequency of the amplifier. So, far we are talking about the mid frequency range operation and if you go to lower and lower frequency then of course, the gain voltage gain it will be dropping.

So, this lower cutoff frequency, one of the candidate to define this lower cutoff frequency is that this coupling capacitor C_1 and parallel connection of R_1 and R_2 . So, we need to be careful that now while you are picking this R_{BB} , we need to satisfy this condition to make sure that circuit is remaining insensitive to beta variation. But at the same time the lower cutoff frequency to keep it low the value of this R_1 parallel R_2 should not be very small.

But whatever it is once we follow all these guidelines the small signal equivalent circuit of this circuit is given there which is essentially very similar to whatever we have seen for fixed bias circuit.

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CE amplifier – ~~Fixed bias~~ (contd...)

• High frequency Small signal equivalent circuits

$R_{in} = r_{\pi} \parallel R_{BB}$

$R_o = R_c \parallel r_o$

$|A_v| = g_m R_c$

Self-bias

$V_{out} = ?$

So, for fixed bias as well as the self biased yeah. So, I should say not only for fixed bias, but even for this is self-bias of course. In fact, I should have written this is self-bias instead of writing fixed best.

So, even for this circuit also the small signal equivalent circuit it becomes like this. Primarily this C E, it is making this node ac ground and in addition to whatever the things we have discussed so far ah, the parasitic components namely C mu and C pi they are also coming into play particularly for high frequency applications. So, for high frequency small signal equivalent circuit of for sale by a circuit also, it will be similar. And of course, we can retain this R BB here since R BB unlike fixed bias where R B, it was very high R BB we need to consider. In fact, R BB may be comparable with this r pi.

And once we connect this CE, I must mention that the input resistance input resistance earlier it was r_{pi} in series with $1 + \beta$ into R_E , but then now this input resistance it is r_{pi} only and that of course, coming in parallel with this R_{BB} . So, that makes the small signal input resistance it is becoming smaller that may not be a good thing, but to get the gain back at the cost of this input resistance, we need to ground this one.

And the other consequence is that the output resistance; on the other hand it is R_C in parallel with R_O . So, we are not having the other big things where we are having R_E and so and so because the emitter node it is getting grounded. So, these are the two changes. Of course, the corresponding voltage gain A_V , now it becomes g_m into R_C .

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Numerical Example: CE amplifier – Fixed-bias

- $V_{cc} = 12V$; $\beta = 100$; $V_{BE(on)} \approx 0.6V$; $R_B = 570k\Omega$; $R_C = 3.3k\Omega$;

Find Operating point, small signal parameters and voltage gain

The slide features a circuit diagram of a common-emitter (CE) amplifier with fixed bias. The circuit includes a DC supply V_{cc} connected to a base resistor R_B and a collector resistor R_C . The base of the transistor is connected to the junction of R_B and V_{cc} . The collector is connected to the junction of R_C and V_{cc} . The emitter is connected to ground. An AC voltage source V_s is connected to the base through a coupling capacitor. The output voltage V_{out} is taken from the collector through another coupling capacitor. A presenter is visible in the bottom right corner of the slide.

So, we need to discuss some of the numerical problems, but today we are running short of time probably in the next class we will talk about numerical problems from both angles the

design wise as well as analysis wise. That is all for now, we will be resuming this class in the next day.

Thank you for listening.