

Analog Electronic Circuits
Prof. Pradip Mandal
Department of Electronics and Electrical Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture - 21
Linearization of non – linear circuit containing MOSFET (Contd.)

Welcome back after the short break on the topic of Linearization of input or output transfer characteristic.

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Linearization w.r.t. Q- pt.: small signal equivalent ckt.

- **Example circuit_1 (Continued)**
 - Drop out the d.c. parts
 - Use new set of parameters:

Small signal model
Small signal equiv. ckt.

So, we are discussing about this a small signal equivalent circuit we are about to start that, and the intension there it is of course, to get the simplified circuit. So, you may recall the equivalent circuit if I consider the large signal V_a V_r , what we said it is and the transistor may be replaced by a current source dependent current source.

And, this current source it is capital I_{ds} which is function of capital V_{gs} and V_{ds} . So, that is what we have discussed and the V_{gs} it is whatever the voltage it is appearing across gate to source and then we do have the so, this is the device part. And, so, this V_{gs} is controlling on this current source and then we do have the R_D part and this is connected to V_{DD} .

So this is the V_{out} the voltage is coming here it is capital V_{out} and, the voltage you are applying here it may be having a small signal part along with the DC part. So, we may we are talking about this is capital V_{GS} and this part you are talking about small v_{gs} . So, these two together it is defining the capital V_{GS} . So, this model it is referred as large signal equivalent circuit and so, this model it is large signal model of the transistor and the whole circuit it is large signal equivalent circuit.

In contrast to that whenever we are doing the linearization what you are doing is we are simplifying this current equation, namely the drain to source current we are considering only the small signal part, which means that small i_{ds} , which is having an expression like one of the 3 expressions or $2 I_{Dsat} K_n (V_{gs} - V_{th})^2$. And, what is the V_{gs} ? V_{gs} is the voltage across the gate to source and this v_{gs} of course, it is the small signal v_{gs} .

And, the at the gate we are considering only the small signal part, whatever the v_{gs} we do have or V_{GS} we do have that part and then of course, we are retaining the resistor. But, then at the end of the other end of the resistor we are connecting these to ground, which means that we are dropping the DC part for the supply we are dropping this is 0. And, also this v_{gs} part we are removing here and then the corresponding whatever the circuit we are getting here it is small signal equivalent circuit. So, this is what the small signal equivalent circuit. And, so, this is the of course, the corresponding output we will be observing here, it is showing only the small signal output voltage.

So, now we can see that this circuit it is; of course, it is simplified and most important thing is that the direct linear relationship between this v_{gs} and the v_{out} it can be directly obtained by analyzing this circuit. So, that is what it is referred as a small signal equivalent circuit; one important thing is that, whenever instead of say this model.

So, I should say this is, this portion is the model of the MOS transistor, this is referred as large signal model whereas, in the small signal equivalent circuit whatever the small signal equivalent circuit we obtain for the transistor, it is the small signal model. So, this model it is small signal model.

Now if you see in this model and this model, here we do have one factor. So, this factor it is it depends on the size of the transistor, it depends on the device parameter, it depends on the operating point also. And, so, if I say that this is one parameter called g_m ; why g_m , it correlates the v_{gs} to i_{ds} . So, it is conductance so, that is why g and m stands for trans mutual from input to outputs.

So, that is why this parameter it is referred as a trans conductance of the and the device, particularly for the small signal. And this is one of the important parameter. So, whenever you are talking about the small signal model of the device, it involves one new parameter or new set of parameters rather and one of them is the g_m .

So, whenever you are talking about small signal equivalent circuit or a small signal model, we require new set of parameter. So, in the while you are going from large signal equivalent circuit to small signal equivalent circuit. First thing is that we need to drop the dc part and also we have to get this model involving new set of parameters. So, let us see what are the other parameters are there along with this g_m . And, very important thing is that this small signal parameter it is a function of the operating point or it depends on the operating point.

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Linearization w.r.t. Q-pt.: small signal equivalent ckt.

- **Example circuit_1 (Continued)**
 - Drop out the d.c. parts
 - Use new set of parameters:

$$g_m = \frac{\partial I_{ds}}{\partial V_{gs}} = \frac{i_{ds}}{v_{gs}}$$

$$g_d = \frac{1}{r_d} = \frac{\partial I_{ds}}{\partial V_{ds}}$$

Handwritten derivations for g_m and g_d are shown below the circuit diagrams:

$$\frac{\partial \left[\frac{k}{2L} (V_{gs} - V_{th}) (1 + \lambda V_{ds}) \right]}{\partial V_{gs}} \approx \frac{k}{2L} (V_{gs} - V_{th}) (1 + \lambda V_{ds})$$

$$\frac{\partial \left[\frac{k}{2L} (V_{gs} - V_{th}) (1 + \lambda V_{ds}) \right]}{\partial V_{ds}} \approx \frac{\lambda k}{2L} (V_{gs} - V_{th}) (1 + \lambda V_{ds})$$

So, as just now I said that the small signal model of the device, which is having these i_{ds} as function of v_{gs} and then it is having this parameter g_m . And, what is v_{gs} , the voltage appearing across this gate to source; so, this is the gate terminal, this is the drain terminal, this is the source terminal of the device.

Now of course, if I add the rest of the things namely the resistance R_D and then if you make the connection to ground, if you make this to ac ground, and then if you also draw the stimulus input signal stimulus, then that gives us the small signal equivalent circuit.

But, if I for generalization if I concentrate say this part, this is one important information, what is referred as small signal model of the device? And, the small signal model of the device g_m actually it is defined in this way, variation partial derivation of the i_{ds} with respect

to v_{gs} . So, if I say that the circuit is linearized; so, you may see that this part it is nothing, but this small signal i_{ds} likewise, this part it is v_{gs} .

So, in fact, this representation and this definition actually they are same. And, the expression of the g_m as you have discussed, one of the form it is square root of $2 I_{DS} K W$ by L . So, this is one form this is also same as $K W$ by $L V_{GS} \text{ minus } V_{th}$ and also it is having another form $2 I_{DS}$ divided by $V_{GS} \text{ minus } V_{th}$.

So, out of these 3 forms, whatever the forms you consider all of them are actually essentially representing this g_m . So, this g_m maybe having different expression; in fact, this expression you can directly you can obtain by considering this definition of this g_m . So, if you put the expression of this I_{ds} and if you say that I_{ds} is changing only with the V_{gs} that is that is the meaning of this partial derivative, and from that you can directly obtain these expressions.

It involves a small approximation, but actually eventually you will be coming to this point. So, say for example, if I consider variation of I_{ds} , and I_{ds} it is $K W$ by $2 L V_{gs} \text{ minus } V_{th}$ square into one plus λV_{ds} with respect to V_{gs} . So, what we will be getting it is you are almost getting this one, except this V_{gs} it will be small V_{gs} whereas, we are using capital V_{GS} and then $1 \text{ plus } \lambda V_{ds}$ part.

So, we can say that this is well approximated by K into W by $2 L$ capital $V_{GS} \text{ minus } V_{th}$ into 2, this 2 it is also coming. So, these 2 and these 2 are getting canceled out one plus λV_{DS} part it is there and this part you can take it to be 1 and small V_{gs} part we are taking capital V_{GS} and hence we are obtaining this one.

Now, if you use the expression of this i_{ds} in terms of V_{gs} and W by L . And, probably then you can replace these V_{gs} in terms of either I_{ds} or this W by L you can replace in terms of I_{DS} and V_{GS} to get this three forms of the g_m . So, what we like to say that the, this new set of parameter g_m , it is important thing is that whatever the forms are there. Each of these forms are having indication that this the value of this g_m it is directly function of this the

operating point either I_{DS} or V_{GS} or I_{DS} and V_{GS} . So, this parameter of course, it may its value, it may vary if you change the operating point.

So, to get a steady operation, steady linearized operation whenever we are talking about linearization of the transfer characteristic with respect to Q point, it is better to keep this Q point constant. So, that the value of this g_m you can assume to be constant, but in case if it is varying slightly over this linear range, still you may approximate that this is remaining constant. Mathematically, you may say that that is equivalent of saying that small V_{gs} and V_{gs} is same as V_{GS} ; that means, at this point and this point whatever the parameters are there essentially they are same.

So, this is one small signal important parameter and we also have another important parameter. So, far we are ignoring this $1 + \lambda V_{ds}$ part, which means the dependency of this current on the output voltage. So, if you consider that, what we will be getting is that of course, this I_{ds} is dependent through this $1 + \lambda V_{ds}$ part, we may get a drain to source one conducting element. And, that conducting element is nothing, but the output conductance or drain conductance. So, it is definition is; so, again here it is partial derivative of I_{ds} , but with respect to V_{ds} .

So, if you use this equation same equation, you can find the expression of this one. So, please keep that in mind and once you consider this G_D , this G_D actually from drain to source one conducting element. So, let me get the expression of this G_D and then again we will be coming back to the small signal model of the device. Probably, I do have a different slide for that, ok.

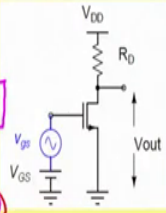
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Small signal Model of MOSFET

- New set of device parameters
- Values depend on the Q-pt.

$$g_m = \frac{\partial I_{ds}}{\partial V_{gs}}$$

$$g_d = \frac{1}{r_d} = \frac{\partial I_{ds}}{\partial V_{ds}}$$



$$\frac{\partial \left[\frac{K W}{2L} (V_{gs} - V_{th})^2 (1 + \lambda V_{ds}) \right]}{\partial V_{ds}}$$

$$= \frac{K W}{2L} (V_{gs} - V_{th})^2 \lambda (1 + \lambda V_{ds})$$

$$\approx \frac{I_{ds}}{(1 + \lambda V_{ds})} \times \lambda \approx I_{ds} \lambda \approx (I_{ds}) \lambda$$

So, it is basically it is a similar thing here, but let me discuss on this. So, we already have discussed this one. Now, we are going to discuss the expression of g_d . And, that we are getting from this definition and this I_{ds} it is having an expression, it is $K W$ by L rather $2 L$ V_{gs} minus V_{th} squared into 1 plus λV_{ds} .

So, this is the current expression and now if I take derivative basically whole thing, so, we are taking partial derivative of this whole thing with respect to V_{ds} . And, if you do so, what we are getting is; so, we are not changing this part we are changing now V_{ds} parts or we can say this part is constant namely $K W$ by $2 L V_{gs}$ minus V_{th} square into λ .

So, you may say that, this is we will approximated by so, this part if I multiply with 1 plus λV_{ds} part and then if I divide by 1 plus λV_{ds} part. Then, what we are getting here it is this part along with this we are getting I_{ds} and then in the denominator we do have

1 plus lambda into V ds. So, this multiplied by lambda. So, in fact, this is normally this part it is considered to be a very small. So, we considered this is in fact, I should say this is not approximation this is exact, whenever we are coming to this approximation we are dropping this part. So, this is I ds into lambda.

And, further to that we may say that this is capital I DS into lambda. So, that is the expression of the output conductance drain conductance lambda into I DS, again here also you can see that it directly depends on the operating point. We may draw the circuit of course, this is the expression of the g d, but we may draw the circuit either in the form of conductance or in the form of resistance. So, then the corresponding resistance it is just reciprocal of this part namely 1 by g d.

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Small signal Model of MOSFET

- **New set of device parameters**
 - Values depend on the Q-pt.

$$g_m = \frac{\partial I_{ds}}{\partial V_{gs}}$$

$$g_d = \frac{1}{r_d} = \frac{\partial I_{ds}}{\partial V_{ds}}$$

So, the small signal model of the device the transistor, we can say that it is the current source which is i_{ds} equals to g_m into V_{gs} , and in parallel with that we do have the conducting element g_d and then we do have the at the input, we are defining the v_{gs} gate to source voltage. So, this is the gate terminal, this is the drain terminal and this is the source terminal.

So, what we are now up to that we started with linearization of the non-linear circuit, and while you are bring the linearization we are dropping the dc parts. And, finally, we obtained different equivalent circuit of the whole circuit and that invites us to consider equivalent circuit of the MOS transistor, and this equivalent circuit involves only the small signals and that is what the completion of the small signal model.

So, this model of course, it is I should say it is a simple enough part then, if this model it is sufficient for mid frequency range and low frequency range, but if you are going to higher and higher frequency, then gate to source there will be some more components namely gate to source capacitance, then gate to drain capacitance. So, those are the additional elements again they will be popping up.

So, gate to source we may be having capacitance. So, it is denoted by c_{gs} like gate to drain there will be another element capacity of element are represented as c_{gd} . So, if I consider this capacity of elements then this model of course, it is also it is small signal model, but this model can I represent this circuit the device behavior in the high frequency range also. So, if I add this capacity of element, then it is called small signal model of the MOSFET in mid frequency as well as high frequency.

So just to simplify it is normally referred as high frequency small signal model, on the other hand if we drop this to capacity of element it is simply said it is small signal model of the MOSFET. So, what maybe it is application of this model? Of course, we can simplify the analysis and then we can find the gain of the circuit so, let us consider one numerical example just to highlight that. So, the same example will be going there, but let us start with this new slide.

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Usage of Small signal Model of MOSFET

- Numerical Problem
 - ✓ Find the Operating point
 - ✓ Find the values of small signal parameters
 - Find the small signal voltage gain

$$I_{DS} = \frac{K}{2} \frac{W}{L} (V_{GS} - V_{th})^2 (1 + \lambda V_{DS})$$

$$= \frac{200 \times 10^{-6}}{2} \times \frac{10}{1} \cdot (2 - 1)^2 \times 1$$

$$= 1 \text{ mA} \Rightarrow V_{OUT} = 10 - 4 \times 1 = 6 \text{ V}$$

$$g_m = \frac{2 \cdot I_{DS}}{(V_{GS} - V_{th})} = \frac{2 \times 1 \text{ mA}}{(2 - 1)} = 2 \text{ mA/V}, g_d = 0$$

$$V_{out} = -R_D g_m \cdot V_{gs}$$

$V_{out} = ?$
 $\frac{V_{out}}{V_{in}} = -4 \times 2 = -8$

So, just to get some number here, let you consider this device parameters and then the dc voltage and the, this dc voltage and the V_{th} and the R_D . And, what we have to do here it is we need to find what will be this v_{out} . So, we need to find what will be the v_{out} in terms of this V_{gs} or V_{in} ? So, this is same as v_{in} also. Now, let me consider what are the parameters are given to us it is a say this is 10 volt, let you consider this is maybe 2 volt and let you consider V_{th} equals to 1 volt for simplicity of calculation. And, then W equals to say 10 micron L equals to say maybe 1 micron and let you consider K the trans conductance parameter K equals to say 200 microampere per volt square.

Then, we can find what will be the output, but of course, we require the resistance value here. So, let me take how much probably 4 k it will be good let us see ok. So, how do you proceed, first of all to get the gain namely, ok; so, what you have to do the statement is that, we need to

find the our objective is to find v out by v in and it is value. So, to do that how do we proceed?

First of all the small signal parameters the depends on the operating point. So, first thing is that we need to find the operating point and to do that what you do let you consider the dc part and let me find the I_D 's. So, I_D 's equals to K by 2 W by L V_{GS} minus V_{th} squared and then 1 plus λ V_{DS} . So, for the time being let you consider λ is λ is very small so, approximately equals to 0 . So, what we are getting here it is 200 into 10 to the power minus 6 divided by 2 into W by L V_{GS} is 2 minus V_{th} and squared this part it is approximately 1 . So, that gives us 1 milliamper. So, that is the I_{DS} dc current.

So, the dc current flowing here it is 1 milliamper so, the drop across this resistance it is 4 volt. So, from that we can see that the V_{out} . So, that gives us capital V_{out} it is 10 volt minus 4 into 1 . So, that is the 6 volt V_{out} the V_{ds} is 6 volt. Now, we do have we can verify whether the device it is in saturation region or not. So, we do have 2 volt here we do have 6 volt here; obviously, the devices having pinch of happening at the draining. So, the device it is in saturation.

Now, then we can find the value of the small signal parameters. So, since we are considering λ is equal to 0 . So, we can see that the, directly we can say that g_d it is a small I mean it is 0 and then g_m . So g_m it is having different expression. So, either we can write in terms of I_{DS} and V_{GS} minus V_{th} say, for example, if we consider 2 times of I_{DS} divided by V_{GS} minus V_{th} . So, that gives us 2 into 1 milliamper divided by 2 minus 1 . So, that is 2 milliamper part volt, right. And, then what is said is that g_d it is 0 ; so, that g_d is 0 . So, that gives us the output voltage v_{out} is R_D of course, with a minus sign R_D into the g_m into v_{gs} .

So from this one what we can find is that this is it becomes minus R_D , that is 4 K and then g_m it is 2 milli so, that gives us the gain of 8 . So, that gives us the final gain of the circuit. So, what we have done is that first we obtain the operating point here and then after that we obtain the small signal parameter value. So, this small signal parameter values are here.

And, then we obtain the small signal voltage gain by considering this one. In fact, we can draw the small signal equivalent circuits whatever we already have discussed; the, this part it is not there. So, this is this I should say g_d equals to 0. So, this is for the MOS transistor and then we do have the R_D part, this is connected to ground and then this is v_{gs} we are applying here, the current here it is g_m into v_{gs} . So, the voltage appearing here appearing at the output node here it is, it is this current multiplied by this R_D . So, that is why we said that v_{out} is this one.

I think probably you can try to make similar attempt in case if the circuit is in different, namely if the circuit is a containing p MOS transistor and then you can see what maybe the corresponding small signal model and so and so. I think that is all I need to discuss, but let us see what are the things we have covered so far?

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Conclusion

- Linearization of non-linear circuit containing Transistor
- Small signal equivalent circuit through linearization
- Small signal model of MOSFET having different set of pars.
- Small signal model is used to simplify analysis & calculation

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So, today we have discussed about linearization of a non-linear circuit containing MOSFET device and then we have discussed about the small signal equivalent circuit. So, whenever you are talking about linearization of the circuit is basically we are translating the actual circuit into a small signal equivalent circuit. And, then in the small signal equivalent circuit whatever the circuit we are having corresponding to the MOSFET, it is referred as the small signal model of the MOSFET, which involves different set of parameters namely say trans conductance g_m and g_d . And, they are values of course, it depends on the center point of the linearization namely there are coefficient point.

And, then what you have seen is that, if you use this small signal model of the device in actual circuit, that will simplify the analyzes and also that will simplify the calculation. In fact, in today's a simple examples, the simplification you may or may not appreciate, but whenever it involves a circuit having many transistors, I am sure that then you will appreciate the utilization of the small signal model of the transistor. I think that is all I do have so.

Thank you.