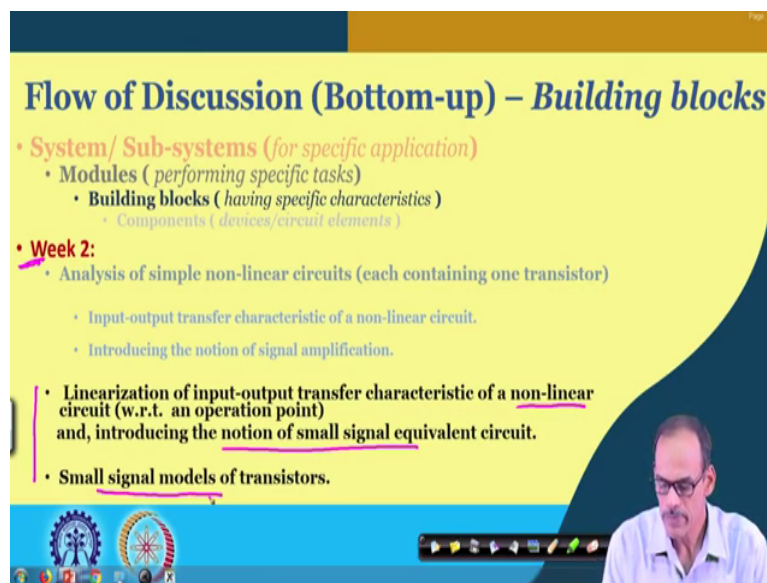


Analog Electronic Circuits
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Lecture - 20
Linearization of Non – Linear Circuit Containing MOSFET

So, welcome back to this course on Analog Electronic Circuits, we are almost to the verge of second week of on this course. And today's topic of discussion is Linearization of a Non-Linear Circuit which contains MOSFET. So, to simplify the analysis, we are considering example having only one MOSFET transistor in the circuit.

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Flow of Discussion (Bottom-up) – Building blocks

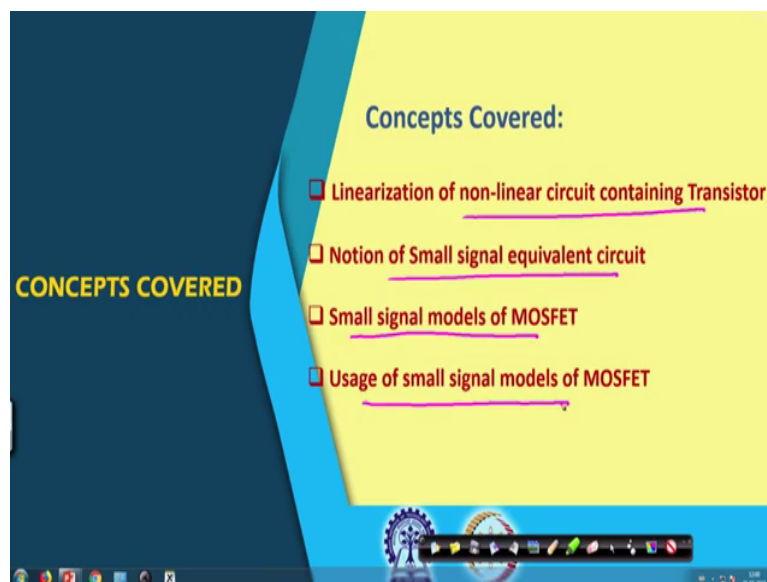
- **System/ Sub-systems** (for specific application)
 - **Modules** (performing specific tasks)
 - **Building blocks** (having specific characteristics)
 - Components (devices/circuit elements)
- **Week 2:**
 - Analysis of simple non-linear circuits (each containing one transistor)
 - Input-output transfer characteristic of a non-linear circuit.
 - Introducing the notion of signal amplification.
- Linearization of input-output transfer characteristic of a non-linear circuit (w.r.t. an operation point) and, introducing the notion of small signal equivalent circuit.
- Small signal models of transistors.

So, let us see what is the where we are in the flow. In our overall flow the in the second week discussion, we are basically linearization of input or output transfer characteristic of

non-linear circuit containing BJT or MOS. And in the previous module we already have seen linearization of input to output transfer characteristic of circuit containing BJT.

Today it will be similar kind of discussion, but then instead of BJT we are focusing on MOS transistor. And then similar to the BJT circuit, we will also be having a notion called small signal equivalent circuit and from that we will see that there is the notion of small signal model of MOSFET transistor. So, and then we will see that how the small signal model helps us to speed up the analysis and helps us to find a gain of the circuit.

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So, that is where we are; so the concept it will be covered today it is linearization of non-linear circuit containing MOS transistor; notion of small signal equivalent circuit and then small signal model of MOSFET, particularly in MOSFET and then we will cover how the small signal model of MOS transistor can be used to solve numerical problem.

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Linearization of Input-Output Transfer Characteristic

- Example circuit_1 (containing a MOSFET)
- Find effect of variation in V_{gs} on I_{ds} and V_{ds} output

V_{out} ?
 V_{in}

C S amp.

• Information known, in saturation region:

$$I_{ds} = \frac{K}{2} \frac{W}{L} (V_{gs} - V_{th})^2 (1 + \lambda_n V_{ds}) ; \quad V_{ds} \geq (V_{gs} - V_{th}) = V_{d sat}$$

At the bottom of the slide, there are logos of various institutions and a Windows taskbar.

So, to come to the example; so here we do have the running example; the here we do have the common source amplifier; and at the gate we do have the voltage we are applying; and at the drain we are applying supply voltage through R D; and then we are observing the output at the drain, namely drain voltage there, which is incidentally the output voltage.

And what we are going to do that, we like to change the input here namely the gate voltage V_g and we like to observe the corresponding effect on the I_{ds} and on the V_{ds} or V_{ds} . So, if you vary the gate voltage V_g , incidentally that we are changing V_{gs} of the transistor; and we can see what is the corresponding effect at the I_{ds} and the V_{ds} , which is the output of the circuit. And this is the information it is known to us; if the transistor it is in saturation region, if it is in saturation region expression of this I_{ds} in terms of V_{gs} and V_{ds} is given here.

So, we do have trans conductance parameter K and W by L aspect ratio of the transistor; V_{gs} minus V_{th} is the threshold voltage of the device square and $1 + \lambda V_{ds}$. And to keep the device in saturation V_{ds} should be higher than $V_{gs} - V_{th}$; this is also denoted as V_{dsat} . Now you might have noted that, I am using a slightly different notation for each of these parameter; say for example, I_{ds} capital I small d s which means that this I_{ds} is subjected to change with time; and so, likewise V_{ds} capital V small d s and then V_{gs} also.

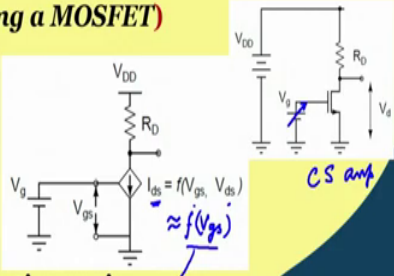
So, if I vary this V_{gs} or V_{gs} , we like to see what will be the variation there. And if I say this is the input. So, if I say that this is the input and if I say that this is the corresponding output; let me say this is also output, but let you consider this is the output; then input to output variation what is known as input to output transfer characteristic. So, we are expecting this transfer characteristic it will be highly non-linear, because the device it is non-linear; and then we will be talking about how the non-linear characteristic curve it will be getting linearized ok.

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Linearization of Input-Output Transfer Characteristic

- **Example circuit_1 (containing a MOSFET)**
- Find effect of variation in V_{gs} on
 - I_{ds} and V_{ds}



$I_{ds} = f(V_{gs}, V_{ds}) \approx f(V_{gs})$

• **Information known, in saturation region:**

$$I_{ds} = \frac{KW}{2L} (V_{gs} - V_{th})^2 (1 + \lambda_n V_{ds}) \approx \frac{KW}{2L} (V_{gs} - V_{th})^2$$


So, let me use new slide for that to go for further discussion, yes. So, here we do have the common source amplifier, where we are changing this V_{gs} and we are observing the voltage at the drain as I said, we do have the V_{DD} . Now here the known information as I said before in saturation region, it may be typically it is well approximated by considering this lambda is very small; namely lambda into V_{ds} it is much smaller than 1.

So, we can ignore this factor and hence we can say that I_{ds} is approximately defined by only this part. So, which is indicating that the I_{ds} current, the I_{ds} current here though it is function of V_{gs} and V_{ds} ; but we do approximate that this is function of V_{gs} only. Whenever it requires we will be considering this factor, namely the dependency of the I_{ds} on V_{ds} ; but for the time being let me assume that I_{ds} is predominantly defined by V_{gs} and its expression is given here, ok.

So, how do we find the transfer characteristic? So, first of all if we vary the V_{gs} or V_g , we know that this I_{ds} it will change and the dependency here it is square law; and then to find the output voltage; so this is either you say this is V_g or V_{gs} . And to find the corresponding output what we do, we consider the load line as well. So, the load line in this case it is defined by this straight line having a slope of minus 1 by R_D and this point is V_{DD} as we have discussed in our previous modules and this point it is V_{DD} by R_D and wherever it intersects, that gives us the sorry ok.

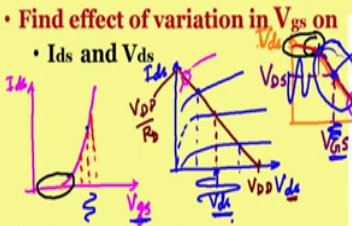
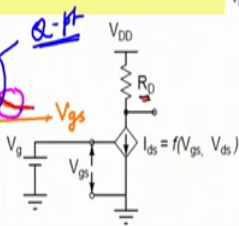
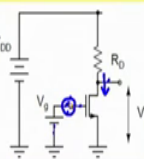
So, let me, I have committed a mistake. So, let me come back to on this point.


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Linearization of Input-Output Transfer Characteristic

- Example circuit_1 (containing a MOSFET)
- Find effect of variation in V_{gs} on
 - I_{ds} and V_{ds}
- Information known, in saturation region:

$$I_{ds} = \frac{K W}{2 L} (V_{gs} - V_{th})^2 (1 + \lambda_n V_{ds}) \approx \frac{K W}{2 L} (V_{gs} - V_{th})^2$$


So, first thing is that we do have the I_{ds} versus V_{gs} . So, that is the square law and then to find the output voltage what you have to do; we need to plot the I_{ds} versus the output voltage. So, for a given value of V_{gs} , so the current it is hardly it depends on the V_{ds} . So,

we may say that the device characteristic namely I_{ds} versus V_{ds} it is constant here in the saturation. And then if you draw the load line, defined by this R_D and the supply voltage; so this is one end of the load line V_{DD} and then the other end of the load line it is V_{DD} divided by R_D .

So, this defines the output voltage or the V_{ds} . So, the V_{ds} here it is given here. Now as we discussed before, if we vary the V_{gs} with respect to some point; then what we are expecting that the device characteristic it goes up and or it may come down. And as a result the intersection point it is changing and that gives us the variation in V_{ds} with respect to variation in V_{gs} .

So, whatever we see here, if we plot the variation in say V_{ds} with respect to variation in V_{gs} . So, then what we will be getting here it is, in the middle portion it is fairly linear; but then if you decrease this V_{gs} and say maybe towards the V_{th} , so it is expected that it will be having a highly non-linear part. And then we do have beyond some point if I increase this V_{gs} , then it may be, the device characteristic it may be approaching towards V_{DD} by R_D and there we will be having another non-linear part which will be the other way.

So, you may say that this non-linear part it is coming due to device entering into triode region. On the other hand the, this non-linear part or rather here it is coming due to device entering into the cutoff region. So, whatever the overall transfer characteristic we are getting here it is as I said that; because of the device characteristic which is fairly non-linear, we do have non-linear input to output transfer characteristic. So, this non-linear input to output transfer characteristic we need to linearize.

So, we have seen that the this portion, middle portion of the characteristic of curve it is fairly linear and whenever you are talking about linearization; if we are keeping our linearization center around middle of this range, then the linearity range it will be bigger. So, whenever we are talking about linearization, probably we need to fix a one point called quiescent point or Q point and with respect to that Q point we may try to linearize. And whenever we are linearizing we can say that, this Q point it is defined by the V_{gs} may be V_{GQ} and the

corresponding output it is say V_{ds} ; and with respect to we may be having some variation.

So, if we are having some variation here, the corresponding effect it is coming here. And in this plot input to output transfer characteristic we may say that, if we change this V_{gs} with respect to this Q point here; we will be seeing the corresponding effect coming here at the output. So, this after doing this linearization we are assuming that this Q point or operating point it will be unchanged, it will be remaining fixed; only the V_{gs} small portion it will be changing centering to this Q point.

So, our the circuit now it will be having supply voltage, having a fixed DC part and the time variant part. So, let us look into this circuit in different way, where the at the input we are giving a DC voltage followed by one time varying component. And then we want to see what will be the corresponding effect coming at the I_{ds} current and also at the V_{ds} voltage.

So, that is basically graphically when we are saying that non-linear transfer characteristic curve we are linearizing; here we are saying that we are changing the V_{gs} with respect to fixed DC voltage. And the variation here it is we are restricting, so that the effect coming here it will be fairly linear with respect to whatever the variation we are imposing on the gate voltage.

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Linearization of Input-Output Transfer Characteristic

• **Example circuit_1 (Continued)**

• **Ids :**

$$I_{ds} = I_{DS} + i_{ds} = \frac{K W}{2 L} (V_{GS} + v_{gs} - V_{th})^2$$

$$= \frac{K W}{2 L} (V_{GS} - V_{th}) + v_{gs})^2$$

$$= \frac{K W}{2 L} (V_{GS} - V_{th})^2 \left[1 + \frac{v_{gs}}{V_{GS} - V_{th}} \right]^2$$

So:

$$I_{ds} = \frac{K W}{L} (V_{GS} - V_{th})^2 + \frac{2 \cdot I_{DS} \cdot v_{gs}}{V_{GS} - V_{th}} + \frac{v_{gs}^2}{V_{GS} - V_{th}}$$

$V_{out} = V_{OUT} + v_{out}$

CS amp

So, let me start with new slide yeah; this is what exactly I was talking about, we do have the supply voltage. So, again this is the same common source amplifier or common source configuration. It is having at the gate we do have a DC voltage, so this is the DC part; which is defining the operating point and on top of that along with that it is having the small signal part.

So, you may say that this is a c part or time varying part. Though pictorially here you are showing it is sinusoidal, but it is not necessarily be sinusoidal; only thing we want to say that, it is average should be 0. So, with progress of time, the coefficient point or the DC point it remains unchanged.

And it will be having impact on the I_{ds} current. So, it may be having a V_{GS} part and of course, it will be having a DC part. So, likewise this output voltage it may be having a DC part called V_{out} and in addition to that it may be having small signal part say small v_{out} .

So, if I see the current here, it is having as I said that it is having DC part denoted by capital I_{DS} and then it is also having a small signal part small i_{ds} . So, the total current flowing through drain to source of the transistor that can be obtained by considering the same equation. Only thing is that the V_{GS} part we are splitting into two part; one is the DC part, another is the small signal part, rest of the things it is same.

So, to know what kind of effect it is coming here, if we expand this equation probably we can get the exact one and also we can get what will be the corresponding linearization. So, if I expand this. So, we do have K divided by $2W$ by L as is and then if we take this part V_{GS} minus V_{th} part separate from this v_{gs} .

What we do have it is V_{GS} minus V_{th} and then plus small v_{gs} . So, we do have square around that, but let you take this part separate and let you take this part out. So, what we have it is K divided by $2W$ by L V_{GS} minus V_{th} square. So, if I take this part out, what we will be having here it is 1 plus v_{gs} by V_{GS} minus V_{th} square.

Now, if I expand this part what you are having is, let me write this part K divided by $2W$ by L V_{GS} minus V_{th} squared 1 plus 2 into v_{gs} small v_{gs} divided by capital V_{GS} minus V_{th} into v_{gs} plus we do have v_{gs}^2 square divided by capital V_{GS} minus V_{th} square.

So, now if I say that we are talking about linearization; which means that we are essentially dropping this second order term. So, if you make this is approximately 0 , whatever the remaining term we do have here up to this point, that gives us the linear linearized transfer characteristic.

So, if I say that if I multiply say one and this part, so that gives us the DC part. So, you may say that this gives the DC part and after linearization if I multiply say this part with say this

part; then multiplied by V_{gs} , so that gives us the ac part. So, we can say that the small signal part or the ac part I_{ds} part it will be K divided by g_m ; 2 and this 2 and this 2 are getting cancelled. So, $K W$ by $L V_{GS} \text{ minus } V_{th}$; so, the square part it is getting eliminated, because you do have $V_{GS} \text{ minus } V_{th}$ multiply this v_{gs} .

So, you can see that it is having the v_{gs} part and rest of the things are, if you see it is unchanged. So, if I say that only small v_{gs} part it is changing. So, the corresponding change we are getting at in the on the current, it is the DC part of the some factor here defined by the DC part and then we do have the time varying part small v_{gs} . In fact, this part it can be rewritten in different form. In fact, it is having three different forms. We do have say form A, form B and then form C all are same; only thing is that if you see here, in this form this part it is defined by W by L and $V_{GS} \text{ minus } V_{th}$.

Whereas for this case it is in terms of I_{DS} and $V_{GS} \text{ minus } V_{th}$ without considering in W by L . And then we do have the third combination it is I_{DS} and the aspect ratio W by L . So, depending on the situation we may take one of these forms, but everywhere we do have the v_{gs} appearing.

So, I should say whether this form or this form or this form all the terms are essentially representing the same thing; namely transferring factor from v_{gs} to i_{ds} . So, now, what we are seeing here it is. So, if you recall the non-linear input-output transfer characteristic, so I will be coming to this non-linear transfer characteristic.

So, let me go to the V_{out} part. So, now, we obtain the current part and now we like to go to the final output voltage. So, you may keep this information in mind and we will be making use of that. So, I say that this is what we obtain, the expression of small i_{ds} part in terms of v_{gs} and the expression of capital I_{DS} we already know.

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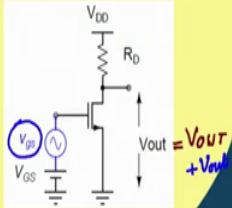
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Linearization of Input-Output Transfer Characteristic

• **Example circuit_1 (Continued)**

• $V_{ds} = V_{out}$:


$$i_{ds} = I_{DS} + i_{ds}$$

$$= \frac{K W}{2 L} (V_{GS} - V_{th})^2 + \sqrt{\frac{2 \cdot I_{DS} K \cdot W}{L}} \cdot v_{gs}$$


$$V_{out} = V_{DD} - R_D \cdot (I_{DS} + i_{ds})$$

$$= (V_{DD} - R_D \cdot I_{DS}) - R_D \cdot i_{ds} \rightarrow -R_D \sqrt{\frac{2 \cdot I_{DS} K W}{L}} \cdot v_{gs}$$

• **So:**

$$v_{out} = -R_D \sqrt{\frac{2 \cdot I_{DS} K \cdot W}{L}} v_{gs} \quad \underline{V_{OUT}} = \underline{V_{DD} - R_D \cdot I_{DS}}$$


So, let us see how do we find the corresponding output voltage. The expression of the output voltage of course, it is we do. So, this is the output voltage which is V_{DD} minus this $I R$ drop.

Now, the $I R$ drop is basically we do have the R_D multiplied by the total current; which is having the DC part as well as it is having the a c part. And from the previous discussion we have already said that the expression of this the total current which is given here; it is having the a c part and then also it is having the DC part.

And the DC part it is expression it is given here, it depends on the voltage here the DC voltage and then the a c part it is having this part. One of those three factors, in fact, this is the form one of those three forms we have discussed, it is basically the form c. So, we can use

any one of them, that is not important right now. But what we like to say that, this is function of v_g s.

So, this I_d s it is having an expression like this and this part it is having this expression. So, which means that, if I multiply this R_D with this capital I_D S, so what we are getting here it is, V_{DD} . So, we are having this is capital V_{DD} minus R_D multiplied by capital I_D S. So, this is the DC part which is I should say that the DC part at the output and then we do have the minus, I should say minus here; we do have minus R_D into I_d , so this I_d s.

And then this i_d s part it is having this expression. So, we can write say this part particularly, this part in the form of minus R_D square root of $2 I_D S K_W$ by L into V_g s. So, as I say that we do have a, this V_{out} is having a DC part $V_{capital out}$. So, this part it is basically the $V_{capital out}$ and this part is the small signal part. So, in addition to that we do have the time dependent part or the part which is coming from this one. So, that is V_{out} . So, that is nothing but this minus R_D square root of so and so multiplied by the V_g s part.

So, that gives us the input to output transfer characteristic considering this time variable part. So, whenever you are talking about linearization is basically we are keeping our focus on this relationship, ok.

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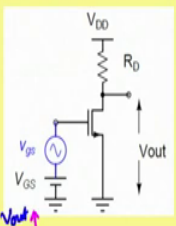
Linearization of Input-Output Transfer Characteristic

• **Example circuit_1 (Continued)**

• $V_{ds} = V_{out}$:

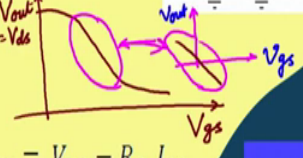

$$I_{ds} = I_{DS} + i_{ds}$$

$$= \frac{K W}{2 L} (V_{GS} - V_{th})^2 + \sqrt{\frac{2 \cdot I_{DS} K \cdot W}{L}} \cdot v_{gs}$$

$$V_{out} = V_{DD} - R_D \cdot (I_{DS} + i_{ds})$$


• **So:**

$$v_{out} = -R_D \cdot \sqrt{\frac{2 \cdot I_{DS} K \cdot W}{L}} \cdot v_{gs}$$

$$V_{OUT} = V_{DD} - R_D \cdot I_{DS}$$



So, what we are going to do now it is; we do have large signal transfer characteristic which is fairly non-linear and then out of that we are keeping our focus here and we are talking about only this part. And whenever you are focusing on this part, the x axis and y axis are essentially the small signal part. So, we may say that whenever you are talking about this linearization, the x axis is small v_{gs} and y axis is small v_{ds} or v_{out} .

And whereas, if I consider the large signal transfer characteristic, the x axis as we know that it was capital V_{gs} in this circuit and then the output it is capital V_{out} ; which is same as V_{ds} , ok. So, whenever you are talking about linearization is basically pulling out only this part, pulling out only this part of the transfer characteristic. So, we are taking this part out here. So, in other words we may say that linearization is basically getting a new kind of a transfer characteristic.

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Linearization w.r.t. Q- pt.: small signal transfer characteristic

• Example circuit_1 (Continued)

i_{ds} vs. V_{gs} and V_{out} vs. V_{gs}

$V_{out} = V_{ds}$

$V_{gs} = V_{in}$

$V_{gs} = V_{in}$

• Notion of small signal equivalent circuit:

So, that is what we are saying; that whenever you are saying it is linearization of the transfer characteristic is with respect to one operating point finding the small v ; I should say this is actually; I want to say it is small v out versus small v g s. So, we are basically talking about the translating the large signal transfer characteristic.

So, we do have V out with respect to V g s; which is incidentally V in and as you say that it is having fairly non-linear behavior and out of that we are extracting the small signal part. And once you pull it out and if we are observing the corresponding transfer characteristic, it is x axis is small V g s or you may say this is small V in. And then we do have the y axis it is V out; which is actually small V d s and this part, this part it is coming here.

So, if you extend of course, it will be having non-linear part. So, whenever we are getting this small signal transfer characteristic curve it is basically we are getting new, you know transfer

the input output transfer characteristic curve. And this curve it may be obtained by some simplifying the circuit through an equivalent circuit; where of course we can suppress the DC part or the quiescent part, it is basically this is the Q point or quiescent point.

That part if you map it here that is equivalent of saying that; if we drop this DC part and if you drop this DC part and also the corresponding and the DC part here V_{DD} , then that gives us the equivalent small signal transfer characteristic curve. So, to get that you may say that the circuit wise what we are doing is; this device whenever you do have this device, we may simplify that circuit in terms of the small signal. So, the current here instead of capital I_D we may say that, this is small i_d as function of small v_g . And we know that the this function i_d ; if you see i_d in terms of v_g , it is having a linear form, namely some factor here multiplied by v_g .


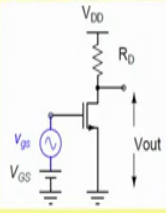
So, that gives us and the v_g is basically gate to source voltage after dropping the DC part. So, this is what the equivalent circuit out of the transistor and then of course, we do have the R_D connected here. So, whatever the circuit we are obtaining by dropping the DC part is nothing but we can see another equivalent circuit; which is used for from small signal transfer characteristic and hence this equivalent circuit it is called small signal equivalent circuit.

And whatever the small, the total circuit it is referred as small signal equivalent circuit; whereas, this circuit it is referred as small signal model of the transistor.

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Linearization w.r.t. Q- pt.: small signal equivalent ckt.

- **Example circuit_1 (Continued)**
 - Drop out the d.c. parts
 - Use new set of parameters:



The slide features a yellow background with a dark blue header and footer. The header contains the title 'Linearization w.r.t. Q- pt.: small signal equivalent ckt.' in blue text. Below the title, there is a red bullet point followed by 'Example circuit_1 (Continued)' and two black bullet points: 'Drop out the d.c. parts' and 'Use new set of parameters:'. To the right of the text is a circuit diagram of a common-emitter BJT amplifier. The base is connected to a voltage source V_{gs} through a resistor. The collector is connected to a load resistor R_D and a supply voltage V_{DD} . The output voltage V_{out} is taken from the collector. The emitter is connected to ground. At the bottom of the slide, there are several logos on the left, a navigation bar in the center, and a small inset video of a man with glasses and a light blue shirt speaking on the right.

So, our next discussion it is this small signal equivalent circuit and small signal model of the transistor. So, whenever you are talking about the linearization is basically going towards that. So, let me take a short break and then we will come back to discuss further about the small signal equivalent circuit.