Analog Electronic Circuits
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Lecture - 19
Linearization of Non – Linear Circuit Containing BJT (Contd.)

Start sir.

So, welcome back after the short break.

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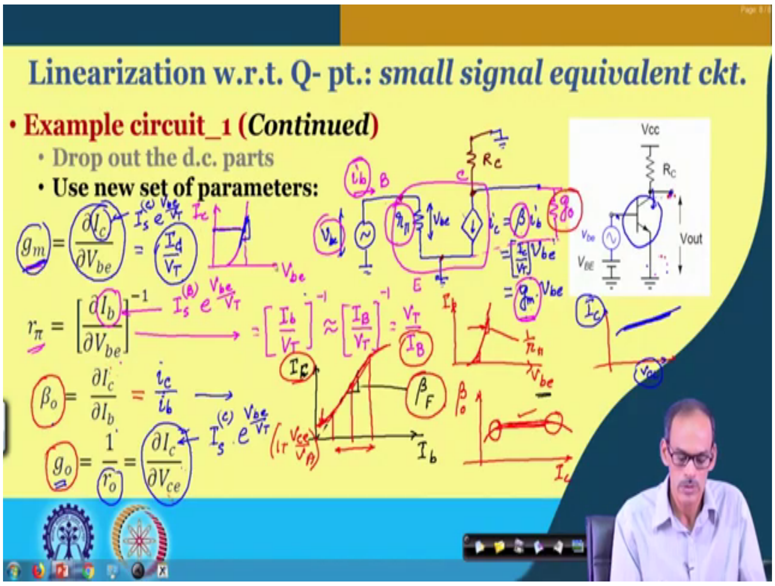

So, what you are discussing is that small signal equivalent circuit with respect to operating point which is basically linearization and we are talking about how do we. Once we have the circuit how do we linearize the circuit and so and so. So, whenever we are considering the equivalent the equivalent small signal equivalent circuit, if I quickly draw the circuit we do have the small signal input and then base to emitter.

We do have something called r pi and then we do have the current source dependent current source. So, either we write in the form of voltage dependent or current dependent. So, this is i c we may write in terms of i b. In fact, there is another way of writing it you know in terms of v be.

So, whatever the voltage it is getting dropped across this base to emitter terminal. So, we call this is V be and incidentally since then there is no resistance here. We may say that this is also V be and then. So, i c it can be written in terms of V be and we have seen that it is having some factor which is capital I c by V T and this is referred as trans conductance parameter called gm into V be ok.

And then of course, we will come back to this point and then we do have the resistance r c and this is and getting connected to ac ground and then of course, we obtain the voltage here. So, what we are saying is that we have seen graphical interpretation of linearization, we have seen in the form of equation and also we have seen in the form of equivalent circuit.

And whenever we are considering this equivalent circuit it is as I said that this equivalent circuit of the whole common emitter configuration it involves the equivalent circuit of the transistor. So, if you see this part, this is the equivalent circuit of the transistor. So, this is the base terminal, this is the collector terminal and this is the emitter terminal.

In this case of course, we do have different elements or parameters involved say for example, we already have discussed about the gm. So, likewise we do have the base to emitter resistance r pi and then we do have from the current gain, base to collector current gain and this is different from beta f this is actually referred as small signal current gain. And so far we are ignoring the early voltage in case if we consider the early voltage depending on the voltage at the collector. So, it will be having some variation of the collector current.

Namely based on the v c variation we may be having some variation in the collector current. So, if you consider that part also then across this current source. So, you may say that it will be having some conducting element. So, across this one there will be another conducting element it is actually it is going to the emitter.

So, this conducting element it is referred as output conductance in this case as you can say that this we refer as output. So, that is why it is output it is referred as output conductance and that is the meaning of each of these terms involve with the this equivalent circuit. So, if you see here, they do have of course, their own unambiguous definition.

So, whenever you are talking about say g m which is referred as trans conductance of the device. So, how is it getting defined? This trans conductance it is representing the relationship between the collector and V be. So, you may recall that this collector current I c versus V be. So, this characteristic curve it is exponential with respect to some dc point or q point we are linearizing it and this linearization is there is basically a linear segment of the line and that line can be represented will represented by the slope of the line.

And this slope of the line is nothing, but the first derivative of this current with respect to V be. So, the trans conductance its definition it is change in collector current with respect to change in V be voltage. Note that this is capital I small c this is capital V small V be V be. So, if we vary this base to emitter voltage whatever the variation of this current we are observing keeping rest of the things constant, this relationship it is getting represented by this g m trans conductance.

Since I am keeping other parameter constant particularly the collector to emitter voltage we are keeping it constant. So, that is why we are considering this is partial derivative and we already have in different way we have obtained this g m. Expression of the g m it is capital I c divided by V t. In fact, if you use the expression of this I c in the form of a I s c e to the power V be by V t and then if you take the derivative of first derivative of this expression what you can get is you will be getting this is I c divided by V t ok.

And this I c of course, it is instantaneous I c which is the slope of course, it depends on the instantaneous value, but then if we restrict our discussion within a small range then we can say that this I c can be well approximated by capital I c namely whatever the current we do have here at the dc operating point.

If you consider this slope of this line probably throughout this one we can assume that slope is remaining constant and hence you may say that this is capital I c divided by V t. So, that is what we obtain here the expression of the g m it is capital I c divided by V t. On the other hand whatever the current it is flowing through this circuit in the small signal equivalent circuit if I say that this current is flowing through base to emitter terminal or rather base terminal is say i b. So, the if I vary this voltage this current is changing.

So, you may say that if I take ratio of these two, then we may say that yes, it is basically the in base to emitter terminal conductance. So, if I see the conductance here namely if I take the if I observe the variation of the base terminal current with respect to V be then equation wise dou I b divided by dou V be. So, that is the input port or base to emitter port conductance, if I take reciprocal of that that represents the base to emitter resistance its r pi.

Later I will see why we do have different subscripts, but at least we understand that the unit here it is ohm. So, we call this is base to emitter resistance. And again if you use a expression of this I b in the form of I s B e to the power V be by V t and then this part the internal part it becomes this is I b divided by V t and then we may approximate that this I b it is with respect to the q point. So, we can say that this is capital I capital B divided by V T inverse.

So, depending on this IB of course, this value of this resistance it will vary. So, in fact, it is intuitive if you see whenever you are observing the let me use a different color. So, whenever we are observing the I b versus v be characteristic curve. So, depending on the operating point here we may get some slope here. So, that gives us one by r pi at that point. Now, if I come somewhere some other point here of course, their corresponding slope here it will be different so; obviously, at this point the corresponding r pi it will be different.

But as I said that if we restrict our discussion or from our experiment with respect to some operating point then you may say that this r pi it is remaining constant. So, based on the at the dc current there I B capital I B we can say its resistance is V T divided by I B.

So, likewise the other parameter if you see here namely the beta naught, beta naught of course, you may say that sometimes we assume this beta naught it is quote and unquote close to or equal to beta f. But strictly speaking it is having its own definition and normally we use beta naught is the symbol which represents the relationship between the collector current variation with respect to base current variation.

So, obviously, this is different from ratio of capital I c by capital I b. In fact, normally what it is; what it is used here instead of writing here dou I c divided by dou I b. The relationship it is fairly linear, but whatever it is normally written in this form of small i c divided by small i b. And I like to say here one thing that, if the beta f it is remaining constant for whatever the range of this i c and i b we are considering then we may say that beta naught it is same as beta f.

But in case if the beta f it is different then; obviously, depending on the operating point their dependency it will be different. So, ideally if we plot say I c versus I b. So, I c along the y axis and this is along the x axis is the I b. So, if you see both of them are having exponential relationship and so it is expected that if this both are exponential both are having exponential dependency on the same parameter called v be then; obviously, it is expected to be it will be linear and slope of this line of course, it will be the beta F.

But in practical case actually if you go to lower level of current it is having a different slope. Here, it may be remaining constant and then again if you go to higher current again it will be having a bend. I will not be going detail of the sources of those in non-linear part, but as long as if you keep the transistor within certain range or if I say that if I consider range of this collector current within sake few 100’s of micro ampere to its maybe 10 milliampere for normal transistor you may say that this range it is fairly constant.

So, if we are within this range. So, wherever we consider slope remains constant and this if I say that, if I extend it if it is since it is going through the origin then the large signal beta F and beta naught they are same. But if you are say somewhere here; obviously, the slope here it will be different which is the beta naught it will be smaller than this one likewise if you consider the upper side.

So, if we sketch the variation of the beta by that beta F or beta naught with respect to say I c what you can say that in the middle range you may say that it is remaining fairly constant. If you go a very low current it may drop and likewise if you go to a higher current it may drop.

This is the violation of low level injection and this is where the recombination current starts also becoming prominent. Somewhere in this range you may say that low level injection is valid as well as the recombination at the base region it can be ignored. And hence both of these curves are having quote and unquote same kind of relationship and hence the beta is remaining constant.

So, that is about the beta, and if you consider on the other hand the other parameter particularly this conductance part. As I said that the source of this conductance is basically the early voltage or you may say if you consider this circuit if I vary this collector voltage namely V be V ce voltage due to early voltage let me use different color.

Due to early voltage effect. So, this I c is having some dependency on the V ce. And in other words if I say that if we observe the collector current while varying the V ce it is having a increment of the current. In other words we may say that if I vary this if I increase this voltage, the total current is increasing which means that the total current it is not just completely defined by the condition at the base.

In fact, some part it is also dependent on this one. And since where if we change this voltage and if you are observing the same terminal current, you may say that if I take the ratio namely change in the current with respect to changing this voltage that gives nothing, but the output conductance.

So, that is why we are saying here the output conductance g naught is defined by change in the collector terminal current versus the collector to emitter voltage. And sometimes either we put this element in the form of conductance or some people are writing in the form of output resistance. So, if I take reciprocal of this one that gives us the output resistance or rather 1 by output resistance is defined by dou I c divided by dou V ce.

So, in fact, if you see again to get the expression of this part you require the expression of this I c and so I sc into e to the power V be by V T. Now, I have to consider these additional factor namely 1 plus V ce by VA part then only I will be getting this part is non-zero ok. The this slide looks very clumsy. So, let me clean up rest of the things I guess you will remember. So, just to have a little more discussion on this one let me clean up the slide here.

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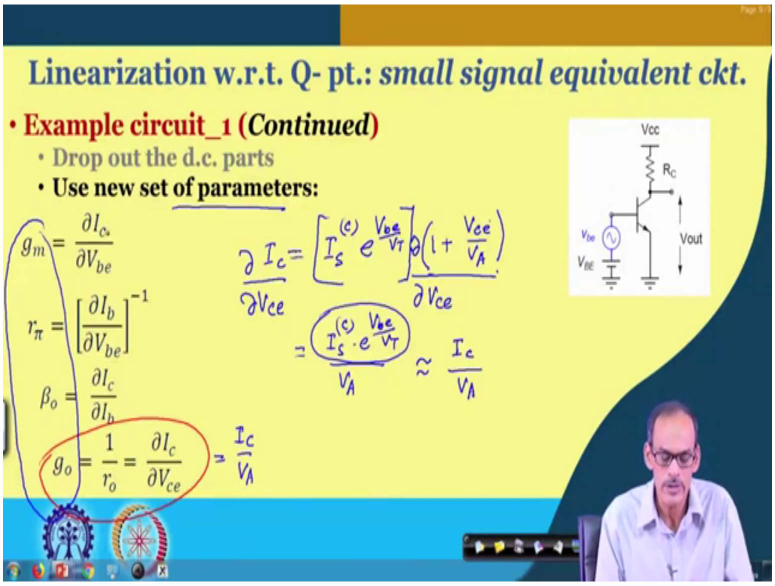

So, we are discussing here and we have to take as I said we have to take the expression of the collector current I c which is having I s c reverse saturation current e to the power V be by VT multiplied by 1 plus V ce divided by V A.

Now, if I take the derivative of this equation particularly, partial derivative with respect to V ce and then of course, since we are changing only this one not this one. So, you may say that this part it is remaining constant, only this part it will be changing. So, if I say that if I take the derivative of the whole thing this part will be taking out and then we do have dou this part divided by dou V ce and that gives us and this I s c into e to the power V be by V T divided by V A.

Now, we may approximate this whole thing by the collector current I c. So, whole thing of course, collector current is having this part, but since this part is appearing as this. So, we may say that this part is very small compared to 1, only while you are taking derivative we are considering this part. And then this part it is I c and this is V A. So, the expression of g naught it is basically I c divided by V A.

So, we can say that all these parameters new set of parameters they are dependent on the operating point and as the operating point we are trying to keep it constant. So, we may say that for small signal equivalent circuit this parameters you may say that quote and unquote remaining unchanged. So, whenever we will be drawing the small signal equivalent circuit first step of course, we need to find this parameter and to get that first thing is that we have to find the operating point.

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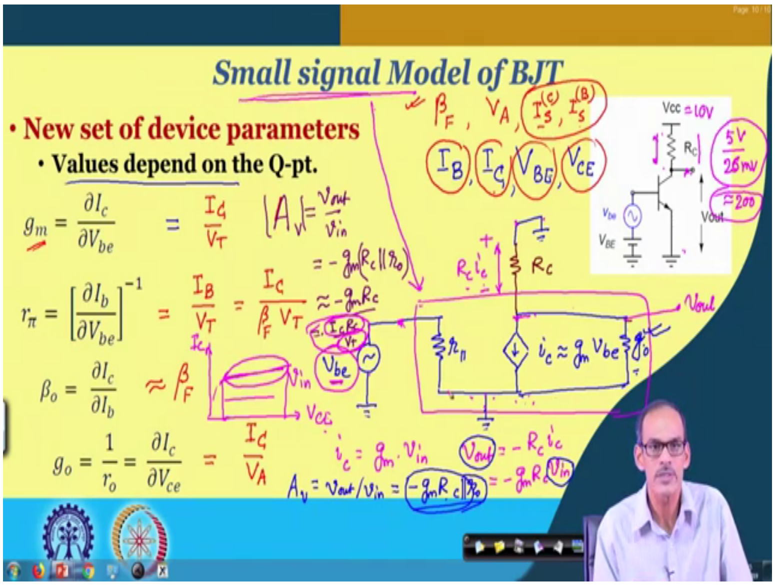

So, let us see what it can summarize here, we already have discussed the expression of this g m. So, what you are summarizing here it is the parameters values, they are dependent on the q point and whenever you are talking about q point basically I B, I C, V BE and then V CE.

And of course, we do have the device parameters namely beta right. So, we do have the beta and then we do have the early voltage and of course, we do have the I s C as well as I s B. And while we are dealing with circuit as long as we are in device we may be more concerned about these parameters.

But once we are in circuit what we will be considering is that we may be focusing on I B, I C and so and so. These two parameters we may still keep our focus, but these two things we may normally we may not be referring back. So, whatever the dependency of these parameters will be discussing here, it will be in terms of these parameters or this parameter, so maybe this also.

So, whenever we are writing the expression of gm what we have obtained it is I C divided by V T thermal equivalent voltage. Whenever you are talking about r pi it is I B divided by V T. In fact, instead of writing in terms of I B we may write in terms of I C divided by beta of the transistor and then V T.

And then beta naught normally we say it is remaining constant. So, we may approximate this by beta F and the output conductance it is I C divided by V A right. And the equivalent circuit as I said quickly for the mos part we do have the r pi we do have the i c which is most of the time we write in terms of g m into v be. And then we do have the output conductance g naught and then we do have the load resistance and of course, we do have the signal source.

So, this is connected to ground, this is ac ground, this is r pi and this is V be. Now, the whole circuit now we are translating into this equivalent circuit as I said that this is referred as small signal equivalent circuit. And this small signal equivalent circuit it contains rest of the things, but important thing is that we do have the equivalent circuit consists of the equivalent circuit of the transistor and this is referred as small signal model of the transistor.

So, this is what we see that small signal model of the transistor. So, if you see just to give you an idea that how useful this small signal equivalent circuit be we will be discussing some numerical example. But before that if you see that if I analyze this circuit and whatever the voltage I will be getting it is the v out. So, if I am applying say input here directly between base to emitter.

So, you may say that this is v in, it is directly giving us the v be. So, then you may say that the corresponding i c equals to gm into v be incidentally that is equal to v in. So, that is the current it is flowing here. And then drop across this resistance due to this current flow it will be R c into whatever this i c and polarity of this current is from this node towards this side.

So, whatever the drop it will be appearing here it will be having rather this side is plus and this is minus. So, the and of course, this is connected to ground. So, if this is ground then voltage here it will be minus R c into i c. So, you may say that v out equals to minus R c into i c and that is nothing, but minus g m sorry minus yeah g m directly, you can write g m into R c into v in.

And so you may say that. So, this is the v out and this is the v in. So, that gives us let me write here. So, if I you say this equation or let me write here itself v out by v in equals to minus g m into R c, but that is nothing, but the voltage gain of the circuit. So, this small signal voltage gain of the circuit is basically directly coming from this one.

And if you see here of course, I have ignored this part, if you consider this part this end of the g naught it is connected to the ground. So, you may say that its equivalent resistance is whatever one by g naught. So, instead of R c part of this current it will also be flowing through this one. So, if I want to consider early voltage also then you have to consider R c in parallel with r naught where r naught it is 1 by g naught. So, that is what the expression of the voltage gain.

So, end of it what you are getting it is voltage gain of the circuit it is let me use this color no this color. So, voltage gain of the circuit it is A v defined by v out by v in. So, that is minus gm into R c in parallel with r naught, typically if I say that this early voltage is very high. So, we may drop this part and we may approximate this by minus gm into R c.

Now, if I consider the magnitude interestingly, if I consider only the magnitude and so I will be having only this part. So, what we have it is g m it is i c divided by V T. So, it will be from that we can say that I c into R c divided by V T. So, in other words the voltage gain, voltage gain it is drop across this resistance divided by V T say for example, if I am having say ten volt V cc and if say the dc voltage here it is say 5 volt and remaining 5 voltage here.

So, this part it is 5 volt divided by VT. So, directly you can say that gain of this circuit voltage gain of the circuit it will be drop across this one which is 5 volt divided by say 25 or 26 millivolt. So, that gives you roughly 200, actually 200 actually this would be 326, it depends on the temperature but. So, approximately a gain of 200.

Which indicates that if I am applying a small signal at the output we can get very large signal. So, even though we are applying here it is a small, but here we are getting large one. So, the natural question is that we are talking about small signal model, small signal equivalent circuit and with a small signal here if we are getting a large signal here then the question is that the validity of the circuit. Is the circuit is still valid for this large signal at the output and the natural question then it will be that how small the signal be to certify the circuit it is small signal.

So, it depends it depends now in fact, if you look back small signal model is basically nothing, but the linearization of the around characteristic of the device. So, the some characteristic particularly base to emitter characteristic it is highly non-linear, because we know that it is exponential relationship and to maintain the linearity the v be should be very small compared to thermal equivalent voltage. Which means that v be should be less than well below than the 25 or 26 millivolt.

On the other hand, if you see the output voltage the input to output I should not say input to output it is basically current to voltage relationship if you see it is fairly linear. So, it is fairly linear over a wide range of the V CE voltage. So, this I c dependency of this I c on this V CE it is fairly linear, even if the V CE voltage is large. In fact, this voltage for this case it can be as large as maybe a 9.6 volt. So, having a large voltage here really is not making any problem to assume the circuit is still linear.

So, since this part of the circuit it is linear, the voltage at this output node can be in the ranges of few voltages whereas, the base to emitter voltage since it is exponential dependency. So, of course, the voltage there it will be small to certify the circuit is small signal ok. So, we are very close to this discussion, but let me cover maybe highlight to one numerical example.

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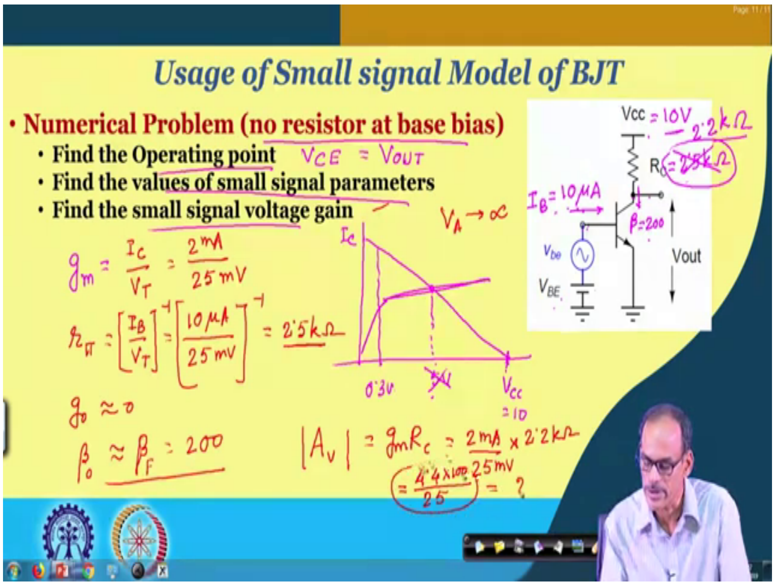

And so, in this numerical example if you see we do have the same basic circuit, the main thing is at this bias we do not have any resistance. So, directly the voltage you are applying both the bias as well as the signal to the base. So, that makes this circuit simple and then we do have say supply voltage is say ten volt and let you consider this V BE such that the current here it is say 10 micro ampere. So, I am giving this V BE in term such that the I B capital I B it is 10 micro ampere and then say beta of the transistor it is say 200, beta a for beta whatever you say.

And so that gives us the cohesion current here it is 2 milli ampere. Now, to get a good linearity range here namely to get good linearity range here we like to keep the operating point will away from this point as well as whatever the saturation limit or active region limit.

So, this is may be the maximum voltage you can have here it is a 10 volt. So, load line it is intersecting there it its V cc which is 10 volt. This voltage it is very small, you may approximate that this is 0.3 volt. So, roughly you even if you are keeping this voltage say at 5 volt we do have good linearity range of this output iv characteristic. So, if we do have 2 milli ampere of DC current is flowing for this base current and this beta then we may say that we may prefer to have 5 volt; roughly 5 volt drop across this R c. So, that gives us this resistance maybe 2.5 kilo ohm.

So, suppose these information's are given. So, what I have done here it is. In fact, there may be two ways of framing the numerical problem, either we give this information and then directly we can ask draw the small signal equivalent circuit, find the values of the small signal parameters and then find the small signal voltage gain. Or probably we can give this bias condition such that this is 10 microampere, this may be given to us then find this operating point and then we can go into this one, but whatever it is just for your practice instead of say let you consider instead of 2.5 which is giving us exactly 5 volt.

Let me consider this is something different may be 2.2 kilo ohm which is a practical value, 2.5 normally we do not get. So, probably you can try out find the operating point namely the I bit is given, collector current is also very straightforward. Then you can find the V CE voltage which is nothing, but V capital OUT also. And then you can find the value of the small signal parameter g m considering the early not early the thermal equivalent voltage to be 25 millivolt.

So, this is I c divided by V T. So, this is 2 milli ampere divided by 25 millivolt. Then the r pi it is not required. So, still you can write r pi equals to I B divided by VT. So, this is 10 microampere divided by sorry this is reciprocal of that. So, 25 millivolt and its reciprocal so, that gives us 2.5 kilo ohm.

By the way the base to emitter resistance where you can see that it is much higher than typical diode on resistance, mainly because the base current though it is having exponential dependency the base current is very small. And then we may assume that early voltage it is very high. So, you may say that output conductance it is 0 and then beta naught you can approximate to be beta f which is 200.

So, from that you can find what will be the small signal voltage gain, the small signal voltage gain it is if I consider magnitude it is g m into R c and gm into R c is 2 milli ampere divided by 25 millivolt multiplied by 2.5 kilo ohm. So, this part it is 4.4; 4.4 volt divided by 25 millivolt. So, that gives us 4.4 divided by 25 into 1000 right. So, whatever the value it is coming.

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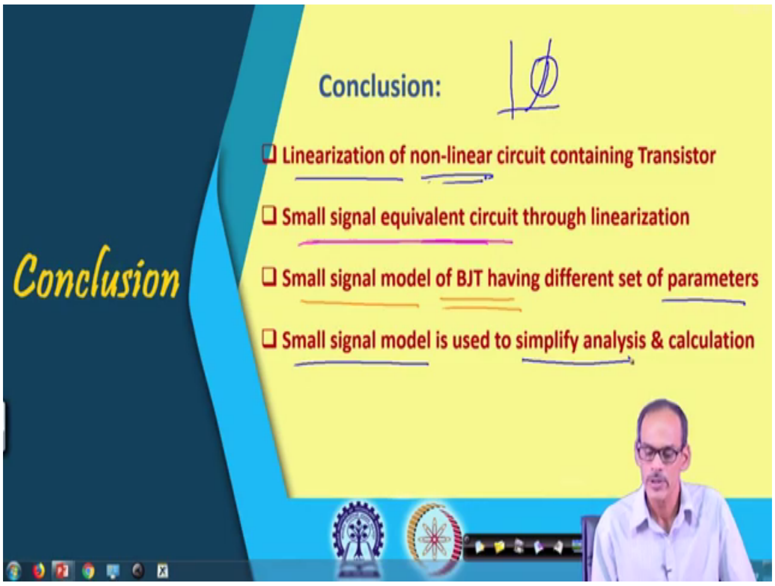

We have observed that linearization of non-linear circuit containing some transistor is important. Because we are looking for linear behavior, then you may say that then why are you looking for non-linear circuit; learning non-linear circuit gives us amplification possibility of amplification. So, the non-linear behavior say like exponential behavior, it gives us highly good gain.

In fact, this is one of the source of one of the factor which is giving us good gain. So, we require non-linear circuit, but then we are looking for linear circuit. So, that the in output signal nature it will be same as the input signal nature and then what we have done is that since you are looking for the linearization through that we are getting something called small signal equivalent circuit.

Which is basically linearization of the total circuit, keeping our constraint namely with progress of time dc wise the operating point should not change only the signal part it will change with time and also the average of the signal should be 0. So, that the operating point or the cohesion point should not change. So, that gives us some different notion called small signal equivalent circuit. Now, whenever you are drawing some small signal equivalent circuit of a given circuit the same notion it can be deployed for the simple transistor also.

So, whatever if I consider BJT in this case if I restrict our signal within some range then the BJT transistor can be represented by a linearized form which is referred as small signal model of the transistor. And why do you go for small signal model of the transistor? That simplifies the analysis.

So, of course, to get the model parameter we have seen model parameter it depends on the operating point. So, it is having different set of parameter, but once we get the value of those parameters and if we know that cohesion point is not changing then we can make use of those parameter value and then small signal model of the BJT or small signal equivalent circuit of the complete circuit it will be basically simplifying the analysis and calculation ok.

This is the heart of analog circuit. So, today we have discussed primarily on BJT. So, similar kind of discussion it will be there in our next class on MOSFET.