Analog Electronic Circuits
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Lecture - 18
Linearization of Non – Linear Circuit Containing BJT

Start Sir.

So, dear students welcome back to this course on Analog Electronic Circuits. Myself Pradip Mandal from IIT Kharagpur, I am associated with the Department of E and ECE. Today's topic for this course is Linearization of Non-Linear Circuit Containing BJT. So, we will be covering what are the topics let us see.

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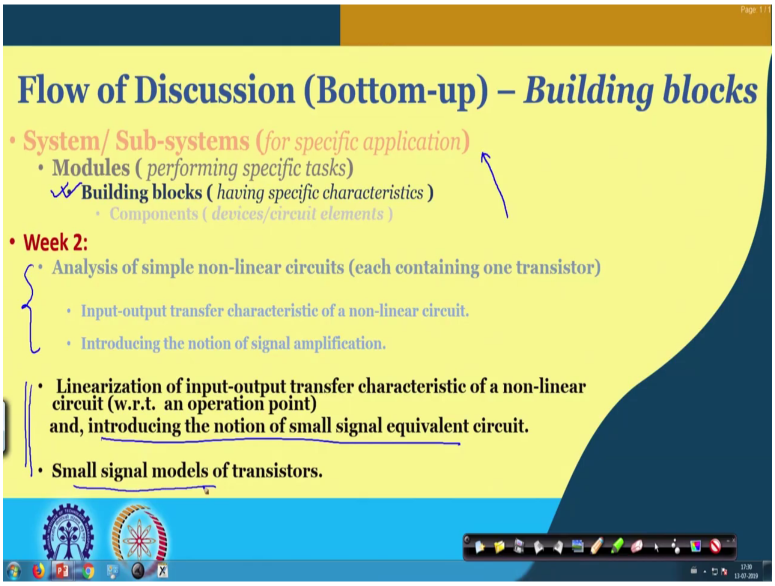

Before, I discuss about the subtopics let me see what is our position so far; as we are mentioning that this is our main flow, from component to building blocks and so and so. And, at present in this week we are the building blocks namely, the basic building blocks and in this week we have covered the analysis of simple non-linear circuits. Today, we are going to start with linearization of input or output transfer characteristic of non-linear circuit containing transistors and this linearization it will be with respect to operating point.

So, then we will also see the notion of small signal equivalent circuit. Then from that we can see that a different model of the transistors namely small signal model of transistors. So, this is what we will be covering today considering the examples having BJT and in the subsequent class we will be covering similar kind of things, but the circuit containing MOSFET.

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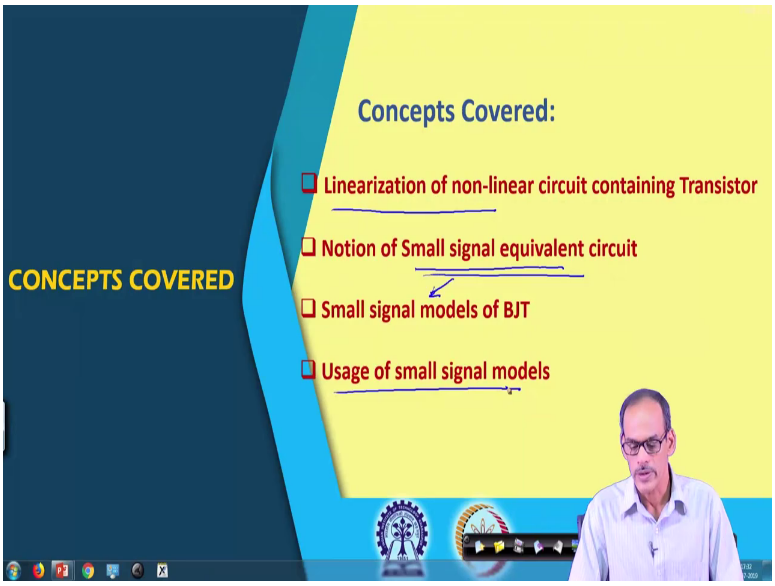

So, what are the concepts we are going to cover in today's presentation? As I said that we will be covering the linearization of non-linear circuit containing BJT, then the notion of small signal equivalent circuit, particularly if you are having non-linear circuit then how do you translate into equivalent circuit particularly the variations of the voltage and currents are restricted, focusing the focusing the discussion within a narrow range; so, that the non-linear circuit characteristic can be linearized.

So, basically when we say an linearization of non-linear circuit we are primarily focusing the narrow range of the input or output transfer characteristic. So, they are related basically the linear linearization of non-linear circuit and small signal equivalent circuits they are related. From that we will see the new model or new concept of models of transistors referred as small signal models. So, we will see that how the small signal models are getting evolved and what is its usage of the small signal equivalent circuits for some practical circuit ok.

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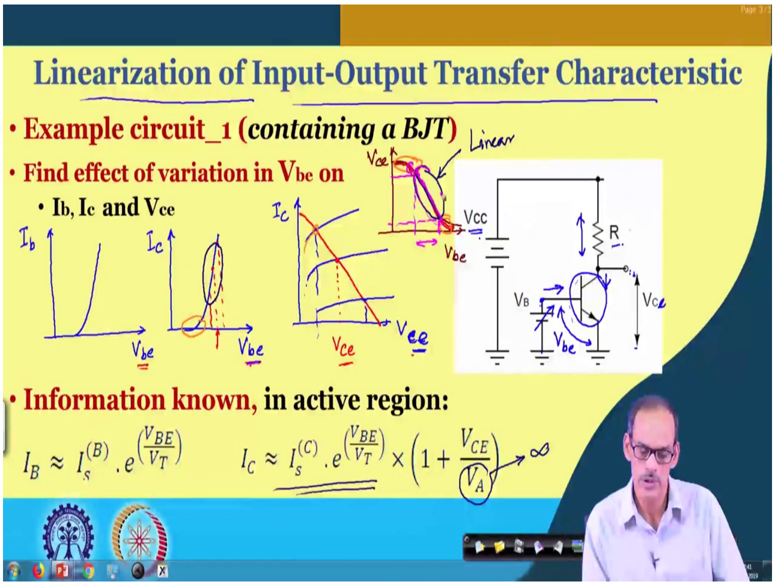

So, let us go to the example circuit now. So, this is the example we have discussed before, but our focus here or our discussion center it will be different. Namely, it will be as I said linearization of the circuit or you may say that linearization of the input to output transfer characteristic of the circuit.

This circuit we have seen before, it contains the BJT NPN transistor and it is having a base bias with a voltage. It is also having a collector bias through this battery VCC through this register R and what we have seen that depending on the voltage here, depending on the value of the resistance and the VCC. And, of course, based on the parameter of the transistor the collector voltage of the transistor; or collector to another emitter voltage of the transistor can be defined, or it can be expressed in terms of those parameter.

And, then if we vary this voltage V B which is incidentally a VBE voltage we may call this is V small B E or capital B B E whatever it is. So, if we vary this voltage it is expected that, if we retain rest of the thing same the voltage at the collector it will change with this variation. So, not only this collector voltage actually this variation it is happening due to the base current variation, then the collector current variation and then the I R drop across this resistance and hence the collector voltage here or emissive Vce.

So, then if I consider that, if we consider this is input and this is the corresponding output, then we do get input to output transfer characteristics. So, that we have discussed for a given value of VBE how we get and also we have seen that if you change this voltage, then how the base current and the collector current how are they changing we have discussed in detail, we will be moving further on so, this is the IV versus VBE characteristic curve. So, likewise we do have if I multiply this curve by beta of the transistor we do get I c versus Vbe characteristic curve.

And, then which is also assuming beta is remaining constant these two characteristic curves are having practically same profile. And, then to find the output voltage here, what we have done it is for a given value of Vbe, we have drawn the transistor characteristic namely the I c versus Vce characteristic, and then also Vce characteristic. And, then also we have drawn the characteristic of the register or the load, in this case whatever the resistance we do have and then the intersection intersecting point it is defining the corresponding Vce.

So, this is we have discussed and formed this one we also have obtain that, if I consider this is input and this is the corresponding output, then if we vary this input and if you see how this intersection point is changing and hence we can plot the input to output transfer characteristic. So, if I just ask you to remember or recall whatever the discussion we are having. So, if we change this Vbe with respect to some point and then the current is going up or down; so, here if we vary this input or Vbe then the corresponding characteristic curve it is going up or down.

And, then accordingly the intersection point is also changing and that gives us the variation of the output voltage. So, that gives the variation of this output voltage. And, from that if we plot the input and input to output characteristic. Namely, if I consider y axis is the Vce and x axis is the Vbe, whatever what we have seen is that the characteristic curve it is. In fact, inverse of this characteristic curve first it is going like this and then I while the device is entering into the saturation region the characteristic becomes like this. So, this is what the we are referring to input to output transfer characteristic of the circuit.

Now, this input output transfer characteristic of course, it is quite fairly non-linear. And, we have discussed about the sources of non-linearity, but just to recapitulate whatever we discussed there, we say that because of the smaller Vbe, because of this portion it is quite non-linear this part it becomes non-linear. And, on the other hand if the characteristic it is going up here towards this point. Then the device it may be entering to the entering towards the saturation region and due to that due to have another non-linear portion.

In between we do have the a fairly linear part. So, we say that this part it is fairly linear, but even then if you if you further zoom here and if you are trying to observe the exact characteristic, it will be compared to this non-linear part it will be linear, but then again depending on the range you are considering there will be non-linear part. Mainly, because you may recall that this part it appears to be linear, but this part it is having exponential dependency.

By the way here in this case we have assumed that this early voltage it is very high and that is why we are considering this is flat. Now, in case if we are restricting our you know voltage variation within this portion or within the middle of the input or output transfer characteristic, then we may say that it is fairly linear. And, so, that is what we are referring to linearization of the input to output transfer characteristics.

So, though the actual characteristic it is not linear, but if you restrict the voltage probably we can we can as preteen or assume that this input to output the transfer characteristic it is quote and unquote linear. So, what are the things we have so, far what we said is that, if we restrict this Vbe then only we will be getting the linear region.

So, we assume that even if you are moving towards this point or this point accordingly if we restrict the voltage variation probably the linearization is possible, but we are looking for big range of linear transfer characteristic. And, hence we prefer that our focus or discussion should be towards the middle of the input to output transfer characteristic.

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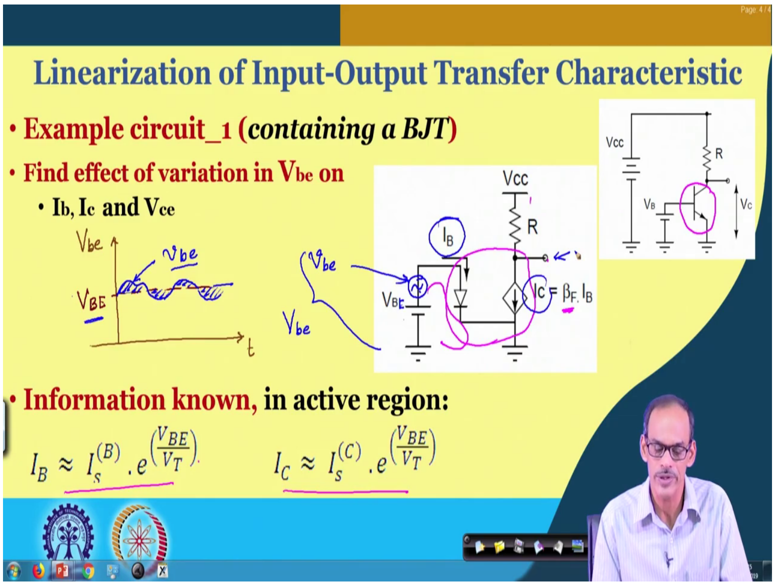

Now, let me go a little more detail of the linearization. So, what we are doing is basically we are making use of these two equations and then we are considering instead of considering this transistor we are considering the small signal equivalent circuit. And, then using this equivalent circuit we are considering this loop and then we are considering beta and then we are finding the corresponding output voltage.

So, whenever we are talking about linearization, pictorially whatever we have discussed, it is also having equivalent equations. And, let us see today we will be going little more detail of those equations. And, let us see what are the equations involved into this linearization, because conceptually graphically it is graphical discussion is fine, but whenever we are looking for numerical solution definitely you have to refer back to the underlying equations.

So, we will make use of these equations and we are considering that the voltage here it will change. And, the variation here instead of thinking it is only DC what will be considering it is having a signal part. And, then it is having a DC part and then this signal part will denote by small vbe and this part it will be capital VBE and this 2 together, we may call it is capital Vbe.

In other words you may say that, with respect to time if we try to sketch the with respect to time, if you sketch the capital Vbe part, it is having the dc part it is having the DC part. So, this is this level it is capital VBE and with respect to that it is having the variable part. So, we may have either sinusoidal or maybe non sinusoidal or whatever it is as long as the average of the small signal part it is 0, then you may say that the DC part with progressive of time it is remaining same as this VBE. And, this part the variable part whatever we see is small vbe that we call it is signal.

And as long as this DC level it is not changing, it can be shown that the circuit characteristic if we translate into linear kind of form the analysis the equations it becomes very simple. So, in the in the next discussion and instead of varying this voltage gradually, we may prefer to restrict our discussion that whenever we are changing this VBE, it is having a quote and unquote constant DC, then on top of that we do have the variation or fluctuation of the voltage, which is carrying the information.

And, we are assuming that it is the average of the fluctuation it is 0. So, it is not necessarily that vbe will be sinusoidal it may be having any other pattern for that matter. And, then if you want to see what kind of effect of this variation it is coming to the base current, and then the corresponding collector current, and then corresponding the output voltage or V C E voltage that is what we are going to discuss now.

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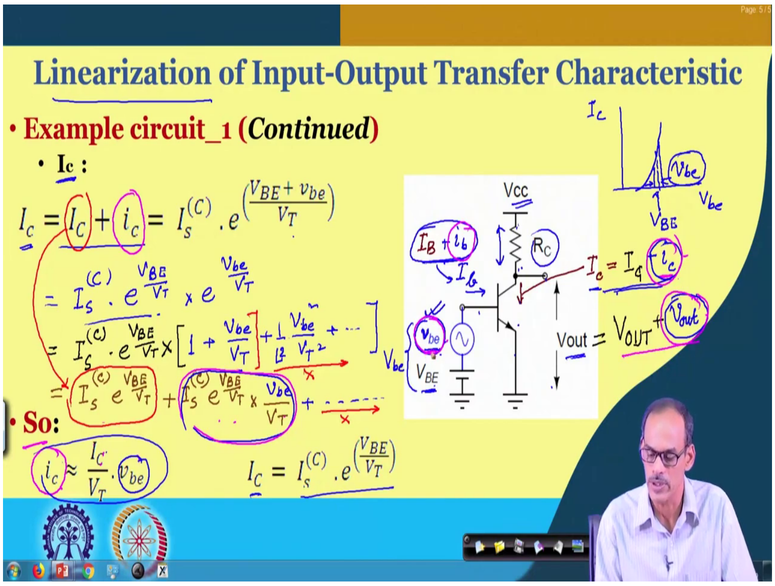

So, the circuit now we will be considering it is as I said the same circuit, but the kind of signal we are giving, it is writing over a DC part. So, as you can see here we do have it is having a DC part, if you have a small signal part and these two together we do have capital Vbe. Now, we do have the basic equations, namely the expression of this current it is again of course, this current will be having DC part and the small signal part. And, hence we will be using different symbol for the base current namely capital I and small b.

And, this is having 2 parts namely it is having I B plus the corresponding fluctuations due to this voltage and that is represented by small I b; so, likewise when we see the current at the collector terminal. So, they are also the collector current this I c, we will be using symbol C capital I small c, which is having 2 parts; one is the DC part capital I capital C, and then in addition to that we do have the small i c.

So, now whenever you are trying to see what is the corresponding effect coming to the collector, either we can find the effect on the base current and then you multiply with beta of the transistor, or probably you can directly use the exponential relationship of this collector current on VBE. And, then you can find the corresponding effect and our main intention is to find the output voltage.

So, it is not necessarily that you have to go through I b effect on I b and then coming to the I c, you can rather directly you can go to the collector current expression in terms of VBE. So, as we have discussed that collector current. So, if we drop the early voltage the collector current expression is the saturation current part and then e power VBE by VT. Now, as I say that this VBE it is having two parts capital VBE plus small vbe.

So, if you see here when we when you talk about linearization in the in the graph, what does it mean in this in the form of equation let us try to see that. So, we do have this constant part I s c part and then e power VBE by V T multiplied by e power vbe by V T ok. And, as I said that this part vbe part it is function of time whereas, this part it is we can see that it is not changing.

So, you may say that this part it is remaining constant. So, the constant part let me write in different color I s c into e to the power VBE by V T. And, this part so this part on the other hand, we can write in the form of 1 plus vbe by V T plus vbe square by V T square factorial 2 plus so and so the higher order terms.

Now, if I say that the vbe, it is we are restricting which means that in our I v characteristic namely I c versus Vbe characteristic curve, if we restrict our variation of this Vbe with respect to capital VBE within a within a narrow range. So, whatever the variation we are talking here it is basically the small vbe.

And, if the small vbe, it is small compared to thermal equivalent voltage V T, then we may drop the higher order terms and then we can find the so, these terms onwards if we remove then we do have only this part. So, you can see that here if I drop rest of the things it is nothing, but linearization of the characteristic. And, if I drop this part what we have here it is the constant part. So, we do have this constant part I s c e to the power VBE by V T. So, that is multiplied by 1 and then I do have this part. So, we can see this plus I s c e to the power VBE by V T into we do have the V T part here and then we do have the vbe part.

And, whatever the other terms if I consider these terms if I assume that vbe by V T is small enough. So, we are going to drop this part. So, we are removing rest of the things. So, that is the linearization. Now, if you see with progress of time as I said that capital VBE it is not changing with time. So, we may say that this is the constant part of the collector current. And, the other part the linearized part after dropping the higher order terms namely this part let me use a different color yeah. So, this part it is basically nothing, but this small I c part.

So, whenever we say the small I c it is changing with time due to vbe is changing with time, then equation wise this I c part it is given by this. So, to say that the small signal collector current it is this part which is nothing, but capital I C. So, that is why you would have capital I capital C divided by VT. So, this V T is there and then we drop the vbe part. So, what we can say that this fluctuation it is having an effect on the collector current and that effect it is given by this equation. And, this of course, the constant current it is remaining the independent of the time and it is remaining constant.

So, now we equation wise we obtain what will be the effect coming to the collector current due to due to the variation here. So, now this current if it is, flowing through this register; obviously, the drop across this resistance it will also vary. And, hence and the supply voltage VCC on the other hand it is remaining constant.

So, if we do have this variation of this voltage with respect to time due to this vbe naturally this will be having the corresponding effect coming there. So, then this output voltage similar to I C and the maybe I B here also it will be having 2 parts; one is the time independent part on the DC part and then also it is having the time dependent part or the variable part. And, this variable part we are denoting by capital V sorry small v small out and whatever the variation you are observing here, it is again coming from this variation.

So, if I say that our source of variation is here that is creating variation here, that is creating variation in i c, and that is also creating variation in this one, which means that suppose this is carrying some information or say signal, this signal it is getting reflected everywhere. And, if these signal somehow if you say that it is stronger than the original signal, then you may say that this circuit is amplifying the signal strength ok.

But, of course, it has to retain the information or you may say that if the relationship of here to here if it is linear then; obviously, variation shape of the voltage fluctuation here, it is getting retained there. So, that is of course, now let us see what will be the expression of this V out we already obtain the expression of i c. And, now let us look into the expression of small v out in terms of this vbe.

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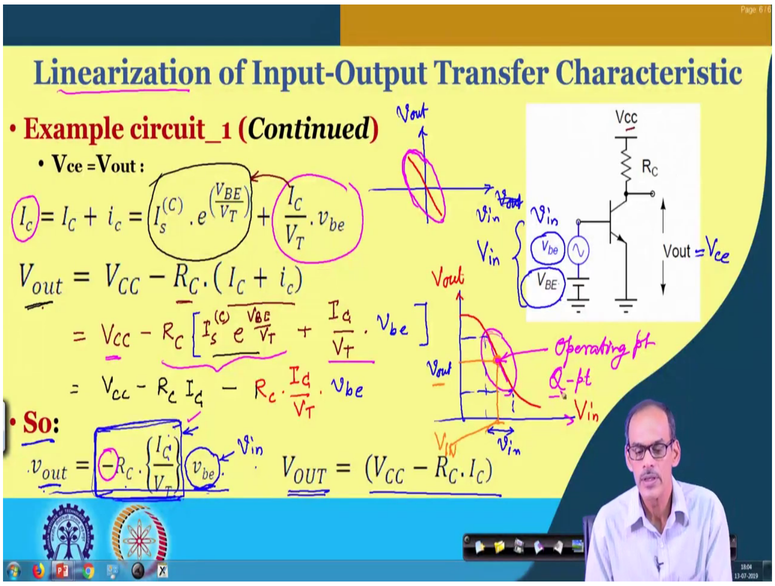

So, let us go to the next slide yeah. So, what we have here it is we already have discussed that, the collector current is this collector current is already having the variable part, namely this part and then the constant part or the DC part the DC part. So, now, we are observing the similar kind of things are the output voltage V out. So, here it is a very straight forward if you see so, if you see the expression of V out it is the voltage VCC minus this drop and this drop it is R C multiplied by whatever the total current we do have and I say that it is having 2 parts.

So, we may write this is V C C minus RC multiplied by I s c into VBE by V T or directly you can write this capital part. And, then we do have the whatever you see I c actually this part it is capital I C. So, this is same as this capital I C here. And, then just to save some space I am using this capitalizing here and then we do have the vbe part and then whole thing it is getting multiplied by RC.

So, now what you can do probably we can consider this part and this part along with this one and we can separate this part. So, what we have here it is the time independent part, it is VCC minus RC and this part it is as I said it is capital I C. So, that is capital I capital C and then we do have and the time dependent part. So, it is having RC let me use this color RC into I C divided by V T multiplied by this vbe.

So, we may say that to summarize then v out v out it is the small signal part or the time dependent part or the information part, it is actually function of this vbe. However, it is having a multiplication factor. So, as long as this multiplication factor is not changing with time, then you may say that whatever the information we do have, that is getting multiplied by this factor and probably if it is more than one then you can say that the strength of the signal it is getting increased. And, of course, the V OUT it is having the DC part. So, the DC part it is given here it is not changing with time.

So, what are the things we do have within this we do have I C by V T and R C. Note that this capital I C of course, it depends on this bias. So, depending on this bias the whole factor here, by which this small vbe it is getting multiplied that is that may get changed. So, if we if you are not really fixing this DC part properly, then this multiplication factor may change with time, then that may create disturbance here.

So, our objective here of course, we need to if we somehow managed to keep this part constant or if you managed to keep this bias part constant, and then we may say that the relationship this vbe to v out relationship it will be remaining constant. By the way this V out it is also it is Vce and incidentally this is V out, likewise this is you may consider this is V in and this part you may say that this is v in. So, if I say this is v in then this gives us the input signal to output signal relationship.

So, equation wise you may say that this part it is representing input to output transfer characteristic. Namely, whatever the input to output transfer non-linear input to output transfer characteristic we obtain, out of that we are probably focusing may be central part of it. And, then the slope of this line it is basically getting represented here.

Note that the slope here it is negative that is captured by this minus sign and so, this is a large signal V in. So, you may say that this is V in and this is V out, large signal V out it is having non-linear effect, but then if we say that we are focusing only here, then you may say that variation here it is v in and then corresponding effect here it is v out.

And, if we say that the variation or the DC part if it is not changing. So, suppose this is DC part defining this point reference point namely this is V capital in. So, if this part it is not changing and if you are looking for the relationship of this v in to this v out or small signal V into V out then you may say that this is nothing, but translating or taking one part of the transfer characteristic and considering only the small signal part.

So, if I take out this the input output transfer characteristic sorry this is v in. So, if I take out this portion of the transfer characteristic and then what we are getting here it is the transfer characteristic it is going through the origin. And, at this part we have discussed some extent, but equation wise this is what we are getting.

So, to further get into that you may say that, we are linearizing this input to output transfer characteristic to get small signal input to output transfer characteristic, centering with this point it is called operating point, operating point or it is called question point or Q point. So, unless otherwise it is stated will be assuming that this point it will be not changing with time of course, our first task is to find this meaningful operating point. And, then only we can say that will be getting good part of the transfer characteristic which is linear and then keeping our focus further subsequent focus around that.

So, whenever you are talking about linearization is basically linearization of the input to output transfer characteristic with respect to Q point.

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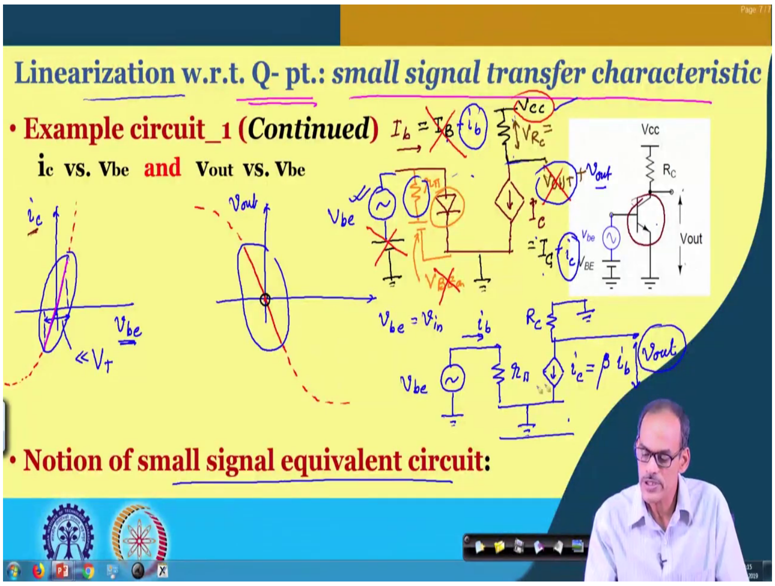

So, that is what we are saying that whenever we are talking about linearization basically it is with respect to operating point or Q point and which is referred as small signal transfer characteristic. So, in the small signal transfer characteristic; in fact, we do have two parts, we already have discussed, but one is a small signal i c with respect to small signal vbe voltage. In fact, this is exponential part exponential behavior.

But, as long as our discussion is restricted within a narrow range of vbe namely if the this variation it is much less than VT. So, if I say that this is much less than thermal equivalent voltage V T, then only you can say that linearization is meaningful. So, similarly if I consider the V out versus Vbe; so, there also either you can say that this is small v out with respect to vbe which is incidentally v in. And, there what you have seen is that the transfer characteristic it is we obtain it is like this.

So, since the both the characteristics are going through this origin and the biggest advantage of translating the transfer characteristic into this form is that the circuit is getting linearized. So, analysis it will be much simpler and if you are having multiple signal influencing the circuit then you can directly apply the superposition theorem. So, that is definitely it will be advantageous and also the overall analysis it will be simplified.

So, here whenever we are talking about linearization is basically extracting the meaningful part of the non-linear characteristic. In fact, you may recall if we extend the non-linear part it is like this and we are focusing around this. And, so, here also the if we if we consider beyond this, it will be actually exponential kind of characteristic. So, if you draw the corresponding circuit now. So, we do have graphical interpretation, we do have equation information and then also we like to have the equivalent circuit. So, that brings up the equivalent circuit something called small signal equivalent circuit.

So, what is that? First of all let me quickly draw this part, the equivalent circuit part. So, we do have the diode and then we do have the either you may say voltage dependent current source or the current dependent current source. So, if we are using the expression of the I c in terms of I b then you may say this is current dependent current source or if you are using I C versus vbe, then you may say that this is a function of vbe.

So, whatever it is we do have the expression of this current capitalize small c and then we do have the capital I small b; that means, it represents both small signal part as well as the large signal part. And, then we do have the small signal component and then we do have the DC part, this is connected to ground and then we do have the register and this is connected to VCC. So, whenever we are you know whenever we are considering the small signal transfer characteristic, as I said that the operating point it is getting mapped on to the origin of this characteristic.

And, this operating point is basically the Q point. In actual characteristic of course, this is not same as the origin in that, but in this small signal context the it is aligned with the origin. So, circuit wise what does it mean. And, so, if you see here the current wise this part this current is having two parts; one is the DC part and then it is also having the small signal part. So, likewise this I C it is also having the DC part capital I C as we have discussed and then also the small signal part.

And, then this of course, it is DC, but and of course, the across the drop across the resistance you may say that the drop across this one is VR C. So, that is having two part; one is DC another is the small signal part. And, likewise whenever we are observing the output the voltage here, that is also having the DC part capital V OUT and then the small signal part small v out.

Now, whenever we are translating the transfer characteristic into this form namely the small signal characteristic here and here. What does it mean is that we are dropping out the DC part, which means that we are removing say this part, we are removing say this part, we are also removing this part and we are removing this part. In fact, if you see this diode.

So, this diode is also it is non-linear it is in fact, highly non-linear rather, but if you are see it is equivalent circuit this can be replaced by piecewise linear, by considering the cut in voltage and in this case cut in voltage is VBE on and then we do have the resistance on resistance incidentally we use R pi to denote that resistance. So, whenever we are translating this one of course, we have to drop this part as well. So, we have to remove this part as well.

So, what are the things are left behind, we do have in this part signal part we may call this is vbe and then we do have this part. And, then of course, the current twice we do have this part and that current if we multiply with beta, then that gives us this collect corresponding collector current. And, whatever the voltage it will be getting developed in that situation, directly it is dropping this part, because vcc is dropped out, this part it is dropped out, this part is also dropped out. So, whatever the voltage will be getting here it is nothing, but this small v out.

So, if I make an attempt to draw the circuit of the left out elements we do have the vbe, then we do have the r pi. And, then we do have a current source either you may say it is voltage dependent current source or current dependent current source, for the time being let me call this is current dependent current source, which is i c small i c beta times i b and this i b, it is basically whatever the current it is flowing through this new newly constructed circuit, this is connected to ground and then of course, the register is there RC it is there and that is connected to equivalent to AC ground.

And, so, we do have RC here. And, whatever the voltage it is getting developed here with respect to this ground of this ground, that it is basically the small signal v out. So, whatever the circuit now we are getting if I compare this circuit and this circuit. So, this is referred as large signal equivalent circuit, where the mass transistor has been replaced by this. On the other hand if you see here all the DC parts are getting suppressed or removed and this is referred as the small signal equivalent circuit.

So, in fact, whenever you are talking about linearization of the input to output transfer characteristic, it is taking us to a situation where the circuit is also getting translated into different form called linearized circuit. And, though it is actually linearized circuit it is primary intention is to use this circuit for signal where linearity is valid. So, that is why this circuit it is instead of calling linearized circuit it is referred as small signal equivalent circuit. So, it is of course, it is having different parameters. So, what you can do?

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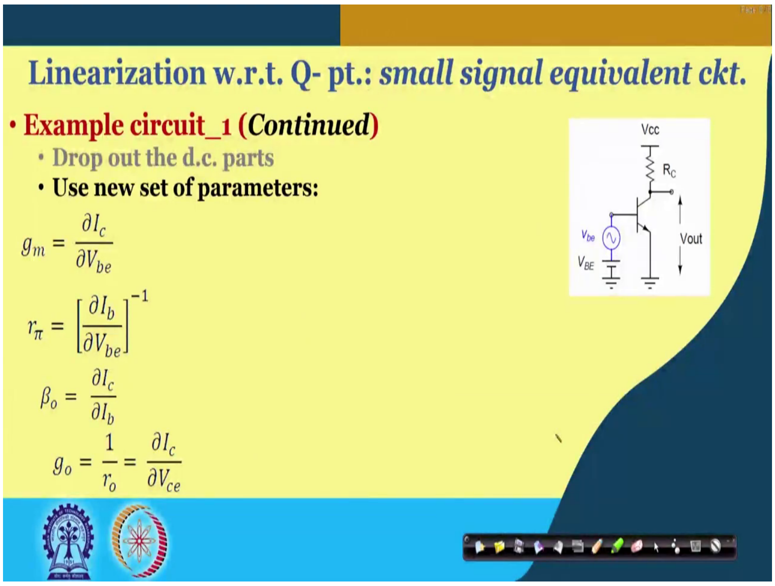

I think in the next slide we will be discussing in detail of that how do we? In fact, we have already have discussed this part we are dropping the DC part and we will be directly going to the new set of parameters. So, yeah so, let me take a break and then we will come back to this discussion of how do we get the small signal equivalent circuit and new set of parameters in this circuit.