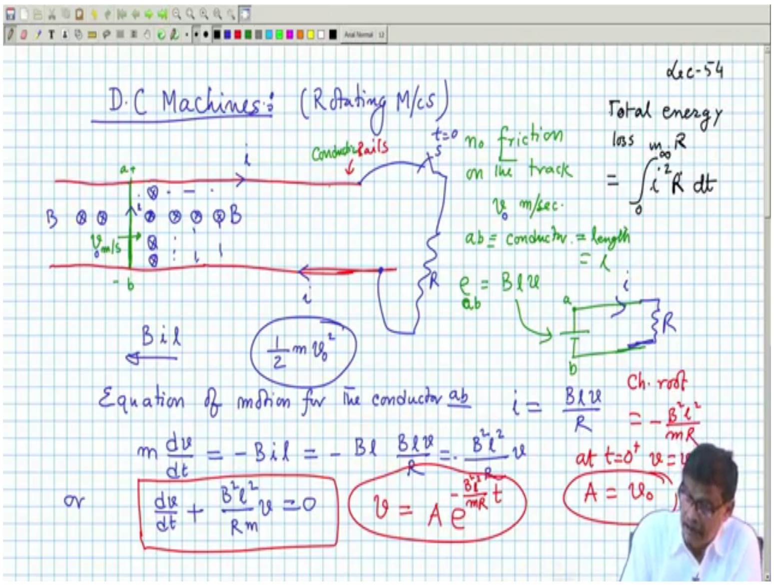
Electrical Machines - I
Prof. Tapas Kumar Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 55
Single Conductor D. C Generator/Motor Operation

(Refer Slide Time: 00:21)



Now, welcome to lecture 55 and before we start lecture 55, we have quick glance what we did in lecture 54. You remember we started DC machines, and eventually we will discussed rotating DC machines, but before that to gate the basic ideas clear what we considered a simple linear DC machines, where a single conductor is only present that is green line and it is suppose moving over rate frictional rails from left to right at a velocity V naught meter per second and suppose this switch is opened initially and since it is frictionless.

So you do not require any force to be applied to the conductor to make it run at a constant velocity V naught meter per second.

But nonetheless it is moving from left to right and also there is a magnetic field perpendicular to the paper here from top to bottom that is why I have shown it crossed B. So, we applied right hand rule and we know that there is a B, there is a V naught there at quadrature, therefore, there will be some induced voltage across the conductor a b and the upper side of this will become plus, and lower side will become minus.

So, it will become a source of emf like a battery that I have shown here, the value of this voltage will B l V naught. If V naught is the velocity voltage available, there will be B l V naught ok. So, initial velocity is V naught ok, and suppose the switch is open open.

Now, if this because we have generated voltage, you must utilize that voltage to deliver power to some resistance say a lan may be. You close the switch suppose at t equal to 0, then what is going to happen because the circuit is closed, and there will be a current i B l v by R. I have written v, because I am not sure whether the velocity will be maintained after it has started delivering power to the resistance.

So, after you have closed the switch at t equal to 0, at any time t, this is the expression of the current in the circuit, and i equal to B l v by R. Now, I have let us imagine that no external agency is pushing this conductor from left to right, because it was not necessary when a even S was opened, because it was moving over a frictionless path.

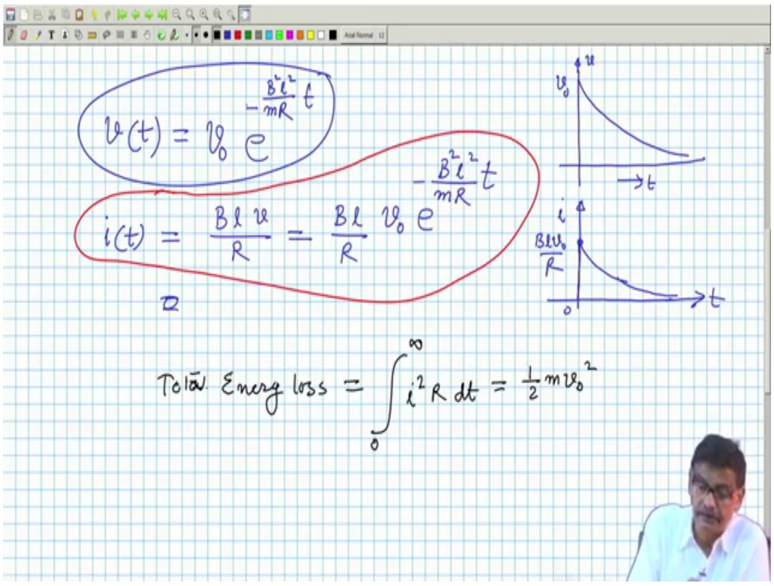
So, without absence of any external agency pushing it from left to right, we expect that it is delivering power to the resistance where from this power will come, it must come from the kinetic energy of this conductor a b, half initial energy stored was half m V naught square from that it is coming and physically we then expect that velocity will gradually decrease and so also this emf will decrease, current will decrease, and finally everything will be quiet, no voltage, no current, initial whatever energy stored that will be dissipated in R.

And, but in between what happens, if you want to write down, this is the dynamic equation applied force m dv dt is the applied force from left to right. But we know that when a conductor carries current i in this direction and if there is a magnetic field here, you have to then this conductor is going to experience also a force and by applying left hand rule, we find that it will experience force in the opposite direction B i l left hand rule you apply, you will get it.

Therefore, equation of motion will be m dv dt is equal to minus B i l and then i is this much. So, you eliminate all these differential equation, you get this as expected velocity will decrease progressively, and A is equal to V naught.

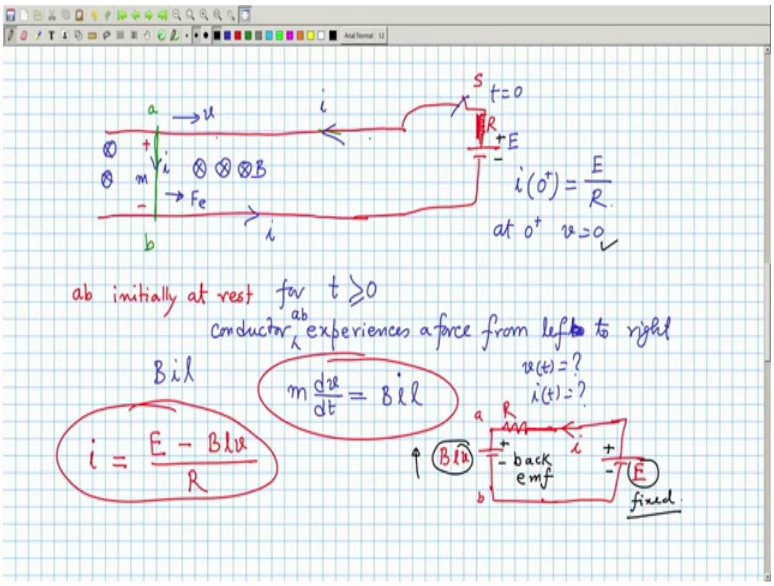
And if you like what I will add here if I add it will remain. So, if I you can verify that the total energy loss total energy loss in R in R will be equal to i square R i square R dt, and integrate it from 0 to infinity i square R dt and I know the expression of both current and current expression is known, next page I must have done it.

(Refer Slide Time: 06:09)



So, expression of the current is known. So, so energy loss total energy loss will be equal to i square R dt in R in the resistance from 0 to infinity time and if you substitute the value of i here, and calculate it you will end up with this quantity and that is what is expected initial kinetic energy, anyway this was the generate generated operation of a single conductor.

(Refer Slide Time: 06:55)



Then we took up the next problem can it act as a motor, that is same structure a b there is a magnetic field perpendicular to the board, and there is a battery with this side plus, this side minus, and switch is initially opened no current in the circuit, no voltage here either B l v is also not there but if you close the switch at t equal to 0.

Since velocity cannot change instantaneously at t equal to 0 plus after you have closed the switch, velocity of the conductor, previously it was 0 it will maintain 0 at t equal to 0 plus. Therefore, current in the circuit will be E divided by resistance of the circuit.

What is the resistance of the circuit? Suppose all the resistance in the series circuit I have shown it by R. So, the current will be E divided by R this resistance may be the resistance of the conductor also. So, E by R will be the current at i 0 plus, at 0 plus velocity is 0.

And then we find that the current deduction through the conductor is this, therefore, I have to apply left hand rule, and I will get B i l that will be the force experienced by the conductor a b from left to right that is a electromagnetic force I will call it, generated because of the interaction of this b and i and l is the length of the conductor a b. It will start then moving as time passes.

Therefore, suppose at any time t, therefore, the conductor is expected to be accelerating ok, because there is a force which will be acting from left to right, and conductor will accelerate, you will get motoring action.

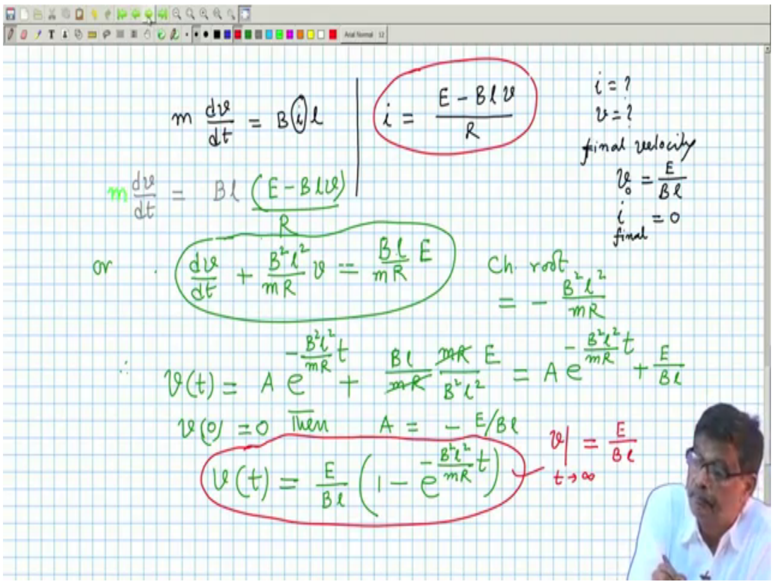
Now, the question is what will be the equation of motion of the conductor now; m dv dt if v is the velocity at any time t at this must be equal to the force. This time you see force is along the same direction of velocity so plus sign and this will be the dynamic equation on the mechanical side ok. Now, let us come to the electrical circuit equivalent electrical circuit here.

Now, one very interesting thing happens, the moment conductor starts moving we have just learnt that if a conductor moves in a magnetic field with some velocity v, there has to be an emf generated across a b across this conductor, therefore, at time t greater than 0, it is not only this external voltage, but there will appear another source of emf across a b which I have shown it by this battery and this voltage is often called back emf.

Therefore, at any time t therefore, the current in the circuit will be E minus B l v by R, and m dv dt is equal to B i l.

So, we got these two equation. So, we start now our lecture 55, we will start from this point. So, these are the two equations we have come across.

(Refer Slide Time: 11:09)



Therefore, it will be what we have got m dv dt is equal to B i l and the second equation is the current expression of the current which was equal to i is equal to supply voltage is E minus the back emf that is B l v and this divided by R, is it not, these are the two equations we have got.

So, our unknown is what is i and what is v as a function of time we want to find out. Before that physically let us see what is now going to happen, it will go on accelerating from left to right and suppose this is of infinite length not that everything finishes here, the this is rail continues infinitely long track.

Now, as it accelerates the back emf, so this B l v is going to rise is it not physically I am trying to examine what is what will be the final set of this thing, we will need to go on accelerating forever.

The answer is no, because of the fact this voltage will go on increasing and this voltage is fixed, supply voltage which is fixed. Therefore, a time will come when this voltage will become equal to this voltage.

And as you can see their polarities are such that they oppose each other, therefore, at that time I expect current will be 0 then is it not, current is E minus B l v by i.

But as it is accelerating, when it this back emf were reaches B l v, this current will vanish ok and then what happens, then no torque on this conductor, no force on this conductor B i l, i vanishes, therefore conductor will attain certain velocity final velocity which will make this voltage same as this voltage and after that no further acceleration and current will be 0; but will the conductor go on moving? Yes, it will, because we have assumed the frictionless track.

Therefore after you have closed the switch, it was stationary, it will accelerate, accelerate, accelerate, and it will attain such a velocity finally, which will make B l v is equal to E, or v is equal to E by B l. When that velocity will be attained current will be 0, and the conductor will be moving with that velocity indefinitely from left to right that is that from left to right. Will it violate any of the physical rule? No, because we know in a frictionless environment to move a thing, you do not require any force to be applied, is that clear?

Therefore, so the before I mean proceeding further, I can by physically examining the system can conclude the final velocity final velocity has to be some V naught say has to be E by B l, it has to be after a long time and final current E i final must be 0, this must happen and let us see indeed this thing happens or not. For that you have to solve this circuit, two unknowns are there.

So, suppose if I solve for velocity, so replace this i in this equation by this expression, and you will be getting here what that is m the m dv dt plus is equal to b l a I have chosen B l and i for i you put this one E minus B l v by R this is the thing.

So, now, in this equation only v is present, I can solve for v but I have to arrange the terms or I will write m dv dt, m dv dt this term plus this term containing v bring it to the left, and it will be B square l square by R into v if I miss a term point of term, this is the thing.

And on the right hand side, it will be B l by R into E, this will be the equation. Then what you do you divide by m both the sides, so that it will look now like this m m divide by m. So, it is a first order differential equation, first order differential equation. And characteristic root, in the same way characteristic root is equal to minus B square l square by m R this is the characteristic root. Therefore, velocity at any time t will be equal to A into e to the power minus B square l square by m R.

Student: plus forces.

Um.

Student: plus forces.

This plus the steady state solution which will be equal to this one B that is you suppressed this term dv dt whatever is v this divided by this is this one. So, B l by m R divided by B square l square by m R, m R by B square l square into E. It will be expected to be the velocity.

Hopefully, it is correct. If I am not correct wrong answer I will get, let us proceed ok. So, this a m R goes and you will be left with this one A into t as I said A into e to the power minus B square l square by m R into t and plus of E by B l. What is E? E is the supply voltage.

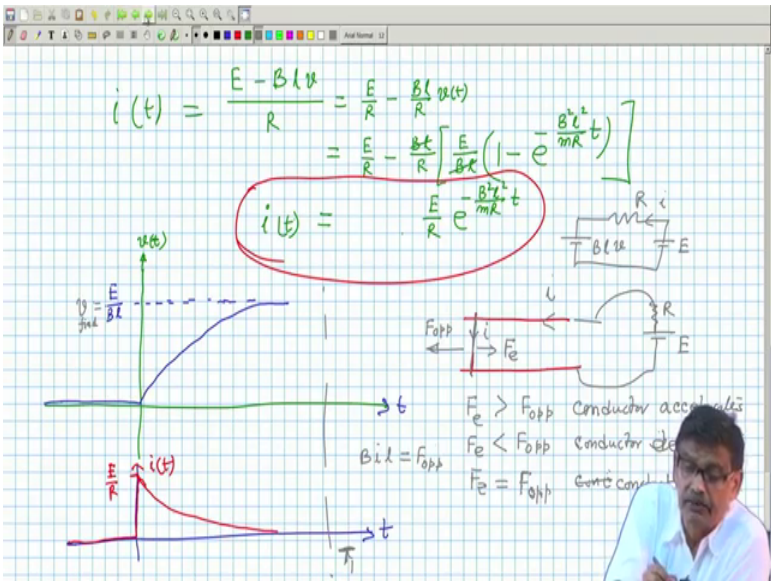
Now, what is the boundary condition? Boundary condition is velocity at time equal to 0 is 0, because it was stationary 0, v 0 minus was 0, v 0 plus is also 0, velocity of a mass cannot change in no time, so that was the argument. So, if you put that, then, then apply this boundary condition, and you will get A is equal to minus E by B l, is it not, put this condition here.

So, I will say v t, v t velocity at any time t will be equal to E by B l into 1 minus 1 minus e to the power minus B square l square by m R into t, it is something like RL circuit sort of thing. So, the this is the final solution for velocity.

Let us verify this. After a long time, after a long time that is velocity as t approaches infinity how much its value will be, this term will vanish E to the power minus t and you will be left with E by B l that is what we concluded physically, is it not, I should ignore that.

So, E by B l this is the thing, and it looks like I mean apparently. So, this is your velocity and how to get current, now put this value of v in this expression to get the expression for the current.

(Refer Slide Time: 22:11)



So, so current expression will be, so current i t will be equal to we got it here it is equal to E minus B l v by R, so E minus B l v by R, R, which is equal to E by R minus B l by R into this v t. So, v t we got it here this is the value of v t this expression E by v l into how to remember this E by v l method. So, so let us write it E by B l. So, this will be equal to E by R minus B l by R, then this velocity is E by B l into 1 minus E to the power minus.

Student: B square l square.

B square l square.

Student: by m R.

By m R into t, is it not, this one. So, you see this B l goes and you will be left with what E by R minus E by R cancels. So, finally, it will be E by R, E by R, this first term cancel plus of E by R into E to the power minus of this thing.

Student: (Refer Time: 24:14).

Um.

Student: E by (Refer Time: 24:16).

That cancels out. This E by R minus this E by R will cancel out, and you will be left with. So, this is the expression of i t, is it not, that is all. So, you know this is the expression of i t. Therefore, if you sketch the as a time how it looks like I will sketch first velocity. If I sketch velocity was 0 till t equal to 0, then it was rising exponentially like this, and finally, reaching this velocity. What was that velocity E by B l.

So, this is the final velocity E by B l ok. This blue curve is velocity and if you sketch, oh below this I will sketch the current. So, current wave form will be in this same this is time, this is time. So, before t equal to 0, current was 0. So, it was there 0. Then at t equal to 0 current is E by R, it shoots up to E by R and then it exponentially decreases, is not it, this is how this is the expression of the current.

Therefore you see in the motoring operation when you make this generator I will just draw a replica of that this was the thing, this was the conductor here and just in short to explain it here, this is was my conductor a b, oh my god, this was my conductor a b which was moving and here in the rail you have connected some resistance was there and some battery, then current was like this.

So, you start from rest, but eventually you attain a final velocity that is this conductor will store some energy which is equal to half m, this final velocity square. If I call that final velocity to be v final, it will be like this. But anyway the motor does not do any mechanical work, is it not.

Let us ask ourselves in this motoring operation that suppose you run a motor to supply is a mechanical load to overcome friction and so on, you imagine that no track is frictionless. Suppose, this conductor has to carry a load, of course, it is not possible to some mechanical load mass put on this, is it not, and suppose the track has got friction.

Therefore, you imagine it is moving train sort of thing on which I can dump load that is heavier things on this one and this track is not going to be in real life frictionless and if you put more weight on this mass, the friction to be overcome will be much more, is it not.

And therefore, this is the direction of the electromagnetic force we have seen. This is your direction of i that is phi. Now, I can easily say that if suppose to this movement of this conductor, I impose an opposing force, suppose some opposing force I impose on the system now after it has attained constant velocity.

Let us some opposing force comes in that will be the that is that is called the load force in rotating machine we say load torque, but load force which will be in opposite direction. Best way to think is that there is friction suppose now appears it has at least this one, but there is now you add friction to the system.

You know in general on the moving conductor, there will be two forces acting for the linear machines, one is the electromagnetic force which is providing the velocity and another is the force in the mechanical system it is electromagnetic force, and it is mechanical force in the opposite direction.

Whenever F e will be greater than f opposition, conductor will accelerate, must accelerate. If at any time F e is less than F opposition opposite force, conductor must decelerate and if F e if these two forces are equal, conductor will move with constant velocity, why I am scribbling, constant velocity.

Let us try to understand. So, it has reached a final velocity v naught, v final and then you suppose impose after it has reached the final velocity sum the conductor faces some opposite force to be overcome, it certain suddenly appears.

The moment it appears what was the electromagnetic force at that time when it has reached final velocity electromagnetic force developed by the conductor was 0. Suppose that that thing occurs at t equal to capital T 1 after long time some at some T 1 I have introduced opposing force.

So, at that time electromagnetic force was 0, because current was 0. So, there was no force from left to right, but I am telling I have imposed an opposite force to the motion of the conductor. The moment you have imposed that at that time velocity cannot change instantaneously mind you.

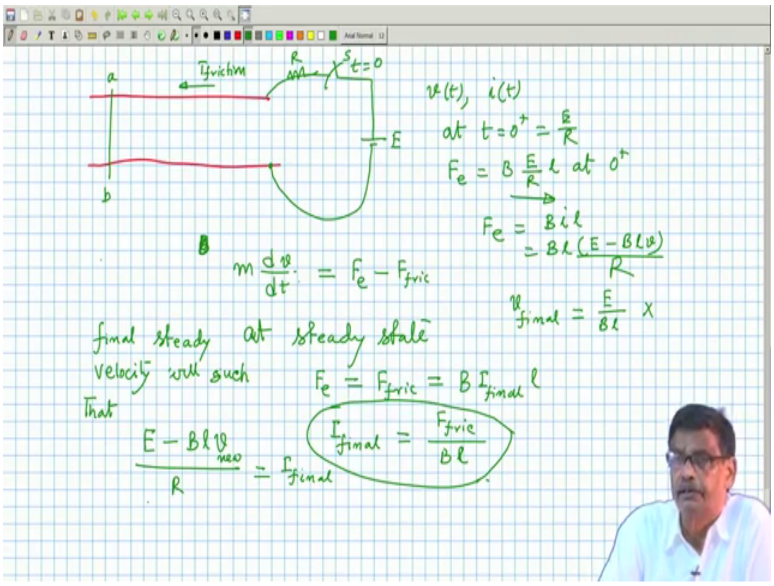
So, velocity at that time T 1 plus suppose capital T 1 plus velocity will still remain v final but the opposite force has appeared. Therefore, electromagnetic force which is 0 is less than F opposite, and conductor will then decelerate got the point conductor will decelerate but the moment conductor decelerates, this E will be greater than the B l v the opposite force, it remember the equivalent circuit is this, this is B l v, this is your R, and this is your the supply battery voltage which is constant.

Therefore, initially when it has reached final velocity, current was 0, this that fine. But if you put a opposite force coming onto the conductor, then it must decelerate, if velocity will start decreasing. If velocity stars decreasing, it will draw some current it will start because this electrical circuit E will be greater than B l v and it will draw current and the moment it draws current, there will be B i l, is it not?

So, what will be the final fate of the conductor? Finally, conductor will run at that speed which will make this B i l is equal to F opposition. At the as the speed decreases, it invites current from the source. If it which was till not absent will now appear and it will increase a time will come when F e will become F opposition but the conductor will run at a speed less than B final, finally it will settle down.

So, you have understood what I am telling. Therefore, so if somebody so as to say that in general for a practical system, I will say that you have a motor, motor mode of operation.

(Refer Slide Time: 36:03)



For example, in this case, this is the conductor. I will reframe the problem and leave it to you to solve it that ok. Here is your switch, here is the resistance R, and there if some battery you will connect to the track ok; and S, I will close at t equal to 0. But this time I am telling that the track is not frictionless, a constant friction force is acting.

So, there is a constant friction force in this direction, T friction, net T friction is this is present on the track you close this switch at t equal to 0.

Then I will once again write let me write that that at any time velocity is v t, current is i t, I want to know the solution, because there will be current at t equal to 0 which is equal to E by R. So, motion will start. What will be the force acting on the conductor at t equal to 0 plus, at t equal to 0 plus current is E by R.

Therefore, force electromagnetic force acting on the conductor is B E by R B i into l and its direction is from left to right. But now the equation of motion should be B and as it starts moving there will appear this back emf, therefore, in general F e, I will write it is at t equal to 0 plus.

So, there is some starting force acting F e at any time t will be B i l as we have seen and i at any time t is E minus B l v by R I am just writing very fast. So, this is the electromagnetic force, but your equation of motion now I should write it like this m d 2 v d t 2.

Student: First derivative.

I am sorry first derivative m dv dt acceleration, this will be equal to this will be equal to the net force acting from left to right which I should now write it like this F e minus F friction there is the mechanical opposite force. I am not going to solve it, but physically I am I am telling you what is going to happen to the final velocity and current.

Do you think the final velocity will be 0? A final velocity will be equal to still E by B l, in this case? The answer is no, because if it is E by l, E by B l then current drawn will be 0, there will be no electromagnetic torque present.

So, what will be the final velocity? Final velocity we will be talking, but before that we must understand if the system reaches steady state at that time at steady state F e must be equal to F friction. So, for this all this dynamic equation need not be solved. One can once again solve this and you will find that this has to be your F friction. So, electromagnetic force is the final I final into l.

So, I will be telling look your I final will be F friction divided by B l. This will be the final steady state current, which of course after solving this differential equations one must arrive at this you can try it.

And what will be the final velocity, how much will be the final velocity? Final velocity will be such final velocity final steady velocity will be such thus will be such that this that this I final current flows in the circuit; that means, E minus B l that new velocity divided by R must be equal to this I final.

See to find out the steady state current and speed one need not go through all these dynamic equation, of course, it is a suggested method of studying at least for one, so that you know there are situations when the motor or generator will run in steady state will be mostly interested in that some few cases of dynamic behavior of DC machines also we will discuss.

But to find out the steady state in DC things, what you have to do it if; it is; it will be running at constant speed, then the electromagnetic and mechanical forces which are acting in opposition they must be same, and electromagnetic force is B i l; from that I will be able to calculate I final and so on.

Anyway we will continue with this in the next class.