

Electrical Machines - I
Prof. Tapas Kumar Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 32
Equivalent Circuit of Auto Transformer

(Refer Slide Time: 00:19)

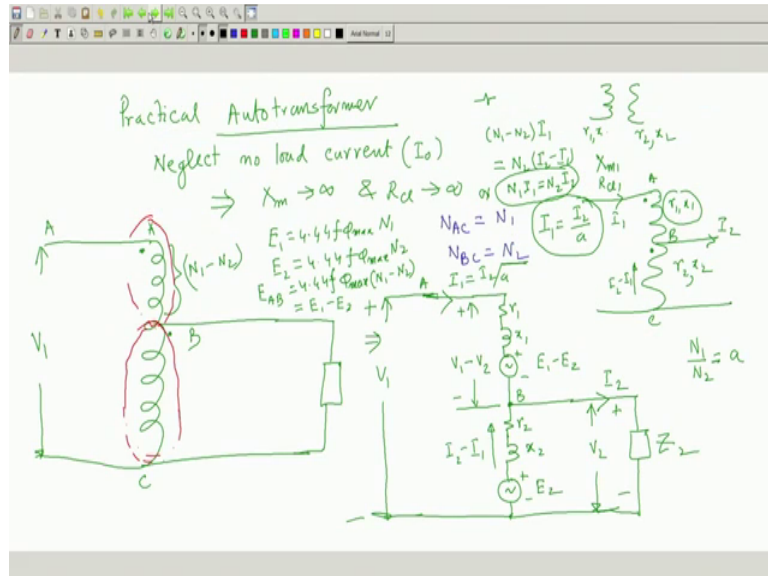
Equivalent ckt of a practical
Auto transformer

lec. 32

$$r_{e1} = r_1 + (a-1)^2 r_2$$
$$x_{e1} = x_1 + (a-1)^2 x_2$$

So, welcome to next lecture on Electrical Machines I. And we have been discussing about the Equivalent Circuit of a practical Auto Transformer. And this equivalent circuit when compared to a two windings transformer is somewhat difficult to obtain and that is why uh we were discussing it how to get it you recall from my earlier lecture.

(Refer Slide Time: 00:51)

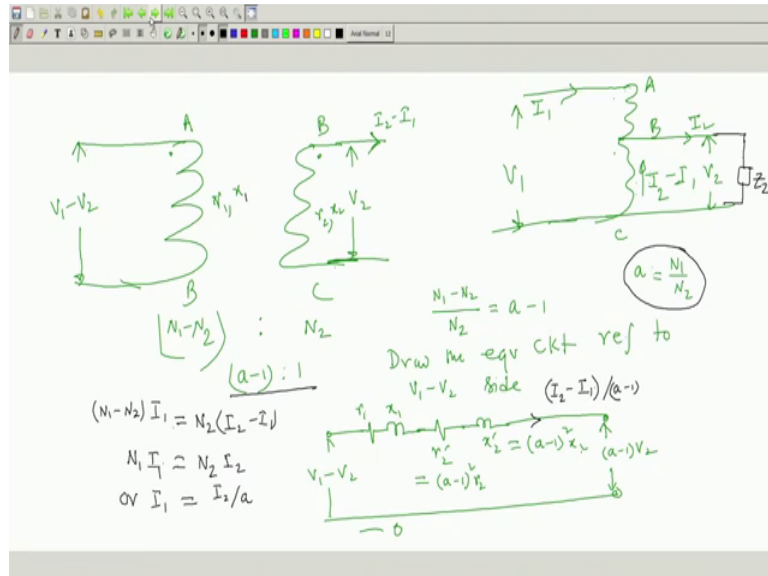


This was the actual auto transformer which is practical and I assume that this these are the visual current distribution I have assumed. Here is I 1 here is I 2. And this part A B which is having N 1 minus N 2 turns. And part B C these are two different coils they have nothing in common between them and this part suppose has got a resistance r 1 and leakage reactance x 1. And similarly this part B C has got a resistance r 2 and x 2 and we know the current distribution and z 2 is the load.

Now, without of course, having an equivalent circuit drawn, this circuit as such is also electrically connected is not by this point. So, it looks like a network problem that way it can be solved that is if you know V 2 if you know I 2 if you know these parameters, then B plus this current into r 2, x 2 these that loop two loops are there two measures these circuit can be handled. But only transformer business which will come in here to note that the voltage induced voltage E 1 minus E 2 here E 2 here this ratio of these voltage is N 1 minus N 2 by N 2 that is the thing, but it can be handled that way.

But we want to find out equivalent circuit and equivalent circuit of this transformer referred to say source side that is this side.

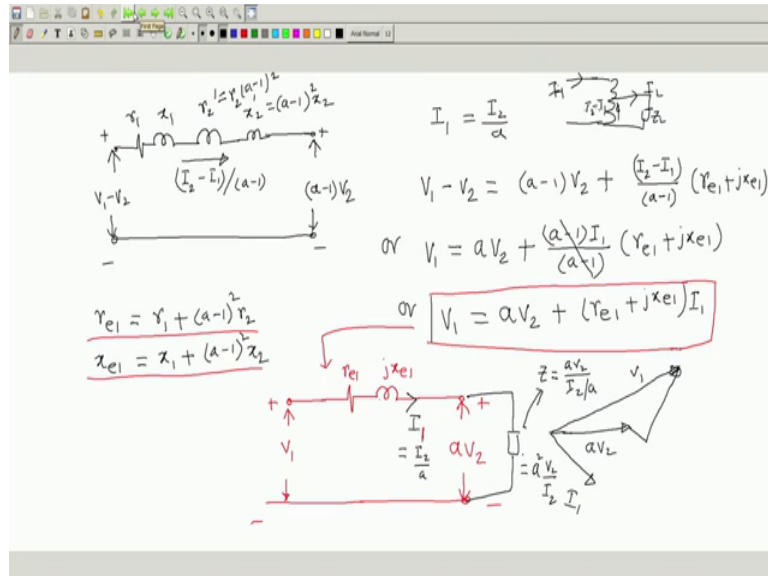
(Refer Slide Time: 02:51)



So, to do this next what we did is that portion A B and B C we noted in my last class is that this two windings are separate; this winding has a turns N_1 minus N_2 this winding has a turn N_2 only. Therefore, and this part is having r_1 , x_1 I am sorry this is r_1 x_1 this is having r_1 x_1 and this is r_2 x_2 ; only thing is the as if the voltage applied to this winding is V_1 minus V_2 and this side it is V_2 . And the current this winding is delivering is I_2 minus I_1 and not this I_2 mind this we understand. Therefore, between these two windings I can apply all the rules that I have applied for a two winding transformers having trans ratio $a - 1$ is to 1, what is a ? a is this N_1 by N_2 that we know.

Therefore, it looks like the equivalent circuit referred to V_1 minus V_2 side of these two coils will be like this, V_1 minus V_2 r_1 minus x_1 then these parameters will be referred to their trans ratio is this. So, $a - 1$ whole square into x_2 r_2 dash 2 will be a minus 1 whole square into r_2 and here you will get this actual voltage v_2 a times V_2 means in this case $a - 1$ times V_2 . And mind you this current here is not I_2 how much will be the current? In case of two winding transformer you know I_2 by a it comes, so it will be I_2 minus I_1 is the actual secondary current that divided by a minus 1 this will be the current. So, what we have got here that I will that will be the starting point here.

(Refer Slide Time: 05:37)



So, I will redraw it on a fresh page that is here is your r_1 x_1 was there, then the reflected value; r_2 x_2 dashed is here what is x_2 dashed? It is equal to a^{-1} whole square into x_2 this is how much actual resistance r_2 into a a^{-1} whole square is not this is the thing.

Magnetizing branch till now we have neglected that can be easily incorporated later. And this voltage was $V_1 - V_2$ and this voltage is how much a a^{-1} times, V_2 you see this voltage is a a^{-1} times 0 this diagram only I have redrawn there afresh so this is the thing.

Now, in this equivalent circuit this plus minus visual plus minus visual and this current is $I_2 - I_1$ divided by a^{-1} . But we have seen that I_1 is equal to also I_2 by a that we have already established is not from this MMF balance recall this we will always use. See the MMF of this portion this portion I will do it here in a this MMF in this portion MMF balance, $N_1 - N_2$ $N_2 I_1$ is same as N_2 into $I_2 - I_1$ this is the fundamental thing from this $N_2 I_1$ gets cancelled and you are left with $N_1 I_1$ is equal to $N_2 I_2$. Or I can say that I_1 can be also written as I_2 by a ; a is N_1 divided by N_2 .

So, knowing these what is our goal? Goal is to replace this equivalent circuit similar to that of the equivalent circuit of a two winding transformer, that is my input voltage V_1 should be present here alone load site there should be V_2 alone present, current some I_2 dash sort of thing should be present things like that we are trying to do.

Now, what is the importance of doing this? If you do this way then this equation by applying $k \cdot v \cdot l$ rule I can write down $V_1 - V_2$ this voltage is equal to $a \cdot I_1 - I_2$ plus of this current that is $I_2 - I_1$ divided by a minus 1 this current into this whole impedance, if I say that r_1 , if I define as r_1 plus a minus 1 whole square $\times I_1 - I_2$ as $\times I_1$ plus a minus 1 whole square $\times I_2$.

If I combine them, then this can be written as this voltage plus the drop here that is a current into r_1 plus jx_1 this is the thing I can write. And from this if you move this V_2 to this side, then we will see it is equal to V_1 is equal to $a \cdot V_2$ very good, plus I want to have I_1 here in terms of I_1

So, what is I_2 ? I_2 is $a \cdot I_1$, so it will be $a \cdot I_1$ divided by a minus 1 . And this is your r_1 plus jx_1 this will cancel out. And now I am happy ok, I have got an equation which is really we are looking forward that is after getting this equation this equation, I will say that this equation prompts me to draw the equivalent circuit referred to primary from this equation. As if a voltage is there V_1 this side and here is the impedance r_2 , r_1 , jx_1 where the description of r_1 is this, description of x_1 is this and then here is I am having a V_2 and this into I_1 mind you I forgot to write that into I_1 this into in this expression let me put it properly, into I_1 here it is into I_1 is not.

So, this current is I_1 is not this equation suggests that, that this is I_1 therefore, $a \cdot V_2$ plus the drops gives you V_1 in the same way as we draw the equivalent circuit referred to primary of a two winding transformer. But there is an important difference what is that important difference ok, terminal voltage gets multiplied by $a \cdot V_2$ fine, but this r_2 I think I made a made a basic mistake here this should be this should be r_1 plus r_2 not $a \cdot 1$ minus this is r_2 .

And similarly x_1 is x_1 plus x_2 please correct that only basic differences the r_2 and x_2 are to be multiplied by not x square, but a minus 1 whole square got the point, so this is the thing. Then that one has to remember at least with refer to primary if you draw then the equivalent circuits will be like this. And then after that everything is fine that you can draw the phasor diagram start with a V_2 here a is a scalar number this is also direction of V_2 then draw I_1 and then draw this drops at and you get V_1 .

So, this is how the equivalent circuit of an auto transformer can be drawn or obtained; and to do that the method there are several other treatment people write lot of equations

try to prove it. But what I am telling this can be very quickly drawn provided you apply the rules of two winding transformer for the windings A B and B C because they are separate winding and. So, as if I say that across AB V_1 and V_2 is applied this one, but only thing is its trans ratio is this therefore, it is to be multiplied these voltage will come like that.

And then, defining this r_1 plus a minus 1 whole square r_2 etcetera as r_1 and so on. You will come here then, from this you write down the $k v_1$ equation of this and that minus V_2 ; V_2 cancels there from this side and soon I find o this is the equation this is the reflected current and so on. So, with respect to primary this is the equivalent circuit. In the same way that is from this equation only if you wish that I am not doing mind you. I will be able to draw, the equivalent circuit referred to the load side also got the point I will be getting the equivalent circuit referred to the load side as well.

Starting from this basic equation here, I will be able to do that clear, but that I leave it to you to ponder over, but better do not waste too much time on that is also true because one side you have got that is fine only one point in this diagram. This I_1 also see I_1 is also equal to I_2 by a is it not I_2 by a , I_1 is equal to I_2 by a .

What do you think the impedance connected here it will be this voltage $a V_2$ by I_2 by a this point is worth mentioning impedance here, load impedance I will at least from this circuit it tells me it is a V_2 divided by I_2 by a , which is equal to a square V_2 by I_2 and what is V_2 by I_2 V_2 by I_2 here it is Z_2 , you know V_2 by I_2 is Z_2 mind you, this current these two isolated current is I_2 minus I_1 and I_1 they must balance the MMF the rules of two winding transformer follows.

But I_2 minus I_1 is not your load current that is why that complication comes in, that is if I say across the output terminals of the auto transformer that is here if I draw it to highlight that; if it is Z_2 , this is I_2 and this is I_2 minus I_1 I_2 minus I_1 and this is I_1 is it? Therefore, the load impedance will be multiplied by a square into a square because of the fact I_2 is not part of this current got the point therefore, this point should be remembered.

So, auto transformer equivalent circuit if you want to draw is slightly difficult means if you are conceptually ok, you can easily develop, but one should be careful. The winding leakage impedance will be multiplied by a minus 1 whole square, but the load impedance

should be multiplied by a square where a is N_1 by N_2 is not. So, the that is all I mean equivalent circuit we have completed.

But only thing is I will just now tell you now suppose, I say that in this equivalent circuit you have not talked about the no load current. In any case I will consider the approximate equivalent circuit therefore, the magnetising current and core loss component of resistance that is the loss component of current; can be shown to be connected right across the supply terminals as two components; one is R_{cl1} , another is X_{m1} . For example, here I will connect R_{cl1} and X_{m1} got the point.

So, if I draw it here that I can always do. So, equivalent circuit of an auto transformer, in fact, a lecture 32 I have written here we were at on the verge of completion last lecture itself, but anyway. So, this will be then V_1 re x_1 and this is V_2 , V_2 or a V_2 we got a V_2 I think a V_2 . And the load impedance whatever was connected will be multiplied by a square Z^2 and this re 1 do not forget it is r_1 plus a minus 1 whole squared $r_2 x_1$ is equal to x_1 plus a minus 1 whole squared into x_2 that is it.

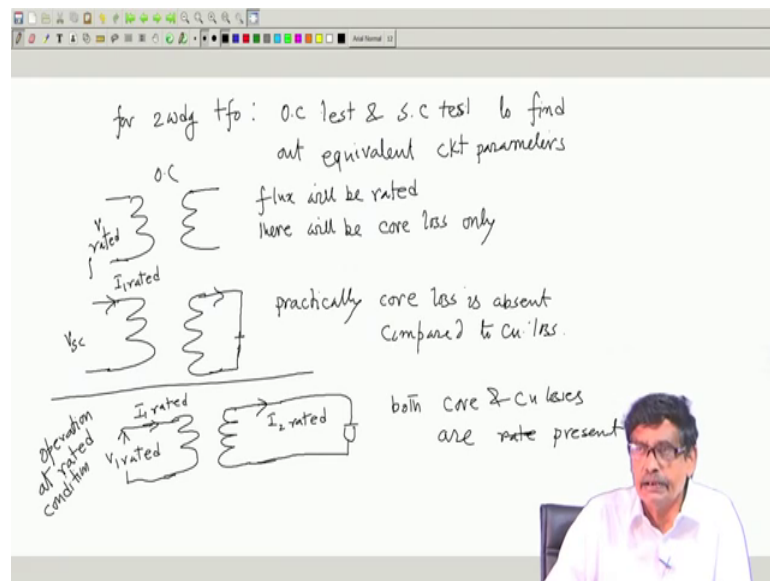
And if you consider this impedance, refer to this side show it here supply side as X_{m1} and R_{cl1} like the two winding transformer and this is I_1 is the current here clear. So, I_1 then I should tell this one perhaps I will tell some reflected current of these primary side that is, I_2 dashed and once again I_2 dashed is not I_2 by a minus 1 it is I_2 by a.

I think we have spent enough time go through it and try to solve some problems on this auto transformer equivalent circuit. We will also give you some problems in the tutorial or in the hand lecture notes some solve problem, I will try my best to include that, but it is very interesting to study the autotransformer; because autotransformers are all both the two winding transformer and autotransformers are used.

Although you can change the level of voltage from one level to another, for any ratios theoretically it is possible, but there are some advantages of auto autotransformer while the voltage ratios are not deferring too much. Then you choose auto transformer, then also isolation problem we can easily see is not that urgent at the same time economically it will be much attractive. And two winding transformer on the other hand gives you the isolation where it is needed.

[FL] After completing this, our next topic will be a 3 phase transformers. But before 3 phase transformer one small thing I would like to tell you a tidbits of see two winding transformer for example, about one test I will just tell. We have talked about open circuit and short circuit test on a two winding transformer mind you, in auto transformer I am not going to those tests, but if this equivalent circuit is with you can easily find out these parameters by doing similar test this.

(Refer Slide Time: 26:30)



But what I am telling is, for 2 winding transformer you recall for 2 winding transformer, we have done open circuit test and short circuit test, to find out parameters equivalent circuit parameters to find out that we have done. And I told you during open circuit test you recall you apply rated voltage V_1 rated at rated frequency rated frequency secondary is open.

So, for this test open circuit test when you connect like that flux will be rated flux will be rated and there will be core loss only core loss only; because the copper loss is negligibly small secondary winding is not carrying any current and primary current is only two to five percent. So, only core loss takes place during open circuit test.

On the other hand during short circuit test, you keep the secondary shorted, here you apply not rated voltage such that rated current flows current is rated here I_1 rated; since applied voltage is pretty small and flux in the core is directly proportional to the applied voltage. So, flux will be very less, but current will be rated. So, in this test what happens

is this practically core loss is can be assumed to be 0 cold loss is absent, as if is absent compared to copper loss all the losses will be copper loss to copper loss ok. And when the transformer will be in use this is during test these I told, but I am once again perhaps repeating, but it is better to repeat keep these points in mind.

When the transformer will be in operation it will be V 1 rated here also and the windings will carry rated current both the windings at full load condition, is not here load is connected rated voltage. And secondary is not silent it is supplying power therefore, during operation this is during operation at rated condition operation at rated condition both the losses are present, both core and copper losses are rated are rated values are rated are present you cannot neglect one from the other that is fine.

Therefore, during open circuit test and short circuit test when you are finding out the parameter values, the temperature rise during this test or that test will never be close to the temperature rise of a transformer which will be put to use at rated condition. Because here both the losses are present temperature rise will be much higher than either of this test and that test. We will continue discussing on a very simple test, I will just mention because that test is so nice and this will also enhance our understanding of transformers. So, I will discuss about sampler test briefly in the next class and then starts topics on 3 phase transformer.

Thank you.