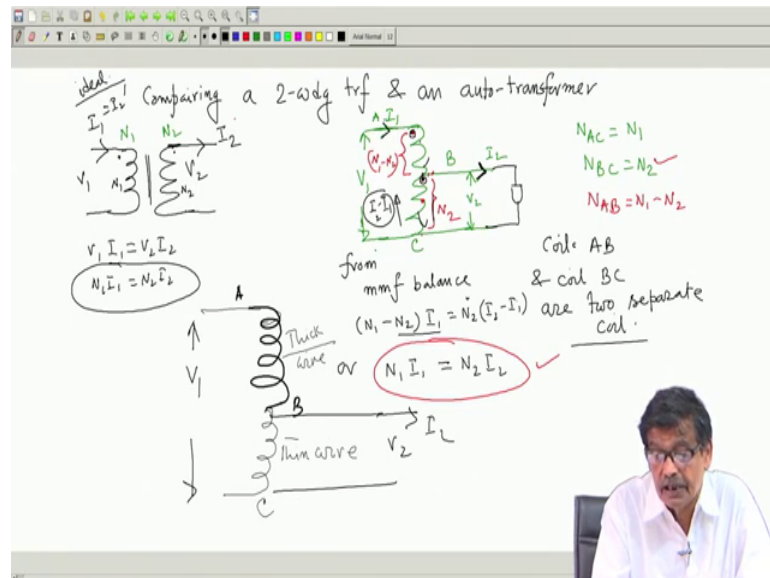


**Electrical Machines - I**  
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**Lecture - 28**  
**Auto Transformer Versus Two Winding Transformer ( Contd.)**

(Refer Slide Time: 00:23)



Welcome to lecture number 28. And we were discussing about Auto Transformer ok. Remember in our last class, so with the help of an autotransformer it looks like I can change the voltage from one level to some desired level of voltage as we have done with the help of it two winding transformer and I could transmit a certain amount of kv here. Therefore, I now have two options. Suppose, I want to change the voltage level from V 1 to V 2 and it will deliver a kv of V 2 I 2 or the transformer will handle a kv of V 1 I 1 or V 2 I 2, they are same. Same thing can be done with an auto transformer.

The moment you have two options, therefore which one to choose if I want to change the level of voltage from one value to another, whether should I go for a for whether should I select it two winding transformer to do this or an auto transformer to do this? So, after looking at these things, what I have assumed, they are same kva rating N 1 turns are same, so that flux level in the core will be same, so they are that way similar.

And then we thought, but here you note the current supplied to the load will be I 2, because what happens in an auto transformer there is a common portion of the winding

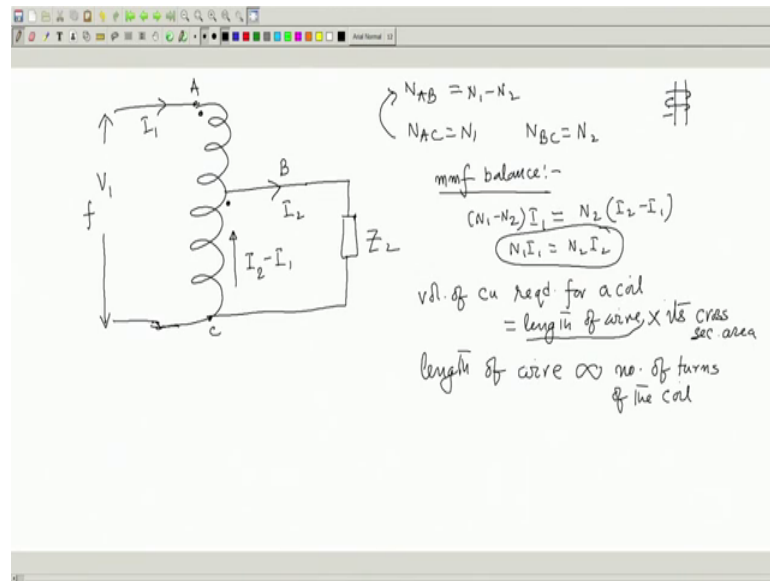
and this is dot, this is also dot here. Therefore, this current should be  $I_2$ . So, we must bring them at this same platform before comparing, so same kva rating, same voltage ratios and so on.

But looking carefully here we note that if this current is  $I_2$  and this current is  $I_1$  that is what it should do like this, then current in the common portion BC is the difference of this two current. And the currents as you know are in phase ideal transformer I am considering, no load that negligible, no load current let us neglect to understand the major thing in an auto transformer, how it distinguishes it from a two winding transformer is that ok,  $I_2$  current then  $I_1$  current must be drawn in and therefore this is  $I_2$  minus  $I_1$ .

Then I know that here once again I have applied voltage  $V_1$  at some frequency, therefore flux level in the core will remain same. And then we argued that this portion of the winding AB and BC this two coils are two separate coils, no common part in between them. Therefore mmf produced by this coil and this coil must balance, through the dot if  $I_2$  minus  $I_1$  is coming out and through the dot if  $I_1$  is imputed in the upper portion of the coil, this two mmf must balance ok. And that thing from this one also it is interesting to note we get that  $N_1 I_1$  equal to  $N_2 I_2$  that is there.

Then I argued without doing any mathematics perhaps this current will be much less difference of these two current  $I_2$  minus  $I_1$  going up and therefore, a thinner coil can be used compared to its two winding counterpart. And therefore, you can save some copper that is what I told. Now, today, we will try to find out some estimate that how much copper will be saved, copper segments, volume of the copper ok. So, we start with this.

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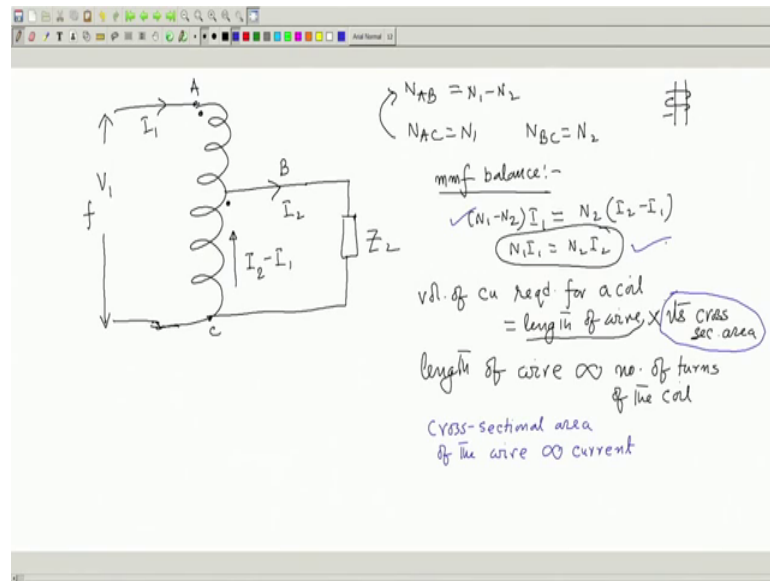


So, I draw the transformer once again. So, this is the auto transformer which step downs the voltage  $V_1$  AC voltage. This point I call A, this point is C and here is the tapping B, and here is the load connected get to secondary load. And this was the current distribution  $I_2$ , this was  $I_1$  and then this current was  $I_2$  minus  $I_1$ . This number of turns  $N_{AB}$   $N_{AC}$  is equal to  $N_1$  this total number of turns  $N_{BC}$  is equal to  $N_2$  turns.

And from these two, we get  $N_{AB}$  to be equal to  $N_1$  minus  $N_2$ . And I told you that these are the dots here. Therefore, if  $I_2$  minus  $I_1$  leaves there,  $I_1$  comes in, so mmf balance. This is what we did last time mmf balance. Keeps me  $N_1$  minus  $N_2$  into  $I_1$  is equal to here the number of turns is  $N_2$   $N_2$  into  $I_2$  minus  $I_1$ , these are all phasors you know. So,  $N_1 I_1$  is equal to  $N_2 I_2$ . This is the thing.

At a volume of copper required for a coil, volume of copper required for a coil, required for a coil will be simply the length of wire, length of wire into its cross sectional area, wire cross sectional area, is it not, very pretty simple, area into length gives you volume. Now, this length of wire; length of wire is proportional to the number of turns of the coil; number of turns of the coil, is it not? This is the core each turn. So, for a given core size, this thing number of turns would be proportional to length because each turn has got a fixed perimeter, therefore, number of turns of the coil it will be equal to 1.

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So, now what I will be doing is this, if this is the case then let me go to the next slide. I will draw the two-winding transformer of same rating  $V_1$ , here it is  $I_1$ , here it is  $V_2$ , here the load current will be  $I_2$ , this is the thing. And here it is your auto transformer, where this is the load same load  $Z_2$ , same load  $Z_2$  here. So, that same current flows  $I_2$ ,  $I_2$ . And here this point is A, C, B,  $N_{AC}$ , let me repeat I mean  $N_1$ ,  $N_{BC}$  is  $N_2$  and  $N_{AB}$  is  $N_1$  minus  $N_2$  that is there. And this is  $I_2$ , this is  $I_1$  and this is  $I_2$  minus  $I_1$ .

Now, what I will be doing is this, I will find try to find out these ratios. Here it is  $N_1$  here it is  $N_2$ , because voltage per turn is  $V_1$  by  $N_1$  and output voltage will be  $V_1$  by  $N_1$  into  $N_2$ . Here voltage per turn is once again  $V_1$  by  $N_1$ , and this turn  $N_{BC}$  is  $N_2$ . So,  $V_1$  by  $N_1$  into  $N_2$  gives you this voltage  $V_2$ , got the point. Now, what I will do, so this two transformers are of same rating. If this two are put in a black box only two terminals are available, two terminals are available, you will not be able to distinguish whether a two-winding transformer is doing the job or an auto transformer is doing the job, so far as the output terminals are concerned ok.

Now, what I will do is this, I will find out the ratio of these two quantities copper required in auto transformer. I am writing it short divided by total copper required in two winding transformer, this ratio I will try to find out ok. In the previous slide, I have told you length of the wire is proportional to the number of turns of the coil. What about cross sectional area?

Cross sectional area, let me write here, cross sectional area of the wire cross section which is forming the coil cross sectional area of the wire will be proportional to the magnitude of the current because magnitude of the current based on that only thicker or thinner wires are chosen, is it not.

So, cross sectional area of the wire is proportional to the current it is carrying based on that I have decided upon the cross sectional area. So, cross sectional area is proportional to the current. And we are talking about rated current. At rated current, based on that the cross sectional areas are chosen.

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$$\frac{\text{Vol of Cu reqd. in Auto tfo}}{\text{Vol of Cu reqd. in 2wdg tfo}} = \frac{N_2(I_2 - I_1) + (N_1 - N_2)I_1}{N_2I_2 + N_1I_1}$$

$$= \frac{N_2I_2 - N_2I_1 + N_1I_1 - N_2I_1}{N_1I_1 + N_2I_2} = \frac{2N_1I_1 - 2N_2I_1}{2N_1I_1}$$

$$= \frac{N_1 - N_2}{N_1} = 1 - \frac{N_2}{N_1} = \left(1 - \frac{1}{a}\right)$$

for step down case  $N_1 > N_2$   
 $a > 1$   
 or  $\frac{1}{a} < 1$   
 Suppose  $\frac{N_1}{N_2} = 2 = a$  then  $1 - \frac{1}{2} = 0.5$   
 Cu reqd. in auto =  $(1 - \frac{1}{2}) \times$  Cu reqd. in 2wdg tfo  
 $= 0.5 \times$  Cu reqd. in 2wdg tfo

$a = \frac{4}{3}$   
 $1 - \frac{3}{4} = 1 - 0.75 = 0.25$

So, it looks like copper required in autotransformer rather volume of copper required, volume of volume of copper required, volume divided by volume of copper required in two winding transformer I am trying to finding. Now, what is the number of turns, length of these wire will be proportional to N BC, is it not and N BC is N 2 into the cross sectional area proportional to the current.

What current this coil is carrying, magnitude of the current I 2 minus I 1, I 2 minus I 1, it is this portion plus this portion plus what is the length of the copper required in portion N AB, number of turns existing between A and B terminals that is N 1 minus N 2, so N 1 minus N 2. And cross sectional area of this portion should be decided by the amount of current discarrying that is the I 1, all these currents are rated values corresponding to that. So, this will be this thing.

Now, because these two transformers are doing the same job come here. Now, here I will write volume of copper required for this portion of the winding will be  $N_2$  into  $I_2$ , volume of copper required for this portion is  $N_1$  into  $I_1$ , that is all. Now, we have already established that  $N_1 I_1$  is equal to  $N_2 I_2$ . Is  $N_1 N_2$  is different in the autotransformers? No, same, same, same thing,  $N_1$ , whatever is  $N_1$  here,  $N_1$  here.  $I_1$ ,  $I_2$  is different? No,  $I_2$  is the current delivered to the load;  $I_1$  is the current flowing in the primary same thing  $I_1$ ,  $I_2$ . So, those things are there.

But we have shown in case of a two winding transfer these already known. In case of auto transformer also we have shown that it so happen that  $N_1 I_1$  is also equal to  $N_2 I_2$ . Although I should not say that  $N_1 I_1$  is the mmf like that which portion I do not know, but I found out very meticulously this relationship is not from this  $N_1$  minus  $N_2$  into  $I_1$  is equal to  $N_2$  into  $I_2$  minus  $I_1$  from that also something was obtained.

Therefore, it looks like  $N_1 I_1$  is equal to  $N_2 I_2$  is also true ok. Then and that constant of proportionality will be same that will cancel out, this proportional mind you and I am taking the ratios. Then what you do is this, the numerator is  $N_2 I_2$  minus  $N_2 I_1$  just open the brackets plus  $N_1 I_1$  and this will be minus  $N_2$  into  $I_1$  and below it is  $N_1 I_1$  plus  $N_2 I_2$  this is the thing, but  $N_1 I_1$  is equal to  $N_2 I_2$  from both this. So, these two can be combined and simply written as  $2 N_1 I_1$  is it not. And this is minus  $2 N_2 I_1$  that is this term and this term. And below it is  $2 N_1 I_1$  I can write is it not?

And therefore, this  $I_1$  business will come out. So, it will become equal to  $N_1$  minus  $N_2$  by  $N_1$  simply which is equal to  $1$  minus  $N_2$  by  $N_1$ . And we were writing  $a$  is equal to  $N_1$  by  $N_2$ . If that is the case, then I will write it as  $1$  minus  $1$  by  $a$ . So, copper required in auto transformer will be this times the copper required in it two winding transformer if this fraction is less than  $1$ . Then there will be savings of copper. And mind you I am considering the step down case.

So, value of  $a$ ; value of  $a$  for step down case  $N_1$  is greater than  $N_2$ , which means that  $a$  is greater than  $1$  or  $1$  by  $a$  is less than  $1$  is not this  $1$  by  $a$  will be a number which is less than  $1$  fraction. So, suppose I say that  $a$  is that is  $N_1$  by  $N_2$  suppose, suppose I say that  $N_1$  by  $N_2$  is equal to say  $2$ , suppose I say then for  $a$  is equal to  $a$ , then copper required in autotransformer will be equal to  $1$  minus half  $1$  by  $a$  is half into copper required in two winding transformer that is  $0.5$  into copper required in two winding transformer.

So, if you require say 10 kg of copper to make it two-winding transformer, you will be requiring only 5 kg of copper to make a auto transformer is it not. You save copper it looks like savings of copper can be easily calculated by taking the difference of these two I am not going to do that it can be easily done. But I will just see how much copper is required in terms of how much copper is required in a two-winding transformer, I find for step down case if turns ratio is 2, it is  $N_1$  is to  $N_2$  is 2 you are stepping down the voltage. Here also you are stepping down the voltage.

Then I find oh my god you will get advantage here because if you require 10 kg copper, there you will be requiring half of that copper required 5 kg copper. So, weight of the transformer will be less initial investment will be less and so on. And this auto transformer will do the same job as this two winding transformer is doing. And it looks like then that no matter what is the ratio if it is greater than 1,  $1/a$  is always to be a number less than 1. And you will be always saving copper, getting?

For example, I want to suppose I say that  $a$  is equal to 4 by 3, suppose I say,  $a$  is equal to 4 by 3, then what will be the savings of copper  $1 - 1/a$  by 3 by 4, how much copper will be required?  $1 - 0.75$ , only one-fourth of the copper will be required. See if you want to change the voltage ratios, in this ratio  $N_1$  is  $N_1$  is to  $N_2$  is 4 is to 3, then I find savings of copper is much more, because copper required will be only one-fourth, it looks like that is it depends on  $a$ , this is the crucial equation copper required in a two winding transformer.

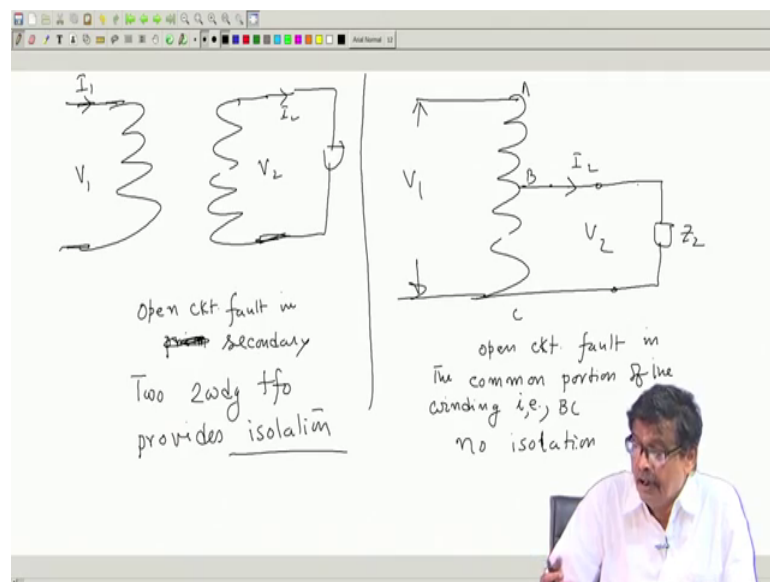
But in any case no matter what is the value of  $a$ , if you are stepping it down, it looks like it will be saving copper. Savings of copper will be more if  $a$  approaches 1. If you decrease the value of  $a$ , savings nonetheless will take place, but at not at high rate as it was doing for  $a$  equal to 4 by 3. So, we understand now that in an autotransformer savings of copper will always take place. Whether it is more or less that depends on the ratio. If  $a$  approaches 1, savings of copper will be more and more; lesser and lesser copper will be required.

So, if that be the case, then we understand then wherever you require a transformation of voltage, two candidates are there you can choose a two-winding transformer or an auto transformer, it then looks like always go for auto transformer, because less copper will be required. Whether that less is really less or not that is secondary issue, but

mathematically we note that savings will always take less, no matter what is the value of  $a$ . What is the value of  $a$ ?  $A$  is greater than 1 here, is it not?

Therefore, it looks like savings of copper will be there in an auto transformer of same rating compared to a two winding, same kva it will transmit, same voltage ratios. Therefore, this exercise then prompts me look whenever you want to use a transformer, out of these two always go for auto transformer because savings are there that is what we get. But the story is not completed, if that would have been the case, then two winding transformer would not have existed is it not, no one would have used two winding transformer, but that is not the case. There are other issues we have to consider.

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For example, I will tell you that suppose you have an two-winding transformer,  $V_1$   $V_2$  you are getting. And a similar rated auto transformer where  $V_1$  and  $V_2$  you are getting that is fine. A two winding transformer, suppose let us consider a situation, it is supplying load ok, it is supplying load, connected rated current it is supplying that is what I would like to operate the transformer because I have purchased a transformer of given kva let it handled given kva why operate it under no load.

Of course, there are situations where things are not under your control like a distribution transformer, load varies we have talked about that. But here let us see suppose they are delivering rated current primary current is also drawn rated, here also doing the same



thing, here it is  $V_1$  like that. And here the supplying load everything is fine, same job is being done two winding and auto transformer.

Let us imagine that an open circuit fault has taken place open circuit fault in the primary. Primary or secondary, say on the secondary, open circuit fault in secondary coil. What has happened is this, you do not know a fault has occurred which creates an open circuit here, excessive current was flowing because of some reason and winding has become opened and which you cannot see from outside. Primary remains energized. Will I get any voltage between these two points? No, because the moment it becomes open circuit current will vanish current cannot flow. But there will be voltage induced in this portion and in this portion, but voltage between these two points will be 0 as there is no current flowing.

Therefore, what will happen if a open circuit takes place here, your load will suffer, it will not get any voltage now, no current flows through the load that is everything on the secondary side will not get power. There this story ends. Let us come here suppose it is doing like this, but what happens is this a open circuit fault takes place in the common portion of the winding that is BC portion B C A. Suppose, open circuit fault in the winding open circuit fault inside the transformer, open circuit fault in the common portion of BC in the common portion common portion of the winding that is BC, in portion BC, a open circuit has taken place.

What happens to this current this time we find that current can flow  $V_1$  is there through this current will flow which was not possible here, and not only that this voltage is no longer it is  $N_2$  by  $N_1$ . This voltage minus this drop. And if this point is closer to this point this voltage will become of same level as that of this one is it not? Therefore, across the load perhaps the voltage will go up which is not in your control via this connection.

And there will be problem on the load if you apply high voltage. Just because of what, because some open circuit fault has taken place which is not in your hand, it may happen in a two-winding transformer as well as in an auto transformer. Then what happens by this ohmic contact in this path, there may be higher voltage coming of the order of  $V_1$  may creep across may come across  $Z_2$ . And if it is a critical load your load goes which requires a more or less constant voltage of level  $V_2$ , suddenly we will see oh  $V_2$  has

gone up. Trying to go up to  $V_1$  via this path and things will be, and this problem is called isolation.

So, an auto transformer does not provide isolation here if such a thing happens it is isolated from the primary. So, isolation to this is most biggest point two-winding transformer provides isolation. Here no isolation. In case a open circuit fault takes place in the common portion of the primary and secondary, therefore autotransformer gives you copper saving no doubt, but at the same time it will never be able to give you a proper isolation across the source side between the source side and the load side, load may be critical you cannot just apply high voltages.

We will continue with this next class. And this is very important and interesting I mean how to choose auto transform, we will continue it is not yet concluded.

Thank you so much.