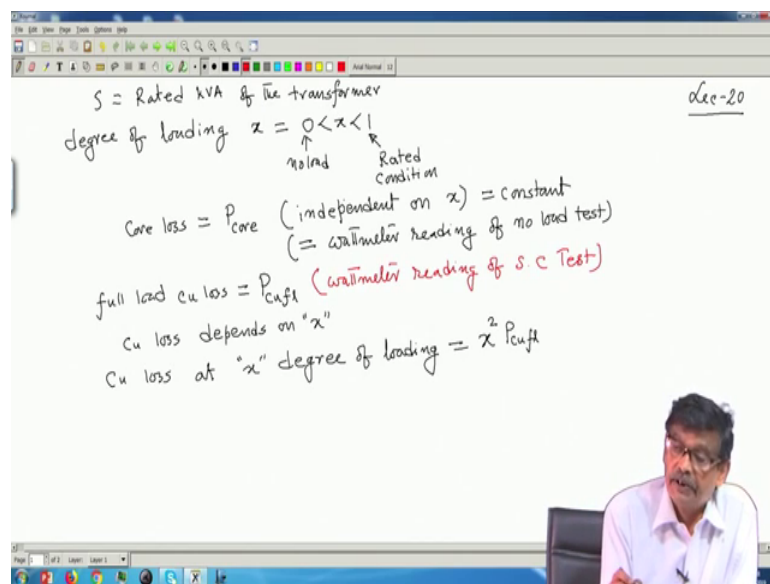


Electrical Machines - 1
Prof. Tapas Kumar Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 20
Efficiency (Contd.)

Welcome to lecture number 20 on Electrical Machines I. And as you know we were discussing about how to estimate the Efficiency of a transformer.

(Refer Slide Time: 00:40)



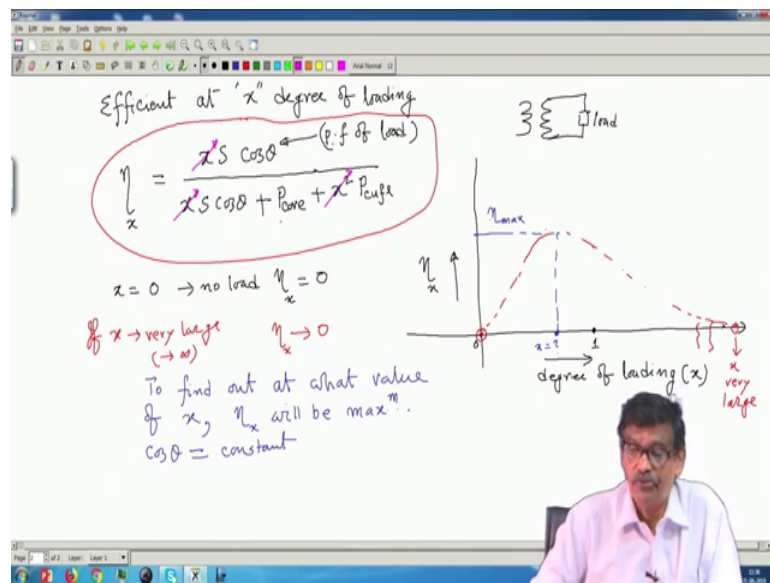
And you recall that I assumed the S to be the rated k VA of the transformer; rated k VA of the transformer and then I defined a factor degree of loading, I tried to represent by a factor x , which whose value normally will be between 0 to 1, 0 corresponds to no load and 1 corresponds to rated condition; rated condition, what is rated condition? Rated condition is rated voltage applied and also windings are carrying rated current ok. Then I explained that there are two main losses in a transformer; one is the core loss core loss and this I denoted it by P_{core} and this is independent on x ; degree of loading that is it is constant, no matter whether no load or full load.

And this P_{core} this can be obtained; this is also equal to the wattmeter reading; wattmeter reading of no load test, so this value is known. And then I defined the full load copper loss, full load copper loss; as I used this notation P_{cufl} ; and this

depends on x; no sorry. Full load copper loss is a constant theme; at full load current when the coils are carrying full load current, what is the total copper loss?

So, this is constant; full load copper loss and then I told you that this is fine P C u full load, then I told you copper loss depends on x the degree of loading. And therefore, copper loss at x degree of loading; at x degree of loading in terms of this P C u full load will be simply x square into P C u full load. So, this must be kept in mind while attempting to calculate the efficiency of a transformer.

(Refer Slide Time: 04:53)



Then efficiency at x degree of loading, will be I denote it by the this x suffix indicates that at x degree of loading, it will be equal to output kilowatt by input kilowatt; what is the output kilowatt? At x degree of loading, the output k VA is excess cos theta; what is this cos theta? Power factor of the load, of load that is; this is the actual practical transformer, this is your load its power factor angle theta. And this is the output kilowatt and input kilowatt will be output plus the losses.

What are the two losses? One is the fix loss P core, plus x square into P copper loss at full load. This is the expression for the efficiency, very important expression. Therefore, I can calculate the efficiency of a transformer at any degree of loading and at a given power factor of the load because this P core is known to me from the open circuit test.

And P_{Cu} full load is also known to me from the short circuit test, because this P_{Cu} full load; here I write is the wattmeter reading; reading of short circuit test got the point? Therefore, this is the thing; therefore, I will be able to suppose I want to sketch the efficiency versus degree of loading, suppose I want to sketch. Here, I will write degree of loading, loading that is x and here I will plot the efficiency; how efficiency changes, as you change x .

What is the domain of x ? It can be under no load condition, as I told you x equal to 0 corresponds to no load, x equal to 1 corresponds to rated condition that is coils are carrying rated current. Therefore, it looks like the domain of x will be from 0 to 1 is not, the value of x ok. If you exceed the value of x greater than 1; that means, the transformer is overloaded. Transformer of course, we will try to see it is not continuously done under overloaded condition; maybe for a big period you can overload it by 10 percent 20 percent, then the rated current the value of x then go up.

But any way this is this point corresponds to full load and this point corresponds to no load ok. Suppose x equal to 0; let us see how this characteristics is expected to look like, if x equal to 0 that is no load; no load. So, efficiency; obviously, will be 0 because there is no output power; although there will be input power. So, 0 by some finite number P_{core} will give you the efficiency under no load condition and that is this point, so efficiency is this one; at x is equal to 0.

Now let us ask our self ok, suppose you make x very large; I want to have some idea how this efficiency curve changes? Suppose x is made very large at least theoretically I will not put the transformer, I will not subject the transformer to this high degree of loading, but just let us mathematically see; what this expression tells me. If x tends to very large; very large ok, on pen and paper let me call it going to infinity how does it matter? Then what will happen to this efficiency term? See there is a x^2 term below; so it will prevail upon all the other x terms. Therefore, efficiency will tend to once again 0, because this terms with respect to x^2 can be neglected and it will be some $x \cos \theta$ by $x P_{Cu} f l$.

So, x tends to infinity means efficiency once again infinity. So, at a very large value of x once again I know efficiency will be x very large; very large it will be also 0 efficiency. But in between, whatever is the value of x ; this will give you some finite numbers

therefore, from 0; if I increase the value of x, efficiency is expected to have some finite numbers; it will grow up, but once again it has come to 0. So, it is expected the; this curve will be something like this, here is not? It has to be.

Therefore, efficiency will rise then finally, it has come to once again 0, and it is a very reasonable expression of x; function of x no complications. Therefore, it will be sort of a smooth curve and I would then expect the efficiency to rise initially and later it will start decreasing becomes 0.

In other words, what I am telling I would expect that there will be efficiency will have some eta max, at some degree of loading got the point, so this is the thing. Now then it is natural to ask that at what degree of loading; x efficiency will be maximum is not? That is the question; so this value of x is how much? Such that efficiency will be maximum; I bound to know that. So, to find out at what x; what value of x x, eta x will be maximum that is what I want to find out, clear?

Mind you, when I am varying x, I will keep cos theta constant, load power factor I will not change; cos theta constant, I will keep it a fixed value of cos theta and only vary x. So, this here in this expression I will vary x; vary x vary x, but other things remaining constant S is of course, rated k VA, so cos theta is kept constant; P C u full load. So, I will vary only x; to find out that one is pretty simple because I will just maximize this function ok.

(Refer Slide Time: 14:57)

$$\eta_x = \frac{x S \cos \theta}{x S \cos \theta + P_{core} + x^2 P_{cu}}$$

$$\eta_x = \frac{S \cos \theta}{\left(S \cos \theta + \frac{P_{core}}{x} + x P_{cu} \right)}$$

$$\therefore \frac{d}{dx} \left(S \cos \theta + \frac{P_{core}}{x} + x P_{cu} \right) = 0$$

$$\therefore -\frac{P_{core}}{x^2} + P_{cu} = 0 \quad \text{or} \quad \boxed{x^2 P_{cu} = P_{core}}$$

η_x will be max^m when variable loss (Cu-loss) = fixed loss (core loss)

So, I will start telling that, η at any degree of loading; I am rewriting this as η is $\cos \theta$ divided by output; once again $x S \cos \theta$ plus P_{core} plus x^2 into P_{Cu} full load; this is the expression. And only under the condition, you are wearing x ; you want to know at what value of x this efficiency is maximum. What you have to simply do is, you have to calculate this you have to differentiate this expression with respect to x and set it to 0 and get the value of x . But that I will do; I will do in a slightly different way, so as to reduce my mathematical effort to get this value. What I will do is I will divide both numerator and denominator by x .

So, it will be this, plus of $x P_{Cu}$ full load this will be the expression; ηx . Now by dividing numerator and denominator by a factor x , the degree of loading which is varying in this case; numerator has become constant. Then I will say ηx will be maximum, when denominator; denominator is minimum, denominator means this factor this is my denominator now; is not? When this fellow is minimum so minimum maximum whatever it is which therefore, tells me that denominator therefore, to have this denominator to be minimum; I should differentiate this like this and set it to 0.

So, if you differentiate this will then become minus of P_{core} by x^2 plus of P_{Cu} full load equal to 0 or x is equal to or I will write another step or you can say that x^2 or x^2 into P_{Cu} full load is equal to P_{core} , this is one very important result; whether it is minimum or maximum? Ok, you calculate these double differentiate; you can show this is the point we are looking for. Only thing you could also differentiate these expression, then in the numerator; there is x denominator x , I want to maximize this means you minimize this one; it is a just one line then derivation that is why I have adopted this you divide by x .

Because, we are looking for some finite values of x for which efficiency will be maximum. And this is the condition; this must be set in now what is this $x^2 P_{Cu}$ full load? This is a variable loss, so in other words I will say in language that ηx will be maximum, will be maximum when variable loss; in this case the copper loss is equal to the fixed loss, in this case the core loss got the point; this is the thing.

(Refer Slide Time: 20:12)

$$\eta = \frac{xS \cos \phi}{xS \cos \phi + P_{core} + x^2 P_{cu}}$$

$$\eta = \frac{S \cos \phi}{\left(S \cos \phi + \frac{P_{core}}{x} + x P_{cu} \right)}$$

$$\therefore \frac{d}{dx} \left(S \cos \phi + \frac{P_{core}}{x} + x P_{cu} \right) = 0$$

$$\therefore -\frac{P_{core}}{x^2} + P_{cu} = 0 \quad \text{or} \quad x^2 P_{cu} = P_{core}$$

$$x = \sqrt{\frac{P_{core}}{P_{cu}}}$$

$\therefore \frac{d\eta}{dx} = 0$
 η will be max^m when denominator is min^m.
 η will be max^m when variable loss (Cu-loss) = fixed loss.

Or I can say that x ; x will be equal to root over P core by P copper loss full load; this is the thing very important formula. Therefore, efficiency will be maximum at what value of x , which is under root the fixed loss divided by full load copper loss' you take under root of that. Whatever number you get it will be; it will give you the degree of loading at which the efficiency of the transformer will be maximum.

(Refer Slide Time: 21:10)

$S = 10 \text{ kVA (rated kVA)}$
 $\cos \phi = 0.8$
 $\text{deg of loading at max } \eta = \sqrt{\frac{P_{core}}{P_{cu}}} = 0.7 \text{ (say)}$
 if the tfo is operated at $0.7 \times 10 = 7 \text{ kVA}$ then max^m efficiency will be achieved.

As an user of the tfo at what value of x efficiency will be max^m

And that is if I go to the next page; as I was telling the efficiency versus degree of loading curve; it is 0 here, it is degree of loading is 1, then efficiency curve will be

somewhat like this; it then decreases ok. And this is your eta max, eta max and I will write the value of x here as $\sqrt{\frac{P_{core}}{P_{copper\ full\ load}}}$ and it will give you a number, it is dimensionless x. Suppose you get this value to be equal to 0.7, transformer rated k VA is 10; then I will say that if you operate the transformer at 7 k VA; efficiency will be maximum; that is the implication of this number.

And I would like to see this number is between 0 to 1, certainly I should not have a number, where efficiency will occur at say 1.5; suppose are you getting me? Suppose I say that suppose the S is equal to 10 k VA for easy calculation it is the rated k VA, rated k VA. And suppose you calculate the degree of loading at maximum efficiency; degree of loading at maximum efficiency how do you will calculate?

You calculate from this P_{core} by $P_{Cu\ full\ load}$ do I know P_{core} ? Yes, from the open circuit test; do I know $P_{Cu\ full\ load}$? Yes, from the short circuit test watt meter reading, take the ratios this these are only two numbers with same dimension take under root and the it will return you a number. If suppose say this becomes equal to 0.7 say this becomes 0.7; then I will say if the transformer is operated at 0.7 into 10; that is equal to 7 k VA, if it is operated then maximum efficiency will be achieved, then maximum efficiency will be achieved, but the point, so this number this is the maximum efficiency.

So, this axis is efficiency and this is eta max; this is general eta x. Of course, I should not forget to attach another conditions, that I have imposed upon the efficiency expression, what is that? This curve is true for a fixed power factor angle; $\cos\theta$ I assumed constant is not? I assumed that is why while disturbing $\cos\theta$ was kept constant; recall that this $\cos\theta$ was assumed to be constant in this whole derivation.

Therefore, I should attach to this curve; a tag saying that ok this curve we have got and this is for example, say $\cos\theta$ is equal to 0.8; leading or lagging does not matter, no point in qualifying it further that is $\cos\theta$ leading or lagging because output power is x into S into $\cos\theta$. So, I must attach another parameter to this curve that this curve has been achieved; when you kept $\cos\theta$ is equal to 0.8. What that does that mean? It means that this is your transformer, actual transformer when you will do this exercise of loading it. What I am telling is, you this load; how keeping $\cos\theta$ constant can I vary the current degree of loading ok, any impedance you know such that you can vary the

current; it is Z magnitude of Z , I will vary keeping theta constant that is what it means; you vary Z that can be done keeping theta constant.

And then you will get different; different current as the value of Z will be different, but theta remaining constant. Therefore, this is the curve and I should not forget at what power factor I am plotting this curve ok. After I get this curve, then I will ask myself; what as an user of this transformer, I would like to have at what value of x efficiency should be maximum is not?

Ok, given a transformer; you do this note down the Watt meter reading from the open circuit short circuit full load reading, very simple these two test; take this ratio and you will come to know about this number. But I am asking as an user of the transformer, at what value of x we would like to have maximum efficiency to occur? As an user of the transformer; of the transformer at what value of x efficiency will be maximum?

So, as an user of this transformer; at what value of x efficiency will be maximum? Do I have a choice for that, to say I have purchased a transformer let its efficiency occur should I say at x equal to 0.5 or x equal to 0.8. So, these are the things which we will discuss very interesting point when you purchase a transformer, you know for what use you will be using the transformer.

And whoever is the manufacturer of the transformer, I will ask while purchasing the transformer; look here you design a transformer, whose maximum efficiency should occur at so much value of x ; certainly it will not be greater than x equal to 1.2 or 1.5 continuous running. And this is the point I will discuss in the next class. I hope you have understood this.

Thank you.