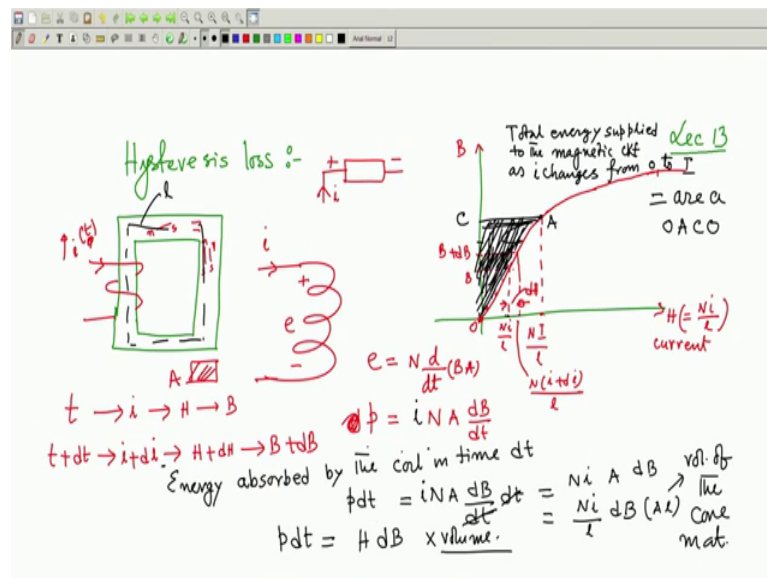


Electrical Machines - I
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Lecture - 14
Exact Equivalent Circuit

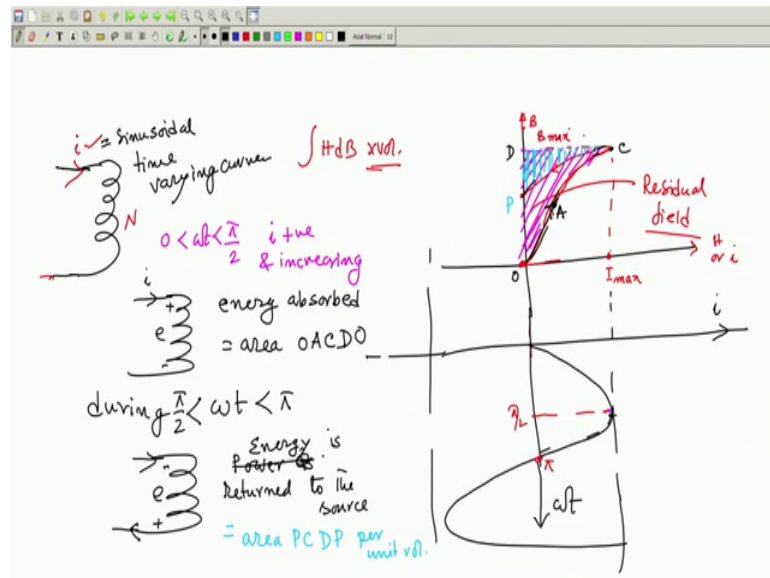
Welcome to lecture number 14 on Electrical Machines-I and we were discussing about core losses. In our last class, we were discussing about hysteresis loss. Before that of course, we discussed about eddy current loss and the factors on which it will depend. I just a quick review of this previous thing what I did was just have this.

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So, while discussing hysteresis loss first what I did that suppose there is a single coil at any time t , current is i , corresponding H is H and B is B . And suppose you are increasing the current from 0 to some fixed value i , then what is going to happen. So, in time dt , if current increases from i plus di , H plus dH and so on, then we showed that the energy will be absorbed by the coil when current is increasing or H is increasing. And the area under this curve BH curve, so $H dB$ is the energy supplied and then total energy supplied can be calculated.

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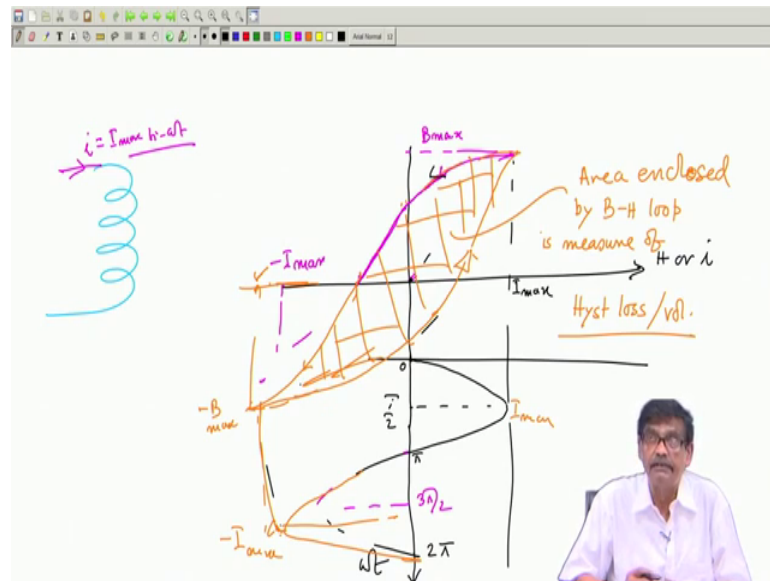


In our case, current will be alternating in nature, I am not saying whether it will be sinusoidal or not, but it is alternating, current will increase will reach a peak, then it will decrease and so on. And then when you first switch on and suppose there was no residual magnetism present in the core, then this is the BH curve, this point will go up. And when it reaches i_{max} , this axis is incidentally also represents current in some other scale. So, it reaches i_{max} and corresponding DP max.

So, during this process you have pumped energy into the system, the area of which will be 0, I mean not 0 O A C D P O. And when you decrease the current that is between $\pi/2$ to π current decreases, but still remains positive. And but di/dt is negative $d\Phi/dt$. So, induced voltage polarity reverses, current direction still remains like this because it is positive current.

And then certainly this coil now delivers energy back to the source. And how much energy is delivered? Once again integral $H dB$, but then the BH curve is like this, this one and this area will be this one. Therefore, the difference of this area represents the energy absorbed by the coil.

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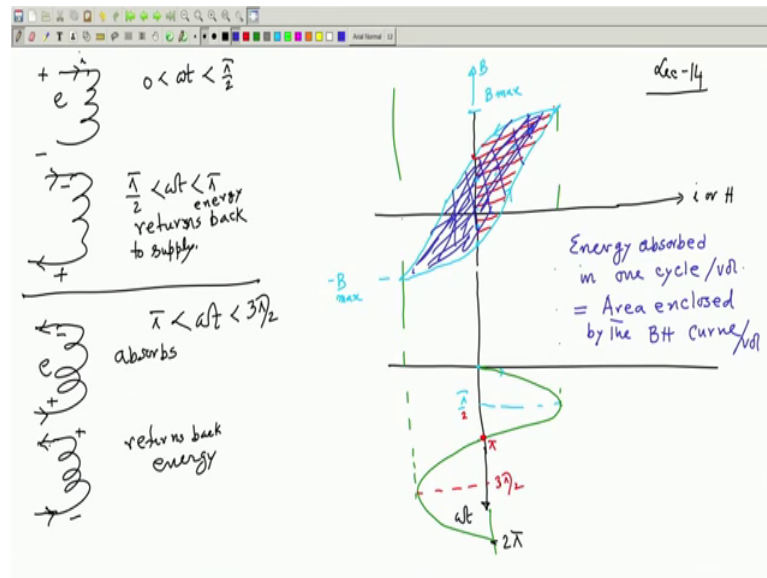


Now, next thing what I did is suppose the current is alternating like this through the coil, then once again if it is alternating, I have not shown this part once again, because after for the first time it will go like this reaches I_{max} , B_{max} . Then when I decreases it will the BH curve will be somewhat different, not the same curve through which it went up, it will follow this curve current decreases B .

So, this is the decreasing thing decreases. Then when current is 0, it will have some residual field then negative current. After π to 3π by 2, it will go like this, then it reaches minus I_{max} ok. And once again so if your code where was initially unmagnetized except for the quarter of the first cycle, it will do like this. So, that portion need not be taken into account, no point in taking into account.

And then we were discussing I was telling that, I will show you that the energy absorbed by the coil when current makes a full cycle of variation from 0 to I_{max} to minus I_{max} to back to 0. The area enclosed by the BH curve will represent hysteresis loss per unit volume that is what we will be doing.

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So, let me draw this curve once again, so that we understand what is going on. So, let me start with this B H curve as usual. So, it will be like this. Suppose, this is my H or i axis i or H and this is the, this axis is omega t to show the variation of the current ok. So, let it be like this. So, this is the variation of the current.

So, you know here I am just like this. So, your B H curve, it will be like this; it will be like this. So, this is B max here and this is this level is minus B max ok. So, this is when the di dt is positive current is increasing, I have to use this; when di dt is negative I have to use this part of the BH curve. This axis is B; this axis is H or i.

So, when the for example, when the current is 0 and increasing in this zone say this is pi by 2, i and H is increasing. So, i equal to 0. So, this is the point when i equal to 0 and it will start from this negative residual field a you just forget me this two are equal ok and it increases like this. So, during this process the coil voltage will be just like this, applied voltage is positive, negative. And why the induced voltage will be positive, because di dt is positive and current is also positive. So, energy will be absorbed. This is between 0 to omega t to pi by 2.

And what will be the energy absorbed by the coil? Energy absorbed by the coil then will be the area under this curve that is if I hatch it, it will be like this. So, this area will correspond to the area absorbed by the coil ok and it will come to this point. Of course

current is changing on its own. So, during $\pi/2$ to $3\pi/2$, it will do like this. And this point is how much? $\pi/2$, this is π sorry I am sorry, this is $\pi/4$.

This is $\pi/2$ and sorry this is $\pi/2$ and this is π and this is $3\pi/2$. So, between 0 to $\pi/2$, current was increasing. And then from $\pi/2$ this, this point will be reached when I have come here is not. I have reached this point so 0 to ωt to $\pi/2$, it is like this. Then what happens between $\pi/2$ and π that is this is the coil and then the induced voltage as you see B decreases, true in this path I am there, dB/dt is negative. So, induced voltage will be plus minus.

And current of course, still remains positive means this is the direction of the positive current. So, this delivers energy. So, $\pi/2$ to 2π returns back energy to supply, returns energy back to supply is it not. And what is that area? That area will be this area that is why during this point at the end of π , it has absorbs so much energy this minus this, this we discussed last time.

Similarly, if between π and 2π , if you consider, it will absorb energy that is let me draw this. So, this is the first positive current cycle, half cycle, then the negative thing will be then say between π to $3\pi/2$. During this portion of this one current is negative means current is actually entering through this and coming out from this; this is the negative current. And induced voltage because current is dB/dt is this one positive, it still remains this one then plus minus. So, it once again absorbs the energy absorbs. This is the e because through the plus current is entering.

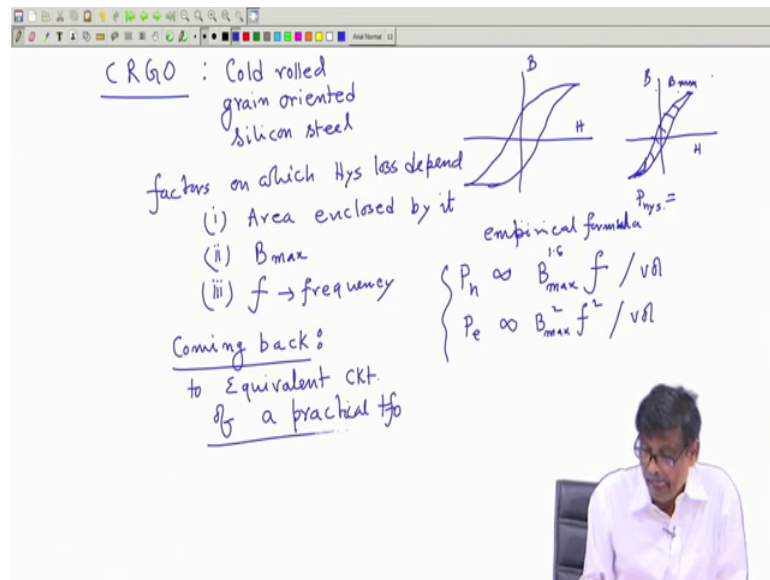
And finally, between $3\pi/2$ to 2π , the situation is current still negative is it not current still negative. So, it will be like this, but di/dt is opposite. So, it will become like this, so, it will returns back supply. So, it returns back energy. So, effectively when current makes a complete full cycle, so that the area corresponding to that will be this.

So, total energy absorbed in one cycle of current variation will be this area plus this area. Therefore, energy absorbed in Joule energy absorbed in one cycle is equal to area enclosed by the BH curve. Therefore, this whole area will represent how much energy this magnetic material has absorbed.

Of course, thinner this area energy absorb per cycle will be also small, because the area is the crucial term enclosed by the BH curve. Mind you this is per unit volume area

enclose energy absorbed in one cycle per unit volume of the magnetic material is the area enclosed by the BH curve per unit volume. This we have establish that earlier, it is per unit volume, one should not forget that so per unit volume the energy absorbed will be this one.

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Therefore, to reduce the hysteresis loss, I must see that the area enclosed by the BH curve should be as small as possible that is for example, if somebody uses a material like this or instead of that if somebody uses a hysteresis loss for this material will be much lesser than this that is the idea. Make the choose the magnetic material in such a way that the area enclosed by its BH curve is as small as possible. This is H; this is B; this is H; this is B. In fact, it depends upon the application in case of transformer application, we will demand that hysteresis this area is as small as possible.

The material used for to reduce to hysteresis loss is CRGO - Cold Roll Grain Oriented, cold roll commercial name cold rolled grain oriented grain oriented silicon steel, silicon steel. This some metallurgical process that is you treat the iron laminations in a particular fashion, so that the grain oriented means the tiny magnets are locally in some groups they are oriented already by that process when you roll this material, anyway we will not go into that. But the idea is you try to see that this area enclosed by the BH curve will be small and silicon if you add to this material it will increase its resistivity thereby eddy current loss will be reduced.

Therefore, in essence this area of course, as you can see it will also depend upon B_{max} . Suppose, the B_{max} is more even if it is thinner area, this area gets increased up ok. Now, all said and done, we do not know the factors on which this hysteresis loss will depend, it will depend upon the frequency factors on which hysteresis loss depends. One is area enclosed by it, area enclosed by it. Then the second thing is it should depend on B_{max} and also it should depend on frequency at which this reversals of frequency of supply, because this area gives you hysteresis loss P hysteresis is a per cycle.

So, more the cycles, more the frequencies, hysteresis loss will increase per cycle per unit volume. So, it will depend on frequency. All this said and done there is a what is that formula called a some after doing some experiments on several material, some people say hysteresis loss in facts it was proposed by Steinways a formula for which there is no derivation ok. What this?

Student: Empirical.

Empirical formula. And it is this some empirical formula, empirical formula, empirical formula for calculating formula is like this that hysteresis loss is proportional to some B_{max} raise to the power some number say 1.6 is a popular number into frequency. Frequency we know it has to be directly proportional to and B_{max} is the peak value of the maximum flux density.

Therefore, and recall that eddy current loss is proportional to $B_{max}^2 f^2$ and thickness square, once the plate is chosen thickness of which may be of fraction of millimetre say 0.25 millimetre or so, I am showing it just proportional to it is also proportional to plate thickness square.

But all these things are once you write frequency here, you then write per unit volume. Volume of what? Magnetic material core per unit volume of the core material. So, this is the at least this results we should remember. And also if you have the BH curve of the material available to you, then you can much more accurately say about the amount of hysteresis loss that is going to take place within the core of the transformer.

We will refer to this particular aspects by solving several problems. So, we will give you in the tutorial. Mind you whatever I have done till now about eddy current and hysteresis

loss, I have written some write up and it will be uploaded. So, read the lecture, listen to the lecture and also go through those lecture notes where somewhat detail analysis of this thing whatever I have done is done somewhat neatly, anyway this is the thing.

Now, coming back to the practical transformer, in fact, we have digressed it from this one coming back to equivalent circuit of a practical transformer circuit, of a practical transformer.

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The whiteboard contains the following content:

- Equivalent Circuit Diagram:** A circuit diagram showing the primary side with resistance r_1 and leakage reactance x_1 . The secondary side is represented by a dashed resistance r_2' and dashed leakage reactance x_2' , which are equal to $a^2 r_2$ and $a^2 x_2$ respectively. A load impedance z_2 is connected to the secondary. A magnetizing branch with reactance jX_m is connected in parallel with the primary winding. The primary current is I_1 and the secondary current is I_2 . The primary voltage is V_1 and the frequency is f .
- Turns Ratio:** $a = \frac{N_1}{N_2}$
- Core Loss Equation:**

$$P_{core\ loss} = P_{eddy} + P_{hys} = k_e B_{max}^2 f^2 + k_h B_{max}^{1.6} f$$
- Maximum Flux Density:**

$$B_{max} = \frac{\Phi_{max}}{A}$$
- Maximum Flux Equation:**

$$\Phi_{max} = \frac{E_1}{4.44 f N_1} \approx \frac{V_1}{4.44 f N_1}$$
- Text Note:** "Value of B_{max} practically remains same from no load to loaded condition"
- Video Inset:** A small video frame showing a lecturer in a white shirt.

Recall that we have got the equivalent circuit of the practical transformer like this. On the primary side, it will be like this r_1 , x_1 leakage reactance small values, then I showed a magnetizing branch here jX_m . And then here there will be r_2' which will be equal to a square into r_2 . Then you have x_2' which will be equal to a square actual value of secondary winding resistance. And whatever load impedance you have connected to the secondary side a square into z_2 . What is a ? a is the ratio of number of turns N_1 by N_2 and this is your V_1 .

So, it is almost the correct equivalent circuit of a practical transformer, but here in this equivalent circuit, how to take now what elements now should add to this equivalent circuits which will represent the core loss; core loss means heating of the core, heating of the core will take place because of eddy current loss and because of hysteresis loss. This is the only source of power which pumps the power into the system. What is the system? Actual thing practical transformer it is like this. You have applied a voltage V_1 and here

is your load connected. In this one, you will never find this resistance reactance is present. Therefore, here is your I_2 and here is your that is what you will observe nothing else.

But this whole thing practical transformer is model by an ideal transformer and these things. Now, eddy current loss core loss is equal to P_{eddy} plus $P_{\text{hysteresis}}$. And P_{eddy} loss will be proportional to some constant k_e into B_{max}^2 , f being constant it will be like this what $f^2 I$ will write plus some constant here into eddy of the loop. Instead of writing that it is proportional to I will say B_{max} may be raised to the power 1.6 into f it is one and the more exact result loop area, but this is like this.

f being constant therefore, the hysteresis and eddy current loss depends upon B_{max} . And B_{max} value of B_{max} we have seen B_{max} practically remain same, remain same from no load sorry to loaded condition or full load condition why because you recall this current is I_2 dashed, this current is I_m - magnetizing current and this current is I_1 .

The voltage which will come between these two points that decides the magnetizing current has the flux or B_{max} . B_{max} is after all ϕ_{max} by cross sectional area of the core, therefore this voltage will change strictly. When no load is connected, suppose there was no load, so X_m^2 , Z^2 is infinity here, it will be open circuit, then the current round will be pretty small. So, this voltage will be this one.

But when the it is loaded, this current will increase no doubt, but this parameters are small. The drop which will take place here the voltage across this magnetizing branch X_m will definitely not be equal to V_1 , but it will only marginally change. So, what is I am just trying to tell is that ϕ_{max} we have seen is equal to it should be strictly this one E_1 by $4.44 f N_1$.

But what i am telling you just approximate these two V_1 by $4.44 f$ into N_1 . So, a little variation of B_{max} will take place from no load to full load, but otherwise it will be there. But any way this then the thing is this loss which depends on B_{max} which practically remain same should be shown by a resistance connected across X_m that is what I am telling this is R_{core} which will take care of the core loss.

And it certainly should not be in series, because it depends on B_{max} . And B_{max} is decided by this voltage. This voltage by this impedance is I_m that decides B_{max} ,

therefore core loss resistance which will represent core loss must be shown connected in parallel with X_m .

We will continue with this. And this complete this is the complete exact equivalent circuit of a single phase transformer referred to the primary. We will continue with this.

Thank you.