

**Electrical Measurement And Electronic Instruments**  
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**Lecture - 40**  
**AC bridges – II**

Welcome back, we are studying AC bridges and in our last class, we talked about product bridge and ratio bridge. The idea is if you make your bridge either in the form of a product bridge or a ratio bridge then the balance equation becomes very simple; simple to derive ok.

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Example 4. To measure a lossy inductor, but we do not have a standard variable inductor. We have to use capacitor. We are going to use product form.

$z_2 z_3 = z_1 z_4 = R_1 R_4$   
 $(R_s + j\omega L_s) = (R_1 R_4) Y_3$   
 $= (R_1 R_4) \left( \frac{1}{R_3} + j\omega C_3 \right)$

$\Rightarrow \begin{cases} R_s = \frac{R_1 R_4}{R_3} \\ L_s = R_1 R_4 C_3 \end{cases}$

product bridge.  
Maxwell's bridge

So, in the last class, we took some example and the last example, was about measuring an unknown inductor; lossy practically inductor, which has some resistance also which could be just the call resistance so, this is not pure. And we found that, the simplest bridge that we can use to measure this is this one. This is the product bridge because, two opposite arms  $z_1$  and  $z_4$  are pure resistance ok. And this is a RC parallel combination, why parallel because, in the equation we have the term  $Y_3$  admittance of this branch and admittance becomes easy to compute, if the elements are in parallel. Because, you can simply add the admittance of the, this resistor and the capacitor to get the total admittance. Whereas, it is easy to compute total impedance, if the elements are in series.

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Example 5 : Suppose the unknown inductor has high Q value  
 $\Rightarrow R_s \ll \omega L_s$

$\theta_2$  will be <sup>very</sup> close to  $+90^\circ$   
 $\theta_2 + \theta_3 = 0^\circ + 0^\circ = 0$   
 $\theta_3 = -\theta_2$   
 $\Rightarrow \theta_3$  is <sup>very</sup> close to  $-90^\circ$   
 $\Rightarrow R_3$  should be very high.

If we do not have a high value standard variable resistance, then what can we do?

Solution:

Now, today let us take another example. So, example 5 ok. By the way just for your information, this particular bridge is called Maxwell's bridge. Once again this is just for your information, you are not expected to remember or memorize this names and this particular structure not at all.

So before, we go to example 5, we will need to analyse this bridge a bit ok. So, suppose, the induct; the unknown inductor is a very close to an ideal one ok. What do you mean by that? We mean the quality factor quality goodness is very high Q factor is very high ok. So, unknown inductor has high Q value; Q factor. What does this mean? This mean simply, the  $R_s$  resistance is small smaller compare to the inductance part ok. So, this simply means  $R_s \ll J \omega L_s$

Now if so, then the angle of z 2 ok; then the angel of this complex impedance z 2 will be close to or very close to 90 degree plus 90 degree because this is the inductance. If so, now, what is the angle balance condition? The angle balance condition says that, the sum of this two angles which is 0 degree plus 0 degree should be same as the sum of this angle and this angle.

$$\theta_2 + \theta_3 = 0$$

$$\theta_2 = -\theta_3$$

So, that means,  $\theta_3$ ; I am not sure whether I was using this angle notice from here; previously or not be consistent to your notation whatever you choose ok.

So, this  $\theta_3$  is close to; very close to minus 90 degree. When is that possible? That is possible only if, this RC combination is almost like a pure capacitance, then only we can have angle close to minus 90 degree. So, this should be almost like a pure capacitance; that means, this leakage if you call so, this  $R_3$  should be very high or the conductance of this branch should be very low ok. So, this implies  $R_3$  should be very high ok.

Now, there could be a problem due to this because, in a previous chapter we have seen that high resistance is high value resistance is have I mean they are difficult to measure. So, they have; that means, it difficult to have variable standard high value resistance ok so, high and very low resistances are difficult to have achieve. So, if this is the case that  $R_3$  should be very high to get this bridge balance and if we do not have a high value standard variable resistance, then what can we do? So, this is the question.

Suppose, the inductance we are measuring is almost pure, which means to get this bridge balance I need this  $R_3$  to be very high and I do not have in my laboratory a high standard high value standard variable is a resistance. So, what can we do? What is the solution?

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Solution: Use a series combination of  $R_3, C_3$  instead of parallel combination

If we find the series equivalent of  $(C_3 \parallel R_3)$ , call it  $\left( \frac{1}{\frac{1}{C_3} + \frac{1}{R_3^S}} \right)$

then we will see that  $R_3^S$  is a small resistance (While  $R_3$  is a high resistance) [Hw, prove it]

So, the solution this is the question. Now the solution is instead of using  $R_3 C_3$  in parallel use a RC series combination, use up series combination of RC  $R_3 C_3$ , instead of parallel

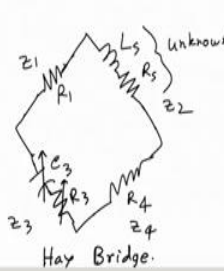
combination ok. So, how would that help because, if you find the series equivalent of these ok; so, if you find the series equivalent of this  $C_3 R_3$ , series equivalent of this thing  $C_3 R_3$  parallel ok. Series equivalent of this call it this, call it  $C_3 s R_3 s$  ok. So, given the value of  $C_3$  and  $R_3$ , you can compute the  $C_3 s$  and  $R_3 s$  such that these two impedances are equal ok.

So, this is what we did yesterday. At the beginning of this AC bridges before we started ac bridges, we discuss how to find the equivalent of these branch; that means, given  $C_3 R_3$  how to find  $C_3 s R_3 s$  such that this two are equivalent. So, if you do that ok so, we did it yesterday. then you can; so, if you find the series equivalent of this, then we will see that  $R_3 s$  is a small resistance, where while  $R_3$  I mean, this parallel combination parallel equivalent is a high resistance. So, let this be a small homework ok. You just have to find the equivalent  $C_3 s R_3 s$  and you will see that if  $R_3$  is high  $R_3$  series will be small.


So, I mean, another way to visualize this quickly instead of doing this full derivation is that so, we want this to be close to a pure capacitor because,  $R_3$  is high. So, this combination is close to a pure capacitor and here also; that means, this combination should be close to a pure capacitor which is possible only if, this impedance that is one over  $J \omega C_3 s$ , is much higher than this impedance which is  $R_3 s$ . The, we want this to be more close to pure capacitor so, that means, here also these two are equivalent, here also it should be more close to a pure capacitor. So, this impedance should be small, if this is high that is easy to see, and you can do this derivation yourself to convince yourself and I would be very happy if you do it ok. So, this is the solution. So, instead of using this two in series use  $R_3 C_3$  in parallel ok.

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then we will see that  $R_3^S$  is a small resistance  
 (while  $R_3$  is a high resistance) [Hw, prove it]



Balance equation  
 $R_1 R_4 = Z_3 Z_2$   
 $Z_2 = R_1 R_4 Y_3$   
 (unknown)  
 Trick:  $R_1 R_4 Y_2 = Z_3$   
 unknown =  $Y_2 = Z_3 / (R_1 R_4)$



So, the new bridge, the modified bridge will have this two in series ok. So, and these are variables of course,  $C_3 R_3$  these are variables. So, now, you can vary this and with the small value of  $R_3$  you can get this bridge balance. So, this is the solution. And just for your information, this bridge is most likely called Hay Bridge. Once again you need not to remember this ok, but did, but then there is a small problem. What is the problem? The problem is, if we write the balance equation. So, you can write

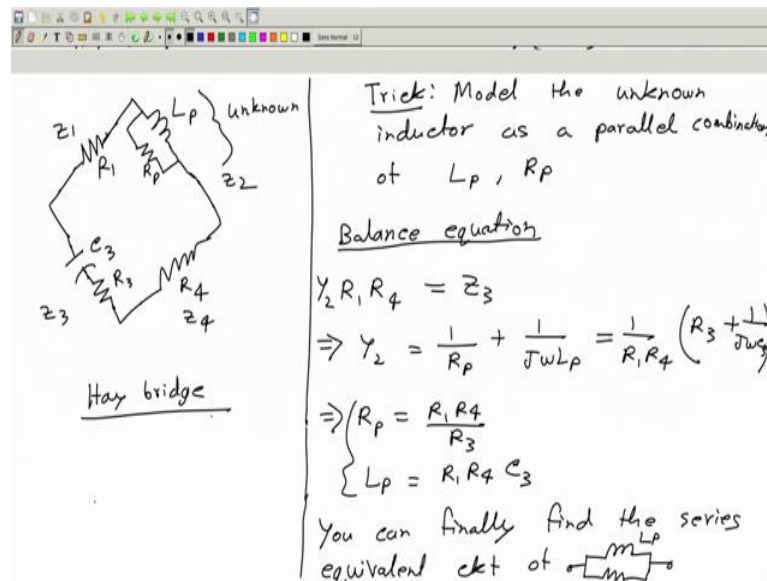
$$Z_1 Z_4 = R_1 R_4 = Z_3 Z_2$$

$$Z_2 = R_1 R_4 Y_3$$

Now, why admittance is difficult to compute when the elements are in series, not impossible not very difficult, but slightly difficult. So, what can we do? Instead, I mean since this is difficult to find and  $Y$  admittance in series and we need this in series definitely because, we do not have a high  $R_3$ . So, we have a small  $R_3$  so, we have to use it in series no other choice, but here it is difficult. So, what is the solution? What is the trick? The trick is write this equation in this form; write it as

$$R_1 R_4 Y_2 = Z_3$$

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So, this trick is model, the unknown inductor as a parallel combination of an inductor  $L_p$  and  $R_p$  ok so, like this. By the way, this is in series due to the reason already mentioned ok. So, let us model it this way and then the balance equation become simple. How? So,

$$R_1 R_4 Y_2 = Z_3$$

$$Y_2 = \frac{1}{R_p} + \frac{1}{j\omega L_p} = \frac{1}{R_1 R_4} \left( R_3 + \frac{1}{j\omega C_3} \right)$$

$$R_p = \frac{R_1 R_4}{R_3}$$

$$L_p = R_1 R_4 C_3$$

So, the main trick is model this unknown inductor in parallel combination. And then if you want, if you really want to represent these unknown inductor in the form of a series combination, you can finally find the series equivalence or equivalent circuit of this  $L_p$   $R_p$ , that trick we have seen yesterday. How to convert a parallel RL combination into its series equivalent or a series combination into its parallel equivalent that trick we have seeing yesterday. So this is called Hay Bridge.

And all the things that we are doing is not a new science, it is not a; it is nothing new. It's all small mathematical and engineering tricks, which makes this things easy and simple ok. So, we are just learning small tricks which make the bridges easy to derive their balance equation that is the point. We are not doing any new science so, in that sense, this is bit

boring too, but if you do it following this trick of product and ratio bridge calculation becomes simple.

It's even simple during the exam because, you can find the; you can design the bridge, you can find its balance equation all in two-three lines ok.

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Situation: We want to measure an unknown High Voltage capacitor

Number of unknowns: 2  
We need two variables/controls.

$\theta_3 - \theta_4 = 0^\circ$   
 $\theta_4, \theta_1$  is fixed

We need  $\theta_3 - \theta_4 = \theta_1 - \theta_2$   
fixed

So we must  $Z_1$  variable

The diagram shows a bridge circuit with a voltage source  $V$  on the left. The bridge arms contain impedances  $Z_1$  (with  $R_1$  and  $C_1$  in parallel),  $Z_2$  (with  $R_2$  and  $C_2$  in parallel),  $Z_3$ , and  $Z_4$ . A diagonal branch contains  $Z_4$ . Voltages  $V_1$  and  $V_2$  are indicated across  $Z_1$  and  $Z_2$  respectively. Angles  $\theta_1, \theta_2, \theta_3, \theta_4$  are shown at various nodes.

So, now let us take another example, example 6, I guess. Let us describe a situation, the situation is that, we want to measure capacitance an unknown capacitor. So, we want to measure an unknown high voltage capacitor. So, a capacitor which is meant to be used in some high voltage application so, some high voltage will be applied across this capacitor and it will work under that situation. So, how it perform; so, how much will be the leakage etcetera under that high voltage under that high voltage stress, that is what we want to measure. Now we if the capacitor is suppose to work under a high voltage stress and we want to measure its performance, then we should also do the measurement under similar high voltage applied across it.

We cannot measure the capacitor with the low voltage across it and get its R; RC values and say that this is the RC equivalent. No, because its performance will change when this capacitor will be used under a high voltage stress in the real scenario. Because, you know I mean we can; I mean simply speaking under high voltage stress leakage current may increase ok. So, therefore, its performance will change. So, to get an actual evaluation or

feeling how will it work under high voltage stress, we should measure it only under a high voltage applied across it.

So, we want to measure an high voltage capacitor so, we will use some bridge of course. So, let us draw a bridge, we have to design this bridge, but now this unknown capacitor R C, if you want to represent in parallel. Let us put it like this, call the Cx Rx unknown capacitor and now we have to design or find what this other parameter are; other branches are ok. Now what can we do? So, one solution that we already have seen is just use the ratio bridge and make it like this pure R R and here put a RC combination. Parallel or series whichever is a convenient. I think parallel combination will be convenient here.

Now, to get it balanced, what we need? We need to vary these three arms; this is z 1. Now we may not need to vary all these three oks, we may choose to vary just may be this two that is sufficient. So, we may or choose to vary this two that may also be sufficient because, we have two unknowns; two unknowns see number of unknowns is two, Cx and Rx. So, we need two controls, we need two variables or two controls at least; so, we need not vary all this Rs I mean this R this C this R this C there are too many variables, we may keep some of them fix, no problem ok.

But definitely we have to vary this capacitor because, if we choose to vary for example, only this two resistances ok, then the number of variables s is two, if we just choose two vary this two and keep it constant. Then number of variables is 2 number of unknown is 2, but we cannot possibly get a balance because, we would not be able to change the face angle relationship. Because,

$$\theta_3 - \theta_4 = 0$$

$\theta_1, \theta_4$  is fixed

We need

$$\theta_3 - \theta_4 = \theta_1 - \theta_2$$

So, we will never be able to change this equation. So, to be able to change this equation I mean get this equation satisfied by changing something, we have to change theta 2 ok. So, sorry, we have to change theta 1 because, then only we can adjust the right hand side and get the right hand side equal to 0 left equal to left hand side ok.



So, we have to definitely change this branch ok. So, we must have  $z_1$  variable ok. Now yes; so we can choose  $z_1$  to be variable and maybe we can choose either of this  $z_3$  or  $z_4$  to be variable we do not need to vary all three, we can choose  $z_1$  to be variable on  $z_3$   $z_1$  and  $z_3$  is fine  $z_1$  and  $z_4$  is fine, but  $z_1$  has to be variable. If we cannot vary  $z_1$ , then we cannot vary the difference between  $\theta_1$  and  $\theta_2$  and we cannot make it equal to 0. That is what we need ok.

I hope this much is clear and then, now the next important point is this. This capacitor is under a high voltage ok, we have to test this capacitor with a high voltage across it. So, if we so, we can put the supply like this, then some voltage drop will happen across this branch, some voltage drop will happen across this branch ok. And the sum of this two phasor sum will be same as this voltage, let me take this as the reference call this as  $V$  phasor. So, this point is at  $V$  and now if this voltage call it  $V$  branch 1 2 3 4 call it  $V_2$  and this voltage branch 4 ok.

$$V_2 + V_4 = V$$

$V_2$  should high

$$V_1 \approx V_2$$

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A Near balance  $V_1$  in  $z_1$   
 $z_1$  is variable and voltage across it is high  
 So it is risky to operate

Schering bridge:  
 Solution:

We will keep  $z_1$  fixed  
 and design a bridge where  
 we need to vary only  
 $z_3, z_4 \rightarrow$   
 $V = V_2 + V_4$

We will have  $V_4$  small, then  $V_2$ , and  $V$  very high  
 $V_A$  will also be close to ground potential

So, this two voltages will be equal and this is a variable arm  $z_1$  is variable. So, that means, we are adjusting this  $z_1$  and the voltage across  $z_1$  is very high near balance and some

body is manually I mean some person is adjusting this voltage. So, this is risky; so, this is risky. So,  $z_1$  is variable and voltage across it is high.

So, it is risky to operate to perform this experiment, to perform this measurement. So, what is the solution? The solution is, what is proposed by Schering this person, this man this proposed a nice solution which is not risky. So, his idea is called Schering bridge, you need not remember this names etcetera just see, how nice the idea is in. Idea is he said, I will have; I will adjust only this branches  $z_3$  and  $z_4$ , so I will have only this two branches variable and keep  $z_1$  fixed.

So, Schering solution is so, this is the bridge. So, here we have  $C_x R_x$ , this unknown calls this  $z_1 z_2 z_3 z_4$  ok. So, Schering said that, we will keep  $z_1$  fixed and design a bridge where we need to vary only this  $z_3$  and  $z_4$  and the supply voltage is like this. So, this is high voltage, this is a very high voltage ok, but we will ground this point this is not just a reference point, this is really grounded earthed ground point ok. So, this potential is really the earth potential or 0 potential. So, this value is really 0 volt with respect to the earth and therefore, this potential is very high  $V$  very high so, this is  $V$  very high and  $V$  is equal to phasor sum is this  $V_2 + V_4$ . I am not drawing  $V_2$  and  $V_4$ , but the meaning is clear  $V_2$  across  $z_2$   $V_4$  across  $z_4$ .

So,  $V$  is very high, but we will choose  $V_4$ ; that means, this voltage across  $z_4$  to be small ok. So, we will have  $V_4$  small then of course, then  $V_2$  and also  $V$  very high ok so, this is small this is high, this is high. So, high plus small is high no problem ok. So, we need this to be high, we will have this high  $V_2$  high, we will maintain this value  $V_4$  small and this is the variable arm, this two are going to be variable arms ok. So, therefore, the person who is adjusting this he is operating a voltage  $V_4$  which is close to ground potential risk is much lower ok. So, this potential call this potential, this value call it  $V_A$  will also be close to ground potential; earth potential. So, risk will be small.

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Schering bridge:

Solution:

We will keep  $z_1$  fixed and design a bridge where we need to vary only  $z_3, z_4$

$$\vec{V} = \vec{V}_2 + \vec{V}_4$$

We will have  $V_4$  small, than  $V_2$ , and  $V$  Very high  $V_A$  will also be close to ground potential So risk will be small.

So, this was Schering's idea that design a bridge such that, we can keep this fixed  $z_1$  fixed and have only this two variables. So, that then we can maintain; we can ensure that the operating person is varying the this  $z_4$  within I mean and this voltage is within the low value; lower value ok. So, now, how can we do that ok; now the goal is to design a bridge such that, we can keep this fixed and we can vary only this two. How to do that, ok.

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Observe: (1) This is a product bridge  $z_1 z_4 \rightarrow$  purely imaginary

$$= \frac{R_4}{j\omega C_1} = -j \frac{R_4}{\omega C_1}$$

(2)  $\theta_1 + \theta_4 = -90^\circ$   
 can be  $\theta_2 + \theta_3$   
 $(90^\circ - 0^\circ) \quad (-90^\circ - 0^\circ)$

(3) by varying  $(C_3, R_3)$  we can vary angle

So, let me draw this, let me give you the solution. So, this is the unknown branch  $C_x R_x$  unknown, this has to be a fixed impedance we cannot change it, this two will be variables. Now, we have to decide, what to put here; resistance capacitance inductance what to put here and what to put here ok. Now this is a small question you may think yourself and find

some solution, you may pause the video at this point and now I am giving solution which Schering proposed.

Schering propose that, let us have a capacitor C 1 pure here and let us put a say resistance ok so, what do we put here then; so, what can we put here? So, this angle is 90 degree minus this angle is always 0 degree, this angle is between some were between 0 and minus 90 so, what can we put here? Such that, the sum of this and this angle is same as the sum of this and this angle. This two sum is minus 90. So, this two sum should also be minus 90 so, this is negative this also has to be negative angle. So, what we can put? We can put here are RC combination, variable RC combination. So, called this is branch 1 2 3 4 according to my notation. So, we will choose this as some combination of C 3 R 3 ok, some series of parallel combination of C 3 and R 3 ok.

Now you see observe firstly, we actually have designed a product bridge, this is a product bridge. Why?

Z1 Z4 purely imaginary

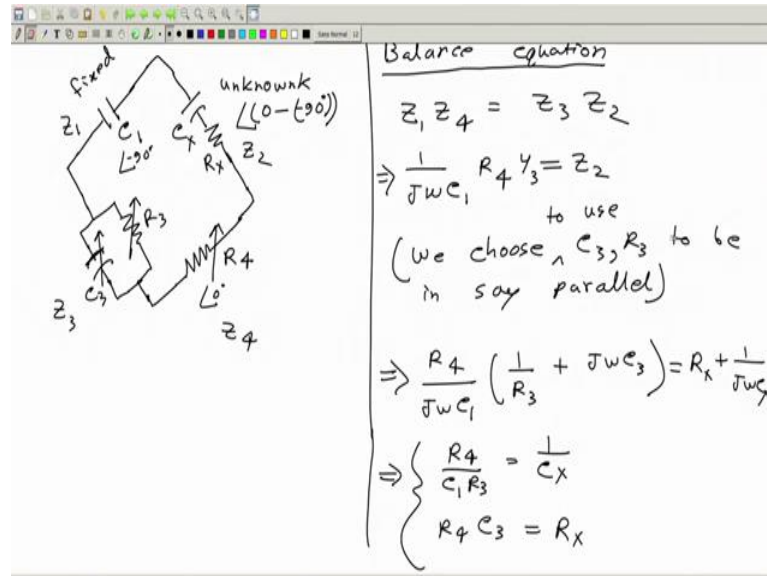
$$= \frac{R4}{J\omega C1} = \frac{-J R4}{\omega C1}$$

$$\theta1 + \theta4 = -90$$

So, this is a purely; pure imaginary number product of two opposite arms purely imaginary and we have seen when the product of two opposite arms are purely imaginary or purely real. We call it a product bridge and such bridges are easy to find their balance equation. So, this is a product bridge.

This is minus sign and this is hyphen ok; so, we can get this balanced. So, this is the nice design, what Schering proposed once again now and point 3 by varying C 3 R 3, we can vary this angle relationship; we can vary angles ok. So, all requirements are fulfilled, now let me finally finish this by finding the balance equation.

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Now, let us once again, this will be very simple. What can we write?

$$Z_1 Z_4 = Z_3 Z_2$$

$$\frac{1}{j\omega C_1} R_4 Y_3 = Z_2$$

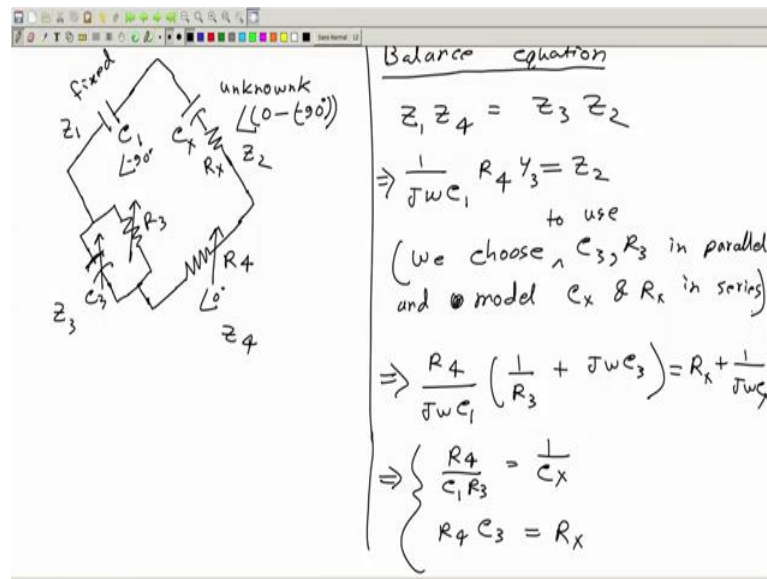
$$\frac{R_4}{j\omega C_1} \left( \frac{1}{R_3} + j\omega C_3 \right) = \frac{1}{j\omega C_x} + R_x$$

$$\frac{R_4}{R_3 C_1} = \frac{1}{C_x}$$

$$R_4 C_3 = R_x$$

Very simple just two lines once again ok. So, and if you had chosen to use a RC series combination, then it would have been easier to model this in parallel form and then if you want to of course, convert one form to the other, you know how to do it. So, here we choose to use C 3 R 3 in parallel ok. So, in the laboratory, we will use a parallel combination of C 3 R 3 and therefore, we model Cx Rx in series ok.

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So, the point is we have initially learnt that any  $C_x$  series combination can be modelled as an equivalent another  $C_x$  parallel combination. Why did we do that? Because, now when we are studying AC bridges; bridges become very easy if you choose sometimes some elements some branches as parallel combination of CR or series combination of CR. So, that depends on the situation and then using what we learnt previously, we can convert one result parallel combination into the series combination or series combination into the parallel combination.

So, this is the; this all we are doing are mathematical tricks ok, how calculations become easy and simple ok. This is not a new science these are all tricks to have calculation simpler and we are giving you practical situations under which this type of scenarios may arise like, when we are measuring an unknown inductor, but we do not have a standard inductor. So, what was the solution? The solution was the Maxwell Bridge which we studied before. Then, we give another practical scenario like, we are measuring a capacitor under high voltage and we want to eliminate the risk of this experiment so, we saw another bridge ok.

So, these are some practical situations and we are applying mathematical tricks, common sense so, I mean do not know how interesting this is may be not yeah so.

Thank you.