Electrical Measurement And Electronic Instruments
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Lecture – 39
AC bridges– I

Welcome back. We are now going to talk about AC bridges. And, in our last class just quick reminder, we have seen the series and parallel equivalence of a practical capacitor or an inductor. That was necessary because that will make AC bridges often very easy to deal with ok. Now, so, we are going to talk about AC bridges.

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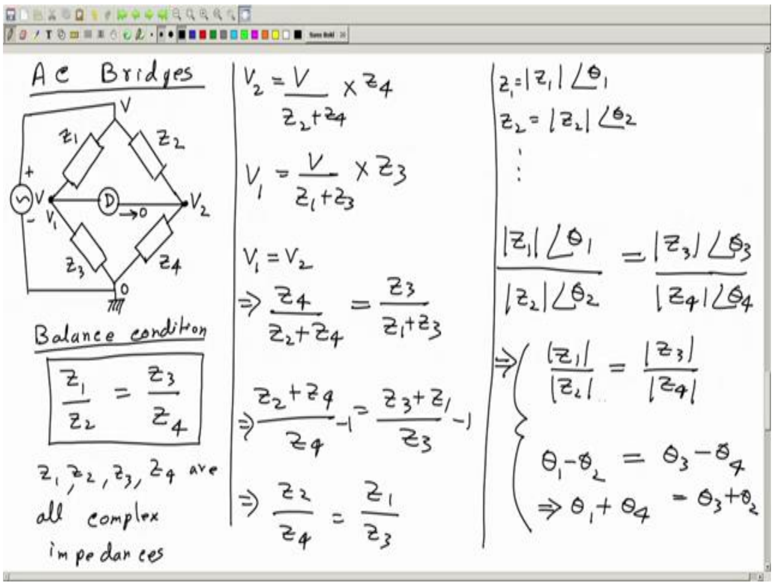

So, this is a generalization of Wheatstone bridge. So, what do you have in a Wheatstone bridge we have four resistances connected in like this. So, in case of Wheatstone bridge all these boxes they denote actually some resistance. I am drawing them as boxes on purpose and then we apply some voltage across two opposite terminals and we measure the current between the other two terminals. This is the detector this is the source ok, this is the structure of a Wheatstone bridge. In case of an AC bridge we just need a AC source of course, so, this becomes an AC source.

This detector should also be a suitable one which will work for AC voltages or currents and this impedance this boxes can now be any impedance other than or even including resistance. So, they are not necessarily pure resistances, they can be inductance, they can be capacitance, they can be anything ok or some combination of that like L C anything they can be in general anything l R C any combination of them.

So, in general so, we will call them as Z; let me call this as Z 1, Z 2, Z 3 and Z 4. And once again we can find the balance condition, what is balance, what is what do you mean by balance condition or the null condition? It is this situation, when no current flows through this detector. And that is only possible if the potential here and here, are exactly same both in magnitude as well as in phase, because now we are talking about AC ok.

In case of DC there is no phase associated with voltage, but now we are talking about AC voltages. So, we need the magnitude and phase of these two voltages to be exactly same. Assuming this is a AC pure sinusoidal supply and all voltage are and currents are purely sinusoidal and which with same frequency no harmonics present ok.

So, then the balance condition once again will mean like Wheatstone bridge is that $\frac{Z1}{Z2} = \frac{Z3}{Z4}$. This is what we have seen for Wheatstone bridge as well there we write as $\frac{R1}{R2} = \frac{R3}{R4}$, now it is general admittance general impedance ok. And how do you obtain this balance condition? Once again it is very simple to derive. So, let us take this point as reference call this voltage as V.

Now, V is the phasor ok. Now V is a phasor. And then this voltage will also be V because this is the difference 0 volt, therefore, at balance if no current is flowing here through the detector ok. So, no current goes out from here or no current goes out or in from here. So, the potential at this point can be found call this V 1 and V 2,

$$V2 = \frac{V}{Z2+Z4} Z4$$

$$V1 = \frac{V}{Z1+Z3} Z3$$

$$V1=V2$$

$$\frac{Z3}{Z1+Z3} = \frac{Z4}{Z2+Z4}$$

$$\frac{Z3}{Z1+Z3} - 1 = \frac{Z4}{Z2+Z4} - 1$$

$$\frac{Z1}{Z2} = \frac{Z3}{Z4}$$

Now, this is not just 1 equation although it looks like 1 equation it is actually not 1 equation, it is 2 equations, why because all these numbers are complex numbers. These are complex impedances Z 1, Z 2, Z 3, Z 4 are all complex impedances ok. Like R plus J omega L or R plus 1 over J omega C or so, other so many other things.

So, these are all complex numbers and one complex equation means two real equations, how? We have to equate the real part of this, with the real part of this, and the imaginary part of this and the imaginary part of this. That is one way of thinking it or you think of this term being represented as in it is magnitude and angle form, this also can be represented in it is magnitude and angle form, then the magnitude of left hand side should be same as the magnitude of right hand side and the angle of left hand side should be same as the angle of right hand side.

$Z1 = |Z1|\ \theta 1$

$Z2 = |Z2|\ \theta 2$

$$\frac{|Z1|}{|Z2|} = \frac{|Z3|}{|Z4|}$$

$$\theta 1 - \theta 2 = \theta 3 - \theta 4$$

$$\theta 1 + \theta 4 = \theta 3 + \theta 2$$

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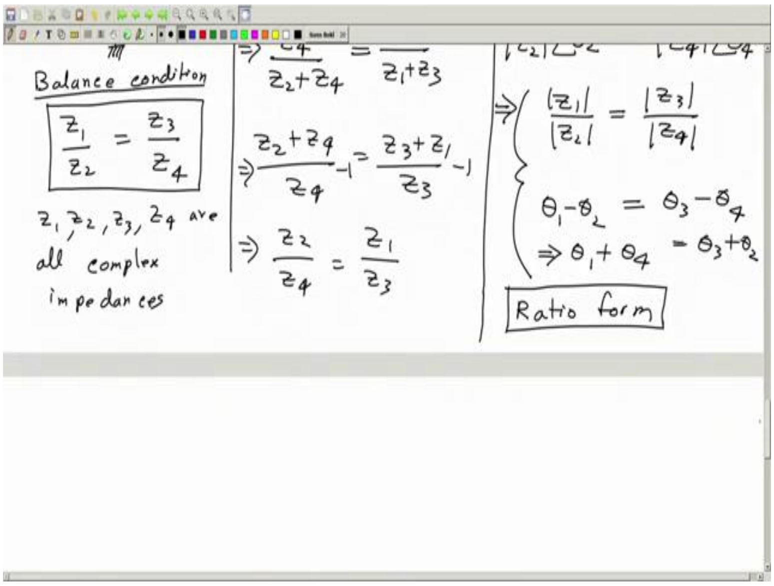

Now, this is the balance condition and this equation we can write this is the balance condition in ratio form, because we are taking the ratio of two adjacent sides Z 1 / Z 2, and the other two adjacent sides Z 3 / Z 4 they are equal.

|Z1| |Z4| = |Z3| |Z2| product form

$\theta 1+\theta 4=\theta 3+\theta 2$

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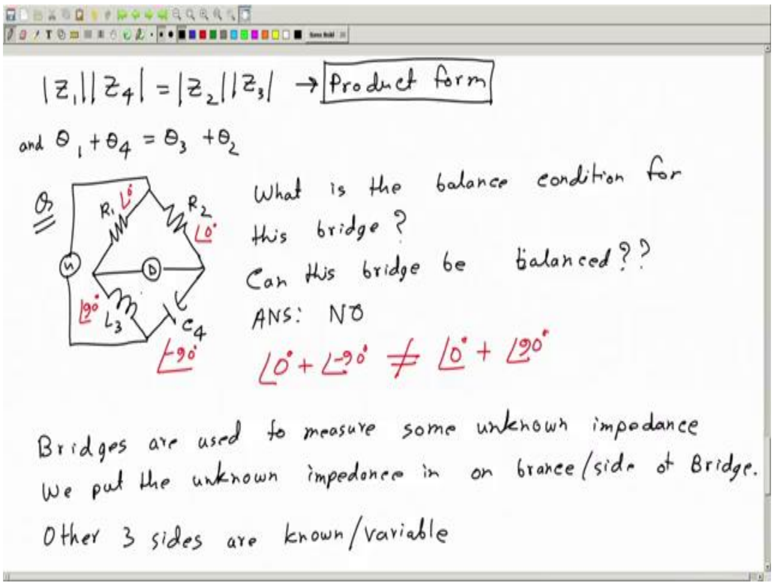

So, this is the product form of the balance equation. Same equation we are writing it in two different forms, why because sometimes it is as we progress you will see it is easy to write things in this form and sometimes it will be easy to write equation in this form ok. So, just a small note that same equation same condition can be written in either form. We can take the ratio of adjacent sides or we can take the product of opposite sides. The ratio of adjacent side should be equal that is the ratio form of the equation and the product of opposite side should be equal that is the product form of the equation ok.

Now, before we progress a small quiz small question for you. If I give you this bridge say I have a resistance, another resistance, here I have a inductor, here I have a capacitor ok. And you can put the detector and the source call this R 1, call this R 2, L 3 C 4, question is what is the balance condition for this bridge?

You may pause the video at this moment and try to come up with the answer or I can pose the question in a different way is this bridge balanceable at all, I mean can this bridge be balanced? Once again you may think yourself by pausing the video or the I discuss the answer the answer is no, why?

Because, if we want to balance this bridge we will definitely need that two condition satisfied that the one is the magnitude ratio of magnitudes like R1/R2 and it is the magnitude of this impedance and the magnitude of this impedance their ratio, they should be equal another condition is the angle condition. Which says the angle of opposite sides they should their sum should be equal. Now, what is the angle here? The angle for a pure resistance is 0; 0 degree. The angle so, the angle here is 0 degree. The angle here is again 0 degree, what will be the impedance angle for a pure inductor?

So, therefore, this bridge can never be balanced. So, through this we learn one fact that all bridges are not necessarily balanceable; that means, I with any choice of R 1, R 2, L 3 and C 4 with whatever value you choose, you can never get a situation where these two voltages are equal and no current flows to the detector ok. Now, what do use bridge, bridges for? Bridges are used to measure some unknown impedance ok.

So, bridges are used to measure on some unknown impedance ok. So, we put the unknown impedance in one branch, in one branch of the bridge or one side of the bridge, other 3 sides are known and variable, not necessarily all three sides are variable, but may be one or two sides are variable.

So, other sides are known variable and then we vary the other 3 sides find the situation where we get a null in the detector. And then from the balance equation Z1/Z2 =Z3/Z4, we can find the value of the unknown impedance, because other because three impedances are known one is unknown so; we can find the unknown one.

Now, in general our task in electrical measurement will be given an unknown impedance to configure or design a bridge with one unknown of course and three known branches, three known and variable branches to find the balance condition. This is our task configure a bridge with one unknown and three knowns and find the balance condition and then from the balance equation find the unknown value.

Now, when we are configuring or designing a bridge, we must keep in it keep it in mind that all configurations are not balanceable, like one we have seen here, this is a known balanceable bridge. So, we should never design or configure a bridge which is not balanceable ok. So, this is one thing to learn.

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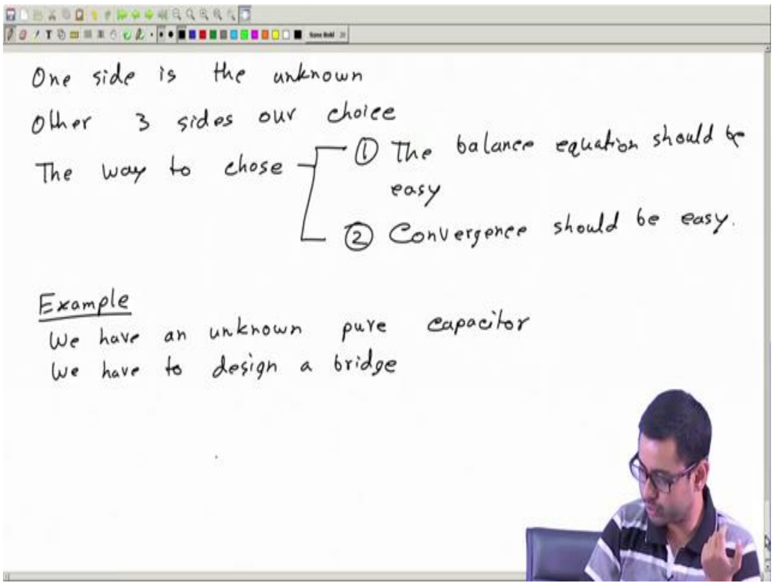

Now, onwards we will learn a lot of different bridges, but the approach you should take is that never try to memorize those bridges or the names of those bridges that ok. This is Maxwell bridge, a Maxwell bridge has a one inductor, one capacitor, they should be in the opposite branches and the capacitor should be in parallel with a resistance, the inductor should be parallel with the resistance, never remember these things.

Never remember that this bridge is called Maxwell bridge, this bridge is called Heybridge Wien's bridge, Candy Foster bridge, never ever try to remember this. Try to understand how this engineers or scientist have designed this bridges, how Maxwell have designed his bridges, how hey or Wien have designed their bridges?

If you understand how to do that, you do not have to remember any bridge. And, even in exam you need not reproduce from memory any bridge; you can design your own bridge. Always the goal is there will be one unknown impedance, which you want to measure, three others branches are known and at our control. Those are variable unknown, we will choose those three bridges such that the measurement becomes easy ok.

So, let us write while studying AC bridge one side is always unknown is the unknown, other 3 sides are our choices, other 3 sides our choice or our choices. And the way to choose is keeping two factors in mind; number 1 the balance equation should be easy, I mean we would like to have a easy looking balance equation, another factor is convergence should be easy ok.

So, we will understand both this factors with the help of examples. So, now, onwards we will only take examples and solve problems. So, example let us start with very simple one. So, we have an unknown, but pure; pure means almost pure capacitor, we want to measure it using a bridge ok. So, we have to configure or design whatever you call design a bridge basically we have to choose what to put on the four arms.

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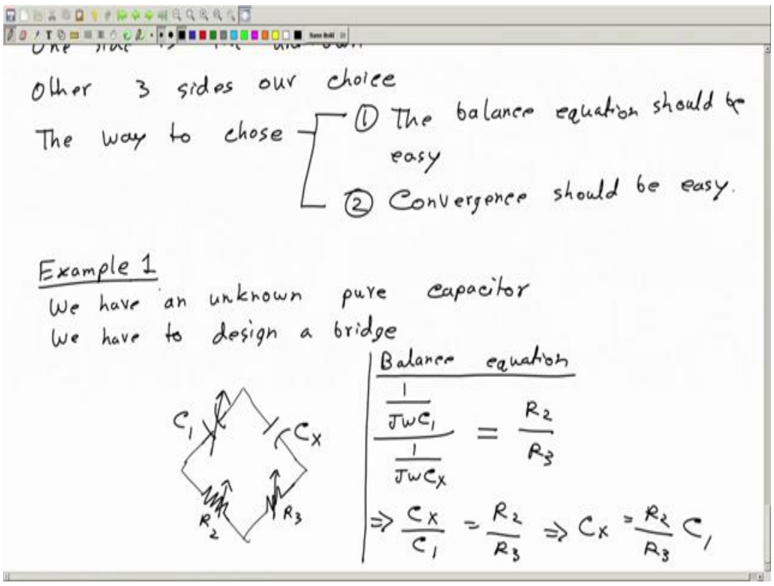

Now, a bridge means always these four arms. Now, one of them will be the unknown the capacitor. So, of course, we can put the unknown one in any branch you like anywhere you like put the unknown one, this is C X and this is nearly pure. So, no parallel or series resistance we are putting here.

Now, the question is what to put in the other three branches that is our task? And we will do it in a way whichever is easiest. And I feel the easiest way could be this just put a variable capacitor here. This is variable C 1 put a resistance, R 2 variable put a resistance, R 3 this can also be variable ok. So, I think this is easiest or you can choose whatever you think easiest ok. You might have chosen to put this capacitor here and these resistance here ok, I mean it is in a way symmetric and why I am saying this is easiest?

So, the balance equation what can we write? We can write say the impedance of this,

$$\frac{\frac{1}{J\omega C1}}{\frac{1}{J\omega Cx}} = \frac{R2}{R3}$$

$$\frac{Cx}{C1} = \frac{R2}{R3}$$

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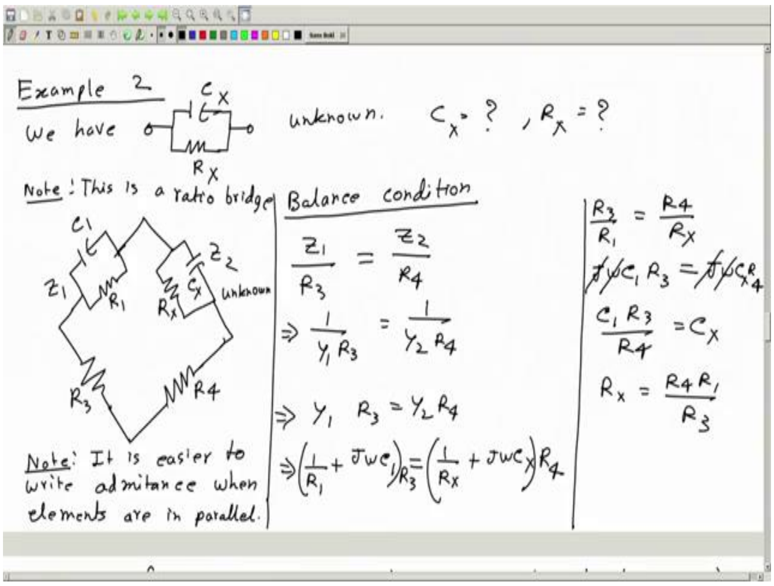

Now, so, this is example 1, now let me take another example, example 2 say we have a practical leaky capacitor. So, which we can always model as we have seen like this C P and R P we so, we have this unknown. So, we have to measure C P equal to how much and R P is equal to how much? This is the task and for that we have to design a bridge. And we will design a bridge which is simple and easy that is the idea ok.

So, once again we start by drawing a bridge which has four arms ok. And I am not drawing the source and the detector ok. It is always there, it is understood that the source and detector is there, but for simplicity I am not drawing it. Another reason I am not drawing the source and the detector, because the balance condition does not depend on the position of source and detector ok.

So, I mean I can have Z 1, Z 2, Z 3, Z 4, the balance condition is Z 1 by Z 2 equal to Z 3 by Z 4 is true no matter, whether the detector is here and the source is here or the detector is here and the source is here. The balance condition will be same convince yourself ok. Since, the balance condition will be same we are not putting the detector and the source to keep the diagram clean ok ok.

Now, task is to measure this, now to measure this ok. Let me put this unknown in one branch or let me call it X X for unknown ok, here also X X X for unknown ok. So, let me put the unknown again same in one branch. So, I put it here. So, this is R X C X and now the question is what to put here, what to put here, what to put here? That is our choice.

Now, we will do it once again in a way which is easiest and which is balanceable that is also important, we have seen some bridges are not balanceable. For example; for example, if we do it like this say we put two resistances and here we put R C parallel combination this is not balanceable, why? Because, the angle condition says the angle of this and this the sum of these two angles should be some of these two angle.

Now, the angle here is 0 degree, angle here is 0 degree. So, the sum of 0 degree and 0 degree is definitely 0 degree the angle of this impedance R C parallel combination will be somewhat between, somewhere between 0 and minus 90. This will also be between 0 and minus 90. Say, this is around minus 45, we do not know, it is it will be somewhere between 0 and minus 90, this will also be between 0 and minus 90. Say minus 45 this so, their sum is minus 90 and these two sum is 90 they cannot be equal. So, this bridge is not balanceable ok.

So, we cannot balance this bridge. So, this bridge is wrong, but let us try by say swapping this two like this put the resistance here and the R C parallel combination here call it C 1 R 1, R 3 and R 4. Now, let us check quickly is the angle criteria satisfied, yes it can be satisfied. Because, this angle is between 0 and minus 90 this is 0. So, their sum is between 0 and minus 90.

Similarly, some of these two will again be between 0 and minus 90. So, by a proper choice of C 1 and R 1 we can get this balance ok. So, this is I think the easiest bridge we can use. So, what will be the balance condition? And this should be easy that is what we want and simple ok. So, the balance condition let us write it this way so, the impedance of this divided by the impedance of this. So, which will be let me call this impedance Z 1 this Z 2 R 3 R 4. So

$$\frac{Z1}{R3} = \frac{Z2}{R4}$$

Y1 R3 =Y2 R4

$$(\frac{1}{R1} + J\omega C1) \text{ R3} = (\frac{1}{Rx} + J\omega Cx)\text{R4}$$

So, this is multiplied by R 4. Observe that, it is easier to deal with admittance, because this branches are this branches and this branches are in parallel ok. And when things are in parallel admittance get added when things are in series impedance get added. So, when dealing with parallel combination it becomes easier to write things in the form of admittance ok. So, small note; it is easier to write admittance when elements are in parallel. Similarly, if the elements are in series it is becomes it becomes easier to deal in terms of impedance ok.

Now, from this you see it is very easy to equate the real part of left side to the real part of right side and similarity imaginary part of left and right sides. So, we can write the real part

$$\frac{R3}{R1} = \frac{R4}{Rx}$$

$$J\omega C1R3 = J\omega Cx\ R4$$

$$\text{Cx} = \frac{C1\ R3}{R4}$$

$\text{Rx} = \frac{R4\ R1}{R3}$

So, what we observe? We observe that, the balance equation is easy if we take this bridge. Once again, the goal is to deal with bridges which has a easy balance equation at the important thing that we learn is that if the elements are in parallel it is easy to deal with the admittance. And then just equate the imaginary part and equate the real part that is all it is as simple as that ok.

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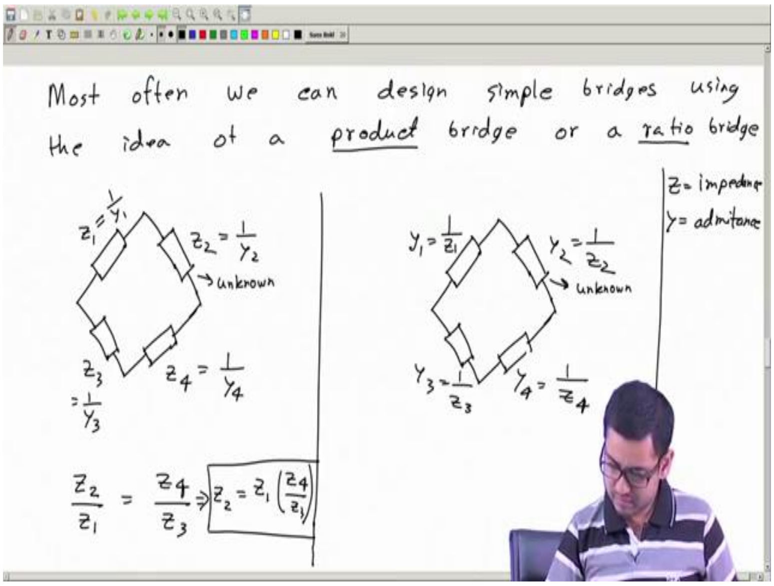

Now, I will tell you a fact a interesting fact most often, we can design simple bridges, simple means whose balance equation is easy and simple to derive. Most often we can design simple bridges using the idea of a product bridge or a ratio bridge.

Now, the balance condition for this bridge you can write in this way say, let me write it as Z 2 by Z 1, same as Z 4 by Z 3, ratio of any adjacent side, you can take it in a anyway Z 1 by Z 3 is same as Z 2 by Z 4 or Z 4 by 3 same as 2 by 1. So, the ratio of adjacent sides, similar ratios of adjacent side should be equal ok. And then if I call one of them say this is the unknown, here also let this be the known ok. So, then we can write the unknown Z 2 as Z 1 multiplied by Z 4 by Z 3, this is the unknown in terms of the known ones.

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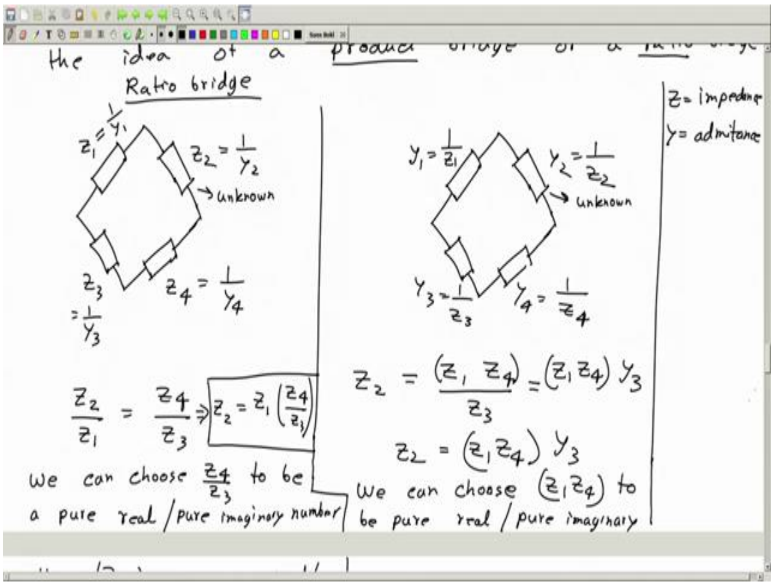

Now on this side, we can again write the balance equation in terms of admittance instead of impedance like, we can write say let us write it first this way, Z 2 = Z 1 Z 4 / Z 3, this is true this is the same equation ok. But, this I can now write in terms of the admittance using, this Y3=1/Z3.

Now, if we choose so, in on this side we choose, we can choose this factor Z 4 / Z 3 to be a pure real or pure imaginary number ok. Then, Z 2 and Z 1 will be a complex number Z 1 complex Z 2 complex, but Z 4 and Z 3 we can choose them to be purely real or purely imaginary.

If we choose them both to be purely real then the ratio Z 4 / Z 3 is a real constant ok. And then the balance equation we will have only Z 2 complex Z 1 complex, that is we can equate the real part of Z 2 with the real part of Z 1 multiplied by this factor and similarly real imaginary part of Z 2 to the imaginary part of Z 1 multiplied by this factor ok. So, if we choose this.

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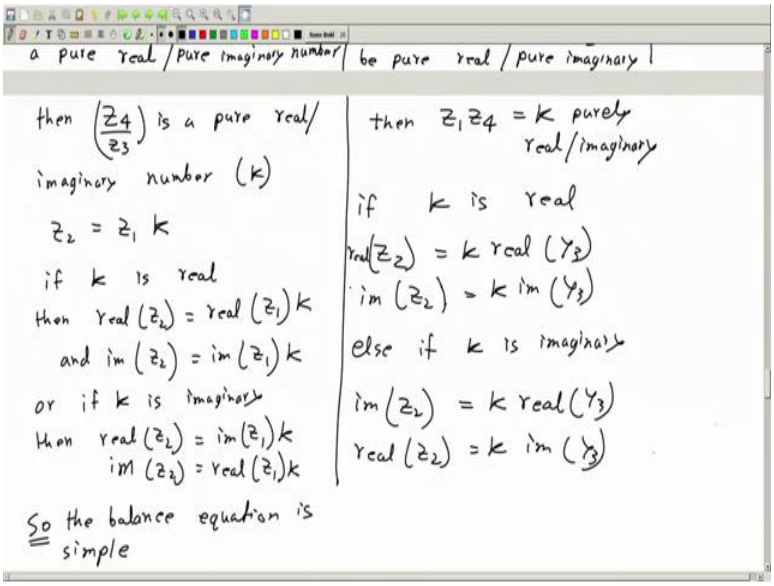

Then, Z 4 / Z 3 is a pure real or imaginary, pure real or pure imaginary number call it k. And then we can write

If k is real

Real (Z2) = real(Z1) k

im (Z2) = im(Z1) k

if k is imaginary

Real (Z2) = im (Z1) k

im (Z2) = real (Z1) k

So, the point is the balance equation becomes easy is simple and easy ok. Therefore, so, our goal was one unknown will be given and three known arms to be design, we can design it in this way keeping this equation in mind. So, we will take if this is unknown we can take Z 4 and Z 3, 2 side by side adjacent arms to be purely real or purely imaginary like pure resistance or pure capacitance or one pure resistance one pure capacitance that is also fine. But, do not mix like series combination of R L parallel combination of R C, no then the balance equation will be difficult ok, that is what we want to avoid.

We will so, this is the way to design, it this is a easy way to design, it then we do not have to remember anything, if we just do it this way. And this we will call the ratio bridge, this form of the bridge we will call the ratio bridge. Why, because the ratio of two adjacent arms Z 4 by Z 3, we will choose it to be pure real or pure imaginary number ok. I think it will be more clear if you just take an example. And the example we have already seen is this ok.

This is the this was the unknown ok and this three are our choices branch 1 3 and 4 was our choice, we have chosen R 3 and R 4 to be pure real numbers, see this is pure real numbers. And therefore, this ratio R 3 by R 4 is a pure real number and that is why the balance equation was so, easy ok.

So, this is this was an example of a ratio bridge. So, this is a ratio bridge where two adjacent sides are chosen to be pure. Now, similarly another type of bridge with that we can design or configure is keeping this equation in mind; Z 2= Z 1 Z 4 Y 3. Now, we can choose any guess? Z 1, Z 4, this factor to be pure real or pure imaginary ok. If we do so, then this factor Z 1 Z 4 is equal to k is a pure real or imaginary number is purely real or imaginary.

And if k is real

Real (Z2) = k real(Y3)

im (Z2) = k im (Y3)

if k is imaginary

Real (Z2) = k im (Y3)

im (Z2) = k real (Y3)

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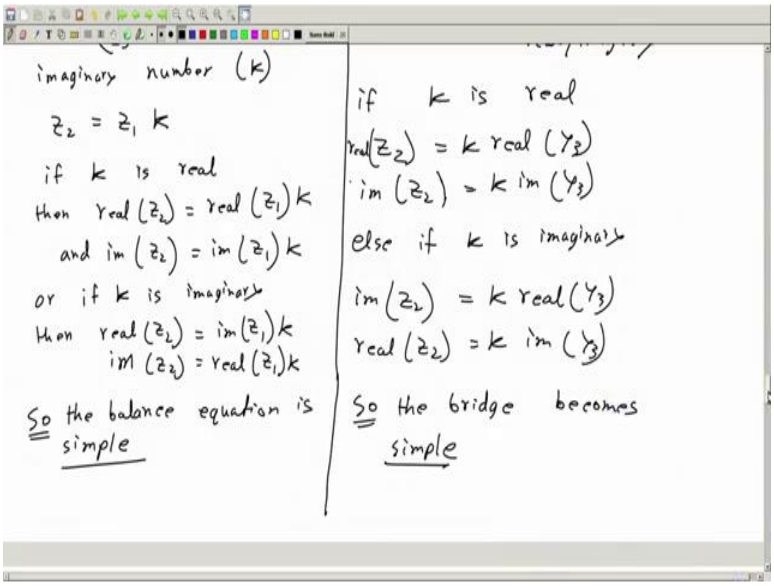

The balance equation becomes easy; once again the bridge or the balance equation becomes simple ok. So, these are the two forms of bridge, this we will call the ratio bridge, this we will called a product bridge, these are the two forms of the bridge that we can often use to design bridges ok. Now, we will take an example, with which this idea will be quite clear I guess.

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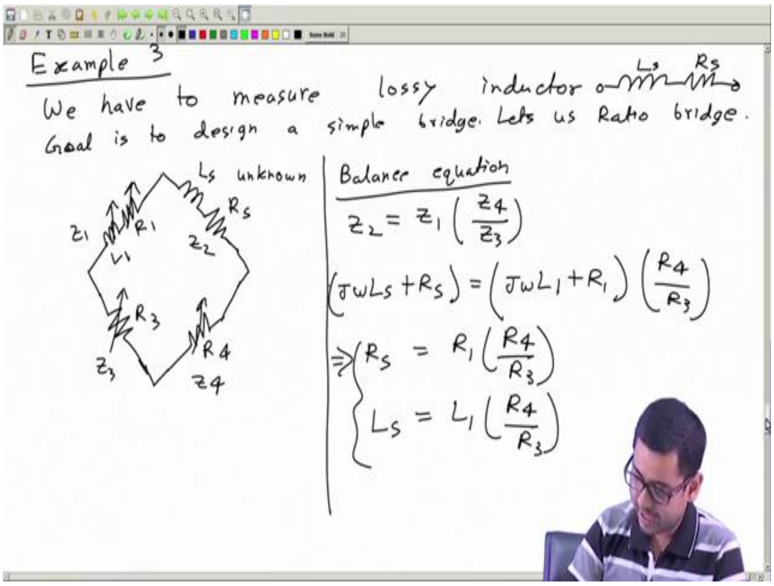

So, now we have to measure say practical or lossy inductor, which we can always denote as a series combination of L s R s ok. So, we have to measure this one and we have to design the bridge. Now, let us take let us draw this bridge.

So, let us first put this unknown one in any branch you like any branch, I like this branch ok. So, this is unknown L s R s, we can call L x also x for unknown, this is unknown and I want to design a simple bridge ok. The goal is to design a simple bridge. And let us use ratio form or ratio bridge, this is my choice I want to use a ratio bridge. What I can do? I can just do this put a known L 1 R 1 variable put a variable R 1 2 3 and R 4 variable ok.

So, then let us see how easy the balance equation is ok. So, we will write this as L s say this divided by this so, J omega L s. So, let us write it this way directly from here, Z2= Z 1 Z4/Z3 ok. So, let us start from there. Z 2 this is two this is Z 1, Z 2, Z 3, Z 4; Z 2 is same as Z 1 multiplied by so, this is 4 by 1, 4 by 3, 2 by 1, 4 by 3 yes.

$$Z2 = Z1\,\frac{Z4}{Z3}$$

$$(J\omega Ls + Rs) = (J\omega L1 + R1)\,\frac{R4}{R3}$$

$$Rs = R1\,\frac{R4}{R3}$$

$$Ls = L1\,\frac{R4}{R3}$$

So, once again the point is to design the bridge in a simple way. So, that the balance equation becomes easy to derive. I just can look at the bridge and also can sometimes write the bridge in one line ok.

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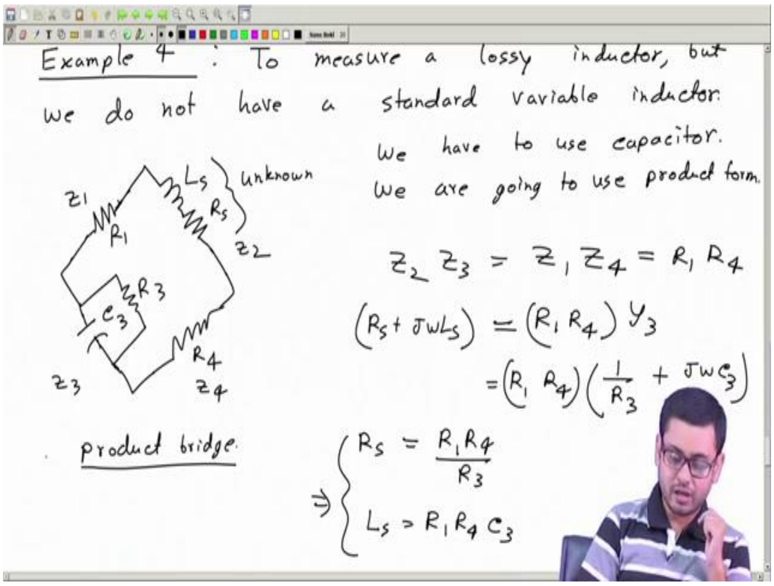

Now, let us take a last example in this class to measure a lossy inductor, but we cannot or we do not have a standard means known standard variable inductor ok. So, in the previous problem we use the variable standard; standard means known here inductors so, L 1 R 1 was known and variable that was used.

Now, we put a constant that we do not have a variable standard inductor that is the constant, we do not have it in the lab how do you do it? This is not a funny question, this is a practical question, because a standard inductors are difficult to prepare difficult to have we will talk about a later why right. Now, you accept that it is difficult to have a standard variable inductor ok.

So, this is a more challenging question, let us do it. So, this is the bridge with four arms and let me put once again the unknown one first. Anywhere you like so, this is unknown put it anywhere you like among the four branches. And now the task is to find, what they should be ok. Now, we cannot use a variable L R as we did here that is not available.

So, what can we do, we have to use can we use just only resistance simple question, can we use only resistance? Like this, no we cannot because then the angle condition is not satisfied, because angle of this is 0 angle of this is 0. So, this plus this is 0, but this angle is something between 0 and 90 this is 0 so, their sum is between 0 and 90.

So, angle condition is not satisfied. So, this is not possible. So, we cannot balance it using resistance only ok. So, what can we do, we cannot use inductor. Can we use capacitor? Because capacitor gives non zero; non zero angle resistance gives only 0 angle. So, can we use capacitor? Let us try let us try by putting a capacitor here let me keep R and R here and let me try to put a capacitor ok.

So, now will this help no once again this will not help, why? Because, some of these two angles is between 0 and 90 ok, but some of these two angle is minus 90, we can never get a balance. So, this is also not helpful, but now we can try another thing, let us try this, let us put the capacitor here and a resistance here. Then, these two angles this plus this is 0 and this is minus 90 this is plus something and minus plus can cancel each other and become 0. So, these two can become 0 for that I if this is not purely 90. So, this cannot be purely 90. So, I may use this ok.

Now, this is between 0 and 90 positive, this is between 0 and minus 90 negative and if I adjust this two, then I can make this the angle of this equal to this. So, they will cancel each other and sum up to 0 and these two they sum up to 0. So, this is possible ok, it is good. So, this is the easy bridge.

And this is also you observe is going to be a product bridge, because you see the sum of two opposite arms here are chosen to be pure call this R 1 2 3 R 4. So, you see according to our condition you see sum of two opposite arms, we have chosen them to be purely real these two R and R ok. So, this is going to be a product bridge ok.

So, that is what we have seen I mean product bridge and ratio bridge this are these are the easiest bridge to design ok, good great. So, we are going to design a product bridge. And so, we have to use capacitor, because we cannot use inductor. And therefore, we are use going to design or use a product form ok.

Z2 Z3= Z1 Z4 = R1 R4

$$(J\omega Ls + Rs) = R1\ R4\ Y3$$

$$= R1\ R4\ (\frac{1}{R3} + J\omega C3)$$

$$Rs = \frac{R1\ R4}{R3}$$

$L_s = R_1 R_4 C_3$

So, the trick is either use a product form of a bridge or a ratio form of a bridge then the balance equation become so, easy you can find it in two lines. And another trick is that when you are using admittance like here in product form use the elements in parallel, then it is easy to write it, express it and if you are using it in impedance form then use the elements in series ok.

So, you see how easy this balance equations are to derive you do not have to do a long one page calculation, if you design the bridges in this way product bridge and ratio bridge. Let me conclude this video we will take more examples in subsequent videos, the idea here is this topic AC bridge is very boring. Often it seems like we have to remember a lot of bridge no, all bridges are designed keeping the simplicity in mind you just design the bridges either product form or ratio form they become very easy.

We will we have seen some examples, we will do more practice in following videos in problem solving and this idea will be clear. Do not memorize, do not remember anything design your own bridge, then this boring topic will be slightly funny slightly interesting.

Thank you.