Electrical Measurement And Electronic Instruments
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Lecture - 25
Kelvin Double Bridge

Welcome back. We are discussing about measurement of resistance and the theme of this chapter is primarily the fact that measurement of low and means very low and very high resistance is not that easy, we need special precautions while doing that. So, in this video we will see what are the problems that we may encounter if we want to measure a small resistance with a Wheatstone bridge and what is the remedy. The remedy is called Kelvin's Double Bridge, we will also see the principle of Kelvin's double bridge ok.

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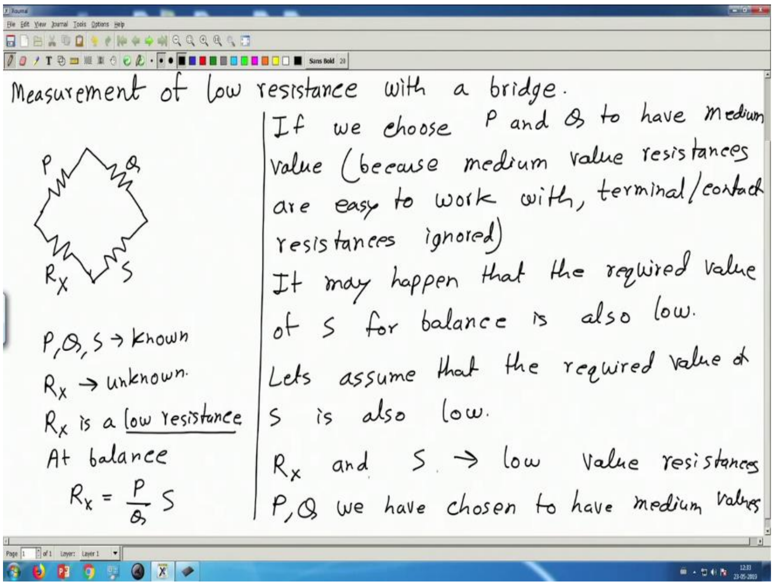

So, let us begin. Suppose I have a Wheatstone bridge with which I want to measure a small resistance. So, the title is measurement of low resistance with a bridge. So, we shall first see Wheatstone bridge. Now so, one among these 4 resistances definitely is the unknown one which you want to measure say this is the unknown resistance call it R X, X for unknown. So, this is the unknown resistance and all other resistances are known call them P Q R and S ok.

So, let us write P Q S are known R X; X for unknown is unknown and R X is a low resistance ok. Now if R X is a low resistance then we may need possibly another resistance which is small why because at balance we will have RX= $\frac{PS}{Q}$ and we will run into problem if any resistance is small. So, we can choose possibly say P and Q to be of higher or medium value ok.

So, we may choose. So, if we choose P and Q to have medium value because medium value resistances are easy to deal with because contact resistance terminal resistance are negligible compared to the compared to the main resistant itself. So, because terminal or contact resistances can be ignored, but if the resistance itself is small then contact resistances are not ignorable. So, we may choose P and Q to be of medium value, but R X is of low value small value.

So, therefore, it is possible that S is also low value resistance that is required to get this balance. So, it may happen that the required value of S for balance is also low it may or may not happen. So, it is it may be possible that with a medium value S this quantity P S by Q comes out to be a of a low value its then its fine not much issue, but it may happen that this is a. So, medium by medium it may be a medium value. So, S we need possibly a small value. So, that the right hand side is again a small resistance because left hand side is a small resistance. So, we may or may not need a small value of S to get this balanced ok.

So, let us assume let us assume that the required value of S happens to be small is also small also low, thus what you have? R X and S these two are low value resistances, R X is definitely low value resistances because we are measuring a low value resistance S may or may not be of low value, but let us assume this comes out to be of a low value and P and Q we have chosen to be to have medium value.

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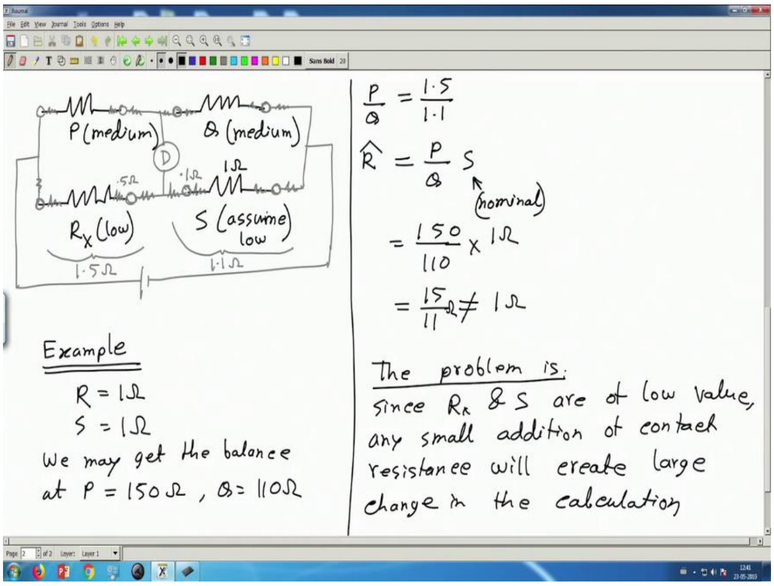

If so, we have let us draw the 4 resistances, P medium by choice, Q medium by choice, R is low because we are measuring low value resistance we have no choice S assume that it also a small. Now we have to form the bridge by connecting these 4 resistances with wires and to their terminals. So, let this be the terminals and we connect these 4 resistances with wires like this in the form of a squared and say then we put the power supply across two opposite terminals maybe this two and say we put the galvanometer or the detector between the other two terminals which is here ok.

So, let or let me just connect this fine. So, I can connect it like this ok. Now this is a low resistance and this is assumed to be low these two are medium value resistances therefore, the contact resistance due to this terminal which causes some resistances here etcetera. So, this resistance the small resistance can be ignored because P is of much higher value this also can be ignored because P is of much higher value similarly this contact resistances can be ignored because Q is of much higher value, but here this contact resistances cannot be ignored because R and S are small or low value therefore, this contact resistance is getting added to this actual value of R and the value of S. Say if now let us take say an example say R is 1 ohm small S is also 1 ohm and say this contact resistance here it comes out to be say like 0.5 ohm and here it comes out to be.

So, this together everything contributes to a 0.5 ohm resistance in series with R and say here we have a 0.1 ohm resistance in series with this with S ok. So, this side will have a total resistance of 1.1 ok. So, here 1.1 ohm here we will have 1.5 ohm and say at. So, so we may get the balance at say wait P equals call it 150 ohm and Q equals 110 ohm ok.

$$\frac{P}{Q} = \frac{1.5}{1.1}$$

R hat= $\frac{PS}{Q} = \frac{150\ X\ 1}{110}\ ohm \neq 1\ ohm$

we know the nominal value of S is 1 ohm because we do not know how much this contact resistances are, we have no idea about this contact resistances these are variable I mean they can be anything we have no control over these contact resistances, we know that the nominal value of S is 1 ohm we have no idea about any of these contact resistances.

So, therefore,. So, we will put the nominal value which is 1 ohm and this will come out to be 15 by 11 which is not same as this value of R which is not same as 1 ohm and this problem is due to this contact resistances ok. Now these contact resistances do not create a much problem because even if we had added a small quantity to 150 like say 150.5 and say maybe 110.3.

These small quantities do not create any problem because the ratio remains almost same. So, this contact resistances beside these medium value resistances they are not of much problem, but this contact resistances are definitely creating problem because this actual value of the resistances they that itself is small, RX is small S small. So,. So, a small change will create a large problem ok. So, this is the problem. So, the problem is since R and S are of low value or small value any small addition of contact resistance will create large change in the calculation ok. So, that is the problem.

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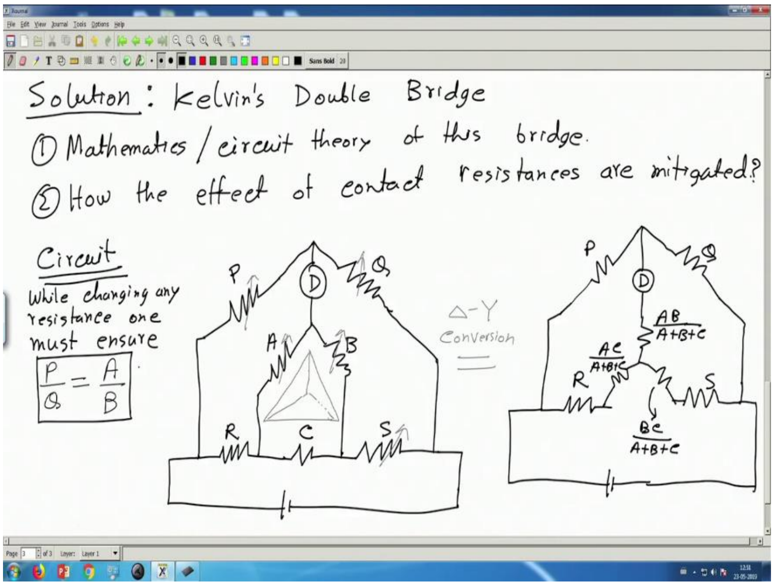

Now, if there is a problem there should be a solution too and the solution is called the Kelvin's double bridge ok.

So, while talking about Kelvin's double bridge, we will divide the talk in two parts ok. So, we will first talk about the mathematics behind Kelvin's double bridge ok. So, we will first talk about the mathematics or circuit theory or the circuit theory whatever you call of this bridge and then we will talk in more detail how the effect of contact resistances or terminal resistances are avoid or I mean are mitigated or how the problem is problem of contact resistance is solved. So, these two topics we will cover while talking about Kelvin's double bridge.

So, first the circuit theory; the circuit. So, this circuit looks like this, it has six resistances ok. So, the bridge looks a bit complicated, but once you understand this bridge the function of each resistance, then you will see it is not difficult to remember this bridge, but right now let us start with this circuit being given.

Let me call this resistance as P this as Q this as R this as S call this as A, call this as B and call this as C ok. So, the power supply is here and the detector is here. So, this is the bridge now we need to find the balance or null condition of this bridge what is the balanced condition when this detector will show zero deflection? To find that we will apply a star delta conversion in this region here. So, consider this delta A B C, consider this delta and we will apply a star delta conversion. So, we will make an equivalent star circuit for this.

So, let us apply star delta conversion or delta to star in this case delta to star conversion. So, what we will have? We will have let us draw first everything else P Q this is P this is Q here we have the detected D and then here is the R and here is the power supply. So, just one small note at this moment please do not think why is this bridge like this. Do not think about how this bridge is obtained focus on the fact that this bridge is given to us and we are finding the balance condition using our knowledge of circuit theory ok. Do not concentrate on the question why or how have we got this complicated bridge, do not go into that question we will talk about that later when talking about this ok.

Right now this is just a straightforward circuit simplification problem ok. So, this is the circuit given, we are now to simplify this circuit using a star delta conversion. So, this delta ok. So, this delta part will now become a star ok. So, this is. So, this A B and C they are converted into three star connected resistances this delta becomes this star ok.

Now, what will be the value of this three resistances? So, the value of this resistance; that means, this branch is given by this is A this is B. So, if you recall the formula for delta star conversion, this is these multiplied by this divided by the sum of this three ok. So, this branch will be $\frac{AB}{A+B+C}$.. This resistance will be how much? This is this branch this will be $\frac{AC}{A+B+C}$. So, this is this. So, this will be $\frac{BC}{A+B+C}$

So, this is the equivalent circuit. Now one important fact in this bridge in this Kelvin double bridge when whenever we change any resistance, so, these resistances are variable. So, you can change P, you can change Q all these resistances you can change whenever you change any resistance change it in a way such that the ratio $\frac{P}{Q} = \frac{A}{B}$ always. So, if you change P without changing Q; that means, P by Q is changed you have to change A or B at the same time. So, this is maintained ok.

So, let me call it while changing any resistance one must ensure this condition. So, if you change P at the same time you have to change A so, that $\frac{P}{Q} = \frac{A}{B}$ or you can if you change Q you may change B so, that $\frac{P}{Q} = \frac{A}{B}$. You cannot change only P or only Q without changing A and B you have to change this together ok. So, this is what you have to maintain while using this bridge. So, this is a condition that must be satisfied that must be maintained ok.

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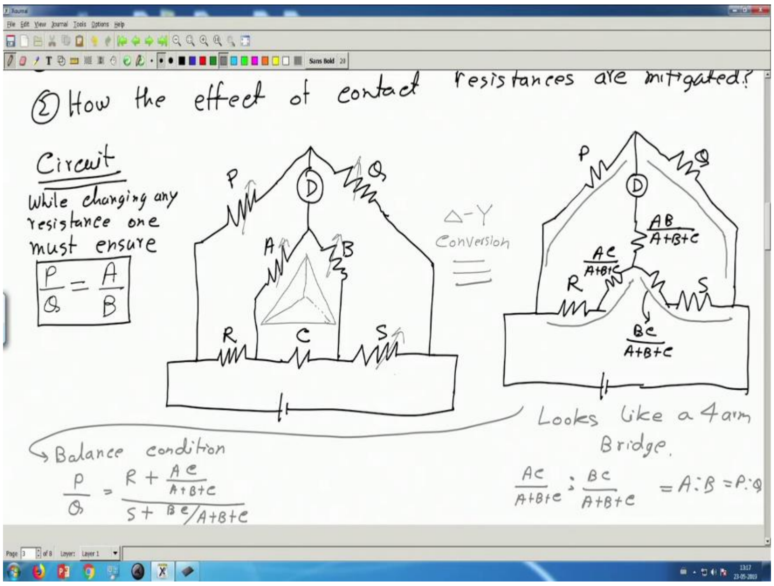

Now,. So, this is the equivalent circuit ok. So, this is the equivalent circuit of this bridge and now you see that this bridge looks like a Wheatstone bridge, it has 4 arms it has. So, this looks like a Wheatstone bridge 4 arm bridge what are these 4 arms? See this is one arm, this is another arm and this two together you can think as another arm and this two together you can think as another arm 1, 2, 3, 4. So, these are the 4 arms of a 4 arm bridge we have the detector between two opposite terminals here and here and the power supply between this two terminals. So, this bridge is now equivalent to a 4 arm bridge and therefore, we can now write the balance condition for this bridge using the balanced condition of a Wheatstone bridge which is so, for this bridge now we can write the balanced condition.

$$\frac{P}{Q} = \frac{R + \dfrac{AC}{A + B + C}}{S + \dfrac{BC}{A + B + C}}$$

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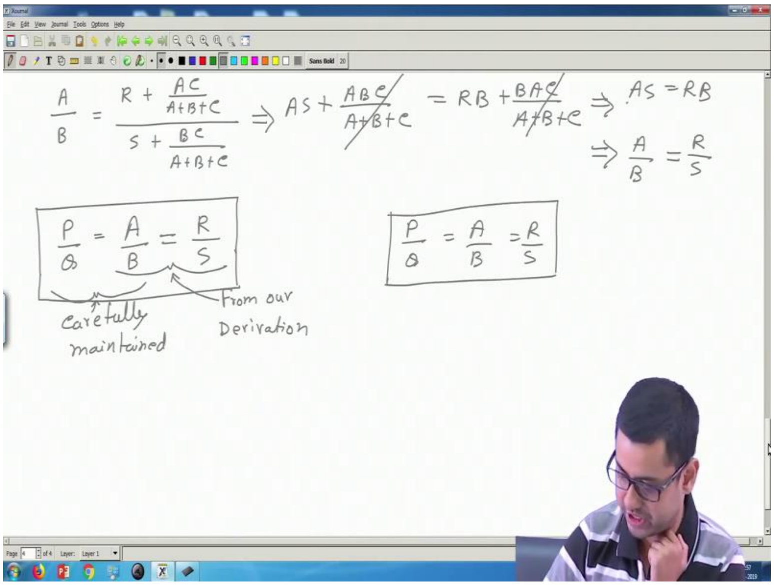

$$\frac{A}{B} = \frac{R + \frac{AC}{A+B+C}}{S + \frac{BC}{A+B+C}}$$

AS = RB

$$\frac{A}{B} = \frac{R}{S}$$

So, $\frac{P}{Q} = \frac{A}{B} = \frac{R}{S}$

So, this condition is derived this part is derived so, from our derivation ok. So, this is the balance condition for the this particular bridge Kelvin's double bridge ok. So, this is the balance condition ok. So, this is the essential circuit theory of this bridge ok.

Now, let us see let us investigate this question, how the effect of contact resistances can be avoided.

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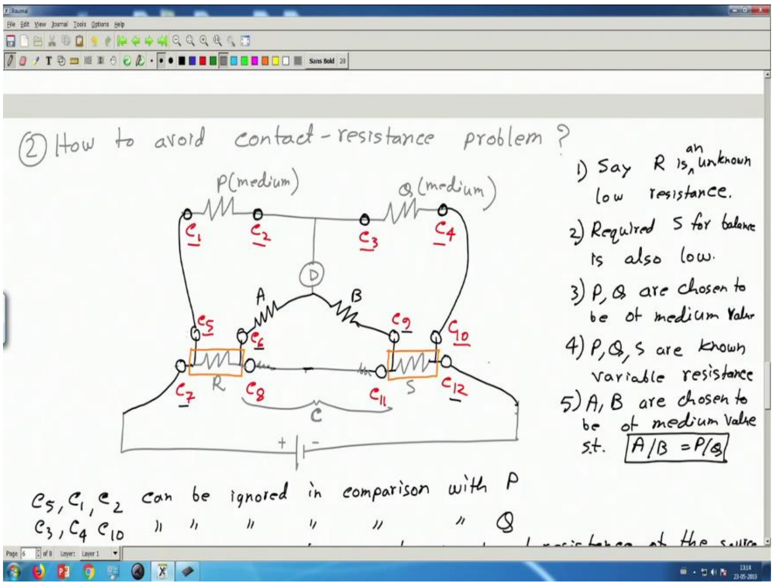

So, how to avoid contact resistance problem? We will start by drawing say Wheatstone bridge. So, this is a Wheatstone bridge with call this resistances P Q R S this is a 4 arm bridge ok.

We can have the detector here and the power supply here ok. So, this is a Wheatstone bridge. Now say R is unknown and a low resistance R is an unknown low resistance and say the required value of S for the balance is also low ok. So, R is unknown required is required is for balance is also low P Q are chosen to be of medium value ok.

And P Q S are known and variable. So, R is unknown, P Q S are known and variable and you can choose P and Q to be of medium value and S suppose its required S to be small ok. So, now, this contact resistance is here this can be ignored no problem. But to avoid the contact resistances of R and S let us use our use 4 terminal resistances as R and S. So, let us use 4 terminal resistance. So, we will connect it this way. So, these are the 4 terminals and we will use ok.

Now, let me connect it here and this here ok. So, we essentially use Kelvin’s double bridge and so, what we will do? We will connect two more resistances A and B and this we will sort and we will connect it here ok. So, we have started with a Wheatstone bridge and modified that into a Kelvin’s double bridge and we are using this as 4 terminal resistances. So, we are using. So, this is in a box this is in a box 4 terminals are outside ok.

Now, so, let us name the contacts these contacts as call these as C 1 C 2 C 3 C 4 ok. So, this terminal can be connected maybe here or here or maybe at like this then C 1 C 2 C 3 C 4 what else C 5 C 6 C 7 C 8 C 9 C 10 C 11 C 12. Now this are the problematic contact resistances this may be this can create some problem ok.

So, now let us analyze what kind of problem can this contact resistances create observe that this C 5 and C 1 C 5 C 1 and C 2 these are small resistances in series with P. C 5 C 1 C 2 these are small resistances in series with P. So, and P is a medium resistance that is a much higher resistance than these contact resistances. So, this is medium this is also medium.

Therefore C 5 C 1 and C 2 C 5 C 1 and C 2 can be ignored in comparison with P because these are in series with P and P is of much higher value. So, even if you add this small resistances to P the ratio say P by Q is not going to change P by Q or P plus these resistances by Q are going to be same. So, this resistances can be ignored in comparison to P similarly these resistances say C 3 C 4 and C 10 can be ignored in comparison with Q is of a larger value than this contact resistance. So, you can ignore this thing ok

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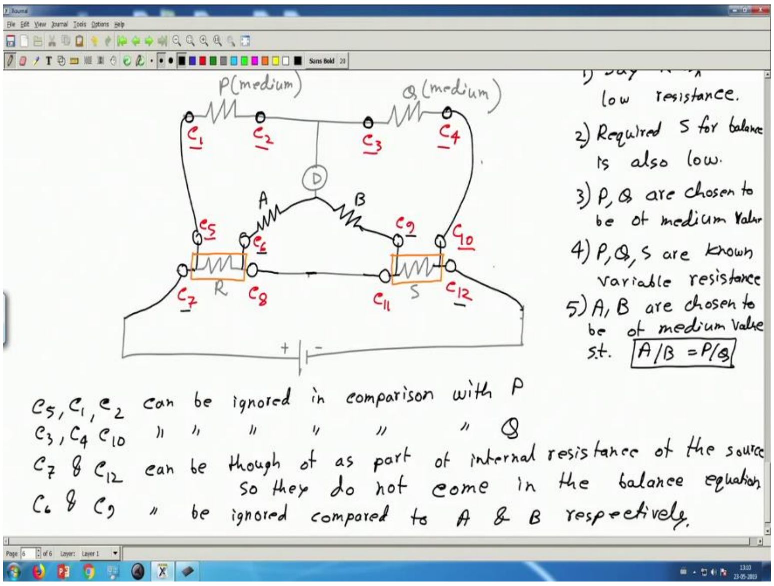

And now then. So, this is solved, this is solved, solved, solved, solved this is not going to create much problem now let us see C 7 and C 12 ok. So, let us talk about C 7 and C 12. C 7 and C 12 can be thought of as a part of internal resistance of the source ok. So, the source also can have some internal resistance and we know that in our balance condition this resistance does not come in the equation. So, this resistance C 7 and C 12 we can think as a part of this source resistance which do not come in the balance equation ok. So, C. So, they do not.

So,. So, they do not they do not come in balance equation come in the balance equation ok now. So, this is solved this is solved C 6 and C 9 now C 6 you can think it is in series with A you can choose A to be of medium value we should choose A to B of medium value and similarly B we should choose to be of medium value. So, let us write A and B are chosen to be of medium value and such that of course $\frac{P}{Q} = \frac{A}{B}$.

So, this is what we maintain now if we change A, we change P if we change B we change we can change Q. So, that this ratio is maintained and A by B A and B are chosen to be of medium value larger value. So, C 6 and C 9 can be ignored in comparison to A and b. So, C 6 and C 9 can be ignored compared to A and B respectively ok.

So, this is not a problem this is also not a problem. So, see almost any of this 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 they do not create any problem much problem 7 and 12 they do not come in the balance equation at all and other resistances they are in series with larger resistances A B P Q. So, they are negligible, but C 8 and C 11 what about them?

So, what about C 8 and C 11th? So, C 8 and C 11 they will have some resistance this resistance call. So, they will have some resistance call this call all these resistances together as C ok. So, C 8 C 11 all these contact resistances together you call a one resistance they are in series. So, you can call them as a common resistance c and you see that this is nothing, but our Kelvin’s double bridge let me zoom out and show you the two circuits.

So, this is the Kelvin double bridge we have say you see P Q R S between R and S we have some unknown we have some resistance c here also P Q R S between R and S we have C and across C, we AB across C we have this AB. So, this two bridges are exactly same and then in this bridge when we have derived the balance equation C does not come in picture at all whatever the value of C it does not come in the balance equation ok.

Therefore in this bridge as well whatever is the value of C 8 C 11 the contact resistances they do not come in picture at all.

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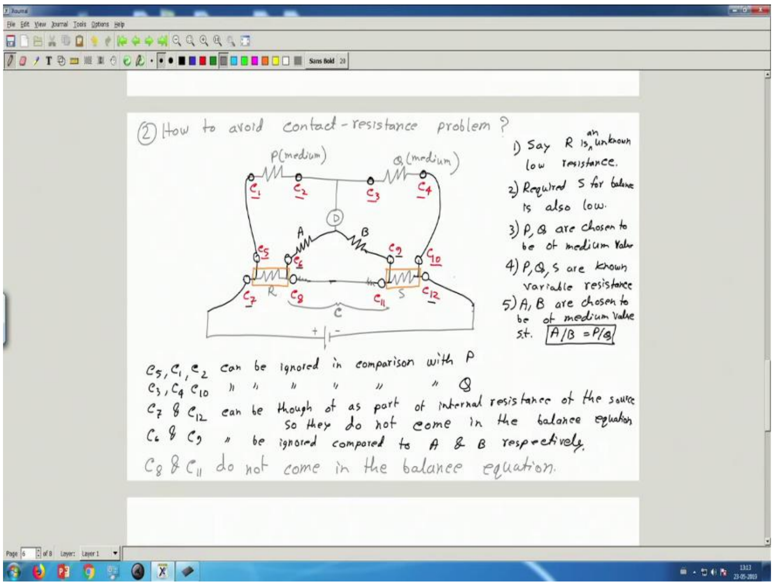

So, this C 8 and C 11 C 8 and C 11 do not come in balance equation in the balance equation and that is so, because of the principle of this bridge this circuit and this is actually true because we maintain always $\frac{P}{Q} = \frac{A}{B}$.

If I do not maintain this ratio then C may not get eliminated then C may come in the balance equation, but if we maintain this ratio then C has no effect and this is how the all contact resistances can be mitigated. So, this is the Wheatstone bridge and once again let me repeat before I conclude that in. So, we have these are 4 terminal resistances.

If we have R and S as small resistance we must use 4 terminal resistance, once again 4 terminal resistances are becoming so, important and we have to connect it in this way, so, that all contact resistances they either go in series with larger resistance and that effect is negligible or they come in this branch of the Kelvin bridge which does not have any effect in the balance equation.

So, this is the beauty of this bridge. So, whoever has thought of this bridge he might be a genius ok. So, he has ignored or he is he is able to mitigate the effect of all contact resistances by this intelligent connection. You see this is not a new scientific discovery this is nothing like equal to mc square, but this is an intelligent connection of wires. So, that these contact resistances are carefully avoided ok. So, this is the beauty of this engineering. Let me conclude this class just by mentioning one small observation about this circuit. So, if you look at this circuit ok.

So, if you look at this circuit. So, one small observation these two resistances A and B they are actually dividing this contact resistance C in the same proportion as A and B why let us see here. So, after this delta to star conversion, the resistance which is here on the left side is $\frac{AC}{A+B+C} : \frac{BC}{A+B+C}$ = A:B = P:Q

So, in a sense this extra I mean this is the second bridge that is I mean this is why its called double bridge it has one bridge like this Wheatstone bridge and another double second bridge which is dividing. So, this AB is dividing this contact resistance in the same ratio as A is to B. So, this part is becoming proportional to A and this part is becoming proportional to B.

So, this is like a I often call it a resistance divider. This is my terminology like we have potential divider this is like a resistance divider. So, this is dividing this contact resistance C in the same ratio as A is to B and therefore, the even in the balance condition P by Q R by S same equation is holding true because these extra zero resistance which is due to the contacts terminals which can get either added to R or S, but this is being divided in the same proportion as R and S therefore, the balance equation is not changed ok. So, this is like a resistance divider.

Thank you.