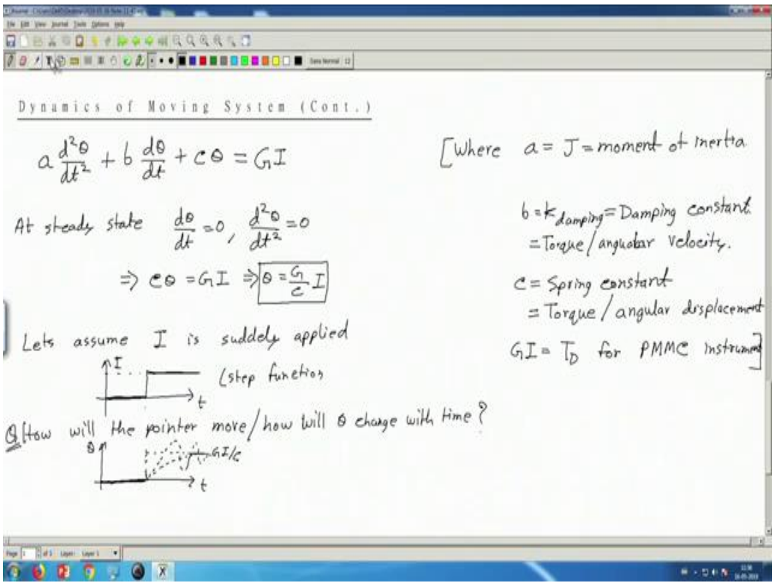
Electrical Measurement and Electronic Instruments
Prof. Avishek Chatterjee
Department of Electrical Engineering
Indian Institute of Technology, Khargpur

Lecture - 11
Dynamics of the Moving Coil and Damping (Contd.)

Welcome again. So, we are discussing about the Dynamics of the Moving System; that means, the pointer, the coil, the frame all these things which move together in an measuring instrument like an emitter or galvanometer.

(Refer Slide Time: 00:42)



So, in our last class we have written the equation for the dynamics, where we had this J $\frac{d^2\theta}{dt^2}$ this is nothing, but the like the inertia times the angular acceleration plus we had the term $k_{damping}$; damping constant ok. So, let me use a simpler notation for ease of writing, let me just write it as a, the next term as b $\frac{d\theta}{dt}$ ok.

So, where a is actually J moment of inertia ok. And, then b is damping constant, then plus let us write c times θ , so where c is basically the spring constant ok. So, this is equal to k according to our previous notation ok, so this is spring constant. And this is equal to T_D that is deflecting torque and if you take a particular example of PMMC instrument this is equal to GI. So, GI is equal to T_D deflecting torque for PMMC Instrument.

So, let us take the example of the PMMC instrument. We can consider any other instrument we will have similar modifications say for example, if you take moving iron or electro dynamic instrument, we have to make this I square and similar modifications can be done, but the basic idea will remain same ok. So, the basic idea is that this is the inertia times the angular acceleration, this should be same as the sum of all the three types of forces damping force, spring torque and the deflecting torque ok.

And, then so, let me also write the dimensions of this variable. So, c is like torque per unit angle or angular displacement. So, it may be like a new say Newton meter per degree or per radian depending on whatever unit you choose. Then, these the unit are the dimension of b damping constant what will it be it is again torque, it is some torque per unit angular velocity. So, this will be torque, but you need angular velocity.

And so, the and this is the dimension of this moment of inertia is like it is like mr^2 you know from the physics, so it is like kilogram I mean SI unit will be kilogram meter square something like that ok. So, this is the; this is the expression or the dynamics that governs the motion or movement of the moving coil at any instant, at any moment.

Now, at steady state; that means, we can also say that this is at equilibrium state where the pointer stops. So, we will have $\frac{d\theta}{dt}$ is equal to 0, you will have also d acceleration equal to 0, at steady state when the pointer stops at completely at some position, this will also be equal to 0 and then, we will have c is equal to GI or θ is equal to $\frac{GI}{C}$.

So, this is nothing, but the sensitivity G by c will be the sensitivity of the instrument. Now, the question that we want to investigate in this video is that, how will the pointer move I mean will it if I apply a current will the pointer reach, it's final position at once or will it oscillate. And, then slowly we stop at that position or then if it reaches that position at once will it reach quickly or will it go there slowly. So, all these questions you want to find out.

And, for that basically what we have to do? We have to solve this differential equation. So

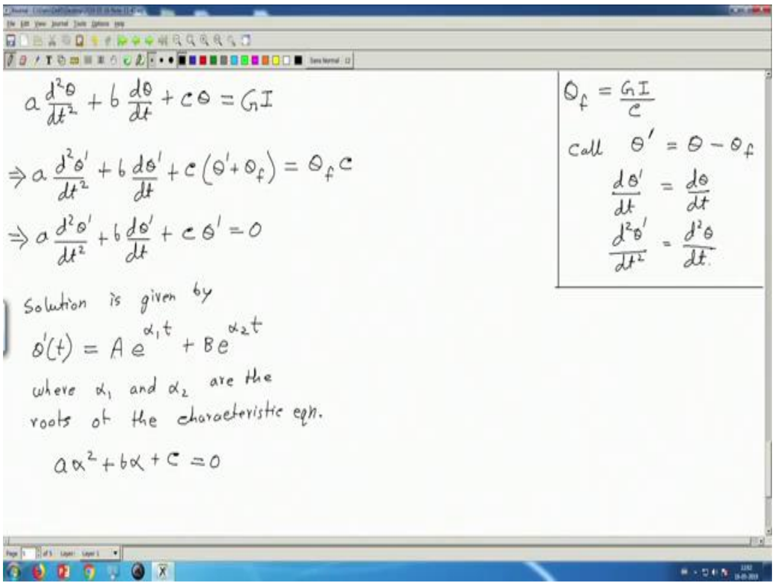
$$,a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\theta = GI$$

this is a differential equation, second order differential equation we have to solve this equation. So, let us say assume I is suddenly applied ok. So; that means, I was 0 for some time initially and then it becomes like this, so this is a step function ok. So, this is time, this is current I and this value of I is applied suddenly ok.

So, then how will the pointer move? So, how will the pointer move or in other words, how will θ change with time? So, this is the question we want to find out ok. So, maybe that θ will, so θ initially should be 0 ok. So, this is let me draw once again θ here, say θ versus time t. So, θ should be 0 up to this position because there was no current so there should be no deflection.

So, it should be 0 up to this point of time and then it will finally, reach some value here which is given by this, so this is θ_f. So, this will be $\frac{GI}{C}$, but will it reach there at once like this, or will it reach there like this or will it reach there after some oscillation? So, this is the question that we want to find out. So, for that we have to solve $a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\theta = GI$ this differential equation and you can solve this differential equation in many ways.

(Refer Slide Time: 09:21)



So, I will adapt a simple way here, so $a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\theta = GI$ is the differential equation we want to solve ok. So, what I will do, I will for simplicity of solution, I will first call say θ_f which is final value of θ, which we know is given as. So, the final value will be given by the condition when these two terms are 0, so c θ_f = G I.

So, we will have G I by of c this will be the value of θ_f final value of θ ok. Now, I will subtract his θ_f from θ, so let me call the new variable θ', which is equal to $\theta - \theta_f$ So, then let us take a derivative. So, we can write $\frac{d\theta'}{dt}$ this will be equal to $\frac{d\theta}{dt}$ ok. So, you are redefining a very new variable and then the double derivative of it will be same as this ok. And therefore, now let us put these values in $a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\theta = GI$ equation ok.

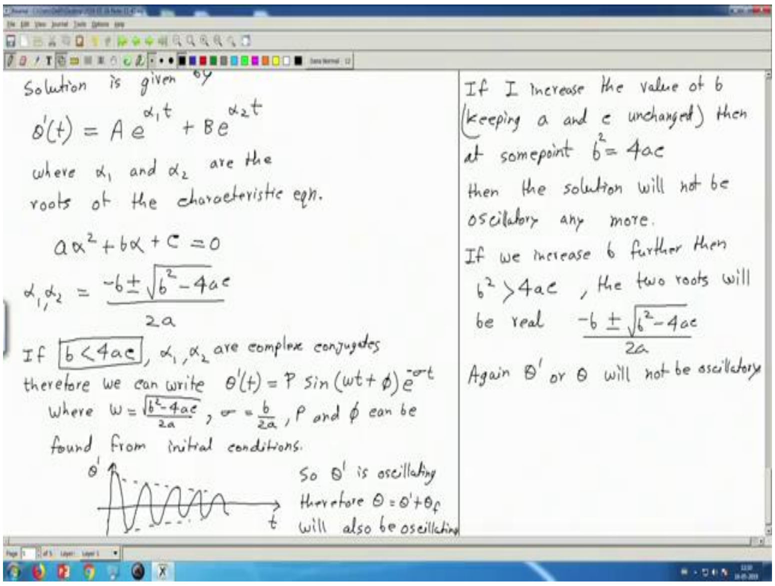
So, now, here we write a $\frac{d^2\theta'}{dt^2}$, because this and then these two terms are same from here, then plus b $\frac{d\theta'}{dt}$ c here in place of θ , I will write θ is equal to(θ'+ θ_f). So, let us write θ'+ θ_f and GI we know GI from this we know GI is equal to θ_f time's c. So, here we write θ_f times c ok. So, this will imply this c θ_f will cancel from both sides. So, I will have a 0 on the right hand side. So, $a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\theta = 0$ is a homogeneous differential equation, where we I have right sides equal to 0.

So, this is easier to solve and we know the solution of this differential equation is given by θ', which is a function of t of course, ok. So, everywhere here I can write θ as a function of t, θ', also function of t, but I am not doing it for clarity of writing I do not want to make it clumsy. So, here so, you can write it with this is also function of t this is also function of t θ if f is constant final value of θ is a constant.

$$\theta'(t) = Ae^{\alpha 1\, t} + Be^{\alpha 2\, t}$$

So, you will get 2 values of alpha; alpha 1 and alpha 2. So, you put them here that will be the solution and where a and b are two arbitrary constants which you can find from the initial condition or the initial value and of θ and the speed etc. So, now what will be these values of $\alpha 1$ and $\alpha 2$?

(Refer Slide Time: 15:06)



So this is a quadratic equation,

$$a\alpha^2 + b\alpha + c = 0$$

$$\alpha 1, \alpha 2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

this is simple. Now, if b < 4ac, then $\alpha 1$ and $\alpha 2$ will be complex and they are complex conjugate of each other ok.

And therefore, you can write this solution you can rearrange this terms a bit and you will find the solution. So, this will be of the form something like, some constant called how do you want to call it a b c, let us call d or let us called P ok times some sin. So, it will be of this form

$$\theta'(t) = P\sin(wt + \emptyset)e^{-\sigma t}$$

now this θ is not, some phase phi ok. So, you can rearrange the terms and it will be of this form ok. And, where omega will be this complex part, I mean the imaginary part ok. And, we will also have another term, so this will get multiplied by e to the power some call it sigma t or minus sigma t. So, and this sigma will be this term this part ok, the real part $\frac{b}{2a}$.

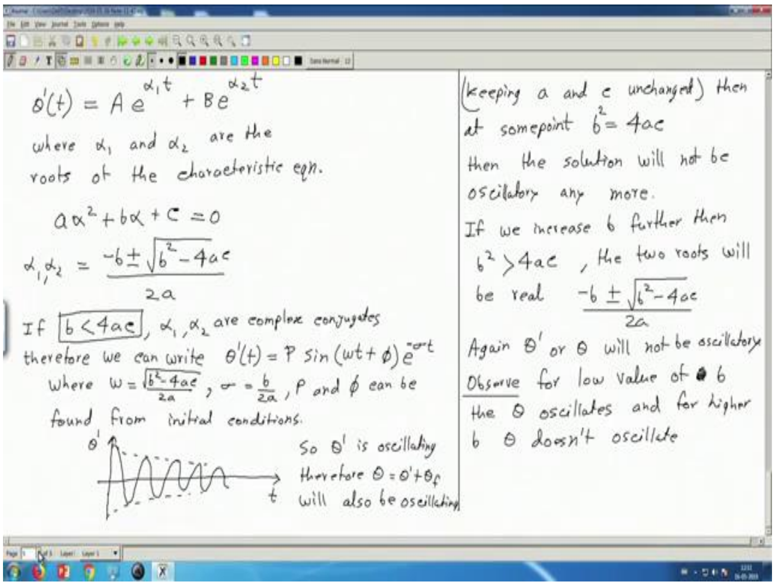
So, I am assuming that you know how to solve this a homogeneous second order differential equation also so you can find the solution of this form. And, the value of P and phi this two can be found from the initial value of theta and derivative of theta ok. P and phi can be found from initial conditions ok.

So, if this is so, b < 4 ac we see that the solution is oscillating, it has a sine term, sin(wt) and then the amplitude decays with because of this exponential term. So, θ' ok, so it is it will look like maybe like this and so on. And, the initial value will depend on this phi, which will depend on the initial value of θ'. So, this is θ' versus t and we know that $\theta$is nothing, but θ'+θ_f ok. So, if θ' is oscillating then θ is just theta prime plus some constant, so θ will also be oscillating.

So, θ' is oscillating therefore, θ which is same as θ'+θ_f will also be oscillating ok. Now, now what will happen if I increase the value of b? So, if I increase the value of b keeping a and c unchanged, then at some point b will be equal to b square will be equal to 4 ac. So, this term will vanish ok, then b square will become 4 ac, and then the solution will not be oscillatory anymore ok.

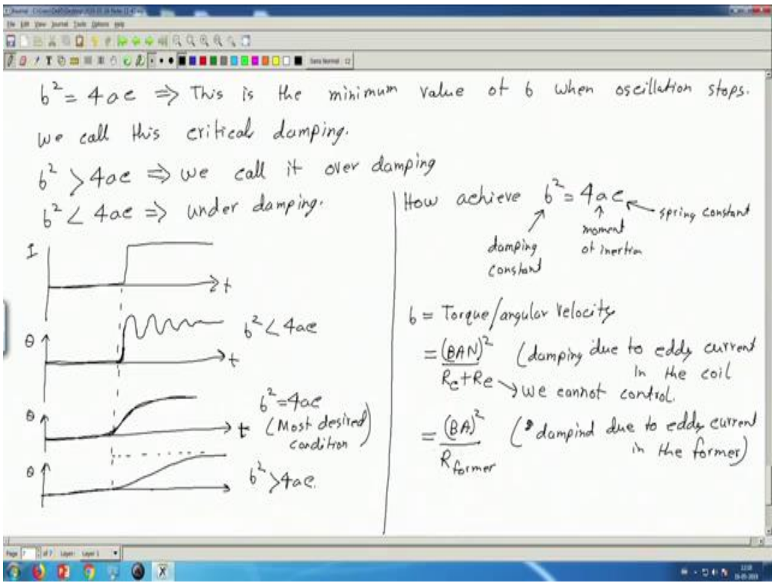
So, when we and if we increase b further, then b^2> 4 ac, and the two roots will be real. So, they will be of the form of course, -b$\pm\sqrt{b^2-4ac}$ now b^2> 4 ac, so this will be a positive number. So, this will be a both the roots will be real. So, again θ' or θ will not be oscillatory.

(Refer Slide Time: 22:54)



So, what we see observe for low value of θ sorry low value of b, the solution is oscillatory or θ oscillates and for higher b, θ does not oscillate.

(Refer Slide Time: 23:39)



Now, b; b^2 =4 ac this is the minimum value of b when oscillation stops ok. So, we call this critical critically damped condition or critical damping ok. So, b^2 =4 ac is the condition for critical damping, for b^2 >4 ac still we will have no oscillation, this we call as over damping. And, if b^2 <4 ac this is under damping, because we will have oscillation ok. So, as I was saying if I apply a current suddenly like this.

So, this is current time theta can be can reach the final value after some oscillation, so this is under damped. So, this is the condition $b^2 < 4$ ac or under damped condition, we can have a situation where theta reaches it is final value like this smoothly. This is the condition for $b^2 = 4$ ac, if we increase b further it will reach the final value again without any oscillation, but it will now take longer time ok.

So, this is the condition for $b^2 > 4$ ac. So, what we want? We want the pointer not to oscillate, because then it is difficult to take reading we and we do not want this pointer to settle down quickly, because we want to take readings quickly. So, this is the ideal condition this is the most desired condition $b^2 = 4$ ac critical damping.

If, we use over damped condition, then it will take longer time to settle down which we do not want and if we used $b^2 < 4$ ac, then it will oscillate, it will also take longer time to settle down, this is also not desired. So, this is the desired most desired condition. Now, how can we achieve this? How to achieve $b^2 = 4$ ac?

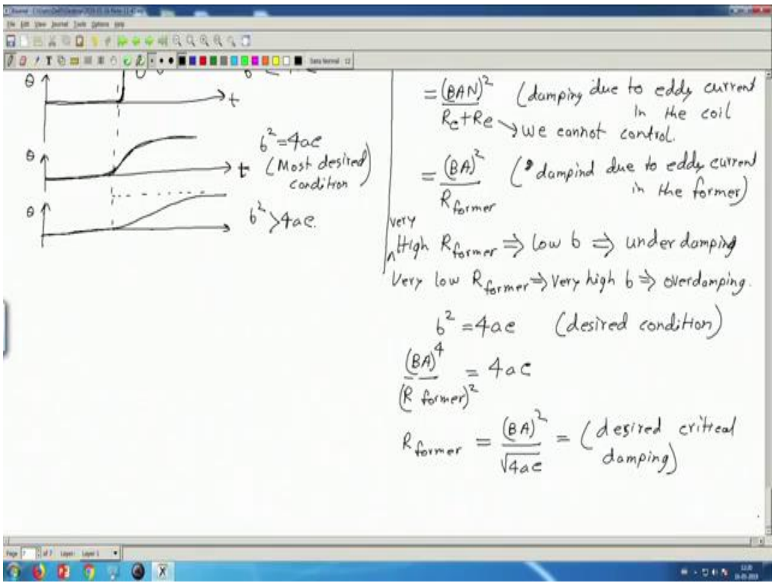
So, note that a and c, so this is the moment of inertia like kind of the depends on the weight mass of the moving system. So, this is the spring constant and b is the damping constant ok. And, b is given by the torque per unit velocity and for say a PMMC instrument we have seen ,

$$b=\frac{(BAN)^2}{R_c+R_e} \quad \text{(damping due to eddy current in coil)}$$

$$b=\frac{(BA)^2}{R_{former}} \quad \text{(damping due to eddy current in former)}$$

this one is not controllable because Re depends on the external circuit. So, we should not rely on this term. So, this term we cannot control, this term we can possibly control and therefore, we should choose a low enough value for R former, so that b is high enough and equal to 4 ac.

(Refer Slide Time: 30:24)



So, we now if we choose, see if we choose low value of R former, let us say we choose first high value of R former; former resistance, this will imply low b may be very low b and may this may imply oscillation or under damped situation ok.

If we choose so, call this very high, now if we choose very low then we will have b very high implying over damping; that means, it will not oscillate, but it will be slow, it will take long time to reach the final position. And, we have to wait before we can take the reading. So, for up for some value of R former, so which I can, so what we want for some value of R former, b square should be equal to 4 ac. So; that means, $b^2 = 4ac$ is the desired condition critical damping and here we will have , so, in place of b^2 we can put the value of this.

$b^2 = 4ac$ (desired condition)

$$\frac{(BA)^4}{{R_{former}}^2} = 4ac$$

$$R_{former} = \frac{(BA)^2}{\sqrt{4ac}} \qquad \text{(desired critical damping)}$$

So, that is the that is about critical damping condition. In our next video, we will talk about few more important things regarding the dynamics. For example, we will so, mathematically that the moment of inertia should be low, the point article should not be heavy, otherwise it will be either oscillating system or a slow system, and we will talk about also few interesting questions regarding this measuring instrument ok.

Thank you.