

Electrical Measurement and Electronic Instruments
Prof. Avishek Chatterjee
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 10
Dynamics of the Moving Coil and Damping

Welcome back in our course.

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Eddy Current Damping (cont.)

Diagram showing a coil with N turns between North (N) and South (S) poles. The coil is connected to an external circuit with resistance R_e .

$$T_{damp} = -\frac{d\theta}{dt} \frac{G^2}{R_e + R_c} \quad [G = BAN]$$

T_{damp} depends on R_e i.e. external circuit.

Eddy current damping due to the Eddy current in Former/Frame of the coil

Frame acts like a single turn coil.

When the coil moves the former also moves so emf is induced in former so eddy current flows in former. Thus a torque is developed which opposes the angular velocity.

Angular Velocity = $\frac{d\theta}{dt}$
 Linear " = $\frac{D}{2} \frac{d\theta}{dt}$

EMF = rate of Flux Intersections
 $= \frac{D}{2} \frac{d\theta}{dt} \times L \times B$ (in one side)

Total EMF = $2 \times \frac{D}{2} \frac{d\theta}{dt} \times LB = BA \frac{d\theta}{dt}$

So, we were discussing about the Dynamics of the Moving System and in particular we were talking about the eddy current damping. So, let us recall it briefly. So, we were considering the PMMC instrument. So, we have these two poles; north and south, between these two we have a moving or a rotating coil like this, and then this coil must have a closed circuit through the external circuit when it is measuring something.

So, let me draw some circuit to which this coil is connected, because we are measuring something; that means, the coil is definitely connected to some circuit in which current flows. And, this circuit can be very complicated in general, but let me take as simple circuit like this. Now, we have seen that the flux lines are like this and in this circuit now there should be no current because according to my diagram, there is no source, there is no voltage or current source which will drive any current. So, this is a very simple circuit, but even now, if the coil starts to move due to any reason if it starts to rotate, then you know

this is a coil inside a magnetic field. So, it is like a motor. It is basically like a motor where we have a coil inside a magnetic field.

So, some emf will be induced, because the sides of the coil are moving through the magnetic field. So, some emf will be induced and this emf will cause some current to flow. And, then using Fleming's right hand rule, we can find out the direction of the emf thereby the direction of the current, and then using Fleming's left hand rule, that is motoring rule we can find out the direction of the force or torque which will act on this coil.

And, in our previous video we have seen that this force due to the induced current is always in the direction opposite to the motion or the velocity or angular velocity of the coil. So, this force always tries to stop the velocity of the coil. And, this current we can call it eddy current, and therefore, we can call this damping mechanism as eddy current damping.

$$T_{damp} = -\frac{(G)^2}{R_c + R_e} \frac{d\theta}{dt} \quad [G = B A N]$$

So, this force acts always in the direction opposite to the velocity or the angular velocity of the coil. So, I can actually erase this mode sign and put a negative sign here to indicate that the direction of the torque is always opposite to the direction of the angular velocity. And, therefore, this is not very controllable, I mean we cannot control the amount of damping torque or damping force at our desire.

So, we will consider now a small variation of this scheme, which is the eddy current damping due to the eddy current in the former or frame of the coil. So, recall that this coil is wound on top of a former or frame, a rectangular frame and that looks like this and this is made up of aluminum, which is a conductor. So, this is a frame and the coil is wound on top of it. So, the coil goes like this and so on. Now, this former itself is a conductor, this is made up of aluminum. So, this is made up of aluminum. So, this is conductor. Now, therefore, this frame can also be thought of as a single turn coil. So, the frame acts like a single turn coil.

So, we have the frame here inside the coil. So, this is the frame and this itself is a closed circuit. So, this acts like a one turn or single turn coil. So, therefore, if this coil is rotating; that means, this former frame is also rotating because the coil is attached to the frame. So, if this coil at the moving system is rotating, then in this single turn coil; that means, the

former will have some emf induced. And, that emf will cause some current to flow through this coil; that means, actually this former, the single turn coil.

So, when the coil moves, the former also moves former means frame. So, emf is induced in the former. So, some eddy current will flow in former. And therefore, this eddy current will now generate a torque which will oppose the velocity or the angular velocity. And, thus a torque is developed, which opposes the angular velocity.

Now, just for our practice let us derive the expression for eddy current sorry the torque due to eddy current in the former, very quickly. The derivation will be very similar to the derivation of the damping torque due to eddy current or due to the induced current in the coil, but we will do it once again very quickly for our practice.

So, let us do it. Assume the flux density to be B , assume the length of this former to be L . So, this is the length of the former, then the diameter or the width of this former, call it W or call it D as per the standard notation. So, now consider any one side, consider say consider one side this side how much flux is it intersecting per unit time? For that we need to know the linear velocity of this side.

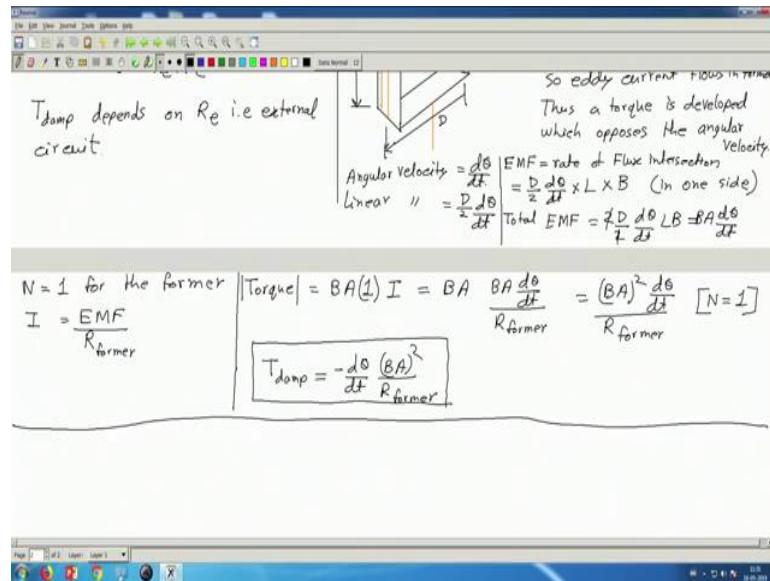
$$\text{Angular Velocity} = \frac{d\theta}{dt}$$

$$\text{Linear Velocity} = \frac{D}{2} \frac{d\theta}{dt}$$

$$\text{EMF} = \text{Rate of flux intersection} = \frac{D}{2} \frac{d\theta}{dt} \times L \times B$$

$$\text{Total EMF} = 2 \times \frac{D}{2} \frac{d\theta}{dt} \times L \times B = B A \frac{d\theta}{dt}$$

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So, now, what will be the current? Current will be emf divided by the resistance. Resistance of what? This will be the resistance of the formers. You see this former is a closed loop it is a closed circuit. So, this closed loop will have some resistance. So, that resistance we have to put here so, resistance of the former.

$$I = \frac{EMF}{R_{former}}$$

$$|Torque| = B A (1) I = B A \frac{B A \frac{d\theta}{dt}}{R_{former}} = \frac{(B A)^2 \frac{d\theta}{dt}}{R_{former}}$$

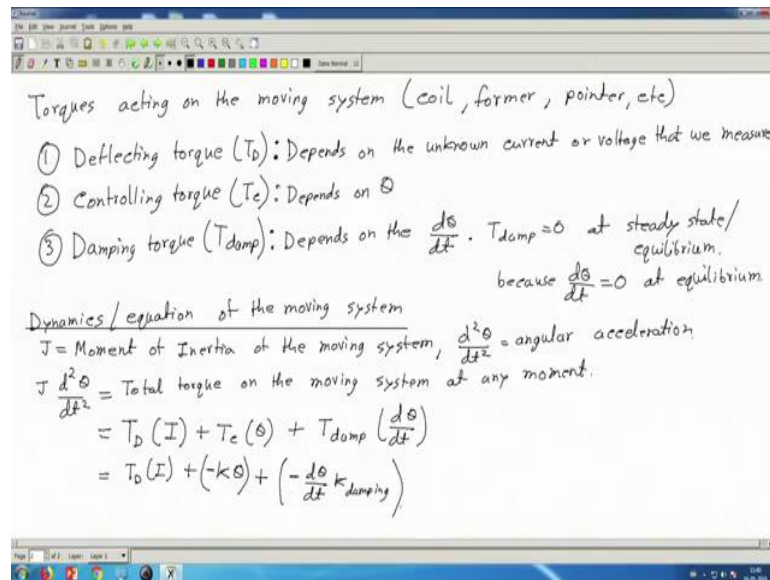
So, that was a very quick recapitulation of the derivation how to find the torque due to the eddy current damping and it once again you see that this torque.

$$T_{damp} = -\frac{d\theta}{dt} \frac{(B A)^2}{R_{former}}$$

So, here we consider only the magnitude of the torque, we did not consider this sign in this in this line of the derivation, but now using the Fleming's rules, left hand right hand rules we can find the direction and we can so, once again that this direction will be opposite to the direction of the velocity.

And, I request all of you to kindly do this small exercise; apply Fleming's right hand rule first, to find the direction of emf and then apply Flemings left hand rule, to find the direction of the torque, do this exercise it will be a fun. So, that is the damping torque. So, what have we seen so far?

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So, we have seen that there are a number of forces or torques acting on the moving system; that means, the coil plus the former frame, plus the pointer and everything else which is attached to this coil etcetera. So, the torques that act on this moving system are. Firstly, the deflecting torque, which we denote as T_D , then the controlling torque, this is nothing, but the spring torque which we denote by T_c and the third important thing which we have discovered now is the damping torque. So, let us denote it as T_{damp} because if I use T_D this two will be confusing.

$$J \frac{d^2\theta}{dt^2} = T_D(I) + T_c(\theta) + T_{damp} \left(\frac{d\theta}{dt} \right)$$

$$= T_D(I) + (-K\theta) + \left(-\frac{d\theta}{dt} K_{damping} \right)$$

$$J \frac{d^2\theta}{dt^2} + K_{damping} \frac{d\theta}{dt} + K\theta = T_D(I)$$

So, this is damping constant and we put a minus sign, why? Once again we know that this torque is always in a direction opposite to the velocity. So, this is the equation for the moving system. Now, we can rewrite it by slight reorganization of the terms.

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(1) Deflecting torque (T_D): Depends on the unknown voltage.

(2) Controlling torque (T_c): Depends on θ .

(3) Damping torque (T_{damp}): Depends on the $\frac{d\theta}{dt}$. $T_{damp} = 0$ at steady state/equilibrium, because $\frac{d\theta}{dt} = 0$ at equilibrium.

Dynamics / equation of the moving system

J = Moment of Inertia of the moving system, $\frac{d^2\theta}{dt^2}$ = angular acceleration.

$J \frac{d^2\theta}{dt^2}$ = Total torque on the moving system at any moment.

$$= T_D(I) + T_c(\theta) + T_{damp}\left(\frac{d\theta}{dt}\right)$$

$$= T_D(I) + (-k\theta) + \left(-\frac{d\theta}{dt} k_{damping}\right)$$

$$J \frac{d^2\theta}{dt^2} + k_{damp} \frac{d\theta}{dt} + k_s \theta = T_D(I)$$

\uparrow \uparrow
 Damping spring
 constant constant

So, this let me write is damping constant and this is spring constant. And, then so, this should be equal to the deflecting torque T_D function of, maybe a function of the current or unknown voltage that we want to measure. So, this is spring constant. So, my computer is getting slower indicating that it is time for me to stop this video. Hence therefore, we will continue on with this topic in our next video.

Thank you!