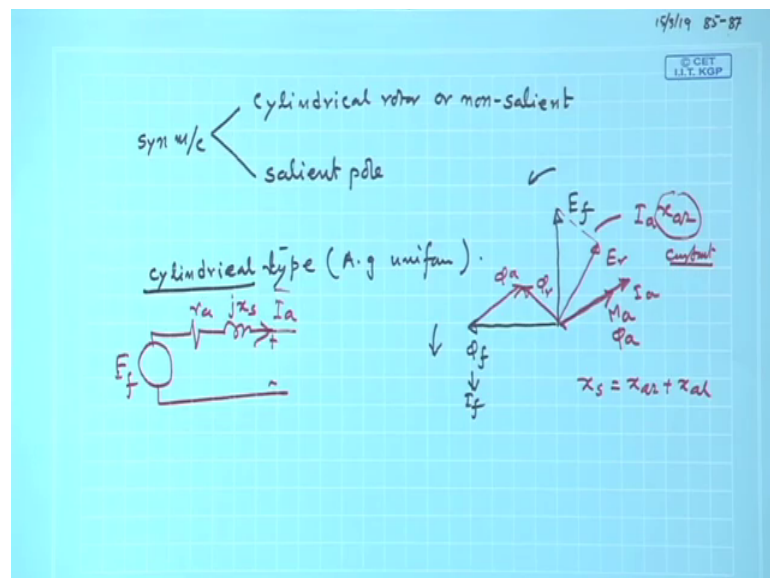


**Electrical Machines - II**  
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**Lecture - 85**  
**Analysis of Salient Pole Synchronous Machine  $X_d$  &  $X_q$**

Welcome to this lecture on Synchronous Machines we have been discussing.

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As I told you that synchronous machine are of two types, one is cylindrical rotor in the synchronous machine; cylindrical rotor or uniform gap or it is also called non salient pole type rotor structure where field winding is housed; based on that it is classified. And the other is salient pole type; salient pole synchronous machine.

Now, the analysis of these two kinds of machine is a different, also it will be reflected in their phasor diagrams. Now, you recall that in cylindrical machine; if you have understood the cylindrical machine synchronous reactance correctly, then there will be not much of a difference recall that in case of cylindrical rotor type synchronous machine where air gap is uniform. Then the phasor diagram was drawn like that, that if this is your  $\phi_f$  then under no load condition; this will be your  $E_f$  90 degree lagging.

All are rotating with synchronous speed in this direction. How this  $\phi_f$  was obtained because  $\phi_f$  is the decided by  $I_f$  the rotor field current this was the thing. But when the

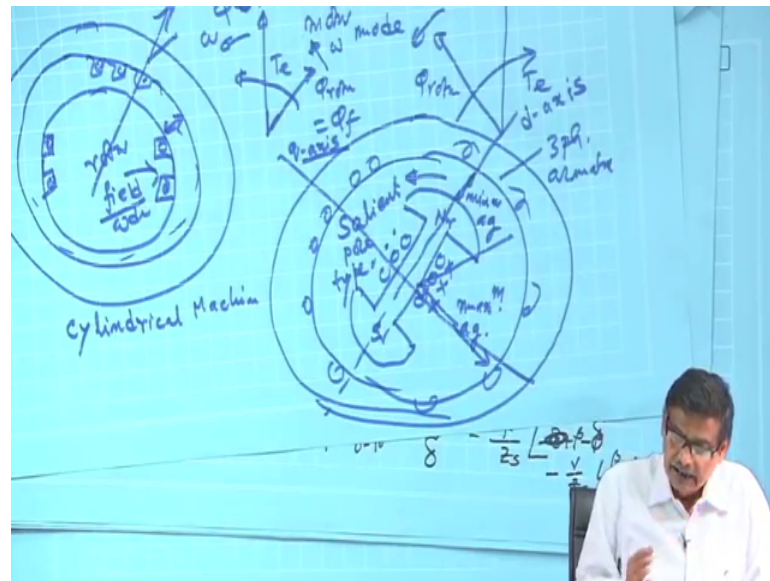
synchronous machine will be loaded, then I told that suppose it is a lagging power factor load then  $I_a$  will be here,  $I_a$  will be here armature current lagging power factor. This angle is called internal power factor angle. Usually denoted by  $\psi$ , but whatever it is this is suppose lagging  $I_a$  and once this  $I_a$  is obtained, then I will say under no load condition it was only  $\phi_f$  existing in the air gap, but when also the armature will carry current balance 3 phase current  $I_a$  armature mmf will also come along this.

And to find out the induced voltage now, you and also  $\phi_r$  we  $\phi_a$  will be along this direction. And these two together we will decide the resultant flux here  $\phi_r$  this is not rotor resultant flux and then induced voltage will be somewhere else here. And then I showed that this change in induced voltage from  $E_f$  to  $E_r$   $E$  can be considered to be a reactance voltage drop called  $I_a X_a$ .

And from  $E_r$  if you subtract the leakage impedance drop, you get the terminal voltage that was the argument. Now in case of cylindrical type machine this  $X_a$  was constant armature reaction reactance and recall  $X_s$  was then the  $X_a$  plus  $X_l$  and then we told that look here you need not calculate  $\phi_r$ . So, far as the modelling of the machine is concerned you simply do like this, an impedance  $r_a + j X_s$  is  $X_a$  plus  $X_l$  and so on and this is your terminal voltage generator mode this is the thing.

So, one reactance could do that. Now, why one reactance can do? Because it the direction of  $m_a$  is decided by the power factor angle of the machine at which it is operating. It could be anywhere, here, here leading power factor; it will be there and so on. But only so armature mmf when armature carries current will be can be in any place in the air gap, but we had the have the advantage that air gap is uniform. So, to calculate  $\phi_a$ , you have to divide this  $M_a$  by a constant reluctance. So, everything was fine. Now, let us see in case of a synchronous machine where this situation is like this I will not draw because last time I drew.

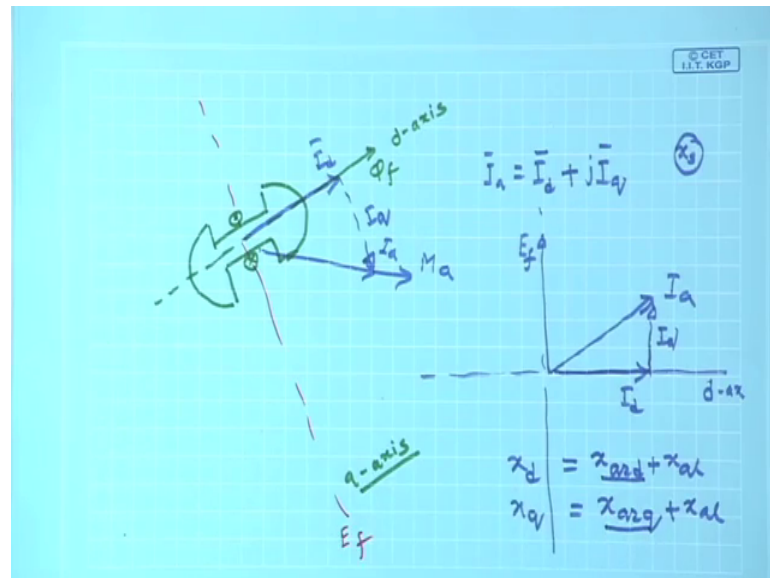
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The starter houses 3 phase winding and it is salient pole structure. So, air gap will be minimum along the salient pole axis; field axis and air gap will be maximum when it is along this and when your armature winding will carry current as I told you armature mmf could be anywhere in space. As the power factor at which the machine is operating will go on changing, the position of M a armature mmf will also go on changing.

As it goes on changing, air gap seen by the armature mmf will be also changing. Therefore, it is very difficult and to calculate the flux for different position you have to divide this mmf by reluctance which is position dependent. Therefore, it is very difficult to model what to do with this, is it then I have to calculate wherever it is I will calculate precisely the reluctance divided by 8, get the armature flux it is not really like that. Blundell proposed an elegant method.

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He told that if this is your only this field winding I am drawing suppose this is the armature rotor field salient pole type, if you draw like this I am not drawing the stator part ok; the field winding will be here and it will have some cross dot. So, field axis will be this one there is no doubt about it and your  $\phi_f$  will be along this line is not  $\phi_f$  will be along this line.

And not only that if this is  $\phi_f$  the no load induced voltage will be at right angles that we have seen.  $E_f$  axis will be will lie along this line is not, 90 degree that is what that if this is  $\phi_f$  90 degree behind  $E_f$  is there. Now, the argument is this axis is there and I just called this axis to be give some name d axis and this line this redline I call q axis quadrature axis these are at 90 degrees.

And along this axis air gap is minimum and along this axis air gap is maximum. Now, your armature mmf or armature current which produces mmf could be anywhere in this space for example, your  $M_a$  is here and in the phasor diagram this is also  $I_a$  like this. So, the interesting thing is a suggested is this the problem is this  $I_a$  will be can be here, can be there depending on power factor angle and so on therefore, reluctances are varying.

So, why not break up this armature current into its d and q axis. For example, this  $I_a$  can be cut off phasor some of these two currents  $I_d$  and  $I_q$  no matter where the  $I_a$  will be  $I$  can always find out its component along the direct and quadrature axis. For example, this

$I_a$  becomes  $I_d$  so so  $I_a$  can be thought of you can replace this  $I_a$  thinking that there are two currents like this exists in the armature. And then the argument is if this is  $I_d$  then  $m_d$  will be also along this line and  $M_q$  will be along this line. So, because of  $I_d$  therefore, the change in voltage drop can be taken as we have taken here. So, I will say that under no load condition when  $I_a$  was 0 if this was your  $E_f$ , now  $I_a$  is here, suppose  $I_a$  is here.

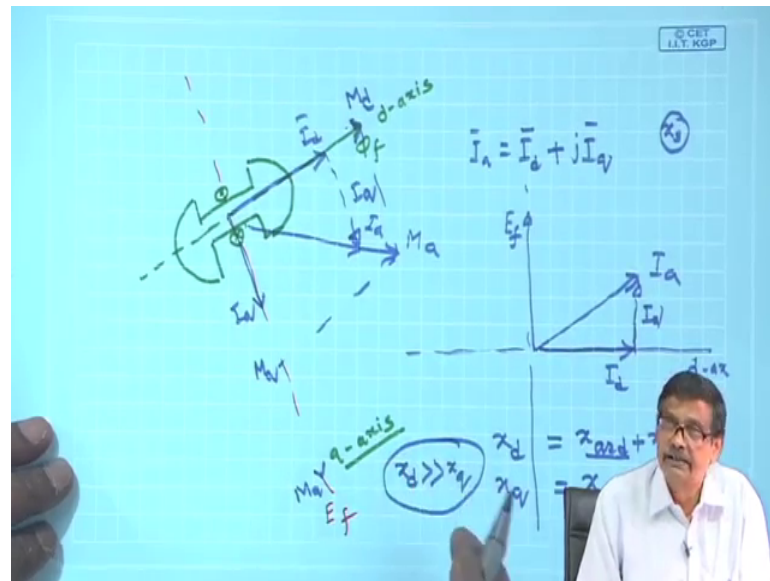
And the this  $E_f$  axis is  $q$  axis, so I will draw a line here that is the  $d$  axis and I will divide this current as  $I_d$  and  $I_q$  this components and then I will forget about  $I_a$ , I will replace this armature current has two components,  $I_d$  and  $I_q$ . Then I will say from because of  $I_d$  there will be a change in voltage and because of  $I_d$  there will be a armature reaction flux  $x_a$   $a_r d$  plus  $x_a l$  this will be some direct axis reactance and  $x_q$  is equal to  $x_a r q$  plus  $x_l$ .

Mind you the leakage reactance is not affected because leakage reactance path will be this as armature conductor and it will complete its path through air gap, ok. So, so leakage reactance remains  $x_l$ ,  $x_a l$  therefore, we have to simply define instead of asynchronous single synchronous reactance  $x$ 's as we did in case of a cylindrical rotor machine it will be then two reactances.

What is the benefit of decomposing the current? Because I know along  $d$  axis the reluctance is constant along the  $q$  axis although maximum air gap, but reluctance along that axis is also constant. Therefore, we have to deal with now two constant armature reaction reactances,  $x_a r d$ ,  $x_a r q$  in the same way as we did in case of synchronous cylindrical type synchronous machine.

So, with this understanding then I can proceed to draw the phasor diagram of this machine and other easily. The crux of the matter is air gap is varying reluctance is position dependent. I must calculate armature flux to add to the no load flux  $\phi_f$  to get the resultant flux, hence the resultant induced voltage, but this is difficult to carry out has reluctance in different different positions will be different. So, the suggestion is armature mmf which is because of stator armature current its distribution etcetera we have seen. So, armature current can be always thought of having two components, one is  $M_d$  another is  $M_q$ , this two components this is  $M_q$ .

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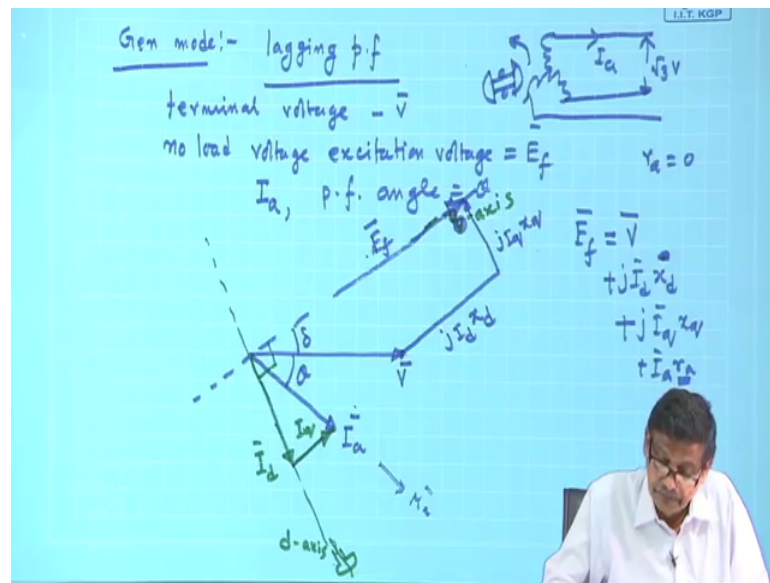


And then  $M_d$  is there. So, I will be able to calculate  $\phi_d$ ,  $M_q$  is known I will be able to calculate  $\phi_q$  resultant flux. But then I will simply say that this  $M_d$  is produced by an equivalent armature current flowing along this direction direct axis  $I_d$  and another current  $I_q$  produces  $M_q$ . Then rest of the things I know what to do. Instead of a single current there now for  $I_d$  and  $I_q$ , I have to define two reactances, and I will add them up to get the resultant force.

See, what I am doing. Suppose let us assume. So, how to draw phasor diagram and it must be understood that  $x_d$  will be always higher may be much higher than  $x_q$ , because of what? Because flux produced along the d axis will be more for a given current because reluctance is minimum along d axis. So,  $x_d$  flux linkage will be more compared to q axis for this  $M$  current. Therefore,  $x_d$  is always greater than  $x_q$ .

Now, let us see how really it will help me to draw first draw the phasor diagram, then we will try to attempt to calculate the or get the expression for power, reactive power this that. First, please listen carefully what I am doing.

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Suppose, I have assumed this generator mode generator mode and suppose it is operating at lagging power factor is that clear. Terminal voltage is  $V$  per phase terminal voltage per phase is  $V$ . And a excitation on no load voltage, no load voltage or excitation voltage is equal to per phase  $E_c f$ . This is the thing.

Armature current is  $I_a$ , this can be measured that is this is the machine salient pole machine, ok. This is the salient pole machine, this is the rotor, windings are here and this fellow is running. So, armature current is this current, terminal voltages if it is star connected per phase voltage  $V$ , root 3  $V$  that is the thing and it is delivering  $I_a$  current balanced current. So, excitation no load voltage  $I_a$  is the armature current and power factor angle is equal to  $\theta$ . And this is the thing. So, this is your  $E_f$  or  $V$ , let me start with  $V$ . So, to  $V$  if I go on adding the drops it is distance drop, then reactance drop, I must get my  $E_f$ .

What is known to me is this two things, this you please be with me for every statement I do here what exactly I am doing. So, these are the things which I can measured that the terminal, what is the per phase voltage, what current it is delivering and at what power factor angle. How do I know power factor angle? Connect somewhat meter how much power it is delivering,  $V I$  is known. So,  $\theta$  is known. These are precisely known to me.

Now, I have to add the drops. Now, in case of cylindrical machine it was. So, simple  $I_a$  plus  $V$   $I_a r_a$  plus  $V$   $I_a j x_s$ 's one reactance, but here I cannot do it, because your armature mmf will be here and its position can be anywhere reluctances will be different the same old story. Therefore, I cannot do anything. I cannot at least think of a single reactance which can be used to at the drop across which can be used to compute  $E_f$ . Instead what will be doing, I will break up this current in the d and q axis. Now, the big question is where is d, where is q axis here? I do not know anything it looks like.

But only one thing I know it is a generator operation therefore,  $E_f$  axis must be above it is not,  $E_f$  axis must be above  $V$  that is why it is generating. In generator mode  $E_f$  will be always ahead of  $V$ , and this angle is delta of course, delta is not known now. So, so q axis which is along  $E_f$  axis will be above it I know I just draw it suppose it is known, suppose to begin with, then what I will do is this listen carefully, I will draw a line perpendicular to this here. This is at right angles, you draw a line. And you say this will be d axis. Means where the poles are there, understood. This is the d axis along this line, ok.

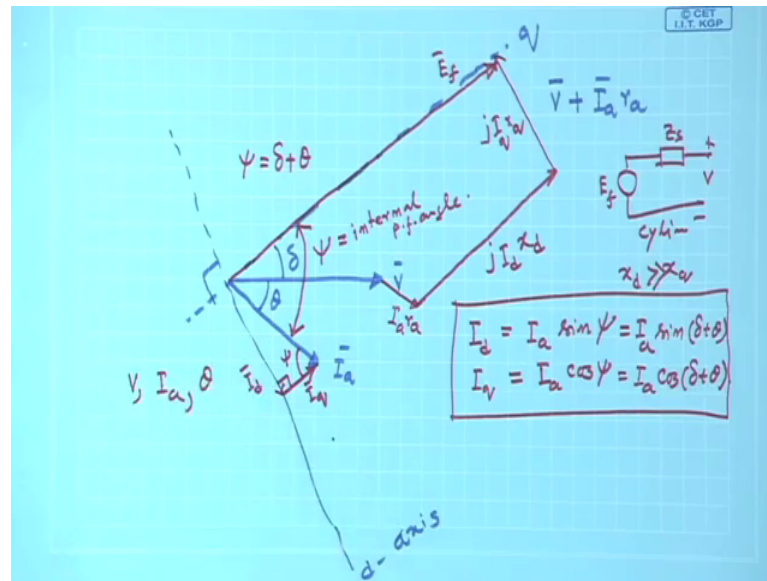
If this is d axis then what will be done is this this armature current I will drop a perpendicular from the tip on the d axis and I will say this is your  $I_d$ ,  $M_d$  will be created here always a constant reluctance and this will be your  $M_q$  or  $I_q$ . This will be the thing. Is that clear? Then what I am going to do is this I will add to  $V$  a voltage  $j I_d x_d$ ,  $I_d$  is here its drop 90 degree ahead.

So, this drop will be parallel to q axis  $j I_d x_d$ , ok. And of course, I have neglected here  $r_a$  ok, suppose  $r_a$  is 0 then I know this much  $I E_f$  should be generator mod  $V$  plus  $j I_d x_d$  plus  $j I_q x_q$  and plus  $I_a r_a$  I will take into account I have assumed  $r_a$  to be 0, so only  $j I_d x_d$  and then I have to add  $j I_q x_q$   $I_q$  is along this. So,  $j I_q x_q$  will be like this. So, you add this to  $j I_d x_d$  and then this will be your  $E_f$  that is what you have to do, ok.

So,  $V$  plus  $j I_d x_d$  plus  $j I_q x_q$  will give you the excitation voltage. So, I am doing once again and this time I will include  $r_a$  also. So, what I am doing? Informations which will be available to you or measurable quantities is  $I_a$  power factor angle theta at which that machine is operating and the terminal voltage.



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So, this time, so in case of generator you first draw  $V$  that is there and this  $I_a$  I can talk with confidence, no problem, and this is the power factor angle  $\theta$ .

The next question is this is  $I_a$ , where is  $q$  and  $d$  axis, ok. I am not certain, but I am sure about one thing your  $E_f$  must lie above this. So, suppose this is your  $q$  axis then because  $E_f$  will lie here and this  $E_f$  line is your  $q$  axis line. And then wherever  $E_f$  will be I am not knowing till now, but I am sure about this thing this angle will be some  $\delta$  angle between  $E_f$  and terminal voltage. Then I am drawing a line perpendicular to this and I will say this is your  $d$  axis,  $d$  axis, ok.

Once I know this, I will be able to break up this current in to drop a perpendicular here this will be your  $I_d$  and this will be your  $I_q$ . Along this two axis is, so that the corresponding mmf will always see constant reluctance along  $d$  and constant reluctance along  $q$ . Then your  $V$  terminal voltage to this if you add  $I_a r_a$  of course,  $I_a r_a$  remains  $I_a r_a$ . So, I will add  $I_a r_a$  here. There is no  $d$   $q$  axis you need not break it up. Of course, you can do  $I_d$ ,  $r_d$  and so on,  $I_a r_a$  u sketch.

Then to this you add  $j I_d x_d$ ,  $I_d$  is a  $j I_d x_d$  and then to this you add  $j I_q x_q$  and you will get  $E_f$  and this will be the thing, phasor diagram is over. But from this phasor diagram one should not say that one can draw an equivalent circuit as we did in case of synchronous machine as  $E_f$ , cylindrical machine a single impedance  $Z_s$  and  $V$  is not for cylindrical machine, this was the phasor diagram, so simple. But here at this between  $V$

and  $E_f$  I cannot put a single reactance to get this, this sort of equivalent circuit. In fact, we will not try to get this sort of equivalent circuit at all it will not be needed. You have to take  $I_d$   $I_q$  separately and get this and  $X_d$  is always greater than  $X_q$ , may be much higher compared to  $X_q$ ;  $r_a$  is of course, very small, but none the less this is the phasor diagram.

So, this angle, angle between  $E_f$  and  $I_a$  is called the internal power factor angle  $\psi$ , it is usually denoted, internal power factor angle. If this is  $\psi$  this is also  $\psi$ . So, you can see from this triangle that  $I_d$  will be nothing but magnitude of  $I_a$  into  $\sin \psi$  and  $I_q$  will be is equal to  $I_a \cos \psi$ , where this is  $\psi$ .  $\psi$  is nothing but  $\delta + \theta$  in this particular generator mode of operation, lagging power factor it is like this. So, this can be written as  $\sin \delta + \theta$  and this can be written as  $I_a \cos \delta + \theta$ . So, these are the relations we have got.

But still ok, if the phasor diagram is completed you have gone up to  $E_f$  then I can do something this way that way I will write down the equations. But the big question is I do not know the value of  $\delta$  because what I know, I know  $V$  terminal voltage, I know  $I_a$ , I can measure with an ammeter, I can measured with voltmeter terminal and power factor angle, I can estimate by connecting and wattmeter. I do not have any knowledge of  $\delta$ . So, we will take up in the next class how to estimate  $\delta$  and precisely get the  $q$  axis, then only you can do all this things unless  $\delta$  is known how can you calculate  $I_d$  and  $I_q$ . Hope you are following me. So, we will continue with this.

Thank you.