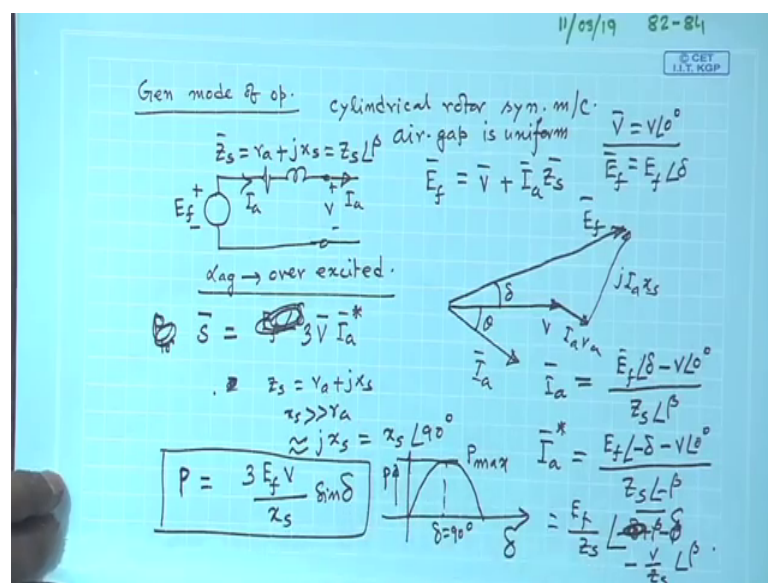


Electrical Machines – II
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Lecture – 82
Synchronous Motor Operation Phasor Diagram and Power Expression

Welcome to this Synchronous Machine lectures and we were discussing generator mode of operation of cylindrical type synchronous machines.

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Cylindrical rotor synchronous machine where the air gap is uniform air gap is uniform and synchronous machine equivalent circuit is much simpler than the case of induction machine because of the fact there is no magnetising branch present here.

So, the equivalent circuit will be something like this plus minus E_f and then r_a plus jx_s synchronous impedance which is often called Z_s and I_a was writing it as some magnitude Z_s angle β , β will be closed to 90° because the value of x_s is much higher than r_a and this is the thing and suppose the terminal voltage. This is per phase all the voltages and currents are here. And, therefore, in case of generator mode of operation suppose the generator delivers a current of I_a that is the per phase current, then it is true that E_f will be equal to V plus I_a into Z_s bar.

And, based on that we did last time very quickly, I will tell this is the terminal voltage, this is generally taken on reference V_0 degree and then suppose it is supplying a lagging power factor load I_a . So, this is the power factor angle of the generator at which it is operating to the external world and this plus $I_a r_a$, $I_a r_a$ plus $j I_a x_s$, $j I_a x_s$ gives you your E_f and in case of generator mode of operation, E_f will be always above V and this gives you δ . E_f the length of E_f is a measure of the field excitation and length of V is a measure of approximately the resultant field ok.

Therefore, a this generator supplying a lagging power factor load is over excited. So, it is lagging power factor, it is over excited we know that over excited. And then I told you if you want to find out the expression of power and torque so, power delivered total power delivered by the machine should be calculated like this S is equal to this is the current delivered to the load. So, it should be E_f this voltage I mean I will do like this V into I_a star this is the visual thing we will do so that we get the expression of real and reactive power simultaneously.

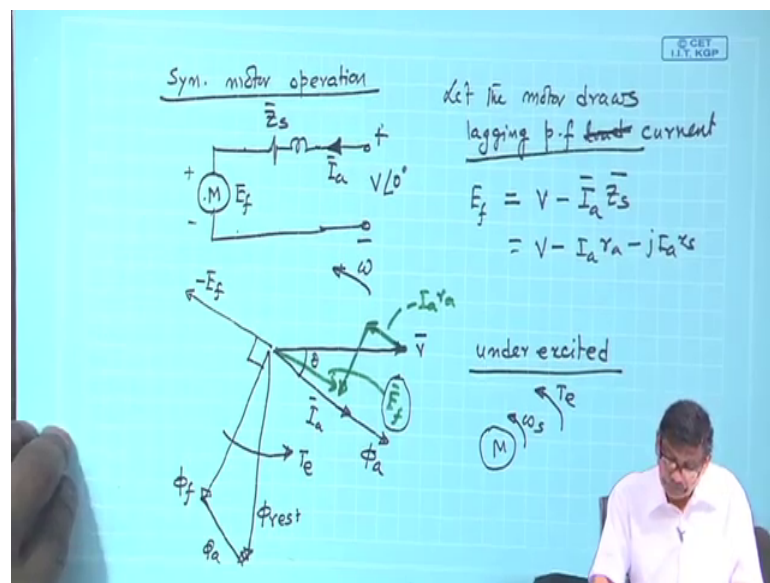
And, since V is V_0 we know that E_f can be written as $E_f \angle \delta$ where δ is a positive number and then what I did, I just wrote I_a ; I_a is this way. So, I_a should be equal to $E_f \angle \delta$ minus $V \angle 0$ degree divided by this $Z_s \angle \beta$ this is I_a . So, I_a star is needed. So, I_a star will be simply E_f , all angle should be made negative minus V_0 divided by Z_s minus β which will be equal to E_f by Z_s and this will be minus δ plus β . No, this will be β minus δ β minus δ this first term and this will be minus V by $Z_s \angle \beta$.

So, this we substitute it here. I will not repeat. So, I have substituted this here. Then, total power will be 3 times this and we get an expression of the complex power delivered to the load. If both the terms become positive; that means, it is delivering really kilowatt which in case of generator, it has to be always positive the real part and imaginary part, if it is positive it means it is delivering a lagging power factor load.

And, then I told you that this Z_s which is equal to r_a plus $j x_s$ x_s is much higher than r_a . So, what people do? Often people use this approximation that this is $j x_s$ which is equal to $x_s \angle 90$ degree. And, if you do that the real power output of the generator is a very famous equation and that is equal to magnitude of E_f magnitude of V divided by x_s into $\sin \delta$.

Since synchronous machine runs at constant speed, this itself will be the expression of the torque if you want to know it in Newton metre divided by 2π this speed of the machine that is MS mechanical speed, you will get the expression of the torque and P delta curve will be sinusoidal which at distinct magnitude P max and P value of P max is proportional to terminal voltage into E f by x s this is delta. Maximum power or torque it can develop is 90 degree, this side is power ok. So, we did this.

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Now, today what I will first; so, we are still dealing with cylindrical rotor synchronous machine, but what will be the expression for power and torque for a synchronous motor operation synchronous motor operation ok. We will first draw the phasor diagram and try to find out the expression of the torque.

In case of synchronous motor operation, I will draw like this, this is this, the impedance is Z_s as usual and here it is bus voltage V which is angle 0 degree volts and since it is operating as a motor I will show that it is drawing power from the source. So, I_a I will assume inconsistent with the supply voltage which is supplying the motor. So, this will be the thing and this voltage will be your E_f , voltage between these two points.

Now, so, let the motor draws lagging power factor load, lagging power factor load, lagging power factor current. It is load is mechanical load on the shaft and it draws at some steady operating point it draws like that. Then, I will say that this E_f should be equal to V minus $I_a z_s$; this will be the magnitude of the rms voltage.

So, how to draw it? So, the drawing of this phasor diagram will be $I_a r_a$ minus $j I_a x_s$ ok. So, we first draw; please carefully follow me what I am doing this is V . Then I have told it is drawing a lagging power factor current. So, this is my I_a ok, then from V I have to subtract this r_a and x_s draw first I subtract. So, minus $I_a r_a$, it will be like this is minus $I_a r_a$ and then $j I_a x_s$ is like this. So, minus $j I_a x_s$ will be 90 degree to this and this length is much higher compared to this length although it is.

So, you will get your excitation voltage E_f and this angle is θ the operating power factor of the machine angle between V and I_a is your θ . So, this will be E_f . One thing I can say; say a motor draws lagging power factor current length of E_f is nothing, but length of excitation EMF means the rotor field this length denotes. V denotes what? The resultant field. Therefore, you can easily see resultant field is more than E_f and it will act as a motor.

So, in motor mode of operation, it is essential to draw the phasor diagram in this way at least this is the convention I will follow. In case of generator when you draw V , it delivers power to the outside world in case of motor, it draws power from the source. So, it should be consistent with the supply. So, I have drawn like that.

Now, in the same way as we have done in case of generator we can suppose I want to know where is my ϕ_f , where is my ϕ_r and where is my ϕ_a . Obviously, this will be the direction of armature flux or armature mmf along the maximum value direction, it will be pointed this is ϕ_a . Now what is E_f ? This point you listen carefully unlike generator.

Now, E_f is the generated voltage inside the machine and its polarity will be like this and it will be inconsistent. If this is EMF is allowed to act alone, it would have driven current in this direction. Therefore, to take that into account your actual E_f is actually here minus E_f you have to take; if you want to get information about ϕ_f correctly. And, then I know this is ϕ_f this will be your ϕ_f , 90 degree lagging the induced voltage and then ϕ_a plus ϕ_a will give you the resultant on net EMF ϕ_r . This r is not rotor resultant.

So, here also I find that ϕ resultant is more than ϕ_f . Therefore, it is under excited. Of course, while solving for problem, you need not it is not necessary to draw E_f ϕ_f ϕ_r all the time, but only one thing I will tell that this is all phasors are rotating in the

anticlockwise direction ω like this. In this case, this is the rotor field ϕ_f this is the net field. Therefore, the torque will be in this direction electromagnetic torque developed by the machine.

And, we know it should be because in case of motor mod, if this is the direction of rotation electromagnetic torque must act in this direction and it is therefore, consistent. In case of generator, I showed you that ϕ_f was behind ϕ resultant. So, the electromagnetic torque was in opposite direction as that of the direction of rotation and we know that therefore, this will be this thing. So, in this case once again, I can find out what is the total power drawn by the motor.

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complex power drawn by the motor

$$S = \bar{V} \bar{I}_a^*$$

$$\bar{I}_a = \frac{V \angle 0^\circ - E_f \angle -\delta}{Z_s \angle \beta}$$

$$= \frac{V}{Z_s} \angle -\beta - \frac{E_f}{Z_s} \angle -(\delta + \beta)$$

$$S = \frac{V^2}{Z_s} \angle \beta - \frac{V E_f}{Z_s} \angle \delta + \beta$$

$$P = \frac{V^2}{Z_s} \cos \beta - \frac{V E_f}{Z_s} \cos(\delta + \beta)$$

$x_s \gg r_a$

$$\bar{Z}_s \approx j x_s = x_s \angle 90^\circ$$

$$Z_s = x_s$$

r_a is neglected.

$$P = \frac{V^2}{x_s} \cos 90^\circ - \frac{V E_f}{x_s} \cos(\delta + 90^\circ) = \frac{E_f V}{x_s} \sin \delta$$

$$P = \frac{E_f V}{x_s} \sin \delta$$

Complex power drawn by the motor will be how much? This is the I will draw it here once again. This is your Z_s angle β and this is your here you do not fumble ok. This is terminal voltage and it is drawing a current like this and also you note that in case of motor mod E_f will be always below V . In case of generator mode, E_f will be always above V . No matter whether it is lagging or leading. So, E_f will be always above V in case of generator and E_f will be below the terminal voltage when it is motor mode.

Therefore, I will assume this voltage E_f to be $E_f \sin \delta$ this is the thing and this is the armature current. Got the point? Therefore, I will say that power drawn should be once again power delivered by the source complex power drawn by the motor is equal to

complex power. Anyway it need not be written complex power delivered by the source it will be equal to $V I_a^*$. This will be the thing [FL].

Now, once again I have to calculate I_a . I_a^* I have to calculate. So, first I calculate I_a . This time I_a is $V \angle 0^\circ$ minus E_f minus Δ by Z_s angle β and this will be equal to V by Z_s minus β minus E_f by Z_s and minus Δ plus β ; this will be the thing. So, minus Δ plus β over I_a have written. So, it is equal to like this; if we this bar confused as you it is like this. Therefore, this will be the thing.

Therefore, I need I_a^* in this one; so, I will calculate it here. So, this is V by Z_s angle β minus E_f by Z_s and this angle will be Δ plus β . Therefore, your S will be complex power delivered will be $V \angle 0^\circ$ into this, that will be V^2 by Z_s angle β minus $V E_f$ by Z_s angle Δ plus β .

Therefore, real power drawn by the machine P part will be V^2 by Z_s into $\cos \beta$ minus $V E_f$ by $Z_s \cos \Delta$ plus β . This is the expression for the real part that is the how much kilowatt it is drawing and that is the work which will it will do on the motor. Of course, after subtracting the rotor copper loss; not rotor copper loss, stator armature loss. This is r_a and Z_s . Total power drawn is real power drawn is this of which a portion will be lost in this one $I_a^2 r_a$, and the remaining portion it will do the mechanical work. That way when you do the numerical, you will come to know.

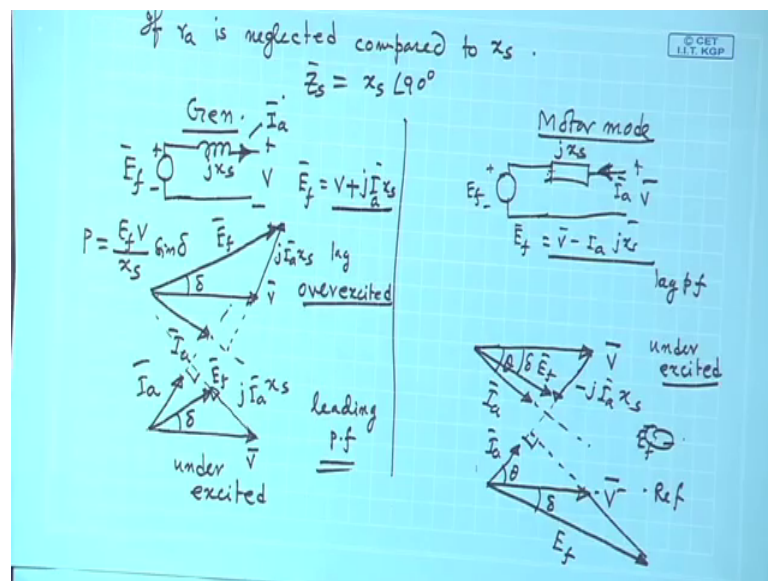
So, this is similarly I can find out Q which I am not doing the imaginary part how much reactive power it is drawing ok. So, this is the thing. Then what I told? These expressions you can memorize; if you have lot of memory, otherwise it is not necessary. What I will do with the numbers I will calculate straight away with a scientific calculator, you get quickly this results, no doubt about it.

But, one result is important which is a special case that is since once again X_s is much higher than r_a . Therefore Z_s can be written approximately as $j X_s$ only and which is nothing, but $X_s \angle 90^\circ$. So, in this case if for this case if r_a is neglected, then the power drawn will be V^2 by Z_s magnitude will be only X_s . So, V^2 by X_s and $\cos 90^\circ$; this fellow goes and minus $V E_f$ by X_s magnitude of Z_s into $\cos \Delta$ plus 90° and $\cos 90^\circ$ plus θ is minus. So, this will become $E_f V$ by X_s into $\sin \Delta$ $\sin \Delta$. Got the point?

Therefore, the expression of the power if r_a is neglected which is often the case, power system engineers will neglect do not bother about r_a . They will simply neglect r_a and represent the power system synchronous machine in terms of x_s and along with the understanding that armature resistance is much smaller compared to x_s synchronous reactance of the machine. And you will get this, same expression as that of generator and motor; the question is where it is.

Therefore, to conclude this so, this a lot of problems you can solve by whether r_a is given or not, whether r_a can be neglected or not; you can solve a lot of problems in this method. So, the expression of the real power which is most important $E_f V$ by $x_s \sin \delta$ is same for motor and generator mode. So, nothing to worry, but only thing you should be clearly knowing whether the machine is running as motor or generator because the direction of armature current, you have to them assume accordingly and calculate; whether it is E_f minus V by x_s in case of generator it was like that and δ was positive and in case of motor current is being drawn from the supply. So, that is the case.

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Now, what I will do? I will once again because this is often done. If r_a is neglected; neglected compared to whom? Compared to x_s . Neglected compared to x_s , then your z_s becomes just x_s 90 degree, is not? Under this condition I will just draw the phasor diagram very quickly. It is a worthwhile exercise to pursue.

See, suppose I will draw it here like this, this side I will draw generator mode and this side I will draw motor mode motor mode. So, in generator mode what is the thing and draw this circuit diagram that is only jx_s nothing else and here is it is V and here is it is your excitation voltage. And, it is generator mode current is delivered to the bus or to the load at a terminal voltage V . So, it must be it is direction must be properly drawn, it is this.

Then, I know that E_f is equal to V plus $jI_a x_s$ is not. Now, if it is lagging power factor, I will draw it like this. Terminal voltage, begin with terminal voltage. Suppose lagging power factor I_a , then there is no $I_a r_a$ now. So, $I_a x_s$ will be perpendicular to this $jI_a x_s$ and it will be your E_f and this angle is δ over and so, this is the thing.

If suppose, the generator is supplying leading power factor load, then what should I do? V is here suppose the generator is supplying a leading power factor load I_a . Whether leading or lagging this equation is true. So, V plus $jI_a x_s$ I have to draw. Here is your I_a direction. So, a draw a line which will be perpendicular to this I_a line and this will be your $jI_a x_s$ and this will be your excitation voltage and this will be angle δ .

So, this is leading power factor load, leading power factor load. So, in case of leading power factor load lagging or leading no matter E_f will be always above V for generator mode. Only thing in case of leading power factor load length of E_f is less than V length of E_f is a measure of rotor field and rotor excitation field that one and V this length is approximately equal to E_r resultant field.

Therefore, this will be measure of the resultant field. So, what should I say leading power factor generator is it over excited or under excited? Under excited because length of E_f is less. So, under excited and this is E_f length of E_f is greater than b . So, it is over excited. I am not going in this phasor diagram to the level of ϕ_f and ϕ_r because length of E_f I know, it is a measure of ϕ_f ; length of V is a measure of resultant field. If I am asked I will be able to draw, but for problem solve solving from electrical sides it is not necessary what I am telling. So, this is the thing.

In case of motor mode, what is the expression of real power drawn by the machine? It will be simply $E_f V$ by $x_s \sin \delta$ that is all I am not writing the expression of q that can be easily found out. In case of motor mode, so, first you draw this E_f , here is your Z

s; Z_s means only jX_s reactance and here is your supply voltage V and it draws a current of I_a . This direction of current assumption is important you can just compare.

Then what I will do? I will once again start from the V terminal voltage suppose the motor is drawing a lagging power factor. So, this was lag, this was lead leading power factor. So, suppose lag power factor. So, motor is drawing a current of I_a like this and this angle is the power factor angle at which it is drawing and there is. So, from V E_f is $V - I_a X_s$. So, V minus. So, $j I_a X_s$ is this way; so, negative of that.

So, it will be what you should do? You should draw a line perpendicular to this and this length will give you minus $j I_a X_s$ and this fellow will be your E_f and this angle will be δ . E_f will be below V , everything will be in place; if this convention is followed. Is it under excited or over excited? Length of E_f is less than less than length of v . So, it is under excited; just opposite to that of a synchronous generator if it is drawing lagging power factor current, it will be under excited.

And, what will be the leading power factor case? V ; suppose this is I_a , this is θ . So, $V - j I_a X_s$ will be your E_f it is written their this is the equation. So, V , so, $j I_a X_s$ is like this. So, negative of that. So, it will go like this such that this angle is 90 degree and this will be your excitation voltage and this is δ . So, if this is the reference excitation E_f is greater than V it is over excited. So, we will continue with this in the next class.

Thank you.