Electrical Machines – II
Prof. Tapas Kumar Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 70
Expression for Starting Torque & Need for Phase Splitting

Welcome to the lectures on single phase induction motors. So, we have seen that a single phase induction motor having only one winding on the stator, will have only running torque. There cannot be any starting torque and then we developed the equivalent circuit of the motor when it runs on single winding. Of course, to get the equivalent circuit we brought an imaginary coil whose magnetic axis is perpendicular to the main winding and assume that current to be 0 while developing the equivalent circuit and hence the expression of the torque.

But, now the question is you cannot have a motor which is not having any starting torque. Of course, physically if you run it will start in any direction you like, but that is not a very technical answer, one should not operate a motor like that.

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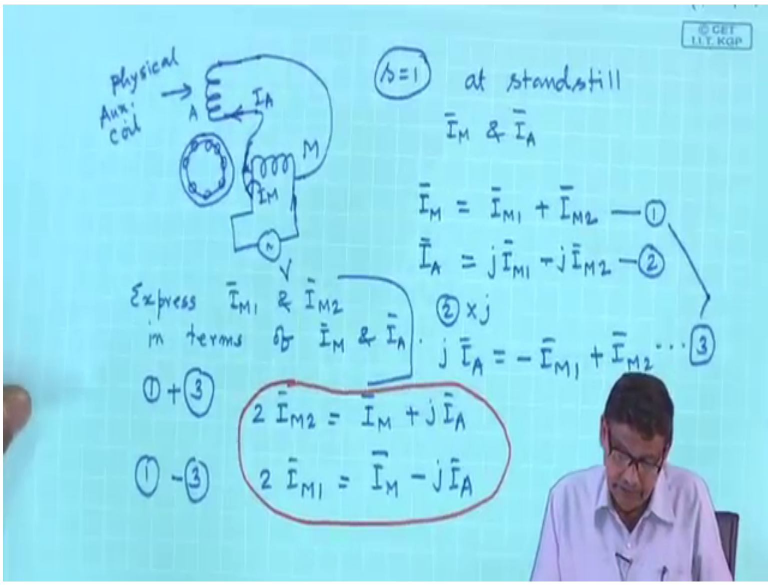

Therefore, to incorporate starting torque what now we have to do is that the rotor is there like this, cage induction motor rotor and here is a main winding as usual. And, now I connect a physical coil. It is now no imagination, nothing like that. This is physical coil auxiliary winding coil auxiliary coil and this coil I will call coil A, and it is now physically present both the coils are present. So, and this two coils will be eventually connected in parallel across a single phase supply, ok.

Now, the question is suppose and everything now I will be dealing with the assumption that machine is stationary or s equal to 1, at that instant what is happening if you switch on this supply? So, at stand still condition because I want to investigate how starting torque can be incorporated in a single phase induction motor with the help of another winding which is called auxiliary winding or sometimes called starting winding. So, these two coils will be eventually connected in parallel and across a common single phase supply like this V therefore, I mean this current is I M current drawn by main winding and current drawn by auxiliary winding is I A.

Now, if the impedances of these two coils seen by this supply at s equal to 1 are different then there will be a time phase difference between these two currents. And, let that current be denoted by I M and I A when you connect it across this supply V at stand still condition, machine is not allow to rotate. In that case once again this two currents can be broken up into a positive sequence current I M can be written as I M 1, these are all phases, plus I M 2 and we have done several times this and the auxiliary winding current. This I M, I A are unbalanced two-phase current be thought of and this can be written as I A 1 plus I A 2, but I A 1 is the positive sequence is nothing, but j I M 1 and this is minus j I M 2 this will be the thing. And, this time I A is not equal to 0 that is what I want to say it is a physical coil it is carrying a current. So, that is there.

Now, from this equation what I am planning to do first is at express I M 1 and I M 2 in terms of in terms of I M and I A that is what I M and I A. That is what will be planning. So, two equations are there that is not at all a problem. In fact, if you call this to be equation 1, this to be equation 2; I mean I am doing this way it can be done several ways. I will multiply this second equation 2 with j both sides. So, that it will be j I bar A is equal to minus I bar M 1 jj minus and this will be j minus and minus plus I bar M 2, let this be called equation 3. And, these two equation, I will now use to get this thing results.

For example, if you add 1 plus 3 add 1 plus 3 if you add it will be 2 I bar M 2, 2 I bar M 2 from the right hand side and this will be equal to I M bar plus j I bar A, it will be like this. Similarly, if you subtract 1 and 3, 1 minus 3 if you do then I M 2 will go off and it will be 2 I bar M 1 this minus this is equal to I M bar minus j I bar A. This is the thing we have got. Why it is necessary it will be just clear, why this knowledge or the expressing I M 1 and I M 2 will be necessary I will tell you that.

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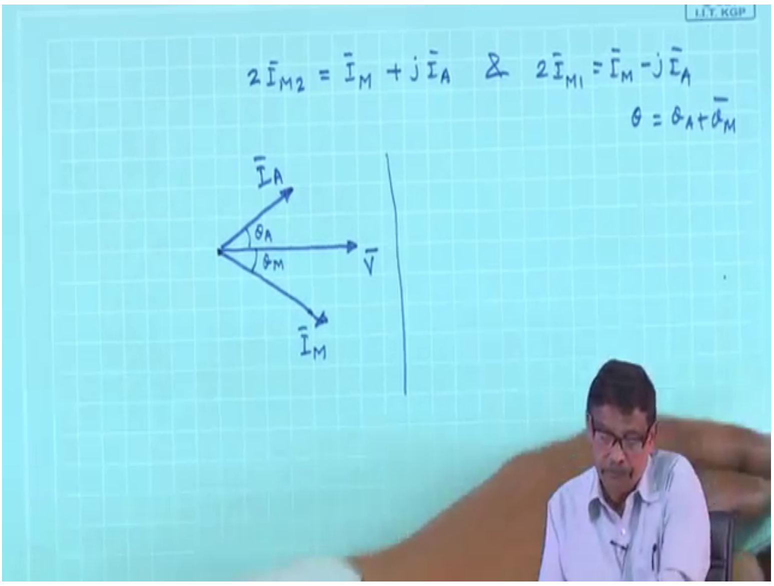

But, now let us see what this means see this equation is I am rewriting here. So, 2 I bar M 2 is equal to I bar M plus j I bar A and 2 I bar M 1 is equal to I M bar minus j I bar A, correct. So, suppose this is the voltage phasor, this is the voltage phasor, is that clear? So, there will be current taken by auxiliary winding and there will be current taken by main winding. Suppose, this is the current of the main winding and suppose this current is the I bar A. Suppose, the impedance of auxiliary winding is such that it is leading.

Therefore, you know this angle will be theta A, the power factor angle of auxiliary winding and this angle will be theta M the power factor angle of this. Let the total angle be theta A plus theta M; suppose theta I M and I A angle between them is theta. So, I will just avoid writing this V what I will simply tell that so, we know suppose the angle between I M and I A phasors is theta and let me only draw this two currents.

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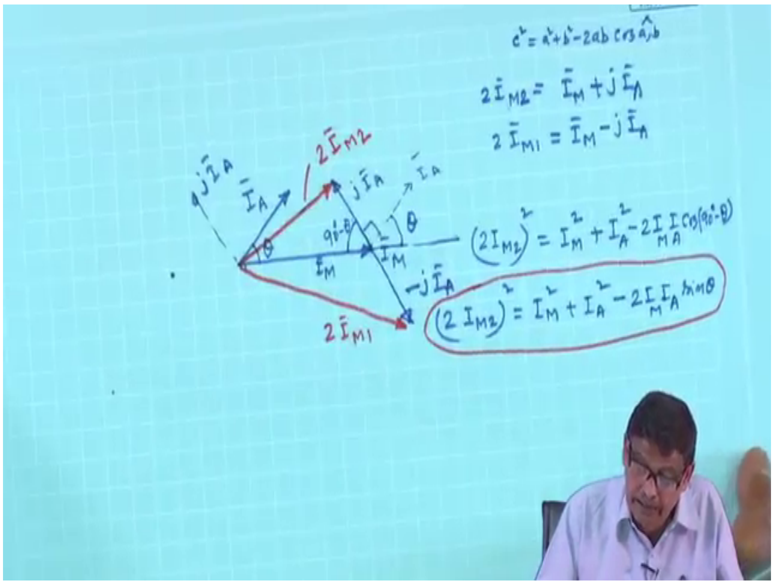

So, this is suppose I M voltage phasor I am not drawing and this is suppose I A and the angle between them is theta like this, V was here for example, in the earlier one. Now, we know that 2 I bar M 2 is equal to I M plus j I bar A. So, to get this negative sequence component of I main winding current I have to add 2 I M j I A. Where is j I A? j I A is, so I A is here, j I A will be right angles to this. This will be j I A is not j I bar A, is that clear? Therefore, you add this is suppose I A, I A parallel to this to this I have to this is 90 degree. So, this is j I bar A.

And, I am telling this will be this then will be 2 I bar M 2 and similarly 2 I bar M 1 2 I bar M 1 is equal to I M minus j I bar A. So, I M is already there j I A bar is this. So, minus of j I A bar I have to add 2 I A bar M. So, it will be this way this is minus j I bar A and if this two are added then this will give you the value of 2 I bar M 1, is that clear? That is what you will get, ok. So, I have got now two triangles recall that this angle the angle between this is the direction of I bar M. So, this is theta, theta this is 90, this whole angle is 180. Therefore, this angle must be 90 minus theta; 90 minus theta must be this angle.

Now, I will go to the properties of triangle consider this triangle that is suppose a sides are a, b, c are the sides of the triangle then any side square for example, c square is equal to a square plus b square minus 2ab and cosine of the angle between these two, this is the standard thing. So, I will use that. So, this side square must be equal to this side square plus this side square this is I bar M. So, this square is equal to this square plus this square minus 2 into this into this into cosine 90. So, I can write, but this time I am writing only the magnitude no phasor business, sides of a triangle their lengths.

So, I will be getting 2 I M 2 it is magnitude square without bar means magnitude are meaning this must be equal to I M squared this length square plus this length square do not put minus I A square, length I A square its magnitude, then minus of 2 I M I A into cosine of the angle between this two. So, angle between this two is 90 minus theta. So, cosine 90 minus theta I will I will write it better. So, cosine 90 minus theta. So, I will get 2 I M 2 square is equal to I M squared plus I A square and this is minus 2 I M I A sin theta sin theta. So, this is one equation important. So, I have been able to express I M 2 in terms of I M I A as I was telling and in terms of the angle between I M and I A.

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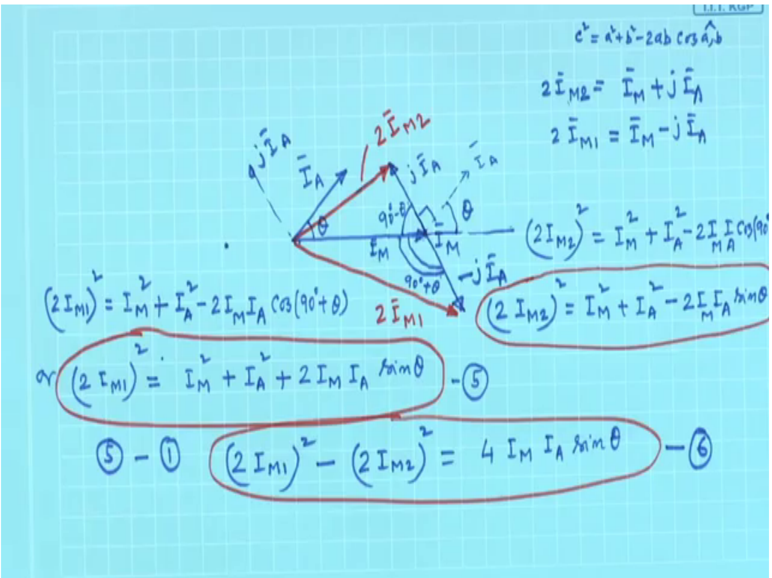

Similarly, you come to this triangle this lower triangle comprising of sides I M 2, I M 1 and minus j I A this sides. So, here once again this is 90 minus theta this whole angle is 180; therefore, this angle must be 90 plus theta. And, here once again you see it is equal to this side square that red side square it is magnitude 2 I M 1 square. This must be equal to I M square the length of this side square plus the length of this side square, this is once again I A square the length of the sides I A square then minus of 2 I M I A, 2 I M I A into cosine of the angle between this two sides and that is 90 plus theta. So, it is cosine 90 plus theta.

Or, it will be 2 I M 1 squared is equal to I M squared plus I A square and cosine 90 plus theta is minus sin theta. So, this will be plus 2 I M I A sin theta. This is the other equation. So, this is one equation and this is another equation and this two equations I will be using to find out the torque in the machine; torque means starting torque because I will not allow s 2 change to any values, stand still condition whether there will be starting torque or not.

In fact, when I will do that it will be necessary to know this one, if you subtract this is suppose 3, this is suppose equation 4 and this is suppose equation 5. So, if you do 5 minus 1. In fact, their difference will be of importance and I could do it later, but anyway since it is in the same paper. So, minus 2 I M square minus 2 I M 2 square this minus this if you do this two terms cancels and you will be left with 4 I M I A sin theta. So, this will be the thing ok. I will this result will be actually utilized say 6 here 6.

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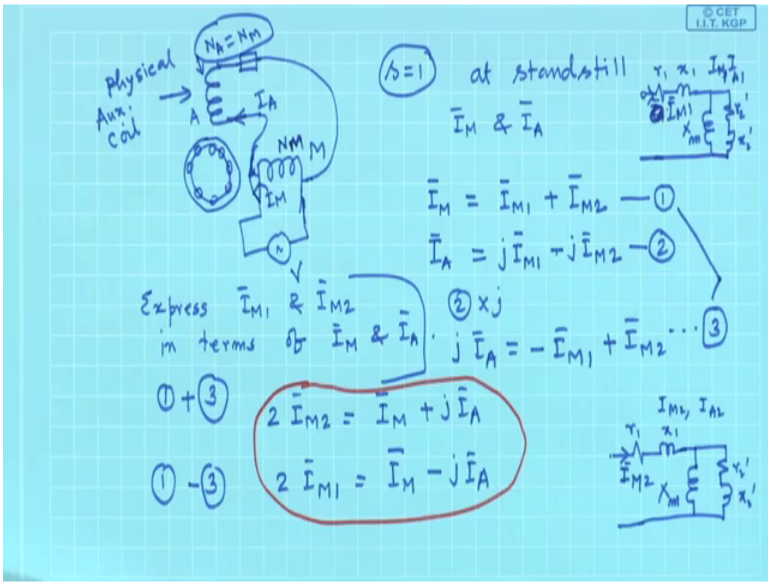

Now, how to find out the expression of the torque? Now, remember that this is the thing I am discussing about this is the machine I A, I M is there. I A, I M I have broken up into positive sequence current and negative sequence current. Therefore, they are going to be two rotating field, is not? Depending upon I M 1, I A 1 and I M 2, I A 2 in the opposite direction.

Each one of them has per phase equivalent circuit. What will be that per phase equivalent circuit? It will be something like r 1 x 1, is not? And X m and r 2 dashed x 2 dashed per phase equivalent circuit of that balanced machine which is excited by I M 1 and I A 1. What will be and this current I am telling it is I bar A at s equal to 1, there is no r 2 dashed by 2 s it is a balanced two-phase machine per phase impedance is r 2 dashed. Similarly, these I M 2 and I A 2 will also have the same exactly same equivalence circuit because slip is 1, X m and this is r 2 dashed x 2 dashed.

And, this current is the main winding current I i am sorry this is I M I bar M 1 and this current is I bar M 2. s equal to 1 in both the case, nothing like r 2 dashed by 2 minus s. No, s equal to 1. So, this is the scenario. When I am telling it is Z a I am telling that if you apply a voltage to this what is the impedance seen by the main winding. Similarly, for the auxiliary winding Z a, I will manipulate this resistances.

Now, in this case if you leave it like that it looks like Z a and Z m will be same is not, but that is what I am telling. Here what I will do in all this analysis I have assumed I have connected something in series with the auxiliary winding, some external impedance apart from its own impedance here. So, that will make that thing change that you keep in mind. But, what I am telling since s equal to 1 whatever the impedance is being shown to this supply by the machine at s equal to 1 that we take into account to describe Z a Z m.

And, another thing I have assumed the number of turns of this main winding is N M and that of the auxiliary winding is also N M I have assumed. So, that the reflected r 2 dashed is same, are you getting? This point is somewhat important because r 2 dashed is what with respect to auxiliary winding, r 2 dashed is what with respect to main winding. In any case this is the scenario and I have assumed N A equal to N M. What is N M? Number of turns of the main winding then it is taking I A it is taking I M.

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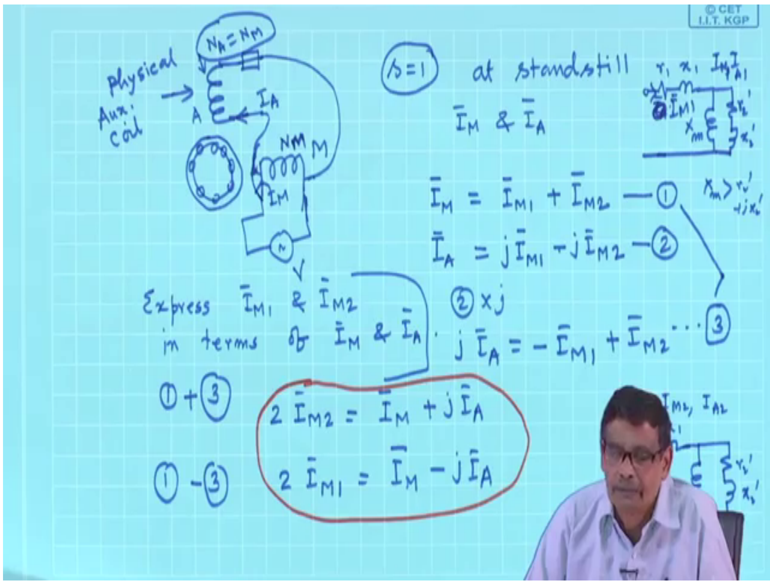

Now, now I will assume since s equal to 1, this X m is because you are only try to guess approximately I will not guess try to get an approximate expression of starting torque. So, I will assume this X m is much higher than this parallel r 2 dashed plus j x 2 dashed whatever it is, are you getting? So, that as if that current is the current flowing through the air gap resistances which represents torque.

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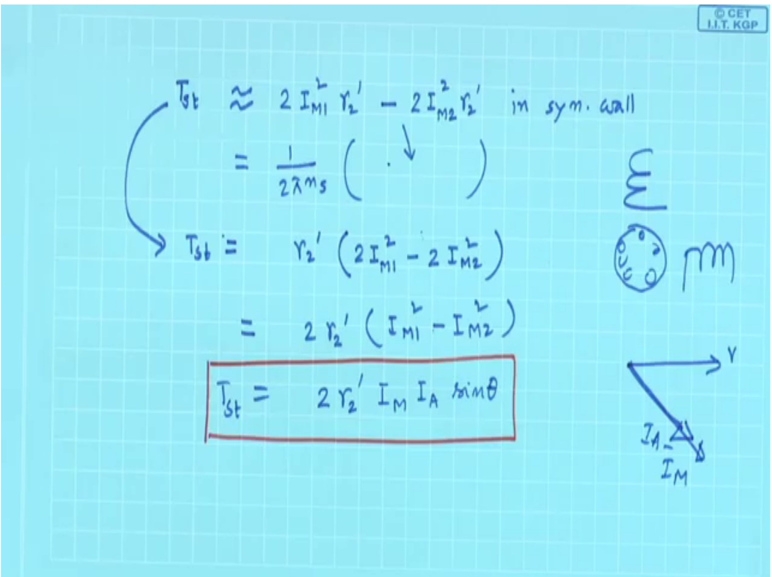

In other words, the whatever I have talked right now, I will tell that the starting torque can be approximated to this I M 1 this current I M 1 is as if flowing through this r 2 dashed because of this assumption ok. I will not distinguish between I M 1 and whatever you call this parallel because r 2 dashed x 2 dashed in this case I know pretty well it cannot be high there is no r 2 dashed by s; X m is high.

So, estimation of the torque will demand that T starting therefore can be simply written as 2 I M 1 square into r 2 dashed in synchronous what, that is all per phase into 2. And, from physical resistance I know the negative sequence is in the opposite direction that is why I have put the minus because that rotating field will rotate in the opposite direction. So, I will write 2 I M 2 square into r 2 dashed in synchronous wall and if you like this will be equal to 1 by 2 pi n s synchronous speed into this same thing here.

Therefore, this is the expression of the starting torque of the of the single phase induction motor having two coils, main and auxiliary winding and whose impedances are not same that is r 1 x 1 here of course, I have written that what I am trying to tell in the auxiliary winding I have connected something. So, here as if I have connected something here that is that auxiliary winding business I have connected like that. So, this is the thing.

So, if that be the case then you know the torque produced will be equal to in synchronous what only I will do T starting then will be equal to r 2 dashed into like this. This will be the expression of the torque. So, difference of I M 1 square minus I M 2 square is coming. So, this will be then equal to [FL] in this previous case recall that I have got this one 2 I M 2 square minus 2 I M n square is 4 I M I A sin theta. So, from this I can write I M 1 square minus I M 2 square is equal to I M I A into sin theta no 2 is missing hopefully.

So, this will then be equal to I will write 2 r 2 dashed into I M 1 square minus I M 2 square and this one is nothing, but I M I A sin theta. So, it will be equal to 2 r 2 dashed I M I A sin theta. This is the expression of the starting torque. See, one thing is clear that your machine is like this; this is main winding, this is your auxiliary winding. If the power factor angle of both this coils are same, then theta will be 0 theta is after all the angle between I M and I A.

So, this equation tells you that the starting torque of the induction motor can be created provided your if this is your supply, if this is your I bar M, and if this is your I bar A in phase then theta will be 0 and no starting torque can be provided. Therefore, to have a starting torque you must phase split this stator impedance of the auxiliary winding with respect to the primary winding.

So, we will continue with this in the next class.

Thank you.