Electrical Machines - II
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Lecture – 07
Analysis of Single Conductor Generator and Motor (Contd.)

Welcome to this next unit of our Electrical Machine - II course and you recall that in our last lecture, I told you about a single conductor motor and generator operations. In fact, we are in the process of analysing that. So, remember the arrangement it is like this.

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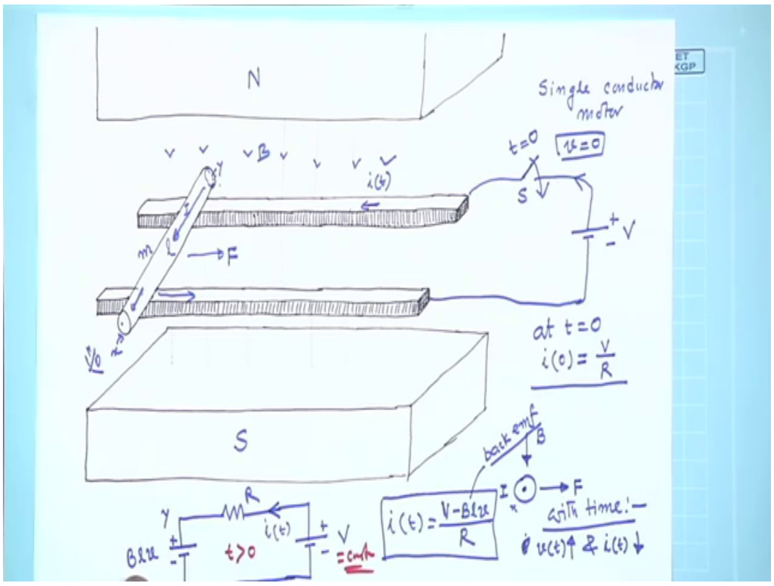

And suppose I say that I will quickly review that. Yesterday, I showed this to be south pole this to be north pole it is your choice, it does not matter. So, for example, I say that this is north pole so that you get the other flavour how to tackle this. This is south pole.

Then these lines of forces will be like this, coming like this and this two are rails frictionless assume to begin with and this is a conductor of length l l metre and it is initially at rest condition. So, the thing is it is single conductor motor ok. So, all the lines of so, this conductor if you look from this end, you will see this end which I called end x and this end is y to distinct ends of the conductor and it is resting and we say that the velocity of the conductor is 0.

Now, also what I have done is this in this rails, I have connected a switch and then suppose a battery like this and the battery voltage is capital V. So, the switch is closed at t equal to 0, I start counting my time from t equal to 0. Before the switch was closed, it was open no current was flowing and this conductor will remain stationary forever. But now what I do; I close the switch. Therefore, there will be now current flowing in this direction will be rail and it will enter through the end y; y is actually this way it will flow is not this is the current and through the rail it will return. It is open circuit here so, it will return like this. So, the current will flow.

So, at t equal to 0, I am sure nothing was moving; the current in the circuit at t equal to 0 was simply V divided by R. What is R? R is the resistance of the rail etcetera; the whole circuit. This was the situation, but the moment this conductor carries current i in the direction shown and this diagram I will if you look from this end can be simply drawn in this fashion. This is your I dot from the end x I am looking at it I and this is the direction of the, this is the thing and length is perpendicular to the board like this length is here.

So, this end is a end x of the conductor and it is carrying a current I. Therefore, a current carrying conductor placed in a magnetic field must experience a force and the direction of the force will be like this. Apply left hand rules B. This is I so, velocity will be I mean force will be in this direction or conductor will start moving to the right from t greater than 0 equal to 0; it experiences a force.

Now, the moment it experiences a force, it has got a mass. Therefore, it is suppose to accelerate and let us assume, there is no friction no opposing force present in the system. So, in a absence of this opposing force, this conductor will start moving. The question is whether the velocity will remain same? Let us argue physically. When it was stationary, it experienced a force the magnitude of which is B into V by R this I into l, but as it start moving; it will have some velocity. Therefore, we also know that a conductor moving with a velocity V and placed in a magnetic field B must generate a voltage across its two ends x and y.

And the magnitude of the voltage is B l V. Therefore, in the circuit this closed circuit now, not only this supply battery voltage V is present, but there appears another voltage here on this side. And, the polarity of the voltage can be easily made out by applying right hand rule and you can easily see the polarity of the voltage will be this side plus this side minus and the value of the voltage is B l V. Therefore, current in the circuit will be V minus Blv divided by R not V by R alone because the moment it has got some instantaneous velocity V, there appears this conductor itself becomes a source of e m f.

So, this whole thing this elaborate thing can be drawn like this; this is the conductor end x and y and there appears a voltage with side plus this side minus. This how do I get? By applying right hand rule and suppose the resistance of the whole network, I show it by R and here is a battery V supply voltage and this is the direction of the current. Therefore, the current which was flowing at t equal to 0 minus which was equal to V by R; I now know that current value must decrease because V is constant, but Blv is the opposing voltage. So, this voltage minus this voltage by R will be this current.

So, i t will be V minus Blv divided by R, this is the current. But unfortunately the expression of the current cannot be found out because V is not a constant thing; I must know what V is as a function of time. So, current which will be flowing in this circuit is V minus Blv, it is sometimes called back e m f. So, this is the most fundamental wave ok; this i t is there. Now if this i t so long i t is present, I am sure about one thing. It will experience a force towards right. Therefore, it will go on accelerating from 0. So, long i t is present B is present so, it will still speed up from 0 velocity when it comes here it will speed up, it will speed up, it will speed up and also I can predict that the i t value will go down and down.

Because B is increasing with time capital V is constant B l R all are constant. Therefore, with time with time, I am confident without doing any mathematics only looking at this expression I can say that i t v t will go up and i t will come down. Now how with time this voltage and current changes? I want to know mathematically. So, that is why we must write down these equations.

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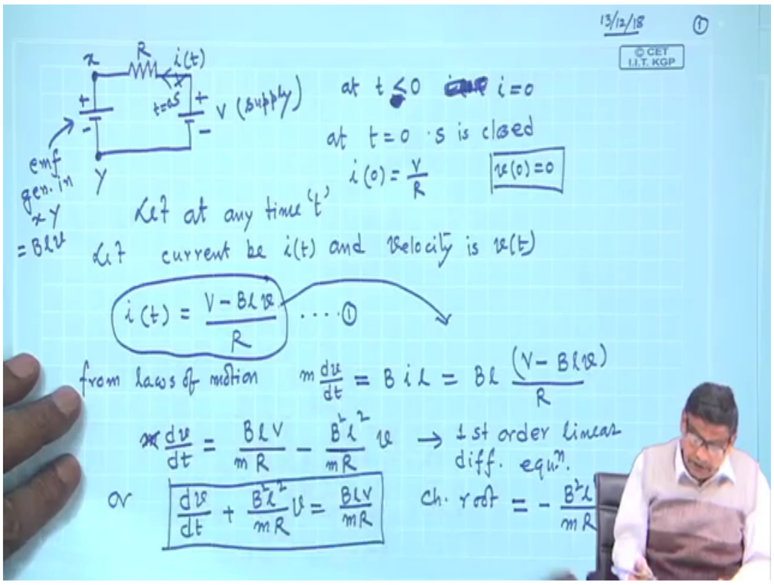

So, one equation is clear. So, this is the essentially the electrical circuit diagram is this R this is capital V, this is my supply voltage mind you V supply and this is e m f generated in the conductor in x y and this is your terminal x, this is your terminal y and this is the current. And as I told you and there was a switch here s which i have closed at t equal to 0. So, at t less than equal to 0 i V i i was 0 i was 0 for all time less than time at including at and that at less than 0 put less than 0. At t equal to 0 s is closed. And therefore, i 0 must be equal to V by R and it will experience a force.

So, let at any time elapses, suppose at any time t let current be i t and velocity of the conductor velocity is V t with the polarity as shown. Therefore, the expression of the current will be V minus Blv divided by R this e m f is equal to B l V. So, this is one expression, but to get the expression of current and voltage for any time t have to generate another equation. And what is that equation? That equation is electromechanical equation dynamic equation of this moving bar. This bar is moving and what let us assume the mass of the bar is m and velocity is V.

So, from Newton’s laws of motion from laws of motion, I know that m d v d t d v d t is the acceleration in to m must be equal to the force switch is acting. What are the force which is acting on the conductor, I have assumed friction absent; no air friction, no rail friction, only force which is acting is this electro dynamic force that is B i l. So, m d v d t is equal to B into i into l at that time whatever is the current into l is the length of the equation

Now, then I will first solve for velocity small v is velocity, no voltage capital V is the supply voltage. So, the this one can written as B into l and for i, you put this num[ber]- expression Blv divided by R. And therefore, your m d v d t is equal to Blv by R minus B square l square by R into c. Now if you divide by m both sides, then this will become m R and m R here. This is constant. So, this is a first order differential equation first order linear differential equation and if you solve it, you will get the expression of velocity. So, how to solve it?

So, I will write it in this way d v d t plus B square l square by m R is equal to Blv by m R. This is the equation and the characteristic root is equal to minus B square l square by m R. I you must be knowing how to solve a first order differential equation with initial conditions. What was the initial condition? V 0 is equal to 0 i 0 was V by R V 0 was 0.

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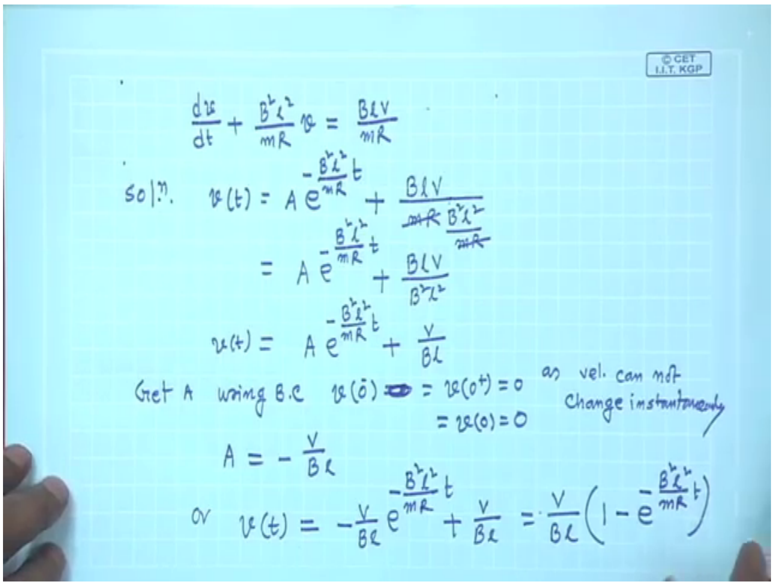

Therefore, your equation solution of this equation d v d t plus B square l square by m R into small v is equal to Blv by m R. Solution can be written as with characteristic root this one is equal to V t is equal to some constant A into e to the power minus characteristic root into t plus forcing function is a constant number. So, so, solution due to forcing function will be this divided by coefficient of this.

So, this will be Blv by m R divided by B square l square by m R. You can also apply Laplace transformation whatever way you solve, solve it. So, this will be equal to then A into e to the power minus B square l square by m R into t plus this m R goes it is equal to Blv by B square l square is not. So, or it could cancel that out. So, this will be equal to A into e to the power minus B square l square by m R into t plus V by B l. This is the thing.

Now, to determine constant, we know get A using the fact using boundary condition that v 0 is equal to 0. See velocity of a mass cannot change instantaneously. So, at t equal to 0 minus velocity was 0. At t equal to 0 plus although there is some current flowing in the conductor, but velocity cannot change instantaneously that is what inertia means. Therefore, v 0 plus is also 0. So, v 0 minus i mean v 0 plus is equal to 0 as velocity cannot change instantaneously. Therefore, this will be (Refer Time: 19:01) or this is this essentially means V 0 is equal to 0 all this things.

Therefore you get A, if you put that A t equal to 0. This will become 1, it will be minus V by l B l or v t will be equal to minus V by B l into e to the power minus B square l square by m R into t plus V by B l which can be written in this way 1 minus e to the power minus B square l square by m R into t. This is the expression of voltage ok. Once I get the expression of the voltage expression of the current can be found out from this, I will put this expression of voltage in this equation. So, let us do that.

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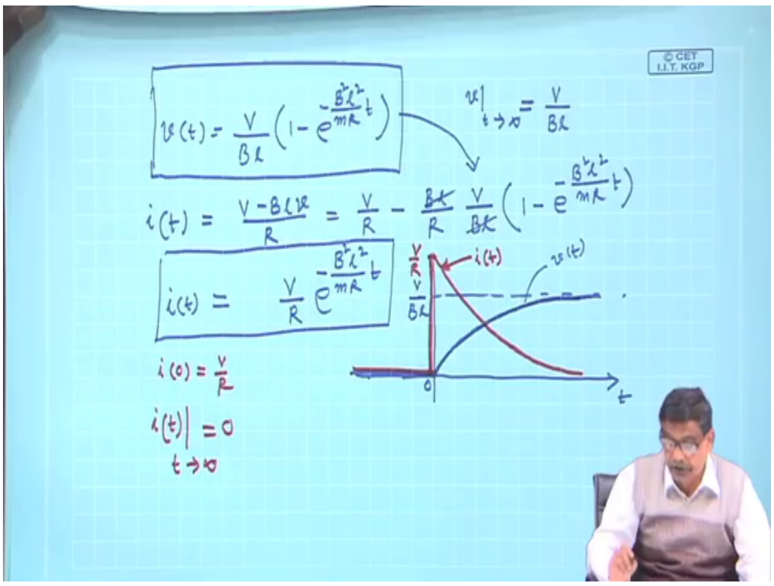

So, v t i am rewriting, I have got it like this; V by B l then 1 minus e to the power minus B square l square by m R into t and we know that i t is nothing, but V minus Blv by R. So, that is equal to V by R minus B l by R into V and for this V i put this one V by B l. It is this one I am putting it here. V by B l into 1 minus e to the power minus B square l square by m R into t; this B l goes.

So, if you simplify this V by R and there is a once again minus V by R. If you open the bracket so, essentially i t will be equal to V by R e to the power minus B square l square by m R into t and this is your expression of i t. So, expression of current how it varies with time I know, expression of the voltage how it varies with time I know. And if you sketch it against time mind, you what is t equal to 0? t equal to 0 when the switch was closed.

So, if you sketch it against i, you will find that velocity will increase exponentially. At t equal to infinity as t approaches infinity, velocity will become capital V by B l; the this level is capital V by B l. What is capital V? Supply voltage exponentially, it will be like this and what will be the current? Current will be before that you, I will also draw for t less than 0 velocity was 0. So, this curve is velocity v t; velocity was 0 then it was increasing exponentially and finally, final velocity is V by B l. What about current?

Current was also 0 prior to t less than 0 because the switch was open and that t equal to 0 current will have a straight jump. The value of current i 0 is equal to V by R put t equal to 0. And since it is a resistive circuit so, current will have a step jump. Step jump is a possibility no problem in a resistive circuit and this amplitude is V by R, it will go and then it will exponentially decay down to 0 because as t approaches 0 i t t approaching infinity is 0. So, this is the expression of the current.

Therefore what I want to tell you is this that this is the physical system. A conductor was there, switch was opened, no current was flowing, there was no velocity. So, this conductor was ideal and it will be sitting at this point forever. Now somebody comes and closes this switch at t equal to 0. If it closes, then current will flow B i l force will appear, it will make the conductor move towards right with a velocity. Velocity cannot change instantaneously because this bar has got a mass. So, velocity at t 0 minus before closing the switch, it was 0 at t equal to 0 plus also velocity has to be 0. Although at t equal to 0, the moment you close the current in the circuit is V by R. In this force will appear.

But the bar is yet to move I mean, it will preparing to move inertia because of. So, velocity at t 0 plus is also 0, then this force is acting on this mass. Therefore, it has to accelerate the moment, it starts moving it acquires some velocity for t greater than 0. The scenario, this circuit here changes. The moment conductor starts moving, Flemings right hand rule will demand that this between the point x and y there must be a voltage because here is a conductor which is moving with a velocity v.

No matter how it acquired that velocity, thing is it is having a velocity it is having a B. It has got a length therefore, it must have an induced voltage of magnitude Blv and by applying right hand rule I can find out the direction of the polarity of the induced voltage which will be this side plus this side minus. Therefore, this whole network for t greater than 0 is this; this is constant supply voltage.

Therefore it will move and it is carrying a current like this, but the point is the current in the circuit is no longer decided by capital V alone. There is another e m f coming here Blv with polarity as shown plus minus by applying right hand rule. So, this V and this Blv difference of this two will decide, what will be the current from right to left as I have shown. So, as time passes velocity increases. Why velocity should increase? So, long this current is there i t is positive, it is having an accelerating force. Accelerating force value will diminish with progressing time progressing time no doubt.

But so, long i t is there, there will be an accelerating force and speed will go up and up rate at which it is going up will of course, slow down as we will go along the time. But the nonetheless this V will go on increasing. So, long i t is there and the value of i t in the numerator, the negative term is increasing; it will decrease. Therefore, I can conclude without having the final result that current value should go on decreasing and velocity should go on increasing.

Now, the question is what is the equations, following which equations voltage and current will a velocity and current will behave. So, to do that we have found out this dynamic equation, first order differential equation from that I found out the value of velocity which is like this and also i found out the current because there are two unknowns V t and i t and then I sketch them. It is velocity will increase exponentially and current will decrease exponentially. So, I continue with this, but try to understand every bit of this lecture.

Thank you.