Electrical Machines - II
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Lecture – 43
Expression for Electromagnetic Torque in Terms of Equivalent Circuit Parameters

Welcome. So, we were discussing about the equivalent circuit of an induction motor. See initially, we just told about how torque is produced in a motor. And try to understand it, physically what is happening that is the rotor field, stator field this that. And the interaction of the stator result in field and rotor field gives you the torque.

Now, we have gone further, and we try to develop the equivalent circuit of a 3-phase induction motor per phase basis. And from that looking at the power flow that takes place in the machine starting from supply source to the mechanical power developed by the motor.

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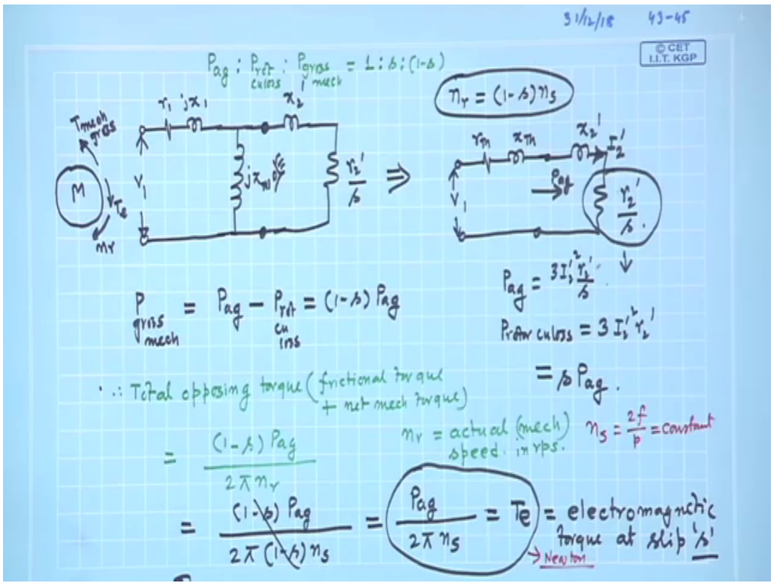

So, I we develop this equivalent circuit and told you that this is the equivalent circuit which is a stator resistance stator leakage reactance per phase. Now, it is similar to that of a transformer, and then you have this magnetizing branch which is j x m. I mean all reactance's are j that is there.

And then on this side, the reflected rotor parameters came out to be x 2 dashed, and this is r 2 dashed by s ok. And slip depends upon the operating point of the induction motors. And slip is a measure of speed as well, because you know actual rotor speed, all speed are mechanical here is 1 minus s into n s; so and n s is this synchronous speed.

So, now this is the per phase applied voltage V 1 this is the thing. Then what I did between these two points, we found out the Thevenin's equivalent. And this circuit was redrawn as you know r Thevenins x Thevenins m series, and then this remains V 1. And then x m goes, and you are having this equivalent circuit ok. And this current is suppose I 2 dashed. When we just considered the power balance, after we have got the equivalent circuit the rest of the work is somewhat routine network type analysis, no field is coming here that is one good thing.

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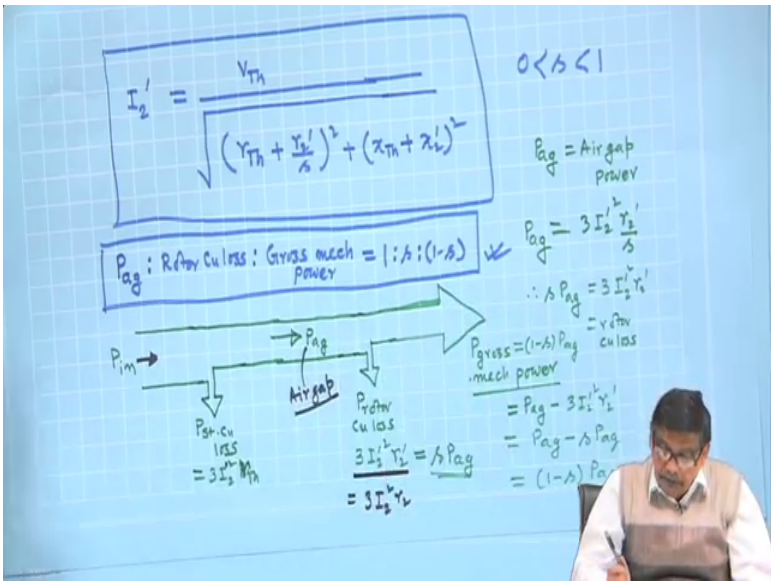

So, the input power to this circuit is and I drew this I will not waste time. Last time we told you that this equivalent circuit means, you are giving some input power drawn from the supply out of which a portion will be lost in the stator coils, as stator copper loss 3 I 2 square r Thevenins that is value will be I mean I 2 dashed squared r Thevenins.

Then rest of the power is as you crossing the air gap, and entering into the rotor that is why, it is called air gap power air gap power for our own understanding ok; we will interpret it like this. Total input power minus this stator copper loss or stator loss whatever is there. There will be cold loss also that we will take care later P a g. And then this power after entering into the rotor circuit, there will be a power loss in the rotor resistance, which mind you is 3 I 2 dew dashed square r 2, which is same as 3 I 2 square r 2 because we have referred everything to there ok.

Now, recall that this equivalent circuit, the power involved in the circuit is the air gap power, because this is the power real power enters. So, P a g is 3 I 2 dash squared r 2 dashed by s. And obviously, rotor copper loss is equal to 3 I 2 dash squared r 2 dashed. So, it looks like rotor copper loss is nothing but s in to P a g.

Now, so from P a g, if you subtract this rotor copper loss if you subtract it, then you get P gross mechanical which will be equal to P a g minus P rotor copper loss. If you subtract, this becomes 1 minus s into P a g. And this is shown in this one P a g comes, then you subtract rotor copper loss, then you get P gross mechanical power whatever mechanical power, it is called gross because a portion of it will be lost to supply the mechanical losses in friction etcetera.

This is not the net mechanical power output, but gross mechanical power output. Because, this gross mechanical power output only decides what is the total opposing torque developed by the machine, put on the shaft of the machine including the actual mechanical rotor and the frictional power. Anyway, so this is the thing.

Now, our target is to find out, what will be the torque developed by the machine. So, this we will do in this way, I have got the P gross mechanical power from which I will find out, what is the mechanical torque present on the shaft of the machine. And we will assume that the machine is running with a fixed value of slip s that means, at a fixed RPM.

Therefore, the if I can estimate the mechanical torque, then the electric torque developed by the machine must be same. So, so remember this visual picture any motor, this is the electromagnetic torque, this is the direction of rotation n r. And the gross opposing torque T mechanical gross, this two must be same, so that it is running with a particular RPM n r or with a particular slip. Because, every n r has got a if you map it to slip, it has got a unique value of slip, so that is the thing.

Therefore, from this I will say and then I told one very important relationship that is the this must be remembered that is P a g is to P rotor copper loss is to gross mechanical is in the ratio 1 is to s is to 1 minus s that we have found out. So, this is the thing P gross mechanical.

Therefore, total opposing torque opposing torque, which is sum of the frictional torque to be overcome fictional torque plus the actual net mechanical net mechanical torque, which is doing useful mechanical work for me that is the thing. Therefore, this must be equal to this must be equal to 1 minus s into P a g that is P gross mechanical divided by 2 pi n r, where n r is the actual rotor speed actual mechanical I will repeatedly not write, but any way actual mechanical speed n r in rps if you want to get torque. So, this is the thing.

Now, what I will do this for n r, I will substitute this thing here. So, it is like this then 1 minus s into P a g divided by 2 pi 1 minus s into n s. And then you see 1 minus s goes, and gross mechanical power developed will be equal to 2 pi n s. And this must be equal to the electromagnetic torque developed, because machine is running at a constant speed or constant value of slip.

So, we are trying to get an expression for electromagnetic torque not in terms of delta or b r or b s, but in terms of equivalent circuit parameters that is the idea here. Because, this equivalent circuit parameters if these are known to me, then I will be able to calculate torque simply by using the equivalent circuit parameter. So, this is the also equal to electromagnetic torque developed by the machine at slip s that is the thing.

So, this is the expression of electromagnetic torque of which you must note this point that this n s is constant n s sorry n s is equal to 2 f by p is constant. So, long the supply frequency is constant, and machine is given so p is also constant. Therefore, the electromagnetic torque developed by the machine, if you divide by 2 pi n s the air gap power, then its unit will be in Newton Newton so much Newton. If this is in watt, and this is in rps.

Now, note that to calculate the electromagnetic torque, therefore if you have estimated the air gap power that itself is a measure of electromagnetic torque developed by the machine why, because 2 pi n s is constant. So, if you have calculated P a g, electromagnetic torque is as such calculated I have to simply divided by a constant number 2 pi n s. So, sometimes P a g is considered to be the torque developed and P a g alone its unit is watt, so it is called a given a name synchronous watt.

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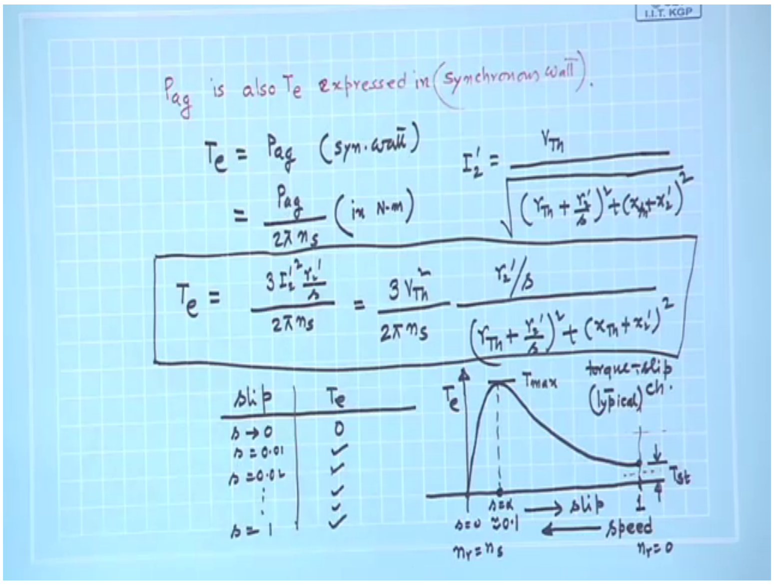

So, P a g is also T e expressed in synchronous watt, this is a term synchronous watt. So, sometimes torque is expressed in sometimes torque is expressed in newton, then so torque T e is equal to P a g you can write absolutely correct, but write synchronous watt its unities or you write P a g by 2 pi n s. And this will be in newton meter that is the thing should just recall.

Now, after I have learned that in torque in Newton meter is this, let us further simplify this expression that is T e P a g P a g is nothing but 3 I 2 dash squared r 2 dashed by s by 2 pi n s. Now, I want to express this expression of the electromagnetic torque developed by the machine in terms of equivalent circuit parameter. And equivalent circuit is this which we will consider the Thevenins equivalent this circuit.

So, so this I 2 dashed is nothing but V Thevenins divided by square root of r thevenins plus r 2 dashed by s whole squared plus x 1 plus x 2 dashed x Thevenins plus x 2 dashed whole square. This is so I have to put this value here. And if you do that, you will get 3 V Thevenins squared by 2 pi n s square it. And in the numerator you have r 2 dashed by s divided by in the denominator it will be a these are all algebraic manipulation r 2 dashed by s whole squared plus x Thevenins plus x 2 dashed whole square. So, this is the expression of the torque in terms of equivalent circuit parameter.

So, if somebody says machine is running, steadily with a slip s equal to 0.05. And suppose the all the parameter values of unknown and I will just those that value of slip, and tell ok. If the machine has to run at this slip as a motor, then the electromagnetic torque will be so much of Newton meter. So, by putting different values of slip, I will be able to calculate, how much torque the machine is capable of developing that is I can make a table here. I will write slip, and value of the slip range is known, and therefore by putting different values of slip varies from 0 to 1.

So, if you can easily see, if you flew put a s equal to 0, this torque will be approaching 0, because r 2 dashed by s is there this will be if you if a s tends to 0, torque will be 0 tend to 0. Because, there is a square term here r 2 dashed by whole square, this will dominate others are negligible, and on the top also there is r 2 dashed by s. So, ultimately the denominator becomes infinitely large, I mean just telling mathematically.

So, torque will be 0, physically I know why the torque will be 0, because its slip equal to 0 means n r equal to n s, and machine cannot develop any torque. Similarly, I will put say s equal to 0.0,1 and I will get some number I will put it here like that 0.02 and so on, and go up to s equal to 1. So, all these entries, I can make. Provided I know the equivalent circuit parameters, suppose these are given to me.

So, I will be able to calculate make a table, where y axis will be torque and slip is vary from 0 to 1. So, this I can make a table like this. And after that what I can do is this, I can plot this for a given machine, and suppose this is slip equal to 0, and this is slip equal to 1, this is this axis is slip. And when you are potting this slip this way, I know this is speed. One and the same way either you tell the speed, and this is n r equal to 0, and this is n r equal to n s you must keep this picture in mind.

Now, if you sketch this tabulated value of torque, again slip which I got from this expression. The typical curve will be somewhat like this. The slip varies between 0 to 1 for motoring mode, and the characteristics typical characteristics will be like this understood this is the shape this curve gives you. And you will note that once this torque slip characteristics, this is called torque slip characteristics torque versus slip characteristics.

And if you get this torque, you will note that there is a slip at which the torque increases as slip value increases, then it attains some maximum value T max at some slip say s equal to alpha. At s equal to alpha there is a maximum value of the torque, and then torque once again decreases, and this is a typical torque slip characteristics typical characteristics.

And also for a well designed induction motor the value of the slip at which the machine is capable to develop maximum torque is valued typically will be about if this is 0 to 1, this will be one-tenth of this about it is no fixed number, I am telling you the expected value of the slip for which the torque attains maximum is about alpha value will be about 0.1.

So, this of course is one-tenth of this one, indicating that the curve go to maximum at a very small value of slip, then it decreases, and this comes here. And this is the value of the obviously, starting term T starting of the induction motor, because s equal to 1.

As we switch on the supply to the stator s equal to 1, machine is capable of developing this much of starting torque. Obviously, frictional torque if it is opposing torque is less that this, then machine will accelerate. If the opposing torque present is this much, oh machine will not start, because starting torque will never be able to match this opposing torque, and it will remain stationary blocked condition. Although, the induction motor has got a starting torque definite starting torque T starting, but it must overcome whatever opposing frictional torque is there, so that it accelerates ok. So, this is the typical torque slip characteristics.

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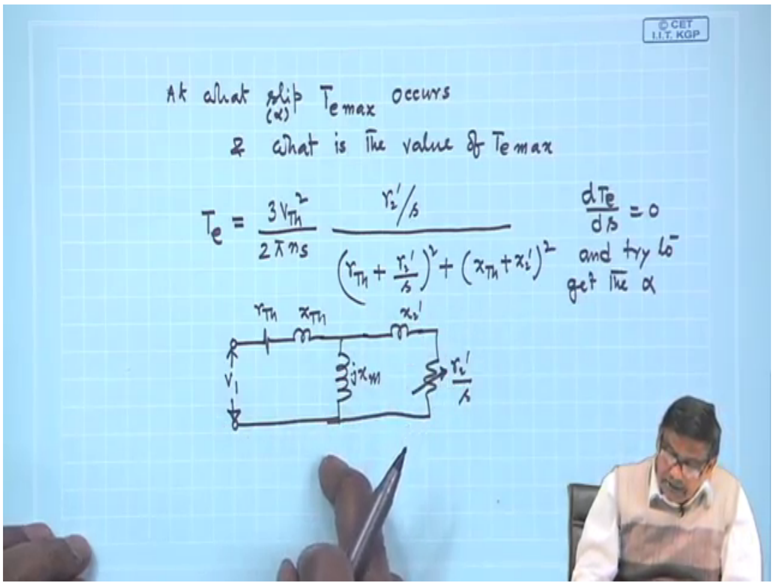

Then the next question comes in terms of equivalent circuit parameters can we tell what will be the value of T max and at what slip that T max occurs. So, so at what slip T e max occurs, and what is the value of T e max can we derive some formula, what is the value of x T e max.

At what slip that slip I have identified as a alpha. So, to do this, what should I do so we write down the general expression for T e that is T e is equal to 3 V Thevenins square, it is to be written actually rewritten it 2 pi n s r 2 dashed by s divided by this is the general expression of the torque r Thevenins plus r 2 dashed by s whole square plus x Thevenins plus x 2 dashed whole square.

This is the general expression of the torque of which is the equivalent circuit parameters are constant V Thevenins is depends on primarily applied voltage, and small r 1 x 1. So, all things are constant except s. And I want to find out, at what slip T e max occurs. So, it comes to our mind that then what we have to do is this, we have to calculate d T e d s, you differentiate this equated to 0. And from that and try to get try to get the value of alpha, alpha is the slip at which maximum torque occurs, but it looks like it will be slightly involved task.

So, we will not calculate in that way. Rather we will try to get that expression in a much elegant way, because I know the equivalent circuit of this induction motor looks like this is r Thevenins, this is x Thevenins, and this is your magnetizing inductance j x m, and this is x 2 dashed, and this is r 2 dashed by s. And sometimes people put some variable resistance sign to indicate that machine may operate at different slip, so it is only this parameter which goes on changing. As you change the degree of loading of the machine, and this is your per phase applied voltage.

So, with this equivalent circuit, the value of the slip that is s equal to alpha, we are looking for which will give you the maximum torque developed can be very nicely obtained, and this we will continue in the next class.

Thank you.