

Electrical Machines – II
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Lecture – 40
Equivalent Circuit of 3-phase Induction Motor

Welcome to the lecture on Machines. And in our last class we defined slip, as slip is equal to s is equal to $n_s - n_r$ by n_s , and n_r of course may change, but n_s is fixed which is in mechanical rps $2f$ by p , p is the number of the machines.

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Slip = $s = \frac{n_s - n_r}{n_s}$ $n_s = \frac{2f}{p}$

$0 < s < 1$
 $n_r = (1-s)n_s$

$s f = \text{freq. of } i_2 \text{ in the rotor}$
 Rotor is stationary

$E_1 = \sqrt{2} \lambda f \phi k_{w1} N_1$
 $E_2 = \sqrt{2} \lambda s f \phi k_{w2} N_2 = s E_1$

$\frac{E_1}{E_2} = \frac{k_{w1} N_1}{k_{w2} N_2} = a$
 E_2 (or s) Per phase

$r_1 = \text{stator resistance}$
 $x_1 = \text{per phase stator leakage reactance}$
 $r_2 = \text{rotor resistance}$
 $x_2 = \text{rotor leakage reactance when rotor is stationary} = \text{at stand still condition.}$

Per phase voltage v_1

$f_r = f$

And the value of slip can be between 0 to 1. s equal to 1 corresponds to lock rotor conditions whenever rotor is not moving you have energized the primary. Of course, rotor circuit is closed then that is slip equal to 1, and extreme case slip could be equal to 0 provided there is no opposing forces, no opposing torque which is of course, is not a very ideal situation. And also, we discuss that if the machine is having some under no load condition, but friction cannot be avoided.

So, machine under no load condition means only opposing torque whichever is present is very little and may be due to friction. When we say that the machines is loaded means we are increasing that opposing torque. And in our last class I discussed how machine will settle down to a final speed after you have started it from s equal to 1, s decreases and so on. And also, the same expression is very useful to write in this way. n_s is constant, so

instead of telling speed directly people would love to tell slip of the induction motor at which it is operating from that of course I can work out what is the value of speed. So, this is another thing.

And then, the important thing we told you is that the stator induced voltage E_1 was equal to $\sqrt{2} \pi f \phi K_w N_1$, and at any value of slip E_2 at any value of slip the induced voltage will be $\sqrt{2} \pi s f \phi K_w N_2$, where $s f$ is the frequency of electrical quantities in the rotor, frequency of v_1 in the rotor in the rotor circuit.

Now, this is equal to s into $\sqrt{2} \pi f \phi K_w N_2$ and at s equal to 1. So, this is s and this quantity is nothing but the induced voltage E_2 at s equal to 1, called the E_2 at s equal to 1 is called the induced voltage per phase in the rotor coils when the rotor is at standstill condition. So, that is this thing [FL].

Now, in today's class what you would like to do, we will try to draw the equivalent circuit of the induction motor per phase per physical valence circuit. Now, it looks like suppose rotor is stationary rotor is stationary, then the in the stator I have applied a voltage suppose v_1 is the per phase voltage applied voltage. And also note that E_1 by E_2 at s equal to 1 E_1 by E_2 is equal to 1 is nothing but $K_w N_1$ by $K_w N_2$ and this can be termed as a terms ratio between stator and rotor effective terms ratio.

Now, what happens is this, I will not first tell like this then let us try to understand. So, at s equal to 1, I know the induced voltage in the rotor circuit is E_2 at s equal to 1 and the frequency of the rotor current is also f . So, at s equal to 1 f_r is equal to f because s into f s equal to 1 stand still condition means at s equal to 1.

Therefore, it looks like at s equal to 1 at least this is just like a transformer, because you have energized the primary with a 50 hertz source and the rotors are shorted at to start at s equal to 1 you have applied some voltage, frequency is f , here also it is f and this voltage induced voltage E_1 and this is E_2 at s equal to 1, this ratio is a constant stator terms is to rotor terms and also the frequency is same. So, it is just like a transformer. At least at s equal to 1 if you can maintain that condition s equal to 1 ok, it will never run as a motor but we will behave like a transformer because transformer what it does, it changes one given voltage to another level of voltage without changing the frequency, frequency remains same on both primary and secondary.

So, at least at s equal to 1 therefore, it looks like the equivalent circuit will be just like a transformer, that is there will be a stator if it is not an ideal widening of course, per phase resistance is r_1 , there will be leakage reactance this widening is having r_1 and x_1 . So, there will be a leakage reactance x_1 . Then there will be a magnetizing inductance x_m , why not, x_m . I will write capital because X_m is much higher. And that is all that is that is all. And here is your E_1 you know in transformer also applied voltage and E_1 .

If r_1 x_1 you will neglect v_1 is equal to E_1 . Mind you, r_1 x_1 are small and X_m is large many times larger than r_1 x_1 , like transformer. In fact, compare to a transformer the magnetizing reactants of an induction motor is much higher because of the fact that there exist an air gap, and to establish a same amount of flux in a transformer or in an induction motor current needed will be much higher, there is the magnetizing current.

Therefore, X_m is much higher current is necessary. So, to establish a given flux, and then here is your E_1 , here is your E_1 . And secondary side what happens is simply this one, this is the secondary coil secondary circuit is closed, so per phase the induced voltage at s equal to 1 will be E_2 at s equal to 1 and the circuit is closed therefore, it will have the widening this r_2 is widening resistance, and it will have a leakage reactance x_2 and this is the widening thing and this is shorted, this is shorted this will be the equivalent circuit of the transformer at s equal to 1.

The frequency of E_2 is f and frequency of any current here is also f . Therefore, it is just like a transformer, the supply voltage is of frequency f after subtracting the drops small drops in r_1 x_1 whatever voltage is available that will be the magnetizing reactants and so on and this is r_2 x_2 . So, these are source of emf in case of transformer equivalent circuit from the circuit point of view we derived at the very first or second lectures.

Therefore, and this is x_2 here. What is x_2 ? x_2 is the leakage reactance. So, let me write r_1 is equal to per phase all per phase, per phase, stator resistance, x_1 is equal to per phase stator leakage reactants stator leakage reactants. r_2 is the per phase no point in writing per phase per phase rotor resistance you know obviously, and x_2 is the rotor leakage reactants per phase.

Now, reactants you see the, this reactants it should be further qualified why because of the fact that this is at s equal to 1 this is fine. So, any reactants is $2\pi f$ into l inductance that is ω into l , ω is $2\pi f$ therefore, in whichever circuit that l is present its

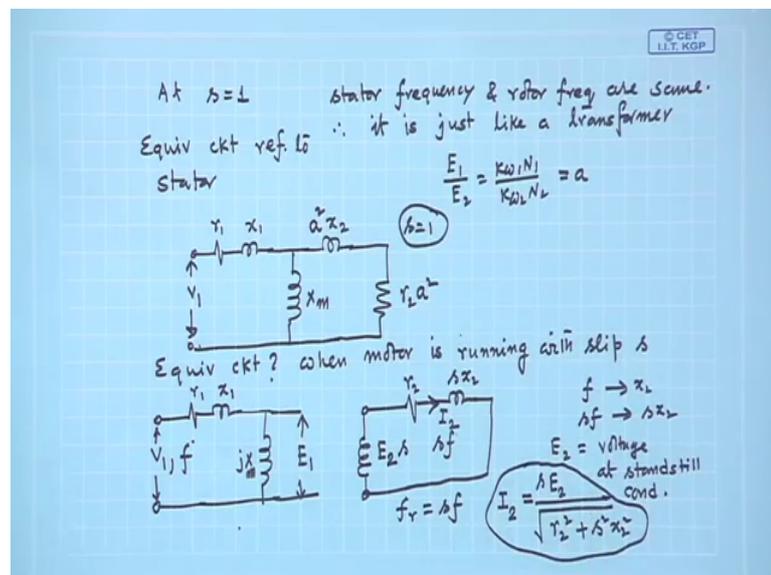
reactants ωL depends upon the frequency. In case of given supply to analyse circuit frequency is fixed everywhere. Therefore, you can that X value whatever you are showing in the circuit that is ωL into L ω is fixed.

Now, here is a different situation, in case of induction motor rotor coil. Why? Because at slip s equal to 1 frequency in this rotor phase will be f equal to the supply frequency, but when the motor will be running the frequency of the rotor electrical quantities will be s into f that we know already.

Therefore resistance of course, will not change because of that change in frequency in this closed circuit; however, the reactance value is then is to modified is not. So, so what we say is this X_2 is the rotor leakage reactants, when rotor is stationary or rotor leakage reactants at stand still condition, rotor is stand still, stand still condition. So, this must be remembered, and this is this equivalent circuit at s equal to 1.

Now, in this case therefore, I will say that from circuits analysis or whatever it is I can then connect this two points, with this two points provided I do this r_2 ; let us do it on a other page so that.

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At s equal to 1 at s equal to 1 only thing I will write that stator frequency and rotor frequency are same, rotor frequency are same. Therefore, therefore, it is just like a transformer because E_1 by E_2 is equal to $K_w N_1$ by N_2 effective

terms ratio and equivalent circuit refer to stator will be simply this is s equal to 1, stand still condition. This will be equivalent circuit provided we have multiplied this with a square, where a is this, that is the terms ratio of primary and secondary this is X_m and this is v_1 absolutely fine. So, this is equivalent circuit refer to stator.

Now, the question is what will be the equivalent circuit is what when motor is running with some slip, slip s which is not equal to 1. Under that condition if I draw the primary and secondary coil separately it will be like this v_1 is the applied voltage per phase on the stator windings, this is r_1 leakage resistance and leakage reactants of the stator and this is your $j x_m$ is not and here is your I mean that coil which is having a voltage E_1 and all frequencies here are f . And on the secondary side there is a source of emf or I just a coil I will draw and here is r_2 here is the reactance and it is shorted.

But the frequency of the rotor electrical quantity is sf which will be less than your supply frequency because s is a number which is between 0 to 1. It is going to be less. For example, if the supply frequency is 50 hertz and motor is running at a 5 percent slip the frequency of the rotor current will be 0.05 into 50 that is 2.5 hertz low frequency compare to this 50 hertz.

Now, in this circuit therefore frequency of primary and secondary coil are different that is one thing and second thing another modification I have to do because the rotor current or voltage frequency is sf . Therefore, this reactance x_2 was at 50 hertz therefore, what will be its reactants as sf hertz at f hertz, this reactants was x_2 at sf it will be s into x_2 ; obviously, $2\pi sf$ into l if l is the leakage reactants you can easily show. So, this should be s . And that is it and this is of course, your E_2 E_2 at some s , which will be also less terms of that. So, this is the scenario at s equal to 1.

Now, the more point is this two points can I, I am slightly in a difficult position now in the sense ok, there is a primary coil, there is a secondary coil, the ratio of the and this voltage $s E_2$ it can be also written as s into E_2 , where E_2 is the voltage at stand still condition, ok. If E_2 is the voltage at stand still condition then the induced voltage when the motor is running with slip s must be s into E_2 that we have seen.

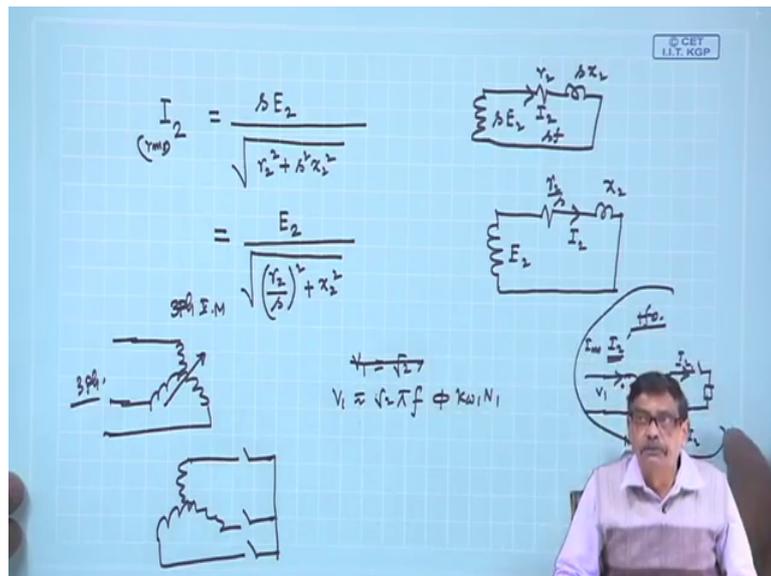
So, mind you this E_2 is the voltage at stand still condition. So, this is the thing. Now, see here also see E_1 by E_2 this voltage by this voltage is no longer simply a terms ratio there is a s factor coming in. So, also the about the current the frequency of that current

and voltage is $s f$ and it is so, frequency of this is $s f$ and this voltage E_2 to s secondary voltage by primary voltage is no longer a constant a .

So, at least I know this much it will be a mistake if I just multiply this with a square and connect it here, ok. This we will not do because the frequency at least must be same, and ratio of the voltages must be properly defined. Now, how to overcome this situation? [FL] Suppose the motor is operating at a slip s and the rotor current is I_2 suppose and this current will be at frequency $s f$ [FL].

What is the rms value of this rotor current? Rms value of the rotor current will be E_2 / s , E_2 divided by square root of $r_2^2 + s^2 x_2^2$ is the reactants at stand still condition. So, that is why the reactants as modified this is the expression of I_2 , fine.

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Now, this I_2 is equal to $s E_2$ divided by square root of impedance that is $r_2^2 + s^2 x_2^2$, and this is the rms value of current, rms value obviously, actual rotor current. So, rotor circuit was like this, there was a voltage source $s E_2$ and it is r_2 and this is s into x_2 . And this current I_2 am talking about frequency here is $s f$ [FL].

Now, let us see this expression of the current, I will manipulate it a bit in which way I will manipulate I will divide both numerator and denominator by s . So, that this will become E_2 / s divided by square root of this is r_2 / s whole square plus x_2^2 . This way I can write it. That is I am dividing this circuit I am trying to draw it like this ok,

there is E_2 and this resistance you divide by s it is r_2 by s and this reactance you divide by s you get X_2 and you close this. This two circuits, so far as magnitude of current is concerned rms value of the current it will remain same this circuit, and this circuit is not the magnitude of the current is to remain same.

Now, a little bit of a talking. See what happens in a transformer, in a transformer, this you listen carefully suppose in a transformer secondary is a open circuits and primary is a energized with a voltage v_1 . Then what happens, it draws only the magnetizing current because that second part of the circuit will be absent because it is open circuited no that twist connected and that is called the magnetizing current and x_m .

Now, this point is important. Now, when you try to draw some current out of the transformer, this is the physical reasoning I am trying to tell. The moment you want to draw current, from the secondary of the transformer by connecting an impedance on the primary side no matter what you do on the secondary side.

Do you think the flux in the core of the transformer is going to change? Practically, there will be no change in the flux which was present when the switch was open or when the switch is closed. The flux in the core of the transformer that cannot change. Why that cannot change? Because the supply voltage the $k v_1$ is to be satisfied here in this widening; applied voltage small $r_1 X_1$ which suppose you neglect. So, the emf here gets fixed by this v_1 nothing doing. No matter whatever you are doing on the secondary side you are shorting, you are closing, some resistance, some inductance, do whatever you like. The net flux in the core will not change. That is why people say mmf balance $N_2 I_2$ is equal to $N_1 I_1$ reflected current.

So, the moment you close it, but it can manage to do that, under no load condition it was drawing only the magnetizing current to establish the flux. The moment you close the switch I am sure there will be a secondary current therefore, the extra emf which is now brought into the system because you have connected a load on the secondary side primary cannot, but draw additional current called reflected current, apart from the magnetizing current whatever it was drawing, drawing. And this the value of this I_2 dash will be such that $N_2 I_2$ is equal to $N_1 I_1$ it is relation to a transformer I am telling. $N_2 I_2$ is equal to $N_1 I_2$ dashed, extra current it will draw in apart from this I_n , such that $N_1 I_2$ dash is equal to $N_2 I_2$. Look at the dot convention they there mmfs will be always

cancelled out therefore, flux in the core will remain only due to $N_1 I_1$ the magnetizing current, is not. That is what it will do.

That is in other words what I am telling if you have a unloaded secondary, secondary of a transformer open circuited. Current drawn by the transformer primary is magnetizing current. If you close the secondary coil with some impedance that is want to draw some current out of the secondary primary cannot remain a silent spectator to this, it will immediately react by drawing extra current I_2 dashed, such that $N_1 I_2$ dash is balanced by $N_2 I_2$. And this two mmfs will be in opposite direction from the dot delivers from the dot it x current such that the total mmf existing in the circuits is once again $N_1 I_1$ the magnetizing current and flux remain constant and everything is now nicely satisfied. That is k v l is satisfied flux as not changed and your mmf balance also takes place, any extra current you draw things will be.

Now, coming this way to the induction motor, see induction motor it is like this. And suppose this switch is open circuited initially. See mathematics is very simple, but try to understand what I am telling it will give you some interest, it will help you to grow some interest about this machine.

Now, here suppose this switch is open induction motor this is a this was transformer. So, three phase induction motor you have energized the primary three phase voltage. And you have closed it, the moment you close it a rotating field results there will be induced voltage. And there is some flux per pole, and here also this $r_1 \times I_1$ of this stator primary if you neglect the flux which will be produced is $\sqrt{2}$ which is approximately equal to $\sqrt{2} \pi f \phi K_w N_1$. So, from this equation we will continue next time.

Thank you.