Electrical Machines - II
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Lecture – 04
Co-efficient of Coupling Energy Stored in Coupled Coils

So, welcome to this next unit of Electrical Machines II in last three units what we actually did is this one.

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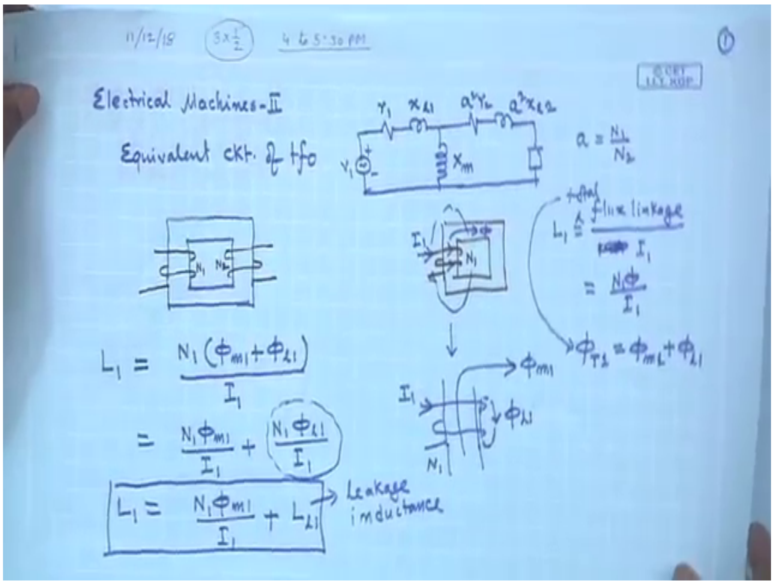

That we tried to found out the equivalent circuit of a transformer from circuit point of view and from the concept of a self and mutual inductances you recall very quickly I will tell that. So, this is 2 coupled coils having turns N 1 and N 2 and then when you pass some current through the coil it produces flux the direction of the flux can be obtained by wrapping your other fingers along the flow of the current and thumb will give you the direction of the flux so, phi.

So, when you pass some current through a single coil it produces fluxes which can be broken up into 2 parts. So, one is the flux component which is confined to the core that I call mutual flux and the other flux which will be leakage flux phi L 1. Suppose coil 1 that is current I 1 then inductances is defined as total flux linkage per unit ampere. So, phi m 1 plus phi L 1 by I 1 in to N 1 gives you the inductance.

This part of the inductance which this flux completes its path through the air gap and this flux causes an inductance of a leakage inductance which is LL 1 and this is L 1.

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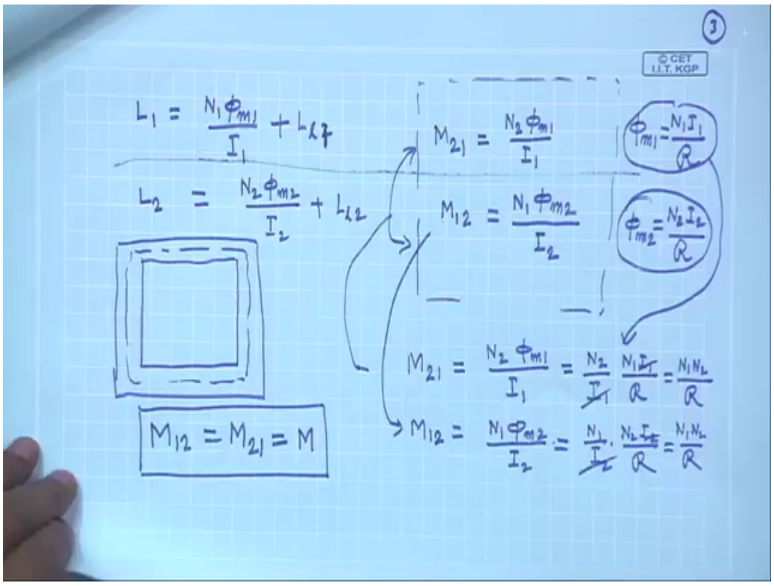

In the same way we did for L 2 and then I showed that M 21 I defined mutual inductance and mutual inductance whether you calculate it from coil 1 or from coil 2 it remains same M 12 is equal to M 21 because, reluctance of the common magnetic circuit is same.

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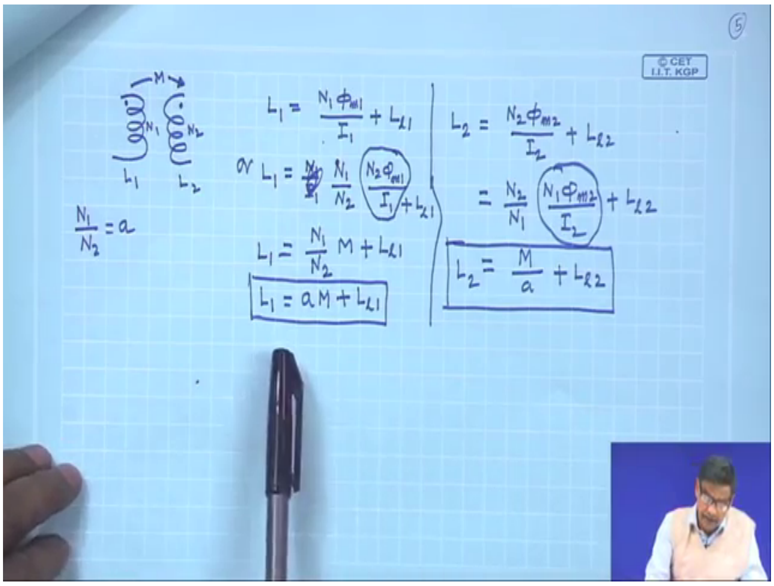

Then the important thing is ultimately we showed that the self inductance of the coil in terms of mutual and leakage inductance comes out to be this L 1 and l 2 is like this.

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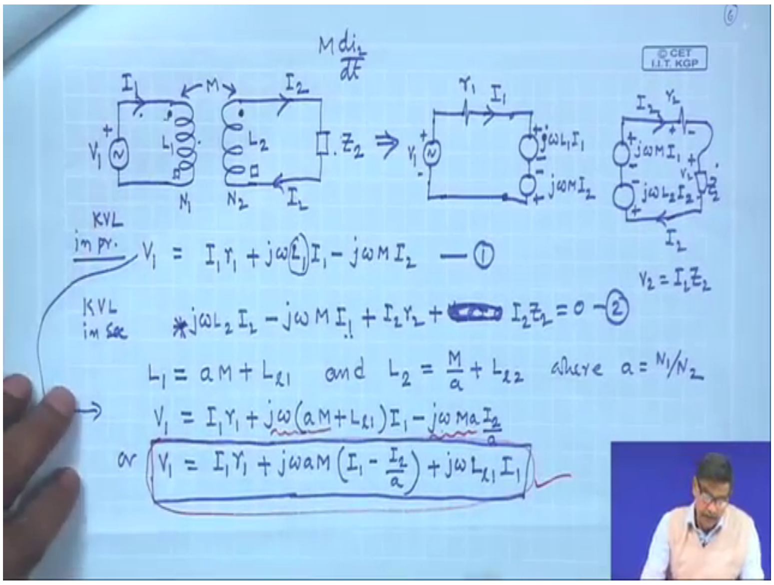

And finally we wrote down the KVL equations from the two sides remembering the fact how to put the dots on these two coils knowing those dots I will be correctly writing the KVL equations with the appropriate polarities of this voltages students often make confusion

But I hope you practices it. so that this will be no problem at all and finally, we found out the equivalent circuit like this here.

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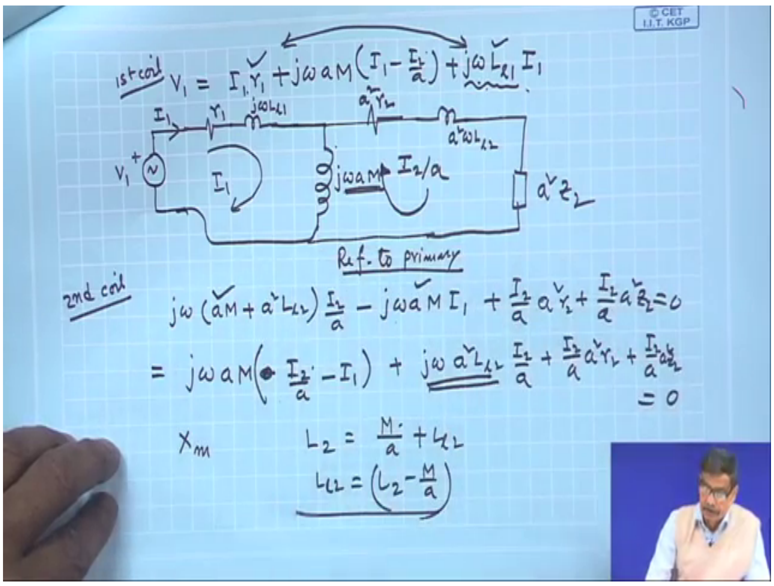

And is it not the same equivalent circuit we got from physical considerations of a transformer? See r 1 and this is leakage reactance this is the mutual inductance which we denote by X m. But in terms of M it is a into a M where a is the number of turns ratios N 1 by N 2 and all the impedances on the secondary side is reflected as a square into r 2 a square into omega l 2 and so, on a square into Z 2.

I will just ask you to find out the equivalent circuit of the transformer referred to the secondary side, I leave it as an exercise to you that is how this equivalent circuit will look like if I want to draw it with respect to this side 2. Anyway so, we spend some time on this, but I just thought that talking about self inductance and mutual inductance sum two three important points it is better I also say that I am talking about L 1 L 2 M.

And then without talking about energy storage and the relationship of L 1 L 2 in terms of M it remains somewhat incomplete. So, I what first I will do is this recall that.

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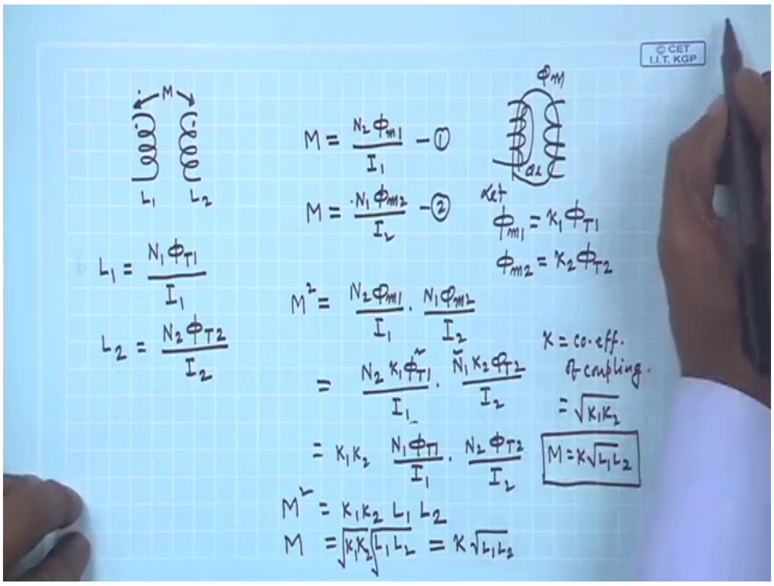

These are the 2 coils having inductances L 1 L 2 and this is the mutual inductance M. And we know that M is equal to say M is equal to I can write as N 2 phi m 1 by I 1 and this M is also same as N 1 phi m 2 by I 2 when the first coil is carrying some current flux linkage per unit ampere and so on.

Now, this fluxes which is mutual flux here this is phi m and these are the leakage flux path phi l. Generally if the coils are very tightly coupled then leakage flux will be little;

in fact, for a transformer coupling should be made very high. So, that most of the fluxes are mutual fluxes and recall that L 1 was defined as N 1 into phi T 1, what is phi T 1? Total flux sum of leakage and mutual flux phi l and phi m divided by I 1 and l 2 was denoted by N 2 phi T 2 divided by I 2.

Now, this mutual flux will be can be expressed suppose let phi m 1 is K 1 into phi T 1 out of the total flux what percentage is your mutual flux? For a transformer it could be 98 percent 97 percent of the total flux leakage flux is little. So, K 1 represents that similarly phi m 2 is K 2 into phi T 2. Now what I do with these 2 equations? We just multiply them so that it become M square into N 2 phi m 1 by I 1 into N 1 phi m 2 by I 2.

Now, you just rearrange the not rearrange now what you do? For phi M 1 it is certain percentage of total flux phi T 1 by I 1 into N 1 phi M 2 is K 2 phi T 2 by I 2 is not then you write it in this way K 1 K 2 you take. And then you write it this terms can be arranged such that it is like this that is you take phi T 1 N 1 together.

And then N 2 phi T 2 by I 2, but this is nothing, but self inductance. So, K 1 K 2 into L 1 and L 2 this is L 1 and this term is L 2. So, M square is this 1 and this K 1 into K 2 is called coefficient of coupling not really that. So, K 1 M M then will be K 1 K 2 square root of that and square root of that. Now, this is expressed as K into root over L 1 L 2. So, K is called the coefficient of coupling which is equal to root over K 1 K 2.

What is K 1 K 2? K 1 is the how Much of this total flux; it is the number fraction maybe 0.95 0.98 for a very tightly coupled coils. And if its value is small it means it is weekly coupled coil if it is 0.5; that means, 50 is mutual flux and 50 percent is leakage flux. Anyway so, this is a relationship which one should know when one is talking about the self and mutual inductances.

The second thing I thought I must touch upon is the energy storage in a mutually coupled coil.

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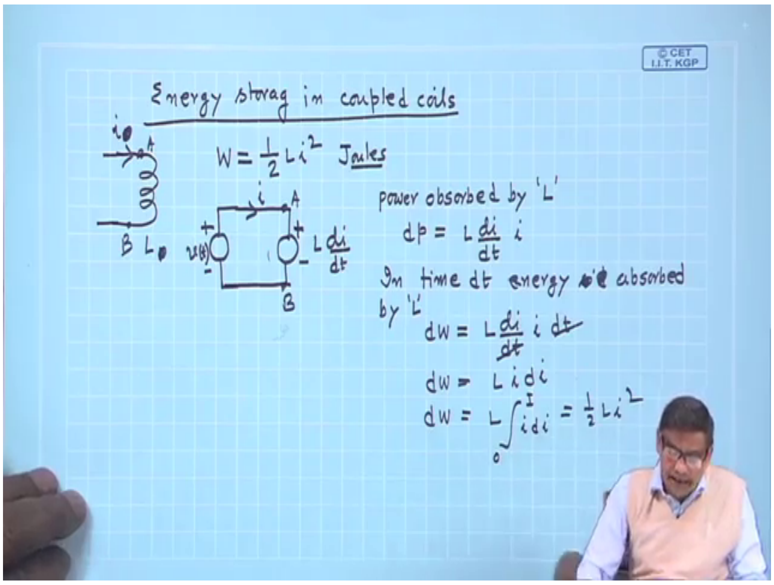

It will take few steps energy storage in coupled coils. The thing is if we have a single coil and it has got self inductance L 1 and this is suppose you are passing some current I 1 through it I because no point in taking telling about L 1 L 2.

Suppose a single coil it has got a inductance l and when it carries a current I we know that energy storage is half Li square so much joules provided l is in henry, I is in ampere and this is denoted by this. So, this is the and how do you get it? Very simply it is like this since this is L this is i. So, as I told you it is better to represent this as a voltage drop.

In this way l di dt is the voltage is not it and here is your say supply voltage which causes this current to flow in this way. Now as you can see with respect to source current is leaving the positive terminal. So, energy or power is supplied by the source and instantaneous power supplied is v into i similarly when you come to this inductor whose terminals are A and B here.

This is also A and B the polarity of the voltage across the inductor this is plus, this is minus L di dt and current is entering through the positive terminal means that it is absorbing power. So, source is delivering and it is absorbing now power absorbed by L is dp is equal to voltage across it into the current i current i. Suppose in time dt dt energy supplied energy absorbed by L is let us call it dw and it will be L d i dt into I into dt dt this dt you can cancel out because they are not equal to 0, but very small.

So, L i d i therefore, you are change if you are changing the current from say 0 ampere to some certain ampere I it will be 0 to I i d i and that gives you half Li square a very familiar results you know that now let us see what happens how do you calculate the energy storage in a coupled coil.

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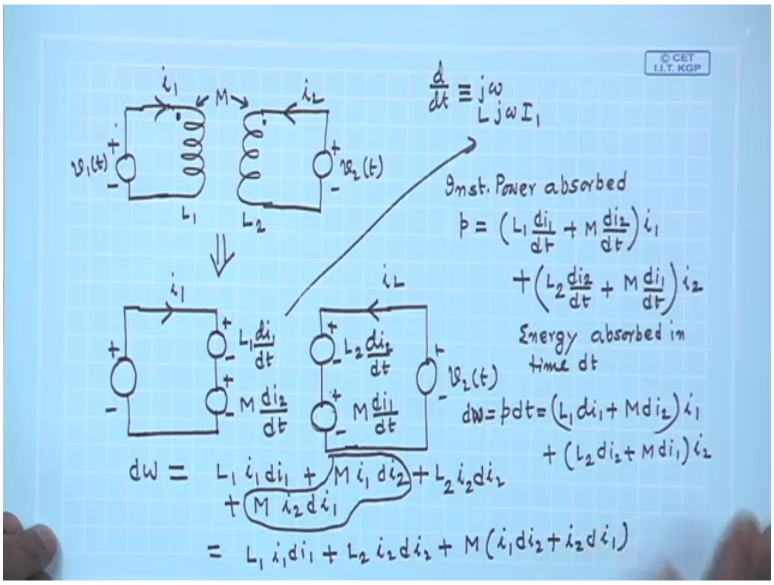

So, we take in the same way these two coils; suppose these are the dots and here I will connect 1 source and let us assume this coils has got negligible resistance almost ideal coil. So, here you connect a source v 1 t and current absorbed by this coil is i 1 similarly you imagine you have connected another source v 2 t here and its current direction is like this.

You can choose the current direction in which ever direction you please, but I have just assumed like that i 2. Now, you see in this system so, these 2 coils are mutually coupled having mutual inductance M and self inductances L 1 L 2. Now, to understand how energy is observed in it what I will do? I will translate this diagram to because between these two points there will be two sources of emf this is 1 this is another, it will also help you to repeat those steps correctly.

So, for example, here there will be one voltage plus minus i 1 is this current. So, for self inductance this is the source L di dt is not. Similarly, for the current in the second coil there will be induced emf here and what will be the polarity? Through the dot it is entering. So, dot will become plus that is the upper side; so, this is plus minus M dit 2 dt.

In our earlier case I assume the supply source to be sinusoidal and then what I did for ddt you just replace it by j omega then you will get L j omega I 1 this term. If the supply happens to be sinusoidal, but in time domain this is fine polarity is like this. Similarly for the second coil there will be two source because between these two points self inductance exists and it has got a coupling with the first coil.

So, so, far as i 2 is this direction and this is your supply voltage mind you M y v 2 T plus minus. So, i 2 is entering through. So, this will be L 2 di 2 dt self inductance and plus mutual inductance will be M di 1 dt; now we have to decide where to write plus minus. Now see through the first coil i 1 is entering through the dot.

Then because of mutually induced voltage in the second coil it should be M d i 1 dt with upper side plus that is the thing. So, once this is correctly drawn then calculations of instantaneous power and from that energy becomes very simple why? See there are now two sources and so, far as this part is concerned current is entered into the positive here power is absorbed and about these two sources also power is being absorbed.

Sources means their inductances; coupled inductances. So, we write instantaneous power instantaneous power absorbed mind you absorbed because both both these coils are absorbing power who decides that? Because I have assumed the currents in such a direction polarity of the voltage is given. So, I can make out whether this fellow is absorbing power or not. So, this is plus minus at the voltages current is entering through the plus it is absorbing.

So, so we write instantaneous power will be then voltage across the this coil first coil plus M d i 2 dt this is the voltage between these 2 points into i 1, but this fellow also absorbs power. So, I have to also consider this coil how much power it is absorbing it is L 2 di 2 dt plus M di 1 dt and the current is i 2 this is the instantaneous power.

So, both the sources are as if pumping power into the system the system is L 1 L 2 and M. So, so this is how it looks like then the energy absorbed in time dt which is vanishingly small this calculus type dx dt like that dt. So, it will be in that small time energy absorbed let us write it as dw is equal to pdt. So, multiply with dt and cancel it out. So, it will be L 1 d i 1 plus M di 2 into i 1 plus L 2 di 2 plus M di 1 into i 2. And this we can write it like this as L 1 i 1 d d i 1 plus M i 1 d i 2 this term plus L 2 i 2 d i 2 this term and plus M i 2 d i 1 this way I can write.

Now, these two terms can be grouped together. So, this is L 1 i 1 d i 1 and then let me let me write the second last term first that is L 2 term L 2 i 2 d i 2 and then this one is M i 1 d i 2 plus i 2 d i 1 like this I get it.

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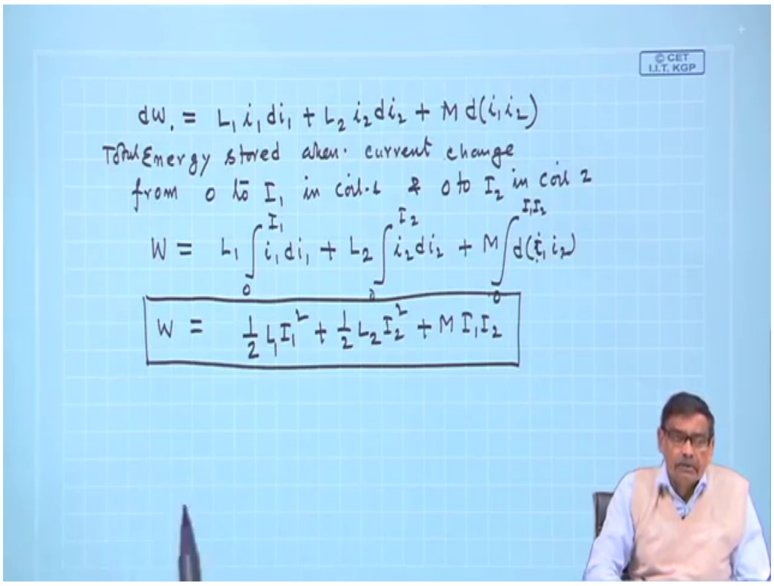

And so dw is L1 i 1 d i 1 plus L 2 i 2 d i 2 and this thing if you look at it carefully this term i 1 di 2 plus i 2 d i 2 can be written as d of i 1 i 2.

So, energy supplied energy stored when current changes from 0 to some current say I 1 in coil 1 and 0 to I 2 in coil 2 2. In that a particular time interval if current changes 0 to I 1 in coil 1 0 to I 2 in coil 2 then total energy stored will be equal to L 1 this is 0 to i 1 i 1 d i 1 plus L 2 0 to i 2 2 di 2 and plus M d of i 1 i 2.

And what should be the limits? Limit should be 0 to i 1 i 2 because this is the differential thing if it is dx limit of x i 1 i 2. So, it was 0 i 1 i 2 was 0 initially and finally, it has reached I 1 I 2. So, product so, if you calculate this it will be half L I 1 square plus half L 2 sorry I 2 square and plus M into I 1 I 2. So, the energy stored in a coupled coils when it carries some current I 1 and I 2 is given by this.

So, this in this you need to what I have done is I reviewed very briefly in the last lecture I will request you you pause the videos of the last three units very carefully and try to understand the main points that is how to tackle mutually coupled coils with confidence and today what I have done in this unit is I just highlighted two things.

One is a about the coefficient of coupling what does that mean? How they are related that is M can be expressed as coefficient of coupling and L 1 L 2 that I have done. And to test complete this is self mutual inductance the energy from the energy point of view I know that if an in a single inductor is carrying some current it stores energy what happens if it is a mutually coupled coils how much energy will be stored in that.

So, I found out that also and remember to do this these are the best way to do given two mutual couple coils b 1 b 2 both the sides I am. So, these two sources are pumping power into the system the way I have assumed currents and voltage polarities it means that it is pumping energy. There is no resistance in the coil where that energy is going? That energy is being stored in the magnetic field. And how to get that? Whenever a coil is there it can be represented by two sources after all flux linking is also taking place in the coil M d phi d t. So, some voltage is induced in the coil this voltage is being balanced by the supplied supply voltage that is different.

But when it is a mutually coupled coils i 1 wherever is entering irrespective of the dot plus will be which side i 1 is entering that we know L 1 d i 1 dt. Now here comes the importance of dot if i 2 is going like this M di 2 dt will be definitely there another source of emf that, but the polarity of that voltage will be through the dot i 2 is entering.

Here also the dot will become plus that is the upper side will become plus like that I do this and then instantaneous power from instantaneous power you get elemental energy integrate it appropriately to get the final energy stored expression.

Thank you.