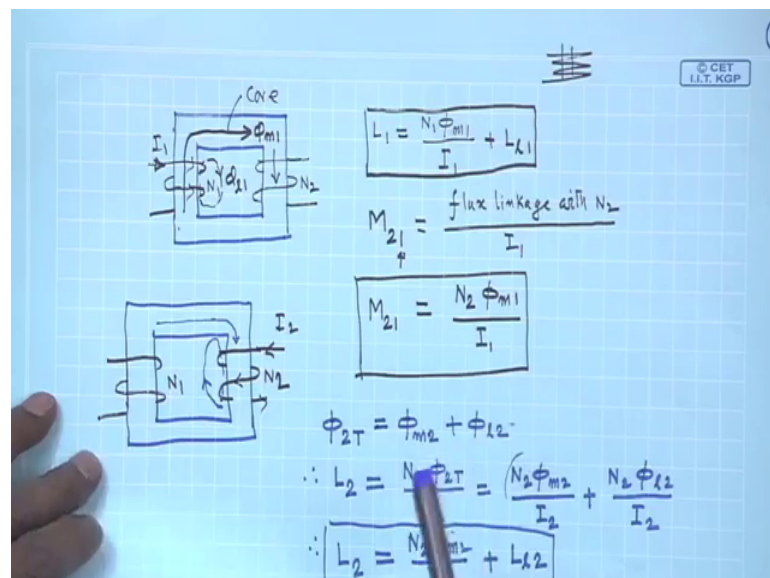


**Electrical Machines - II**  
**Prof. Tapas Kumar Bhattacharya**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture – 02**  
**Relationship of Inductances in Transformer**

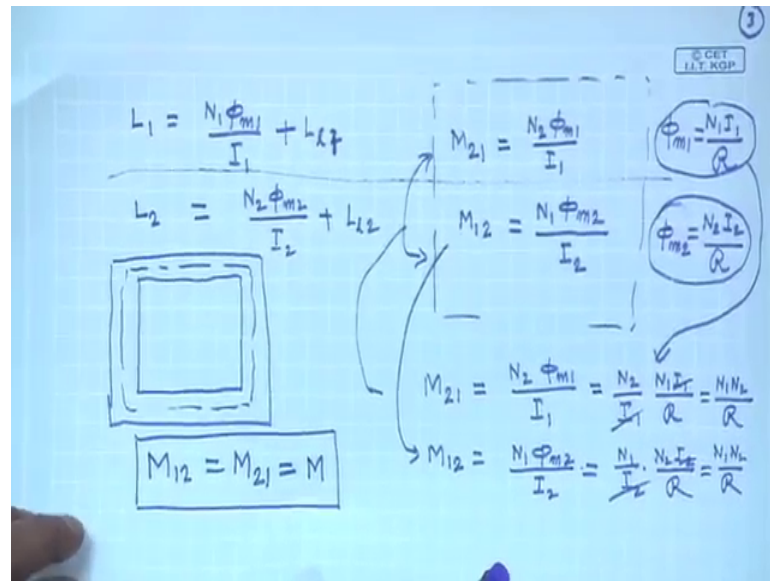
So, let us continue with the self and mutual inductance of coupled coils and we have taken a simple coupled coils that is two coils and they are linked with a common magnetic circuit like this core N<sub>1</sub> N<sub>2</sub>.

(Refer Slide Time: 00:39)



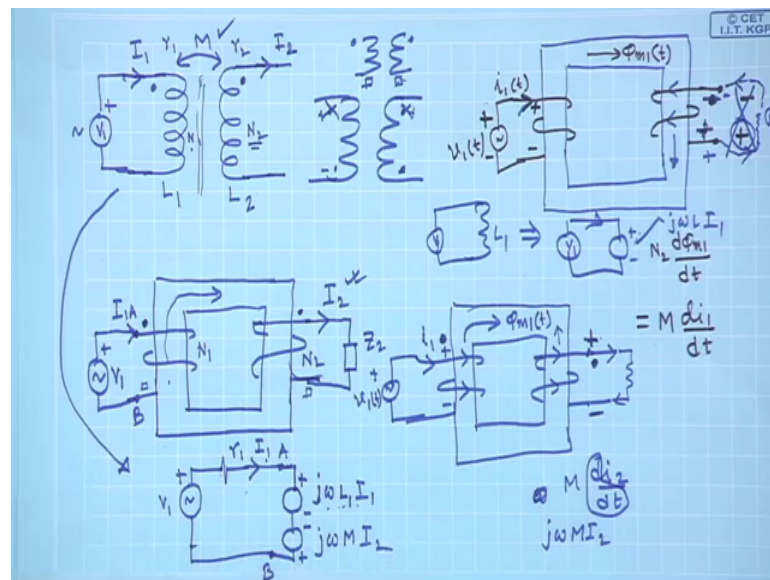
And last time we found out L<sub>1</sub>, M<sub>21</sub>, M<sub>12</sub>, M<sub>12</sub>.

(Refer Slide Time: 00:53)



And eventually in my last class this was the thing we got  $M_{12}$  and  $M_{21}$  are same and it is equal to capital  $M$ , now what happens is this.

(Refer Slide Time: 01:17)



So, we will now view draw the circuit in a much more simpler way that there are two coils, I will not draw the magnetic circuit and I will draw it like this. two coils having self inductances  $L_1$  and  $L_2$  and mutual inductance between them is  $M$  this is how these two coils can be depicted  $N_1 N_2$ . With one important thing we have to also show in coupled coils that is known as dot; dot convention.

So, listen carefully what I am telling. Now, suppose you have got two coils like this and this is; this is the sense of the winding it is like that. You know if you pass some time varying current instead of DC current based on which we have defined the inductances  $L$  now could be anything, but anyway if it is time varying current.

What happens is if it is dc current for example, there will be mutual flux, constant value, leakage flux constant value and across the secondary terminals you will not get any induced voltage because those fluxes will be fixed with respect to time. So, flux linkage with this in the second coil when it becomes a function of time then only you get a voltage across the other terminals.

So, therefore, this mutual flux which is created by coil 1 must be a function of time. How it can be function of time? Provided your  $i_1$  is also a function of time is not. So, if  $i_1$  changes with time  $\phi_{M12}$  will change with time and therefore, Faraday states that there will be across the two terminals of the secondary coil you will expect a voltage  $N_2 \frac{d\phi_{M12}}{dt}$  and this can be shown to be equal to  $M \frac{di_1}{dt}$ .

This is one and the same thing, I leave it to you to show that whatever things I have done. So, this is an exercise for you try to express  $\phi_{M12}$  in terms of  $\phi_1$  and whatever  $M$  I have defined. So, this the voltage induced across the second terminal can be also represented in terms of inductance, mutual inductance. If there is a time varying current here there will be an induced voltage here.

If there is a time varying current in the second coil there is bound to be additional induced voltage across the primary current and you know that  $m \frac{di_1}{dt}$ . Now, the question is that if at any instant of time and how this current can be made time varying, it can be made time varying provided you have connected a time varying source on the primary side  $v_1(t)$ .

If you make  $v_1(t)$  time varying  $i_1(t)$  in turn will be time varying if  $i_1(t)$  is time varying  $\phi_{M12}$  will be time varying and the mutual flux is time varying there will be an induced voltage across the second coil. Now, suppose that at a given instant that this is plus this is minus the question is at that particular instant which one of these two terminals will become plus and which one becomes minus. That is the instantaneous polarity of the secondary voltage with respect to known polarity of the primary coils at the same instant.

Suppose that this instant it is plus, then which of these two will become plus that is the question being asked. Now, recall that the answer would be either this is plus or this is minus or this is minus this is plus only we have got two options. Out of this I have to choose the correct options this time polarity is now known to me it is plus it is minus.

I have to decide about the polarity of the secondary coil voltage at that particular instant and I know that only two possibilities are there. So, I will take one possibilities at a time and try to establish which one is correct that is the thing I am going to do, if possible let us assume this is the correct thing. Now, if this is correct, then what is going to happen? You imagine that the secondary circuit is connected with some resistance like that loaded.

If this is the voltage applied across a say load connected here then what will be the direction of the current in R at that instant, this is the voltage this is the this has become a source of voltage with this plus with this minus. So, current here has to be like this at that particular instant if this is the direction of the current in the second coil the flux produced in the second coil will be like this and its direction will be in this way.

So, the cause for which voltage is induced in the second coil is now strengthened. Why because I have assumed that  $\phi_{m1}$  is positive flowing like this and it is increasing with time that is why it is  $d\phi_{m1}/dt$ . But Faraday says that the polarity of the induced voltage in any coil should be such that it will try to oppose the very cause for which it is due.

What is the very cause  $\phi_{m1}$  was increasing in this direction at that instant and if this is to be correct then the secondary coil too produces flux in the same direction as that of  $\phi_{m1}$ . Thereby strengthening it this is wrong, the polarity of the induced voltage in this second coil should be such that it will try to, given a chance it will try to what is given a chance you have connected a resistance.

So, you are giving a chance to that induced voltage to act to act means to pass some current ok. If this is the polarity assumed then I find no it cannot happen in that case flux will simply get strengthened then define Faraday's law. So, this cannot be this cannot be this is plus this is minus, let us try the other one. So, what is the thing assumed here this is the primary coil here you have connected voltage at that instant this is the voltage  $v_{1t}$  and this is  $i_{1t}$  and  $i_{1t}$  is increasing plus minus so that  $\phi_{m1}$  created by 1 is.

It is polarity and directions are known  $\phi_m 1$  and secondary second coil is like this and since  $\phi_m 1$  is a time varying flux therefore, it is going to induced voltage in the secondary coil and we have right now seen that below 1 plus and upper 1 minus is out of question it cannot it will define Faraday's law.

So, only other alternative left is where as this is plus and this is minus whether let us see it is consistent with Faraday's prediction or a Faraday's law is it consistent with that. So, once again this is this has become a source of emf at that instant with this voltage it is like a source of emf, with this side plus this side minus.

And let us now try to connect some resistance here, imagine that resistance is connected then this emf will act sending some current here and it has to flow at that instant in this direction current. So, the current direction in the coil will be like this and then the secondary coil current is flowing in such a direction that the flux produced you know you have to put your all the other fingers along the direction of the current to get the direction of the flux as we have done here.

So, similarly here this will be the direction of the flux secondary flux and we find this is fine because the secondary flux now tries to oppose the primary flux which was growing in the in the clockwise direction. That is what Faraday says that the polarity of the induced voltage at every instance should be such that it will try to oppose the very cause for which it is due. What was the very cause very cause was  $\phi_m$  was increasing with time there by increasing the flux linkage with the second coil.

The polarity of the induced voltage will be such that when it is allowed to act it must oppose that very cause and that can be only done provided this is plus this is minus. Therefore, what they do is this instead of this plus they will put a bullet here or dot mark and here a bullet here.

So, convention of dot polarity is a nothing, but showing the instantaneous polarities of the primary and secondary side voltage ok. Sometimes because the all things are time varying this voltage will reverse it is polarity this will become plus this will become minus this will then also become plus this will also become minus.

Therefore, instead of showing plus why they show dot to indicate simply that at whatever be the polarity of this terminal at a given time the same polarity exists here. If it is plus it

has to be plus if it is minus it has to be minus and so on. So, you understand that the dots simply indicate the instantaneous like polarities of these two coils if it is plus it will be plus it is minus it will be minus and so on. So, this is the thing when you show two mutual coupled coils merely writing  $L_1$ ,  $L_2$  and  $M$  is not a complete thing.

You must show mutual inductance, but this dot convention is customary to be shown. Then you are conveying also additional things if you connect an ac voltage here, if at any instant this fellow is plus this fellow too will be plus if at any instant this is minus this too has to be minus and so on.

Therefore, we show the instantaneous polarity of the voltage, that is why plus or minus is not written only dots are written. So, anyway this is the correct way of representing a coil. Now, what we do is we will now as I told you my goal is to obtain an equivalent circuit of a transformer and transformer I will believe it has some inductances, self inductance  $L_1$ ,  $L_2$  and mutual inductance  $L$  and this is having turns  $N_1$  this is having turns  $N_2$ .

So, if I want to and there this core material I am not showing, it will show like this to have clarity in the picture I have now understood what it is and also I am now draw from this diagram you cannot find out the sense of the winding. Anyway dots are provided I am happy with that ok.

So, I will now consider this problem that is this is one coil, this is another coil  $N_2$  turns and here you have connected a AC source of voltage which is  $V_1$  phasor and here you have connected load on the secondary side whose impedance is  $Z_2$  I am not putting this bar. You must bear in mind these are all complex numbers get to and this is this scenario and it is expected that since this is AC voltage current share will be also AC.

Let this current be  $I_1$  phasor and let this current be  $I_2$  all are AC quantities and; obviously, this is dot this is dot this I have just established by arguing with the help of Faraday's law, to be consistent with Faraday's law this is done. So, this is the circuit so what we can do and this is  $N_1$  turns.

Our plan will be I will write down the KVL equation of the primary side and KVL equation of the secondary side and from that I will simplify those equations in order to get some equivalent circuit perhaps referred to the primary side. Now, before I start that

you in the light of self and mutual inductances let us try to understand this suppose this is the thing it is carrying a current  $I_1$  and this is carrying a current  $I_2$  like this.

And how to and this is the source  $V_1$  AC source minus to plus. Now, across this coil what are the things which are present now this diagram I am coming here. This is your supply voltage  $V_1$  fine then let us assume this windings has got resistances  $r_1$  and  $r_2$  then I will show  $r_1$  as a lump resistance outside the coil. Generally, windings are made of copper with low resistance, but anyway let the  $r_1$  be the resistance of this coil, where I have connected supply.

Now, what is there between these two terminals see it is carrying a current  $I_1$  now here in the primary also the induced voltage will be there because this primary coil is subjected to a time varying flux and that induced voltage balances this supply voltage you know that. But what I am saying telling in terms of inductances this is also another source of voltage plus minus  $j\omega L_1 I_1$  self inductance of the coil.

If  $I_2$  is 0 then if this side is open circuited you could complete this  $V_1$  is equal to  $j\omega L_1 I_1$ , but now I have drawn a circuit where both the coils are carrying current and it is like this. Now, between the two terminals let me call this point A this point B. So, this is A, this point is B.

So, I am certain about one thing,  $I_1$  is the current  $L_1$  is the self inductance. So, there will be a voltage drop here plus minus  $j\omega L_1 I_1$ , but I also know that these two coils have a mutual inductance  $M$  between them. Therefore, there is bound to be another source in this coil there will be another sheet of emf because of secondary coil is carrying a current  $I_2$ .

What will be the value of that voltage  $j\omega M I_2$  what will be the polarity of the voltage polarity of the voltage I know that if current enters through the dot other dot become plus. So, in this case and you must agree with me another observation I am just trying to tell so that you can quickly write down this equation.

Suppose you have been given these two dot, what I told at any instant of time if this is plus this is plus; obviously, the other terminals which have not been marked there will be also be like terminals. So, if I like I can erase these two dots and put dots here also, this is equally correct or I will do it slightly like this is dot this is dot.

The other terminals two are like terminals and put some other symbols say square at any instant if this is plus this is plus if at any instant this is minus this is minus. So, these two are equally good candidates to put some marks to indicate that these are the light terminals. Therefore, you can see I can also put a square square mark here to indicate that these two are also like terminals. Therefore, current is entering through the square then the square is to be plus.

So, below side will be plus plus minus. So, this will be the thing B 1 is winding resistance and between A and B, had there been a single coil with no mutual coupling I have shown it like this B 1 this is L 1 only matters. And this circuit would have been this is V 1 and there is another source of emf it is  $j \omega L I 1$  whichever side current is entering that is plus simple.

So, here this coil is having a self inductance L 1. So, plus minus current is entering through this I 1 top one is plus, but I know this coil has a mutual coupling with the second coil with the second coil. So, in the second coil second coil is carrying current therefore, the first coil must be having some induced voltage that is  $j \omega m d I$  to  $d t$  and  $ah$ . So,  $j \omega m$  so,  $m d i$  to  $d t$  you know by this time if you replace this  $d d t$  by  $j \omega m$  you get the phasor, these things you must be knowing.

So, that is why  $j \omega M I 2$ , but the big question is where to write plus minus. Since these two have been dots already declared, other two terminals are also will have same polarities that you must understand spend some time and think about it. So, I have marked it as square square. So, current is entering I 2 here so, the polarity of the induced voltage because of I 2 in this coil has to be lower side plus and you will be getting this equivalent circuit.

So, please this is very important for the next class you pause the video and try to understand what I have told I have been telling it rather slowly I believe, but you on your own please try to understand totally whatever I have talked in this particular lecture class before I start the next.

Thank you.