Electrical Machines - II
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Lecture – 01
Inductance, Self and Mutual

Welcome to the course on Electrical Machines II, in this course we will primarily focus on three phase induction motors starting from its basic operations and then single phase induction motors and then synchronous motors. Anyway, before going to three phase induction motors it is essential to have some knowledge on windings on electrical machines, that we will be doing before I start three phase induction motors. And before that of course, we will discuss about the basic principles of any rotating machines.

Basic operating principles of any rotating machines, we will see that there is a some common underlying principles which govern the operations of all the kinds of standard electrical motors. Be it DC, motors be it three phase induction motors, be it single phase or three phase synchronous motors. So, the study we will centre around, the steady state performance analysis of those machines in somewhat detail. Before of course, I thought that essentially what you will see that when we will study any type of electrical motors, we will try to draw its equivalent circuit and then analyze that circuit to predict the performance of that particular machines.

Since the course is on electrical machines II it is presumed that you have undergone a course on transformers and DC machines, all though DC machines is somewhat difficult in at its constructional level. But, nonetheless generally the electrical machine I course centres around transformer and DC machines. If time permits I will also touch upon some interesting aspects of DC machines as well.

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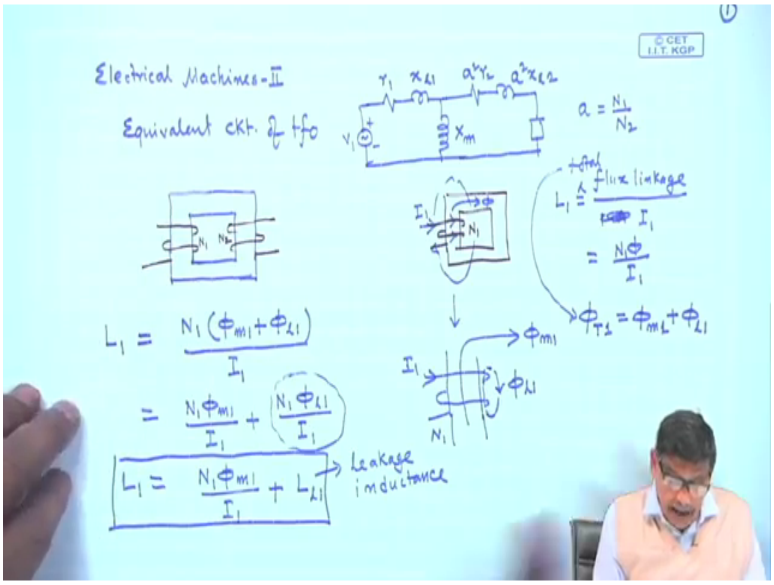

Therefore the course begins with electrical machines II and as I told you equivalent circuit is one of them. So, let us not leave a transformer in its totality. Because the equivalent circuit of the transformer you must be remembering, we will start from that; assuming you have undergone a transformer course, but I will try to arrive at the equivalent circuit of a transformer in a from the viewpoint of circuit analysis.

And before that if I want to derive the equivalent circuit of transformers, which you must be knowing it is drawn like this and this transformer let us assume it has got no core loss. So, that parallel resistance I have not drawn. And it’s this is how it is drawn r 1 x l 1. This is the what is known as magnetizing inductance j X m and this is a square r 2 and this is a square x l 2 where x l 1, x l 2 are called leakage reactance’s of the transformer; r 1 r 2 are winding resistances of primary and secondary side and a is the ratio of number of turns N 1 by N 2.

Where, primary side is that side where source has been connected that is here V 1 ok. This equivalent circuit must be familiar to you and it has been derived based on some physical considerations of the leakage flux, mutual flux and so on. And this impedances r 2 is the actual secondary winding resistance, x l 2 is the actual secondary leakage reactance. These are to be multiplied by a square in order to draw the equivalent circuit referred to what is known as primary side.

Similarly, the equivalent circuit can be drawn referred to the secondary side as well, but now as I told you this equivalent circuit we will try to arrive at by purely from the consideration of circuit analysis; after all a transformer is nothing, but a collection of two coils which are magnetically coupled is not. So, this is the transformer core, this is the primary coil say, this is the secondary coils. And, these two coils have got their separate identities with terminals brought out and if you look at this configuration; you will immediately notice that these two coils have mutual coupling between them.

Because, if one coil passes current flux will be created, it is going to link the others and thereby, inducing a voltage in the other coil; the number of turns of the primary the N 1 and that of the secondary is N 2 it is like this. Now, a coil has got also a self inductance and how to find it out, suppose you concentrate on one coil; this part I will do rather quickly. Suppose this is a coil and I want to find out its inductance. Now, to find out the inductance of a coil we will start from the fundamental definition of inductance; inductance of a coil is flux linkage with the coil per ampere per 1 ampere.

If it is carrying current I divided by I. Now, what is flux linkage? Flux linkage is if you pass a current I 1 through coil 1, it is going to produce a flux like; this is phi. So, this N 1 into phi divided by I 1 is called the inductance of the coil. Now, in a transformer what happens is this, the flux which will be created when a coil carries a current. If I draw this portion in a bigger way it will be like this, this is the code this is the turns. There will be some flux which will be confined to the core and that I call phi m mutual flux and there will be some flux lines which will complete their path through the air gap and this is called leakage flux. But the definition of inductance says that it is the total flux linkage total per unit ampere. So, if I say that I have energized coil 1 by current I 1 and total flux created by 8 I will write it like this phi T 1 is the total flux.

Then this total flux will comprise can in fact, it constitutes of two pieces: one is the mutual flux phi m 1 plus phi l 1, the leakage flux. So, if it is coil 1 discarding I 1 current. So, phi m 1 and phi l 1 together is your phi t 1. This is the total flux linkage, this one. Therefore, by definition inductance self inductance of the coil will be flux linkage with the coils. So, N 1 into phi m 1 plus leakage flux phi l 1 divided by I 1 and this can be broken up into two terms as N 1 phi m 1 by I 1 plus N 1 phi l 1 by I 1, is it not. So, this is the thing.

Now, this part which is not confined to the core that is the I 1 material of the arrangement is called the leakage inductance, this is also an inductance because N phi by I. So, this is written as N 1 phi m 1 by I 1 plus this I will write it as L l 1 that is a leakage inductance of the coil, leakage inductance. So, this is what this L 1 means. Therefore, self inductance of a coil if you want to find out, you have to consider the total flux which consists of two parts. That is phi m 1, the flux which will be within the magnetic material and phi l one is the flux which process or closes its path largely through the air gap ok. So, this is the leakage flux.

So, I think we have understood what self inductance means so, this is page 1. Now, coming back to the transformer we have to we have now got two coils instead of a single coil. So, once again I mean referring to this coil, this is one coil, this is another coil.

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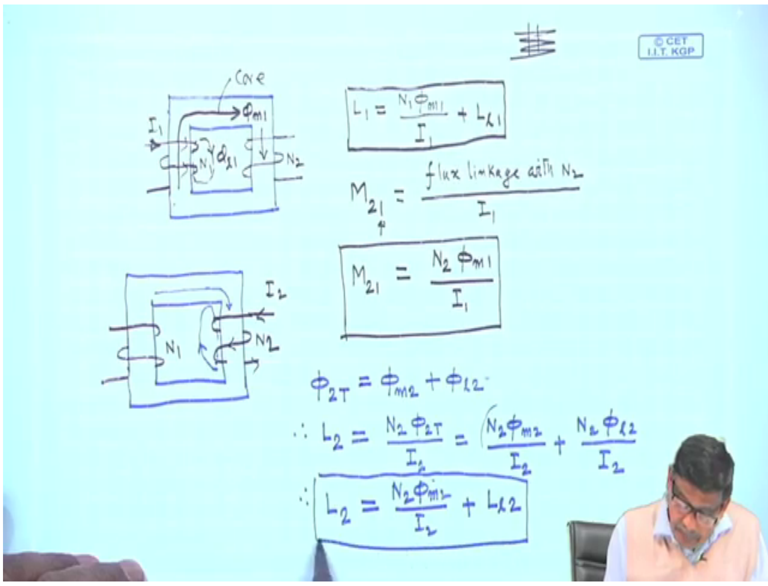

So, suppose we say that let me re draw it this is better this is the core and these are your coils. See, while drawing the coil I have drawn it rather in detail showing its sense of the winding I have not drawn it like this, never draw like that better draw in this fashion. And this is the another coil this has got number of turns N 2 this has got number of turns N 1 and this is the core.

Now, let us imagine I have only energized the primary coil with the DC current I 1 ok, then as I have told you there will be flux created which will have two components phi m 1 and there will be flux lines linking only the this winding that is called leakage flux phi l

1 and based on that I have defined the in the previous page L 1 to be N 1 phi m 1 by I 1 plus L l 1.

You know this is the self inductance of the coil. Now what is the mutual inductance when there are two or more coils sharing a common magnetic circuit, then if any of the coil carries current that is going to be flux linkage with the other coils. So, in this case is very simple, two coils are there one coil is carrying current and it is this mutual flux phi m 1 that is going to link the second coil N 2, phi l 1 after all cannot link the second coil.

And this is therefore, I can define mutual inductance like this M 2 1 as flux linkage with the second coil with N 2 divided by 1 ampere flowing through coil 1. Once again the unit of M will be same as L, Henry that is flux linkage Weber turns per ampere. So, this is flux linkage with N 2 divided by I 1, second coil I have not touched nothing is flowing through the second coil. So, this is called the mutual inductance of the second coil, the second suffix indicates that the first coil is carrying current.

So, this will be flux linkage with N 2 by I 1 and this is; obviously, the flux linkage with N 2 because of I 1 is nothing, but N 2 phi m 1 divided by I 1 is it not, this will be the value of M 2 1. So, this is the mutual inductance when coil 1 carries current, coil 2 nothing is being and you get M 21 and L 21 these are the two things you get.

Now, let us say I am once again redrawing the circuit and I am telling that this time what we will do is this I will pass some current through the second coil. Let it be constant current DC current I 2 and N 1 and this coil I keep it open nothing is connected this is N 2 sorry this is N 1 and you pass a current like this. If you pass a current in this direction as you can see the flux produced you can show the direction of the current if you draw in this particular way as I told you and decide about the direction of the flux.

So, flux will be once again clockwise and it will be like this is not and this flux once again the total flux created by secondary coils second coil I will write it as phi 2T and in the same way can be broken up into two terms. One is the mutual flux phi m and this time I will write 2 because coil 2 is carrying current and it has created the flux.

So, phi m 2 is the mutual flux which is confined to the core and there will be a component of flux which will completes its path largely through the air gap and that will be termed as phi l 2 in the same way, as we have done in the case of the first coil. So, same explanation goes this is the total flux created by coil two it is carrying a current I two and the total flux is nothing, but sum of mutual flux plus the leakage flux.

So, this is this therefore, the inductance self inductance of the second coil will be once again the flux linkage with the second coil total flux linkage phi two t per unit ampere that is I two self inductance of this second coil and this can be after you put these two things here it can be broken up into N 2 phi N 2 divided by I 2 plus N 2 phi l 2 divided by I 2.

So, the self inductance of the second coil will be this term N 2 phi m 2 by I 2 and this term I will call leakage inductance of the second coil L l 2 this is how these things comes, now this is this. So, L 2 is this one. So, let us go to third page.

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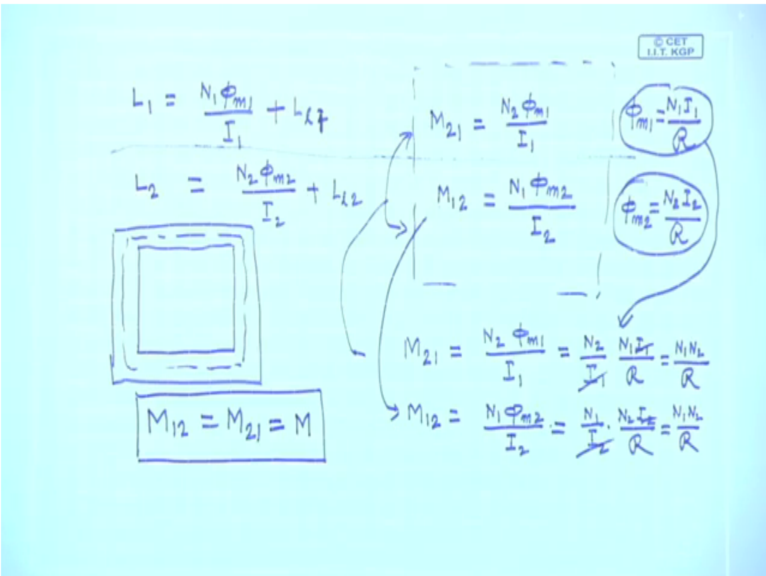

So, recall that now I have not going to draw, but what results I have got till now is L 1 is equal to N 1 phi m 1 by I 1 plus L l 2 and l 2 self inductance of the second coil is N 2 phi m 2 by I 2 plus L l 2 this is sorry 1, leakage inductance 1 these are the two things. And I told you inductance mutual inductance in this case M 21 is nothing, but N 2 phi m 1 by I 1, similarly I can define the mutual inductance when second coil is carrying current. So, I will write it as M 12, this is the flux linkage with the first coil N 1 and the mutual flux this time is phi m 2 and divided by per unit ampere flowing in the second coil divided by I 2, this will be the thing fine.

Now, let us concentrate on these two equations here your this two coils mind you they are sharing a common magnetic circuit ok. So, what will be the flux, we have done magnetic circuit analysis if a coil is energized how to calculate the flux inside the code? If I assume it is a linear magnetic circuit phi m 1 will be nothing, but mmf divided by reluctance.

In this case when coil one is energized mmf is N 1 into I 1 and divided by the reluctance; reluctance of what? Reluctance of this flux path which completes its path in the code; so, this curly r is the reluctance similarly phi m 2 will be nothing, but mmf in this case I am only energize second coil in the second row. So, this is N 2 I 2 by the reluctance. So, this is the thing where reluctance I am assuming it to be constant and assuming linear magnetic circuit.

So, I can always calculate the reluctance of the magnetic circuit you know it is like one by mu 0 mu r l min by a cross section I am not going into all those details, but I know that flux is mmf by reluctance; mmf is n into I divided by reluctance. So, I have got this; now let me play with these two equations what I will do is this M 21 I will it is already obtained as N 2 phi m 1 by I 1 this is the thing is not.

N 2 phi m 1 by I m 1 and M 1 2 is this one. Now what I will do is this I can write it as N 2 by I 1 and phi m 1 from this I will plug in this phi m 1 here N 2 by I 1 is N 1 I 1 by reluctance, where I 1 goes. And it is nothing, but N 1 N 2 by reluctance similarly M 12 if you calculate it is from this it is N 1 phi m 2 by I 2, but phi m 2 is nothing, but this

So, put it here. So, you will be getting N 1 by I 2 into phi m 2 is N 2 I 2 by reluctance I two goes and it is equal to N 1 N 2 by reluctance. So, what is the conclusion M 12 is equal to M 21 this is a very important thing there is no reason to distinguish between these two, provided these two coil share the same magnetic circuit.

So, and therefore, the mutual inductance can be represented by a single number m not M 12 no suffix I will write of course, for self and mutual inductance we have to use L 1 and L 2 therefore, what we have done till now in this unit as I have told you my target goal will be to derive the equivalent circuit of a transformer. Looking at it as a as in terms of self and mutual inductances and try to get this equivalent circuit where I have assumed that the transformer is having no core losses that is hysteresis and eddy current losses.

Whether, such an equivalent circuit can be arrived at using the circuit concept and using the concept of inductances; so more in the next unit.

Thank you.