Networks, Signals and Systems
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Lecture - 06
Locus Diagram - Introduction to Signals

Good morning friends, today we will be discussing about a new topic locus diagram. I will just give you a brief idea before we start about the importance of this locus diagram sometimes we are interested in studying the behavior of the current in a circuit when say one of the parameters or one of the elements of the impedance is varied over a wide range, this is applicable in many circuits specially those who were you have studied about induction machines will know induction motor parameters the in the equivalence circuit you have some variable resistance. So you can play with this, you can have maximum torque transfer condition, maximum power transfer condition and so on.

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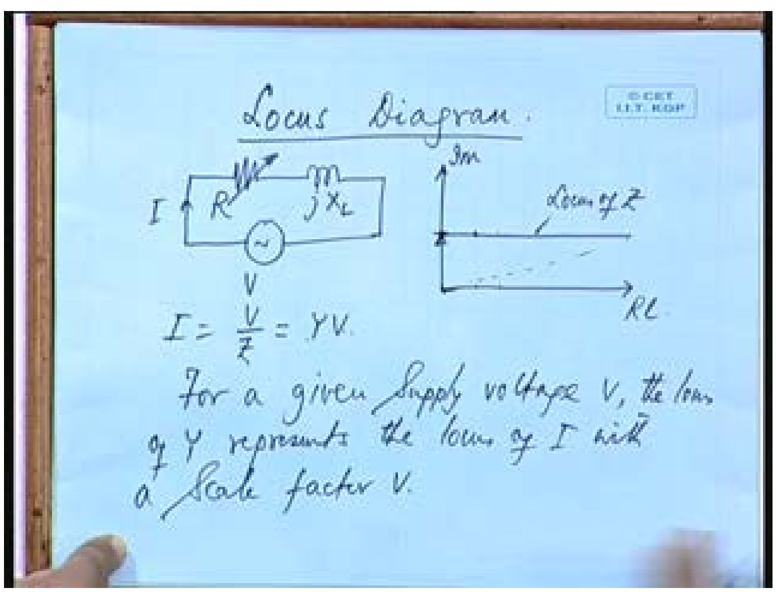

So let us see the basic principle involved in this suppose we have a very simple circuit consisting of a resistance and then the inductance, the impedance is j X L and resistance is R, this is the voltage that is fixed and one of them either R or XL is varied, say R is varied what would be the locus of the current. Obviously, the total impedance value is changing the changing so the current will be varying. So what will be the nature of variation of the current, so if we see the impedance diagram suppose this is the real part, this is the imaginary part then it is j X L which is fixed and we are taking different values of R.

So R plus j X L will be say R 1 plus j X L, R 2 plus j X L. So the impedance varies along this line is it not its minimum when it becomes purely inductive so that time its value is X L okay resistance is 0 and its maximum value it tends to infinity when R is infinity and it will have an angle will find this angle will tend to 0 so at a very very large value of resistance, it will be practically resistive with an infinite magnitude okay. So this is the locus of Z all right. Now what will be the current, current is V by Z for the time being let us forget about the angle part at this moment. So you can write Y into V where Y is the admittance okay. So if the locus of Z is known what will be the locus of Y since V is fixed, so the locus of Y will be a scaled version of current I is it not. So how is Y changing with different values of R, okay? So for a given supply supply voltage V, Y represents the locus of the locus of Y represents the locus of I in some scale Y is having the dimension of mho all right, unit of mho, I is current ampere.

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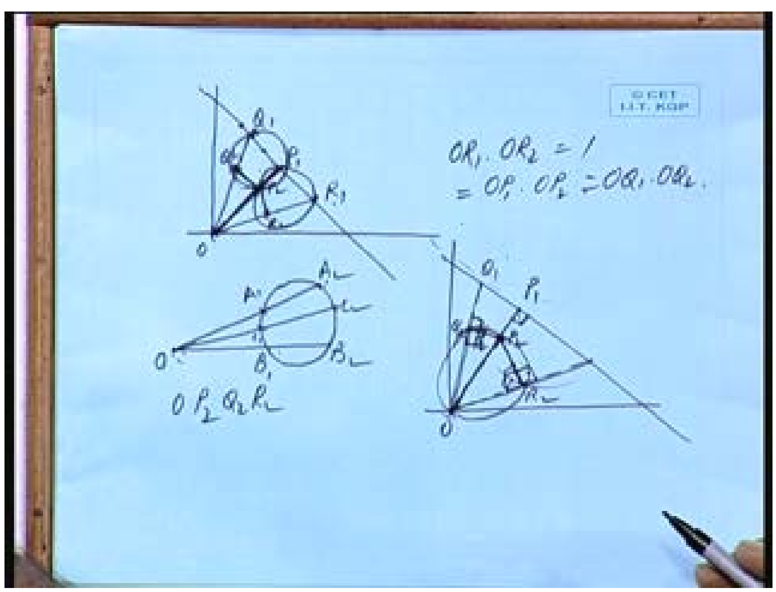

So obviously, it will be only a scaled version with a different unit the locus of I with a scale factor V, we can change the scale that is not a problem. So our first task is to find the locus of a straight line like this inverse of a straight line. Let us take any straight line and find its inverse, what is the inverse of this straight line, this is the origin, this is the straight line I want to know for any point on this straight line what will be the corresponding inverse. So can you tell me what will be the inverse of this, what will be the inverse of this?

Suppose we have O, R 1, Q 1, P 1, I have taken 3 arbitrary points one of them is a perpendicular all right. I have taken 3 points on this straight line, I would like to see what will be the positions of their inverses that is R 2, P 2 and say Q 2, what will be the location of R 2, P 2, Q 2 and how are these points changing their positions okay. Now OR 1, OR 1 into OR 2 is equal to 1, is it not because OR 2 is 1 by OR 2, OR 1 and that is equal to OP 1 into OP 2 its equal to OQ 1 into OQ 2, is it not, is that all right. Now P 1 is a perpendicular all right this is a perpendicular that is this the shortest line.

Now from here, consider these points P 2, R 2, Q 2 if you remember in geometry you have studied if I draw 2 lines like this say O, A 1, A 2, B 1, B 2, so O, A 1, A 2 is a line cutting the circle at 2 point C 1 A 2 similarly, this one cutting at B 1, B 2 then OA 1 into OA 2 is OB 1 into OB 2 is that all right. So if I find OP 1 into OP 2 is equal to OR 2 into OR 1 is equal to OQ 2 into OQ 1, what does it prove? These points A 1, A 2, B 1, B 2, C 1, C 2 these are all lying on a circle. So Q 1, Q 2 let us take these 4 points Q 1, Q 2, P 2, P 1 these are lying on a circle okay. Similarly these 4 points will also lie on a circle is that all right?

Now once again I redraw it for this is a perpendicular all right, this is 90 degree, OP 1 is perpendicular all right. These 4 points are lying on a circle, so opposite angles of a quadrangle inside a circle will sum up to 180 degrees. So if this is 90 degrees, this will also be 90 degree all right similarly on this side I have taken Q 1, Q 2 this will also be 90 degree okay. So if this is 90 degree this is also 90 degree if this is 90 degree, this is also 90 degree.

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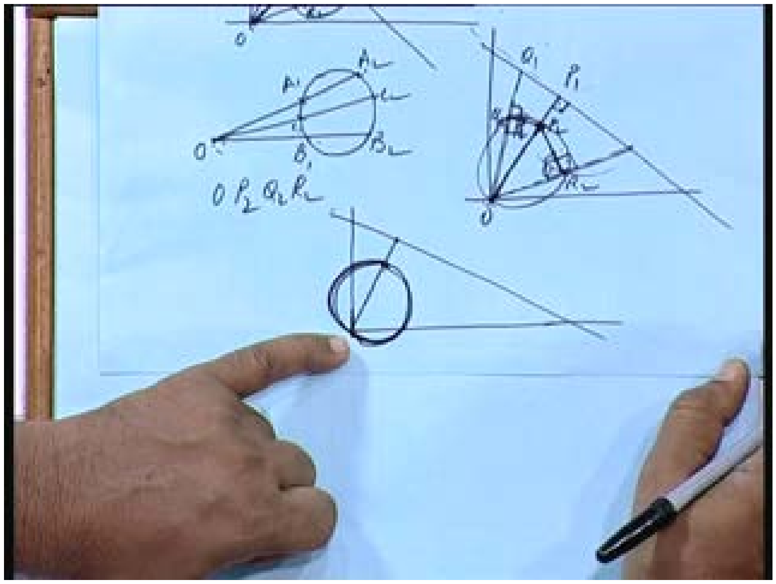

So O, P 2, R 2, Q 2, O, P 2, Q 2, R 2, in this quadrangle opposite angles are 90 degree so these 4 again will form a circle is that all right that means all the inverse points that is inverse of the points lying on this will lie on this circle touching the origin and OP 2 is the diameter because this is 90 degree. So the semi-circle this will be a semi-circle. Similarly, this side also it will be a semi-circle. So OP 2 will be the diameter okay therefore given a straight line. Now I know what will be the locus of its inverse what will be its inverse draw a perpendicular take its inverse and then draw a circle including with this as the diameter.

So the origin is included all right sorry, so this will be the circle is that all right. So the inverse of a straight line is a circle that will include the origin. Conversely, the inverse of a circle where the origin lies on that circle will be a straight line okay. So if I am given a circle, what will be its inverse, how to determine that straight line draw a diameter through the origin then extend it take the inverse of the diameter whatever be that you mean and then drop a perpendicular on this line at this point. So this will be the straight line which will be the inverse of this circle all right. So inverse of a circle is a straight line if the circle touches the origin okay.

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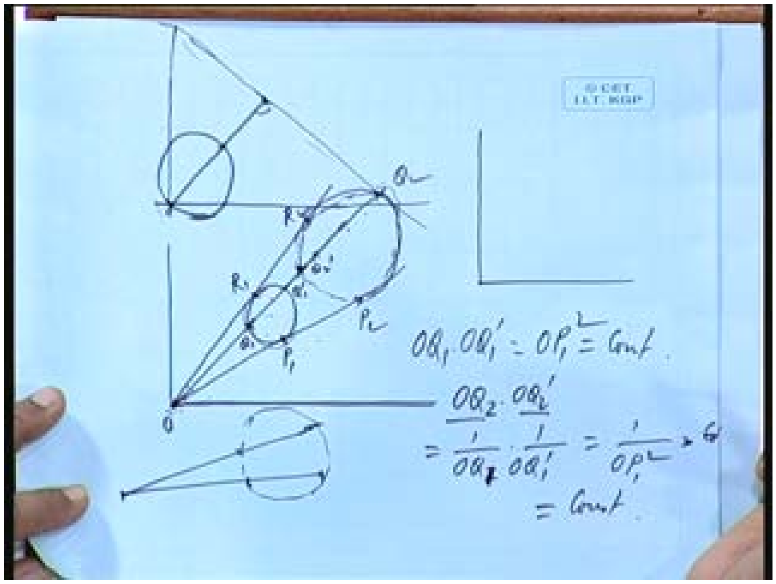

Now let us take another situation what would be the inverse of a circle lying outside the origin that means the origin lies outside the circle will it be again a straight line so you join this okay you can take any other line so inverse of this say OP 1 is may be OP 2 O similarly, because the tangents are of the same length OR 1 is OR 2 okay. So these are the inverses, suppose this is Q 1 and Q 1 dashed then Q 1 will have its counterpart Q 2 and Q 2 dashed somewhere here. We will find, we will prove it they will again form another circle that is inverse of a circle inverse of a circle when the origin is outside the circle is another circle whereas the inverse of a circle when the origin was on the circle is a straight line is that all right. Should I leave it as an exercise or would you like me to prove it okay I will give you the hints. See OP 1 into OP 2 okay OQ 1 into OQ 1 dashed is equal to OP 1 squared or OR 1 square either way okay because the tangents are of same length OP 1 squared this is a constant.

Similarly, OQ dashed OQ 1 dashed into Q 2, Q 2 dashed OQ 2 into OQ 2 dashed what will it be equal to OQ 2 dashed into OQ 2 will be just inverse of this 1 by OQ 1 into 1 by OQ 1 dashed and 1 by a constant OP 1 squared which is again a constant okay. So what does it prove, the product of these segments is also a constant whenever you are having a straight line drawn from a particular point and there are 2 points the product of these 2 segments is constant then these points lying on such straight lines will also lie on a circle okay. So, all these points will lie on a circle, they are all inverses of this okay. So the inverse of a circle when the origin is outside that circle is also another circle, we will make use of this relationship for the computation of currents, the locus of currents in different situations.

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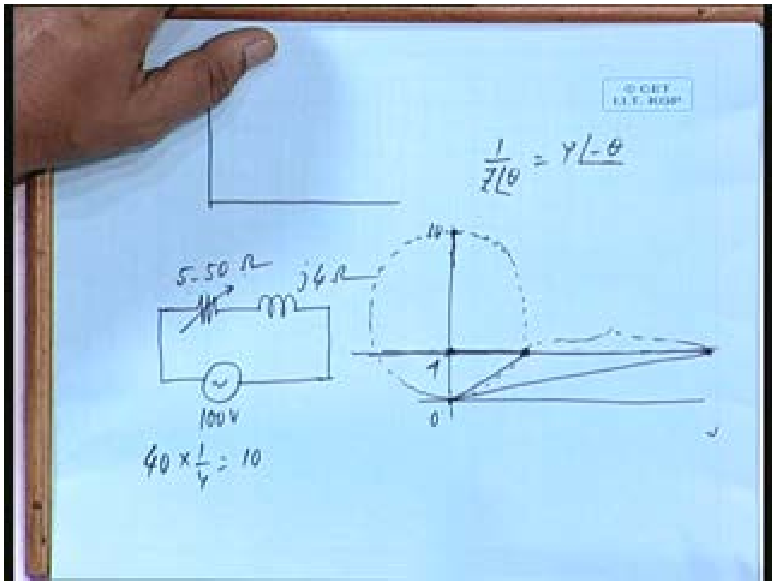

Now so far we have not taken care of the angle part all right if we have in electrical engineering we deal with phasors or impedances with angles. So when you write V by Z or 1 by Z the angle associated with Y will be just negative of the angle associated with Z, 1 by Z theta will come out as Y with an angle minus theta. So whenever we are inverting a circle here if these represent the phasors or impedances then this should come in the other segment other quadrant. So, first quadrant quantities will be reflected in the 4th quadrant.

Similarly, if it is R minus J X kind of impedance function 4th quadrant quantities will be taken in the first quadrant all right. So this is to be taken care of when we handle a new problem on circuits. Now we will also take the scaling factor suitably to get the circles or the other loci of proper sizes. So we will take a suitable scaling factor and then take care of them while making the final computation okay.

Suppose, we have a resistance and then inductance, the resistance is varied say between 5 and 50 ohms okay and inductance is say given some particular value 4 ohms j 4 we are giving a supply of 100 volts, what would be the location of the current that is what would be the locus of the current and what will be the values of the currents at these 2 values say 5 and 50 ohms or you can take any other intermediate value, what will be their specific magnitudes and angle.

So let us draw first of all the impedance Z, what would be the locus of Z, this is fixed 4 j, so a unit of 4 taken on this side, mind you the scales chosen for reactance and resistance should be identical otherwise a circle might be distorted into an ellipse unless the scales are identical both are ohms so both of them should have the same scale. So this is 4 and this is varied 5 ohms on this side will be somewhere here. So this will be 5 plus j 4. Similarly, 50 ohms down the line here somewhere here I say, so 50 ohms which has to be chosen to scale we are not really following any scale here, it has to be drawn on a graph sheet with scale.

So this is say 50, so this is 5 plus j 4, this is 50 plus j 4. So this is the location this is the portion of our interest okay had it been from 0 to 50 then this would have been the segment, corresponding segment is it not. So we are interested in the inverse of this portion. So we will take the entire straight line what will be the inverse of this straight line this is 0, this is 4. So 1 by 4 is a very small quantity all right now, they are 2 different entities this is 4 ohms, 1 by 4 will be more I can choose any skill 1 by 4 of admittance can be of much larger magnitude it all depends on the choice of the unit okay they are not same units.

So I can choose a suitable scale for fixing of the magnitude say if we take 40 as the scale factor I have taken as scale factor say 40 then 40 into 1 by 4 will be 10 units I can choose the units that way. So I can choose 10 units here and choose this as 1 by 4 all right, what is the location of the, what will be the circle lie corresponding to this Z line a horizontal line that means this itself is a perpendicular all right. So the circle will be with this as the diameter you draw the circle like this okay. I will its getting clumsier here, I will draw it once again as I told you this is may not be to the scale and a free hand drawing and making.

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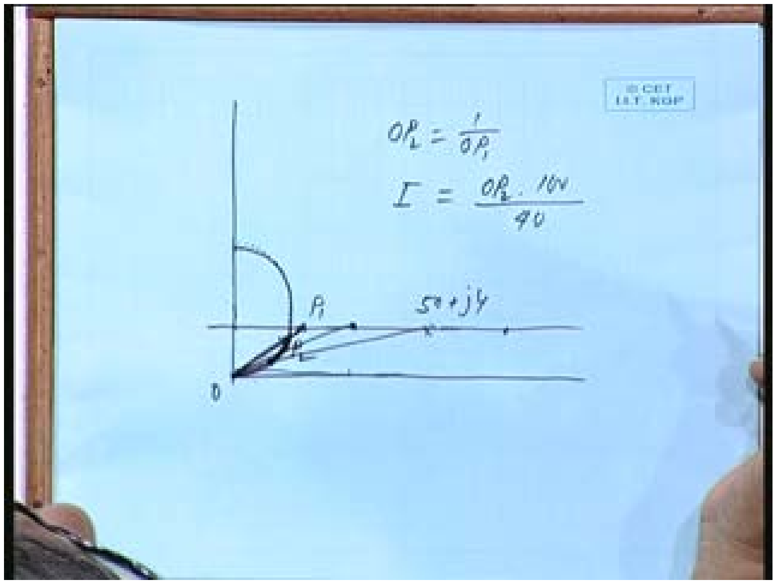

So this was 5 plus j 4 okay OP 1, so its inverse will be on this line actually it will be in this quadrant if we remember that the angle has to be taken care by transferring it on this side, this will be the magnitude, is it not. So this point let it be P 2, so OP 2 is 1 by OP 1 in magnitude. Similarly 5 plus j 50was here say, say say here 50 plus j 4, so its inverse would have been here all right wherever this circle is met with this Z line. So this is the portion which represents that segment 5 to 50 of the resistance plus j 4 okay. So this is represented by this segment inverse of that is represented by this segment.

So you can now multiply for any particular location. So if I want to calculate for 10 plus j 4 what is a current for 10 plus j 4 suppose this is 10 plus j 4, this is a point. So draw a line wherever it intersects this is the length of the vector which is already magnified 40 times I have taking taken a scaling factor of 40. So that has to be divided by 40 is that all right? So whatever length you get on this semi-circle divided by 40 that gives you and multiplied by the voltage 100 volts that give you the current.

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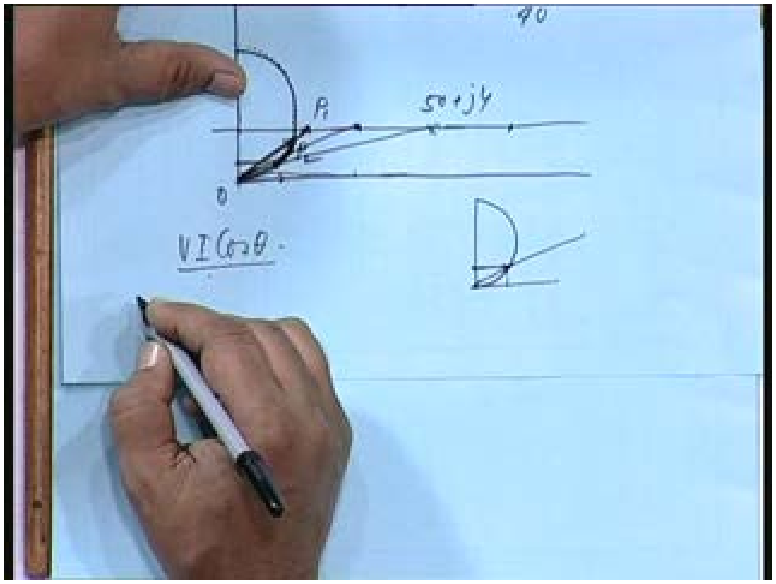

So the current I is say for example for P 1, OP 2 into 100 divided by 40 the scale factor once you know that you can calculate the current also power also, what will be the corresponding power dissipated VI cos theta okay. So this is the real axis, so cos theta means if you drop a projection, if you drop a projection cos theta into yes, this is our semi-circle. Suppose for any position this is the current then what is I cos theta this one, is it not. So measure of this distance multiplied by voltage 100 is that all right.

So VI cos theta will be the horizontal segment of that point on the semi-circle multiplied by V 100 of course that divided by 40 scale factor, what I am measuring as I is actually 40 times I. So you have to take that into account. Let us take another example, we have a resistance and a capacitance, this capacitance is varied, this resistance is fixed. There is another problem extension of this, this is j 20, this is 10 and this is C. The question is first of all show the locus of this current, next see this locus is determined, next when is the total current having a power factor of 0.4 lagging, for 0.4 lagging power factor total current for the total current, what is a value of C, question is for what value of C will this total current, total current I have a power factor of point 4 lagging have you understood the question.

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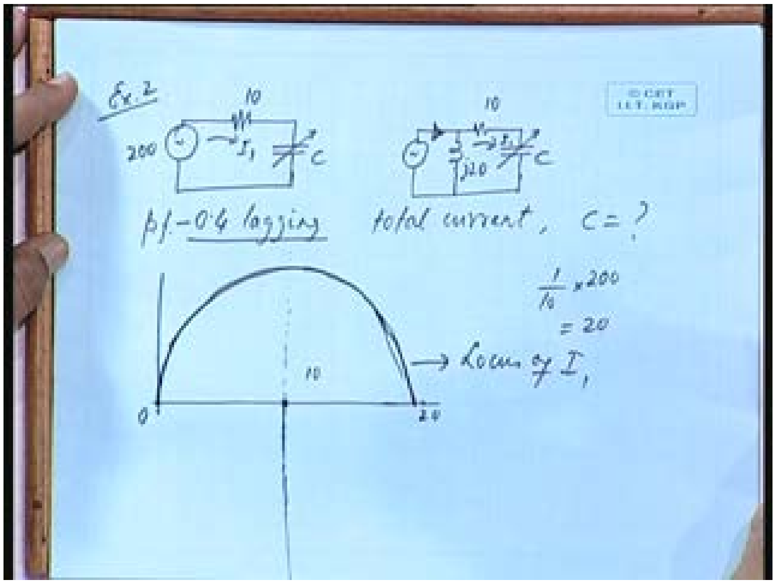

So let us try to find out the locus of this first all right R is fixed 10 ohms C is varied. So XC is varied from 0 to infinity if it is not specified then the variation can be over the entire range. So this will tend to infinity, so this is the straight line of which only this is the relevant portion because X is not varied from plus infinity to minus infinity, it is from 0 to infinity, 0 to minus infinity all right. So it is 0 to minus infinity, so what will be the location of the what will be the locus of the admittance function it is 0 to 100 and 10. So one 10^{th}, one 10th of this 1 by 10, I can choose any scaling factor all right. Let us choose a scale factor of 200 all right, this voltage is 200 or straight away we can multiply by 200 because after all it is a voltage multiplied by the admittance which gives me the current.

So if I take straight away a scaling factor equal to this voltage then that represents a current. So 200 into 1 by 10 that gives me 20, so 20 amperes I will choose 20 units will represents with this 20 as the diameter, I will draw a circle I need not draw a circle, I can draw only a semi-circle because inverse of this will come in the first quadrant, this is in the 4th quadrant. So inverse will come in the first quadrant, so this is the had it been varied over this range also X is varying with an inductive to capacitive range then this entire circle would have come into picture okay. So this is the locus of current I call it I 1 because I will refer to this current here also I 1 is that all right what about this current I 2, it is 100 ohms sorry 200 volts divided by j 20 it is a fixed current.

So 10 ampere all right 200 by j 20 what would be the location minus j 10. So that current is minus j 10 is that all right? So this much this is the location this is the locus of I 2, this is the locus of I 1, I 1 varies like this with respect to the origin, I 2 is fixed like this. So what is I 1 plus I 2, I 1 plus I 2 shift this entire thing downward by 10. So say minus 10 suppose, this is I 1, I 1 minus 10 will be somewhere here, so minus j 10.

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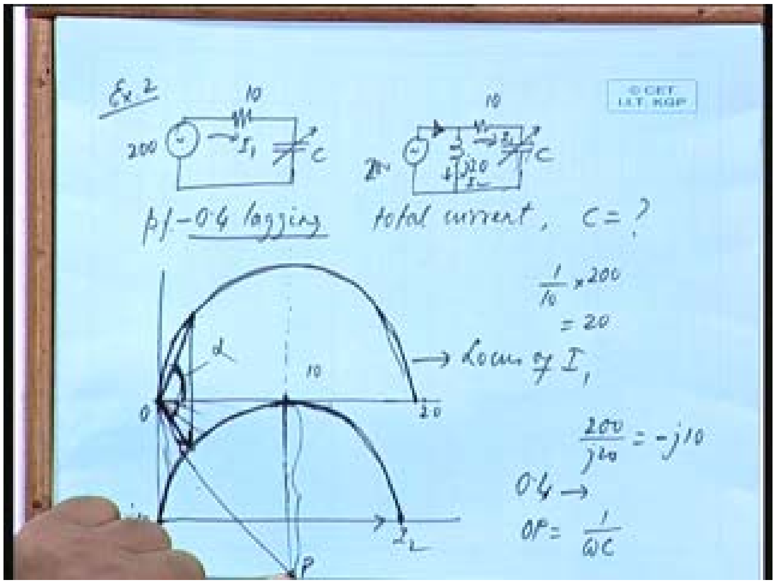

So the same semi-circle you reproduce here in this particular example it so happens this is 10, this is also 10, so their that means diameter was 20. So radius was 10 and this side the shift is also 10, so it so happens that it is touching this foot of the semi-circle it need not be so in all the cases okay. Now what is the question, question is for point 4 power factor lagging what is the value of the capacitance for this current, now the current locus is with respect to this origin, this is the current locus, is it not, total current, the origin is fixed. So when the current I 1 is 0 the total current is just minus j 10 when there is no current through this this is minus j 10 and so on.

So this is the current for point 4 power factor lagging for 0.4 lagging power factor what is the angle cos inverse of point 4 whatever be that angle okay. So how much is it approximately 76, 71 about 75 degrees okay sin inverse of point 4 is 4 into 6 to a approximately 23, 24 degrees all right. So 90 minus 24 it is about say 76 degrees very crude approximation. So draw 76 degree here that is tan inverse is a cos inverse point 4. So this is the current is there any other possibility for lagging power factor is there any other possibility, suppose we okay let me complete this first, so this is the total current.

So what is the corresponding current in I 1, when the total power factor is point 4 lagging, what is the current through I 1 I know that time what is the total current this is the total current. So how much is the current I 1 shift it vertically wherever it hits, this is the current I 1 is that all right measure of this angle, suppose that angle is alpha, how do we get alpha, first of all you have got the given power factor corresponding angle you draw here and then find out the current from there you draw a vertical line you reach here. So that gives you the angle alpha of this current all right then how do I get the corresponding impedance.

So draw an angle alpha here angle alpha here whatever be that all right and then stretch it, this is the point X, is it not. If you remember from the current that is from admittance if you come back to impedance the same angle first quadrant angle will become 4th quadrant and 4th quadrant angle will come to first quadrant. So Z to Y or Y to Z if you keep on interchanging, so this was the location of I that means corresponding to Y, corresponding to admittance this was angle. So corresponding to Z it will be minus alpha, so at minus alpha you draw a line that will give you the corresponding reactance it is 10 plus j X.

So once you get this height that is 1 by omega C, so C is suppose this is P then OP is 1 by omega C, so you know that is add C is that all right. Now my question, next question was suppose I give you a power factor of point 8 or may be point 9. So you draw when the total current is having a power factor of point 9 lagging, power factor of point 9 lagging means this much this is one solution this also could have been another solution. So long as it is within this semi-circle there can be 2 possible values one is here, one is here all right whereas for the other one it may not have a feasible value because it is on the other side of the semi-circle which is not there for a capacitance reactance okay. So there could have been 2 possibility…… we will first discuss about simple signals and then at a later stage we shall be using these signals for different analysis, network analysis or representation.

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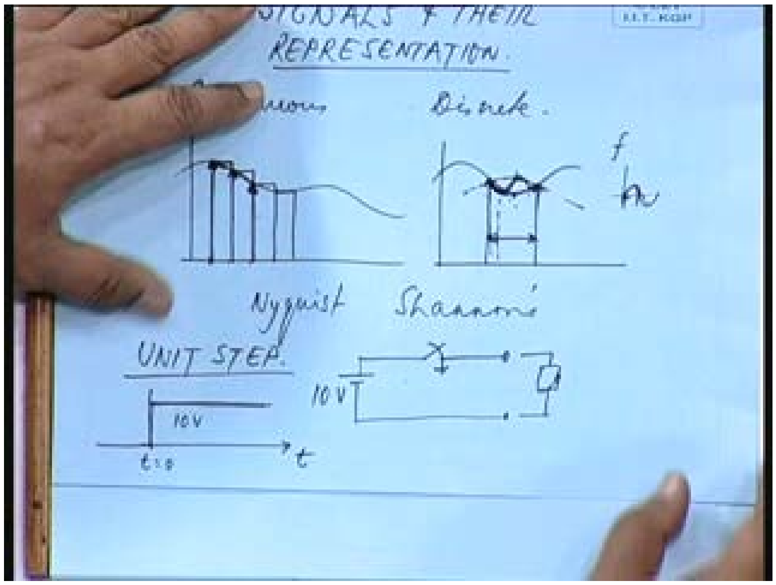

Now signals that we we shall be discussing are of 2 types, one is a continuous, the other one is discrete. There are some signals which are by nature continuous most of the natural signals are continuous okay. For example, if you take the record of voltage if you have a pen recorder you get continuously recorded values with respect to time it is recording the value continuously but there are certain signals which are integrated over time and then they are registered at regular intervals for example your energy, energy meter records okay or the population that you count after every 10 years so these are all integrated signals by nature they are integrated and hence you take stock, you take the count after certain time. So they are all or say sale of tickets every day sale if you count so after every 24 hours you see how many tickets have been sold. So most of the records maintained in our offices, most of the records are of discrete nature they are not continuous.

Now even continuous signals when you use in computers you have descritised versions. So discrete signals are also equally important, signals the discrete signals are say this is a continuous signal and if we take at regular intervals the values recorded this will be the descritised version okay. So for example you measure the power of a particular substation say at Kharagpur, the substation power you measure at every 1 hour is at 12' o clock, 1' o clock, 2' o clock you record the values, in the log book they enter these values. So they are descritised version not that between 12 to 1 there was no power power was there we assume this to be some average of this either the value at 12' o clock is maintained till the next value is registered or sometimes for convenience if somebody wants to make any mathematical calculations, any computation then you can take the average of the 2 okay somewhere in between.

So you join these by straight lines okay subsequent the consecutive values you join by straight lines or you have blocks like this, this is an approximation of the curve that is representing the actual signal okay. Now how fast, how fast can we measure these descritised values or should we measure these values, how quickly should we take the samples. Let us take a sample here and a sample here, does it represent the signal very correctly, it has gone through a trough. Suppose there is another signal like this.

So I cannot distinguish between this signal and this signal if the signals are taken with intervals so widely separated all right the samples so widely separated that means with large intervals. I will be missing out this bottom or the top point, this is that means with the signal if the largest variation largest variation that is taking place is represented by some frequency, this represents half that variation. Suppose this can form at the most half of a cycle, half of a sinusoidal function okay this a maximum possible change and if I take half that value then I will be missing out the trough or the peak. So the sampling frequency should be such that I do not miss out this, so half the length means twice that frequency that is present.

So maximum frequency that is present that means maximum number of changes that may occur in the original signal, twice that frequency if I sample the original signal at that frequency then only I will be able to just recover this point okay. There can be some more changes okay, there can be a peak and trough both then that means it is varying at a much higher frequency. So I must reduce the sampling time further to trap that change. Otherwise, the sample value will not be representing the original signal is that all right? So this is very famous theorem Nyquist or Shannon's theorem, sampling theorem okay that means the sampling rate should be atleast twice the maximum frequency present in the signal okay, right now we are not going to discuss further details about this we will be taking up this thing further when we discuss about discrete signals later on.

Now what are the standard signals that we come across, mathematically a unit step is a very convenient signal, it can be tested on simple system, simple networks suppose you have a battery of some fixed source voltage 10 volts, you are switching it on if the battery is ideal one then there is no internal drop 10 volts supply will appear here after the switching instant and that remains constant.

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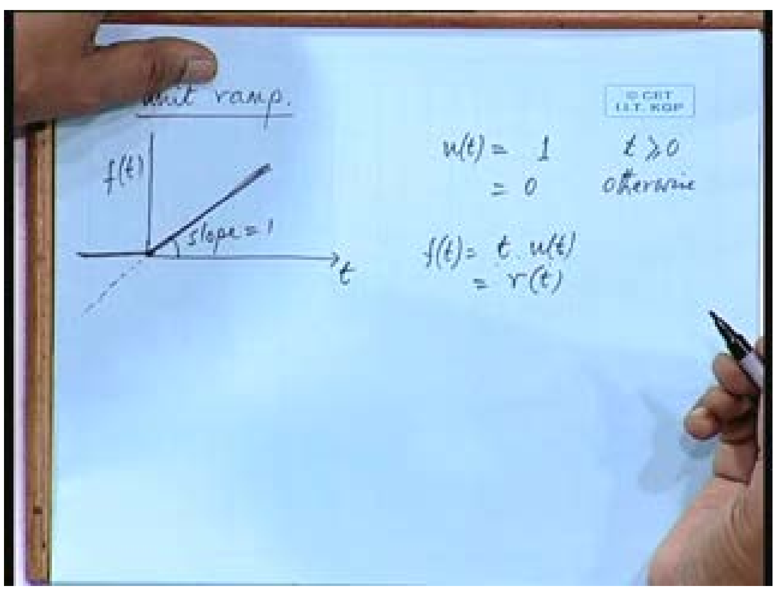

So before the switching, it was 0, if we count the time from t equal to 0 at the switching instant then this represents a step function 10 volts when the magnitude is 1, it is unit step okay. This is unit step then we have unit ramp for unit step, we use this notation ut that is equal to 1 volt when t is greater than equal to 0, equal to 0 otherwise or in the negative region of time okay this is for the unit step, unit ramp is a ramp function with unity slope. So ft equal to t into ut, y into ut the function t is also having values in the negative region of time but I want a function which will start from t equal to 0 all right before that it is non-existent. So if I multiply this by a unit step by a unit step this type then on the left hand side if gets 0.

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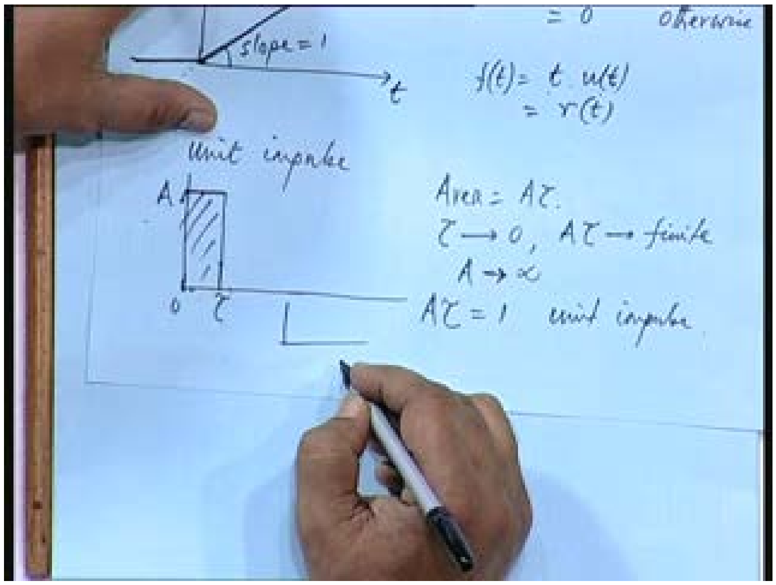

So the usual notation for this will be t into ut, you will find in many books they also write rt, rt means it is a ramp function like this 0 and then it is like this with a unit is slope if it is having any other slope then it will be k into rt or kt into ut okay where k is the slope then we define unit impulse, what you mean by unit impulse if you, if you have a step function like this applied for some time tau if the magnitude is A okay this is a pulse of width tau, what is the area of the pulse A into tau all right.

Now if we make tau 10 into 0 that is the duration of the pulse is very very small 10 into 0 but the product is finite A tau is finite then A must tend to infinity. So you are having a situation A is very large not measurable tau is very small not measurable but their product is measurable that is finite and when that finite quantity is equal to 1, we call it a unit impulse okay. You hit a cricket ball, you hit a cricket ball with a large amount of force all right P is tending to infinity, the duration of contact is very very small tau is very small but P into t, P into tau that is equal to change in momentum that is finite. So momentum change in momentum is this product A into tau something like A into tau where you can measure the magnitude of the impulse all right. So the magnitude of the impulse can be measured in terms of the product that is area under this curve neither by the magnitude of the force A nor by the duration of contact tau.

So this is something like if you switch on a battery and then immediately switch off, you give a kick to a circuit in a galvanometer; you just apply a voltage and then withdraw. Now the amount of voltage say it may be very large or suppose it is 10 volts and you apply it for say 1 milli second then 10 into point 1 that will be representing a very small strip that will be almost equivalent to an impulse of that magnitude okay we will stop here for today and we will discuss about this in the next class, next class is now, thank you okay.

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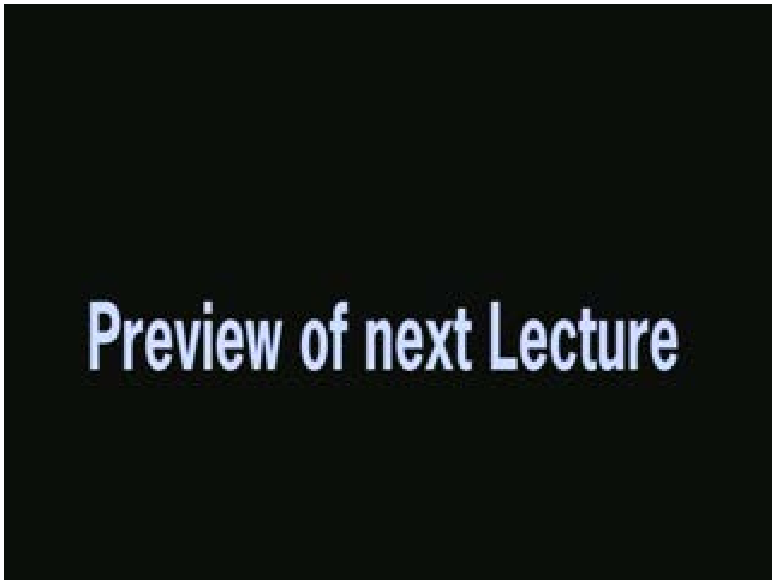

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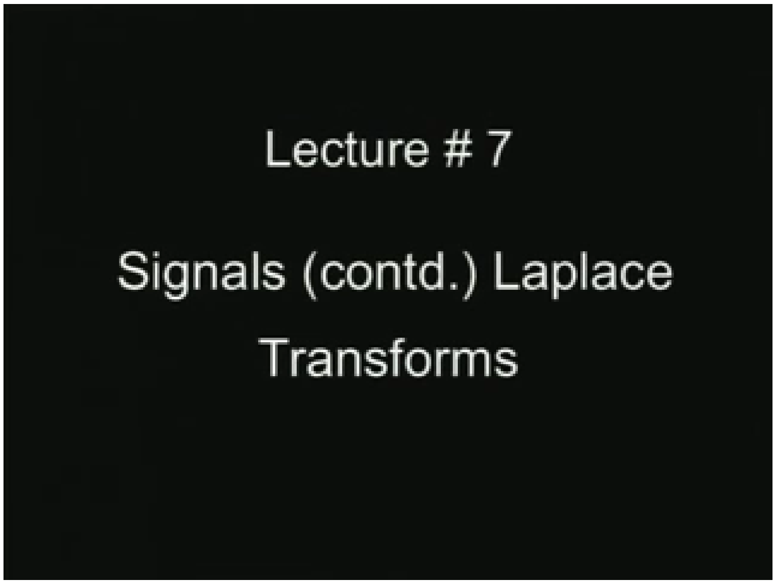

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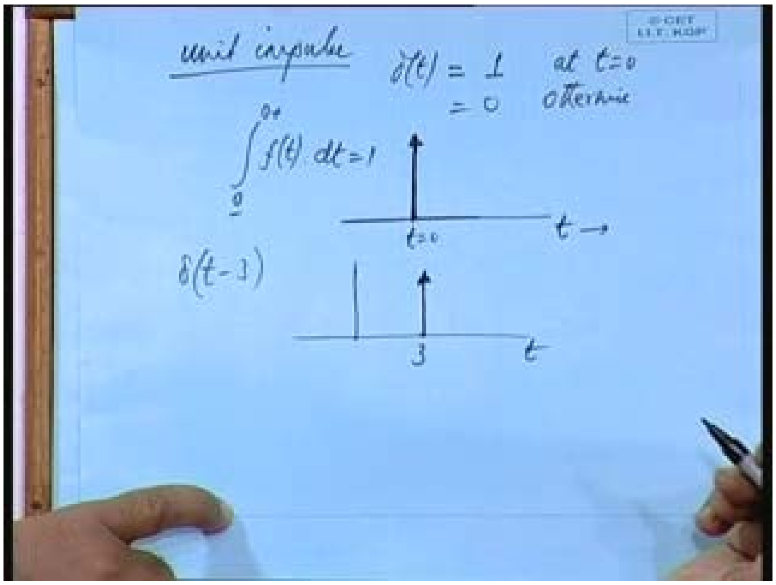

Okay, so gentlemen we will continue with the discussions on signals. Last time we were discussing about impulse functions, the unit impulse will be denoting by delta t, this is a function whose value is 1 at t equal to 0 and equal to 0 otherwise where this function is representing the force, so this is from 0 minus to 0 plus or you can write infinity, it really does not matter that means the function is coming here its magnitude is very very large and duration is very very small and then it is going and this thick line represents something equivalent to this area all right.

So the integral is finite when this is 1 it is a unit impulse okay this is time t and this is the function we normally show it by an arrow and a vertical line, impulse appears at this moment t equal to 0. So delta t minus 3 represents what, this is a shifted version, shifted version of the same impulse that is at t equal to 3, this function appears okay.

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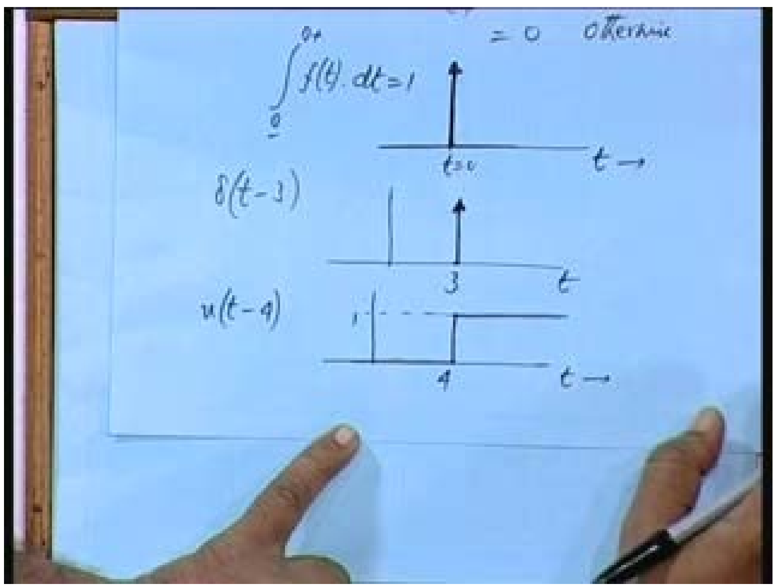

Similarly what will be ut minus 4 it is the shifted version of unit step okay ut minus 4 represents a function which starts after t equal to 4 or at t equal to 4, it is unity after that before that it is 0. So any function if we shift ft is a function, so like this then what is ft minus tau if I want to represent the same function which is starting after an interval tau if it is identical okay then what will be the mathematical representation of this function ft minus tau does it represent this cosine bt plus j sin bt okay.

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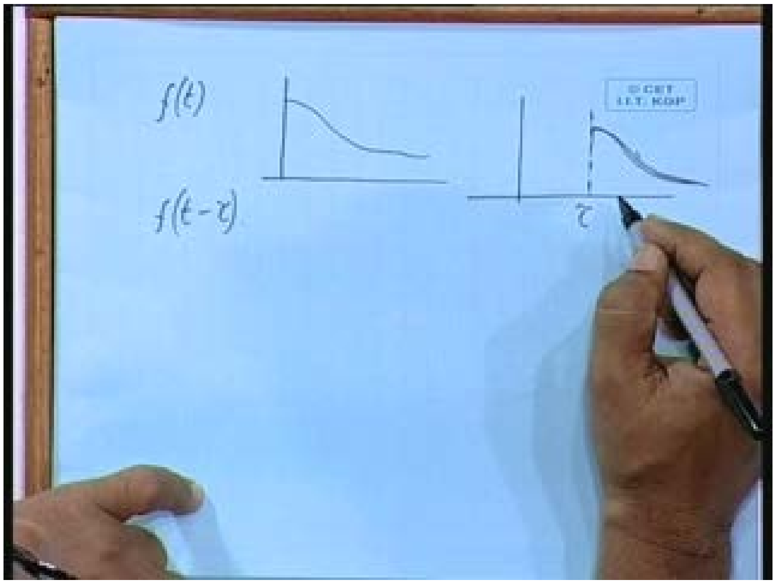

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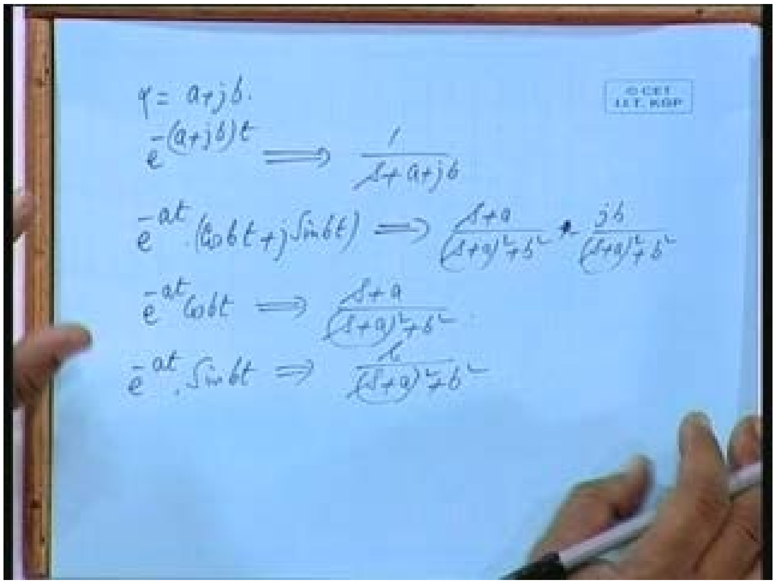

So on this side if I separate out the real and imaginary parts will be s plus a by s plus a whole square plus b square minus jb by s plus a whole square plus b square equate the real parts and the imaginary parts then e to the power minus at cosine bt will give me s plus a by s plus a whole square plus b square e to the power minus at sin bt will be b by s plus a whole square plus b square c to the power minus at into cosine bt is a damp sinusoid starting with a maximum value, this is e to the power minus at cosine bt.

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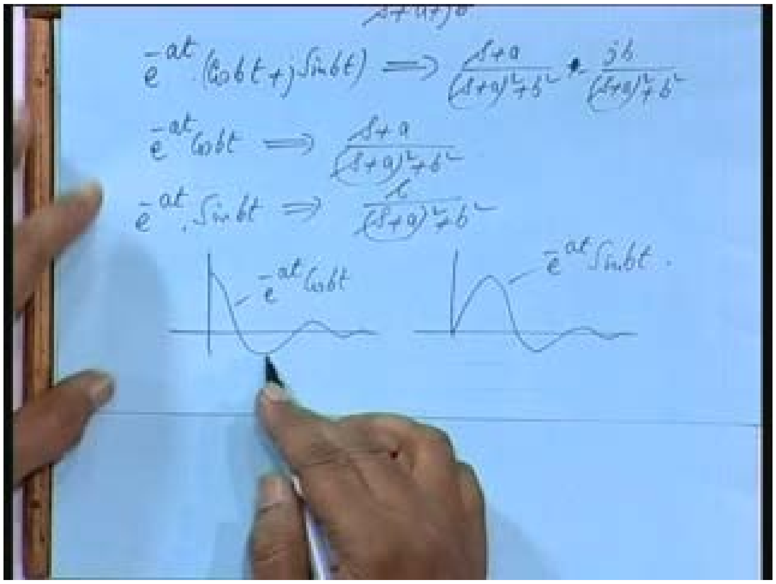

Similarly, e to the power minus at sin bt, it is like this you take a pendulum give it an oscillation all right give it a displacement and then let it oscillate, it will oscillate and die down is it not and you have a recorder all right. You just record the image of this point the bob then it will be damp sinusoid if you start your camera when it is in the maximum position then it will be a cosine function all right you move that you move the paper it will be describing this if you start from the central position then it will be this one okay so both are representing basically a damp sinusoid. So today we will stop here for today will continue with this in the next class.