Networks, Signals and Systems

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Lecture - 32

Synthesis of 2 - Port Network

Good morning friends, today we will be discussing about 2 port network syntheses.

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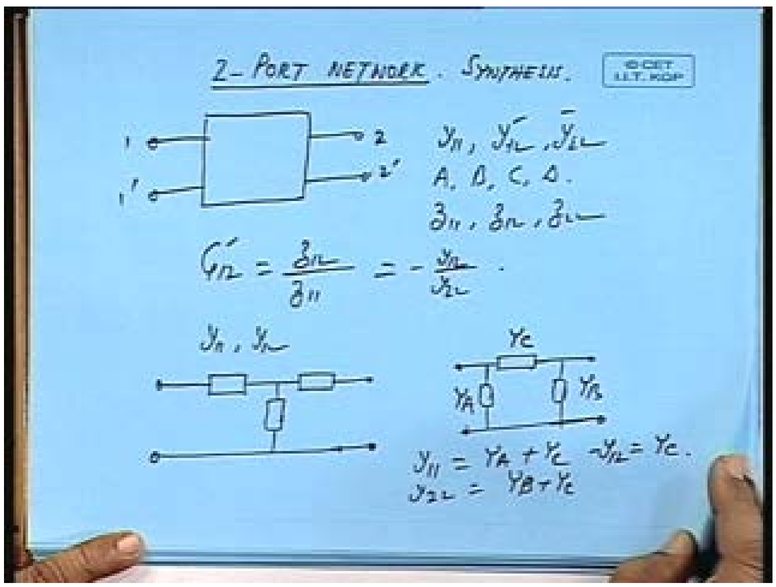

You know in case of a 2 port you have various specifications now, for a 2 port network Y_{11}, Y_{12}, Y_{22} these are the parameters or may be A, B, C, D or Z parameters and so on. These are the 3 commonly used sets sometimes you may be given only a gain function G_{12} which you know is given by this or minus Y_{12} by Y_{22}, so if this voltage by this voltage this is the gain function only the gain function is given then you are asked to realize a network to give that particular gain function all right.

So these specifications can be many of many types unlike 1 port network where we are given either Z or Y(s), Y(s) is just inverse of Z(s). So you have 1 specification here the specifications can be in terms of say Y_{11} and Y_{12} or Y_{22} and Y_{12}, any 2 can be given or may be just the gain function all right. So how to realize a network to meet these specifications? So before we go into that we just see the network functions how they are related to this 2 port parameters network elements.

Let us take a t network okay or a phi network from a t, you can always find a phi, this is start star to delta or delta to star conversion all right. Normally if you are given the specifications in terms of Z parameters you go for an equivalent t this is easier. Similarly, for Y parameters you go for phi network suppose this is given as Y_A, Y_B and Y_C. Let us take this first what is Y_{11}, Y_A plus Y_C very good Y_{22}, Y_B plus Y_C and Y_{12} okay minus Y_{12} is Y_C okay.

Now these elements these elements Y_A, Y_B, Y_C they can be any RLC combinations okay. Let us see Y_{11} what are the poles of Y_{11}, the poles associated with Y_A and with Y_C will be the poles of Y_{11}. Similarly, poles of Y_{22} will be the poles of Y_B and Y_C so Y_C is the common element for the poles of Y_{11} and Y_{22} and also minus Y_{12}, so the common poles in these 3 will be the poles of Y_C.

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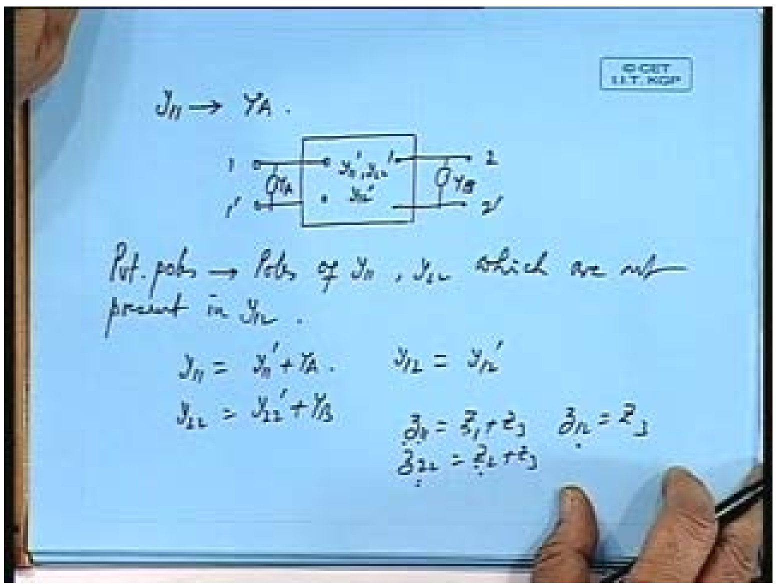

So in Y_{11} there can be some poles of Y_A which are not included in the other 2 functions all right. So poles of Y_A will be present only in Y_{11} nowhere else, similarly poles of Y_B will be present only in Y_{22}. So I can remove these poles still have such poles can be realized as separate elements, I am not given Y_A and Y_C, Y_B, Y_C separately, I am given Y_{11}, Y_{22} and Y_{12} so out of which I find out the poles which are not present in Y_{12} but present in Y_{11}.

Similarly, the poles which are present in Y_{22} not in Y_{12}, I call them private poles of the 2 ports private poles of 1, 1 dashed and 2, 2 dashed okay poles of Y_{11} and Y_{22} which are not present, which are not present in Y_{12} okay. So the poles which are present in Y_{12} will be present in Y_{11} and Y_{22} but the converse is not true okay. So if I have a network which is having Y_{11} dashed Y_{22} dashed and Y_{12} dashed as the parameters then Y_{11} as seen from this side will be Y_{11} dashed plus Y_A, any addition of an admittance here will be adding to this value only, is it not. If I have another element another additional admittance here that will be changing only Y_{11} that will not be affecting these 2, is it not because it is only Y_A which is getting modified. So Y_{11} will be this additional element plus whatever is a Y_{11} dashed here. Similarly, Y_{22} will be Y_{22} dashed plus Y_B but Y_{12} will be Y_{12} dashed is it not that remains unaffected.

Similarly, for the Z elements Z_{11} will be suppose this I call as Z_1, Z_2 and say Z_3 then Z_{11} will be Z_1 plus Z_3, Z_{22} will be Z_2 plus Z_3 and Z_{12} is Z_3. So here also you see the poles of Z_3 that is Z_{12} poles of Z_{12} will be present in Z_{11} and Z_{22} but the converse is not true Z_{11} and Z_{22} will have additional poles coming out of Z_1 and Z_2 okay. So if I have a series addition of such elements this is some Z_1, Z_2 suppose this is having Z_{11} dash Z_{22} dash and Z_{12} dash then overall Z_{11} from this side I should write small z small z_{11} will be Z_1 plus z_{11} dash, is it okay.

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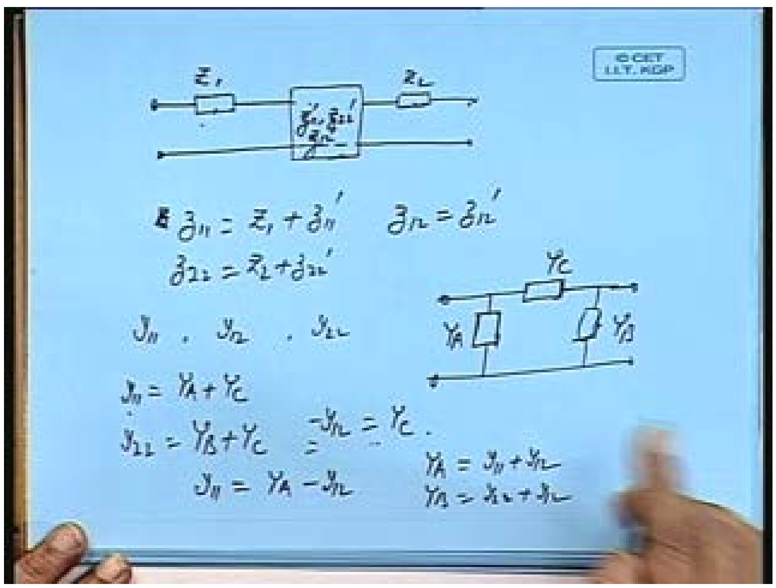

Similarly z_{22} will be Z_2 plus z_{22} dash but z_{12} remains same. So once again the privacy poles of an impedance, so this is the series and shunt separation of private poles. Now let us see what are the requirements for the impedance functions? All right, what are the requirements for the impedance functions to be realized? Now let us take a y_{11}, y_{12}, y_{22} once again I draw the, now what is the y_{11}, Y_A plus Y_C, so this Y_A, Y_C, Y_B, y_{22}, Y_B plus Y_C and y_{12} minus y_{12} is Y_C okay.

So y_{11}, y_{11}, Y_A minus y_{12} is it all right or Y_A you can write as y_{11} plus y_{12} okay similarly, Y_B will be y_{22} plus y_{12} and Y_C is minus y_{12}. Now given any network, given any network you can have a very complex structure of a 2 port network and so all right that can be bridge elements also whatever be the type I can always reduce by repeated star delta conversion, repeated star delta conversion this entire set to either t or a phi, is it not. Now in this repeated star delta conversions what are the elements? How do you calculate the elements of a star delta set? Say in terms of impedances Z_1, Z_2 plus Z_2, Z_3 plus Z_3, Z_1 divided by something all right divided by Z_1 or Z_2 or Z_3 the operations are all positive, there is no subtraction okay.

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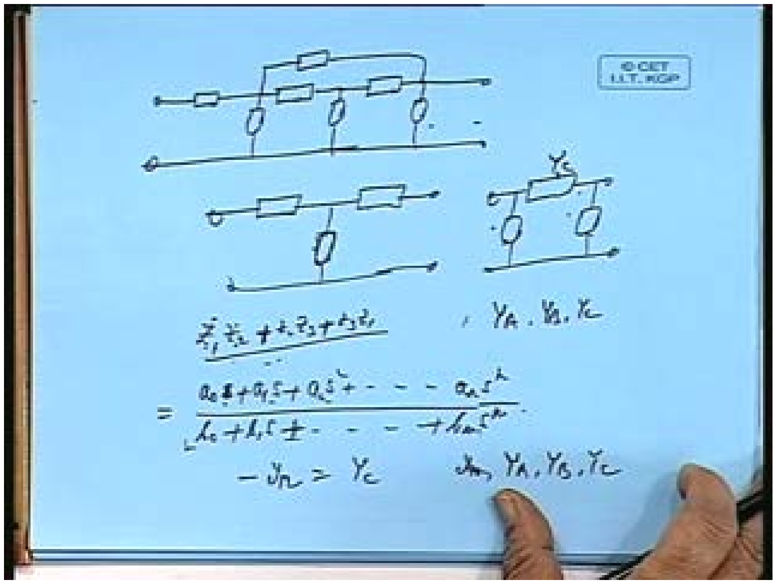

Now Z_1, Z_2, Z_3 these will be R plus L, S plus 1 upon SC or their parallel combinations series combination, so there all having positive signs and these after multiplication also the signs do not change then again you are having additions. So all the elements will be finally added there is no subtraction, there is no chance of a negative sign, is it all right. So all these elements finally Z(s) or Y(s) elements here it is Y_A, Y_B, Y_C or Z_A, Z_B, Z_C these will be having the forms some a_0 s to the power a_0 plus say a_1(s) plus a_2(s) square and so on divided by say a_n s to the power n divided by b_0 plus b_1(s) and so on.

So b_n s to the power n okay these elements Y_{AA} these will be all positive real functions, so far as Y_A, Y_B and Y_C are concerned these 3 elements are positive real after all they are obtained after reduction of some positive real functions, star delta reductions. So this will be having expressions like this. Similarly, y_{12} minus y_{12} will be Y_C, so y_{11} sorry Y_A, Y_B and Y_C are all having positive coefficients of polynomials in the numerator and the denominator. So let us see y_{11}, what was was our y_{11}, Y_A minus y_{12} okay.

So suppose you are given minus y_{12} as a_0 sorry otherwise I have a small slip, there was a small slip check y_{11} is Y_A plus Y_C okay y_{22} is Y_B plus Y_C all right and y_{12} minus y_{12} is Y_C okay therefore y_{11} it may be given, let us write the denominator may be some polynomial, numerator can be any polynomial that will be a common polynomial if there is no private poles okay. We can consider a common polynomial for the denominator then this can be say a_0 plus a_1(s) plus a_2(s) square and so on, y_{22} is b_0 plus b_1(s) plus b_2(s) square and so on and y_{12}, c_0 plus c_1(s) and so on okay.

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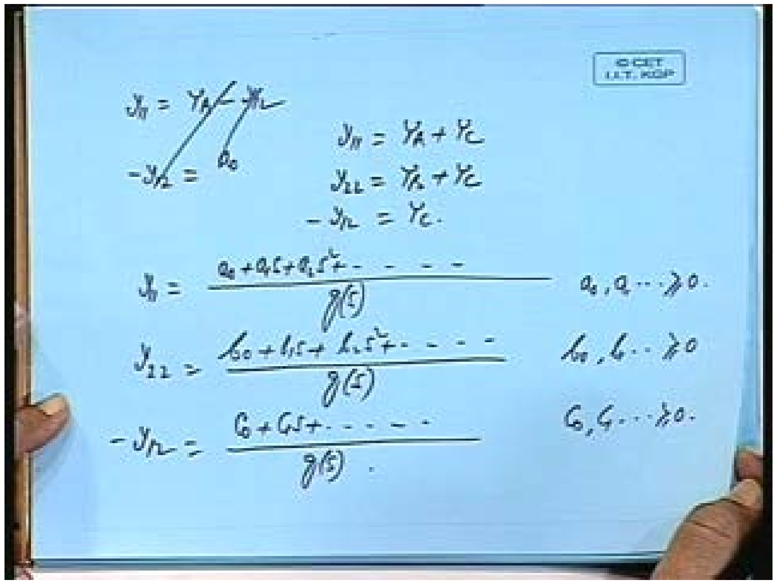

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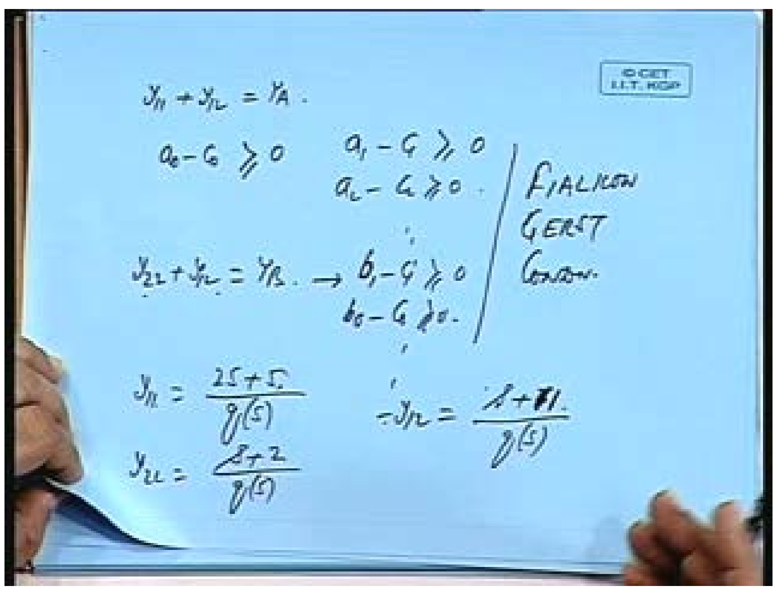

So one of the conditions is a_0, a_1 these must be greater than equal to 0 that is what we have seen all of them should be positive similarly b_0, b_1 must be greater than equal to 0, c_0, c_1 etcetera must be greater than equal to 0 all right, what is y_{11} plus y_{12}? Y_A, is it not, y_{11} plus y_{12} is Y_A is it all right now Y_A has to be with all positive coefficients. So if I substitute here y_{11} plus y_{12} means a_0 minus c_0, a_1 minus c_1 all right into s so that gives me a_0 minus c_0 must be greater than equal to 0 a_1 minus c_1 must be greater than equal to 0, a_2 minus c_2 must be greater than equal to 0 and so on okay.

Similarly, by the same logic y_{22} plus y_{12} is how much, Y_B and that also we discussed after a repeated reductions Y_A, Y_B, Y_C must have all positive coefficients in the numerator, so y_{22} plus y_{12} also must be having numerator coefficients as 0. So b_1 minus c_1 is greater than 0, b_0 minus c_0 is greater than 0 greater than equal to 0 and so on, all right. So these are the coefficient conditions to be satisfied when you are given the specifications is it all right. So y_{11} suppose is given 2s plus 5 divided by some denominator, y_{22} is given s plus 2 divided by same denominator and y_{12} minus y_{12} is given s plus 7 by the same denominator. Is it possible to realize this, let us see y_{11} and y_{12} the coefficients first coefficient 5 minus 7 is negative, so it is not possible all right.

So this must be less than 2 if it is s plus 1 yes, it is possible suppose it is 2s plus 1 then this is all right but this is not satisfied okay. So if you are having the coefficients of the numerator given like this the polynomials are given then by applying this rule you can check whether it is realizable or not, all right whether the network can be realized. So this is known as Fialkow Gerst Condition.

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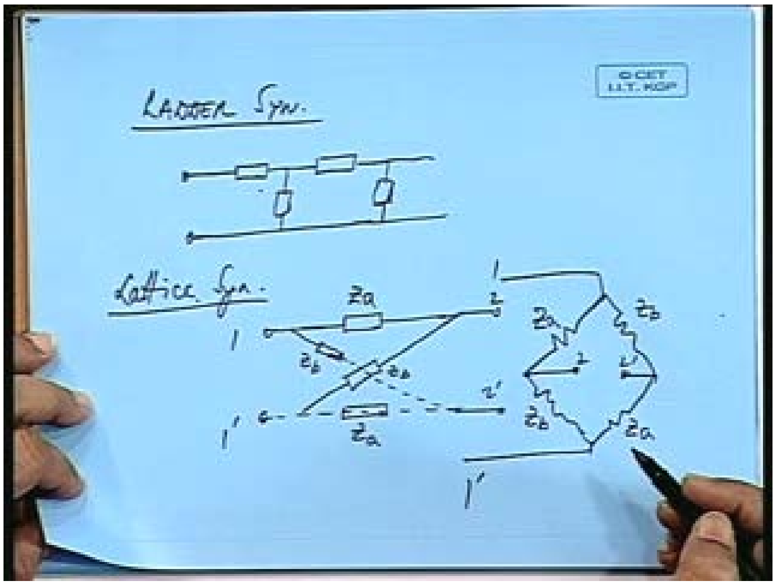

So before going for any synthesis we must see that these coefficient conditions are satisfied. There are 2 ways of realizing a 2 port network, one is Ladder Synthesis that is you try to go for realizing the 0s of transmission in steps from one end or there is a there is another one Lattice Synthesis which will be taking up later on that is a very interesting synthesis. It is a structure like this okay if you want you can have repeated block surface, so this is a Lattice network they are Z_a, Z_b, Z_b, Z_a it is like a bridge basically this is Z_a, Z_b and you take the output from here, this is 1, 1 dashed and 2, 2 dash this 1 as 1, 1 dashed, 2, 2 dash okay.

We shall see what are the 0s of transmission and how to realize them okay. Whenever you are having a_0 somewhere what do you mean by 0s of transmission transmission 0s means what I give a signal at the other end nothing is received, is it not. Now in how many ways that can be ensured I give a signal you take a water pipe line all right. In what are different possible ways, the say delivery of water at the other end can be disrupted, very good there is a leakage so water is grounded basically the pipe is grounded the entire pressure is grounded is there any other way of blocking.

So if you have some blockade in the series path or a short circuit in the shunt path then you have a 0s of transmission that means how do you create a 0s of transmission by short circuiting at some frequency suppose at some frequency I find s squared plus 9 in the numerator that means at s equal to J_3, this s square plus 9 will be vanishing that is a_0 at that particular frequency. So at the frequency I want there should be no transmission of signal and I want short circuit at that particular frequency, no, short circuit means what it should basely is resonance if have a series resonance at s equal to J_3 and that element is put in shunt okay. So you are having an element like this there are many such elements in the ladder I create this element is say an lc combination.

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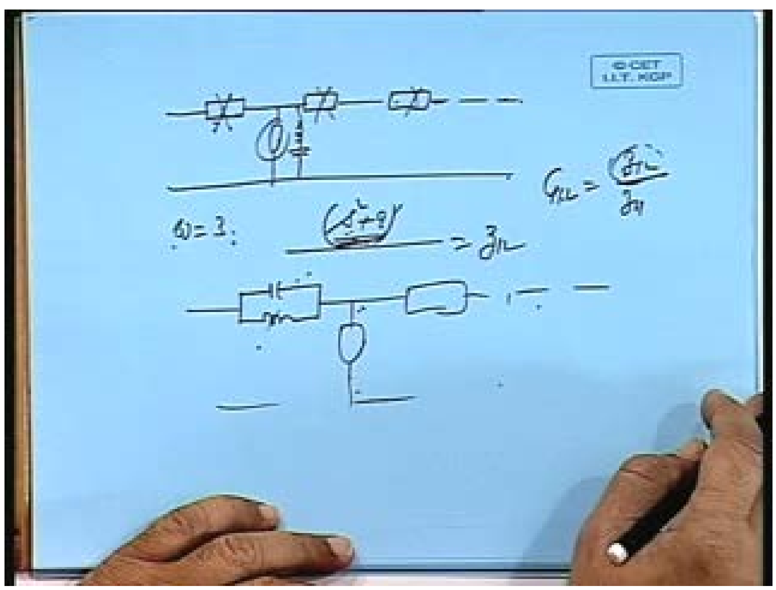

Suppose this is resonating at omega equal to 3 then s squared plus 9 will be giving me omega is equal to 3 this is resonating frequency, this will be the numerator okay. So in z_{12} if I have a numerator of this kind s squared plus 9 my target will be to create a_0 in the shunt element such that the elements are having a resonance frequency at omega equal to 3, is it all right or alternatively I open any of these that means there is no transmission after this. So how do I create that, how do I create that at omega equal to 3, now how to create that parallel resonance, anti-resonance.

So if I have an anti-resonance element any of these at that frequency and then there are others then also it can be blocked. So a_0 of transmission can be created 0 of transmission means because G_{12} is z_{12} by z_{11}, so it is the 0 of this which will be the 0 of this transfer function okay, so output will be 0 at that frequency. So that can be created either by a parallel resonance here or a series resonance is here in the shunt element, is it okay. So you keep on repeatedly removing all the 0s by adjusting these elements okay. So this is the technique of 2 port synthesis when you are given the transmission 0s you convert them into poles and then try to realize the pole any 0 can be realized in terms of a pole, is it not? How to do that?

Suppose you are given z_{12} how do remove that, how do, how do I remove a pole a_0 you take inverse of that function if it is in the admittance form then you convert it into impedance and then make partial fraction corresponding to that pole remove that okay.

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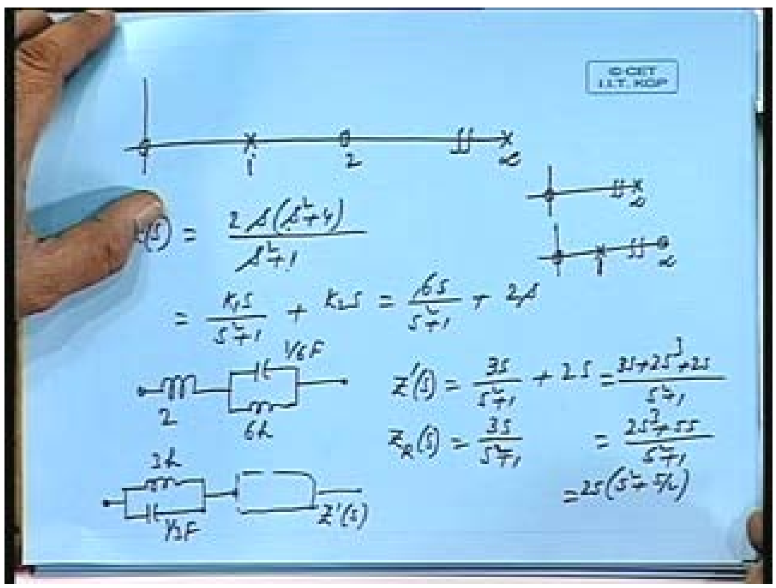

Let us take before we go further let us take a function Z(s) equal to say any function you tell me it is a 2 into s into s squared plus 4 by s square plus 1 okay I can make partial fractions K_1(s) by S square plus 1 plus K_2 into S okay, K_1 comes out as if I multiply by S square plus 1 divided by s and make s square plus 1 equal to 0. So it will be 3 into 2, 6, 6S by S squared plus 1 plus K_2 into S, how much is K_{22} into s all right. So that means 2 Henry, how much is this 1 by 6 farad and 6 Henry okay. Now if I remove 2 s from here from here if I remove this K_2 totally, if I remove 2s whatever is left over will content only the other pole all right. If I remove this I am left with only 2s now in this the 0s are at 2, 1 and this is a pole, this is a pole 0 configuration. 0 then pole at 1, 0 at 2 and pole at infinity okay.

Now suppose in this I, when I remove this I remove the pole at this one, what I am left it only 2S, a 0 here and a pole here okay for this function if I remove this what will be the pole 0 configuration for 2, s_0 here and a pole here at infinity if I remove this 2s then 0 here, pole at 1 and a 0 here okay. So if I remove this pole totally this 0 also gets eliminated all right. If I remove okay, if I remove this pole, this pole okay means this is having a pole at infinity, is it not, if I remove the pole at infinity then infinity becomes a 0 and this pole remains where is this pole. So this pole remains at 1 but the other pole has created a 0 here okay.

Now if I do not remove the poles or the poles of either this or that totally if I retain some part of it what happens that means instead of 6S plus s square plus 1 suppose I take 3S by s square plus 1 okay. Then I call that balance admittances Z dashed S as 3S by s square plus 1 plus 2S okay and some Z removed S is 3S by s square plus 1 that means 50 percent have removed 50 percent have retained. So if I remove 50 percent of this means what, 3S by s square plus 1 means allow 3 Henry, 3 Henry and 1 third Farad. Suppose I take out an element like this whatever is left over is Z dash into S, is it not what is Z dash into S, what will be the poles and 0s of this, poles will remain same I am not disturb the pole what about 0s.

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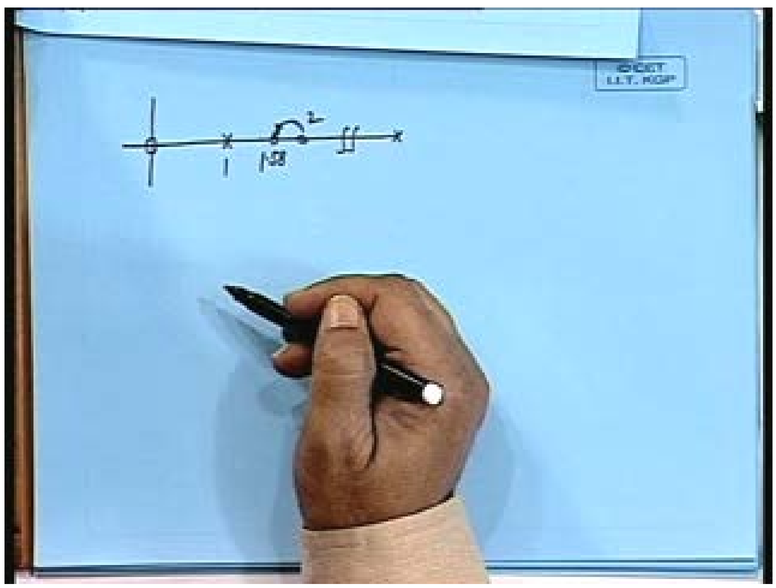

Now this is not totally removed, it will have a new 0 all right, what I have done is I have not totally removed this pole, I have weakened the pole, I have removed the part of that residue residue was 6S all right. So I have taken a part of it, so if I remove a part of that pole that is called part removal of the pole then what do I get in the oral distribution of poles and 0s poles remains as they are they were but what about 0s. So this will be 3S plus2 S cubed plus2 S all right that means divided by s square plus1 that means 2 s cubed plus 5 S in the numerator so S into if I take 2S common then S squared plus 5 by 2, correct me if I am wrong okay.

So the new distribution of pole and poles and 0s will be 0 at the origin is remaining intact all right, pole at 1 is again appearing because I have not removed it completely, what about this 0, this 0, it is now root over of 5 by 2, it has earlier it was at 2, now it has drifted towards this. Pole at infinity that also remains as it is okay. So this 0 at 2 has now shifted to root over of 5 by 2 okay approximately 1.58 okay. So if I weaken a pole I can shift a_0 to a desired position I have taken out only 50 percent of this 6S, this I said you I could have taken any percentage and hence

I could have shifted this to a desired location, is it all right. Therefore, by partial removal of a pole I can create a_0 at a desired position some of the 0s can be shifted, had there been many other floating 0s means between 0 and infinity, had there been some more 0s they would have all drifted towards the pole all right. The drift will be by different amounts all right.

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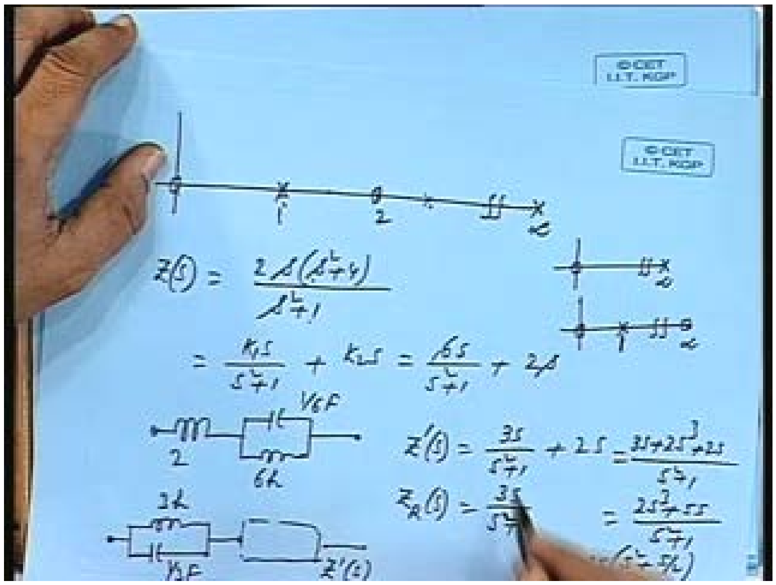

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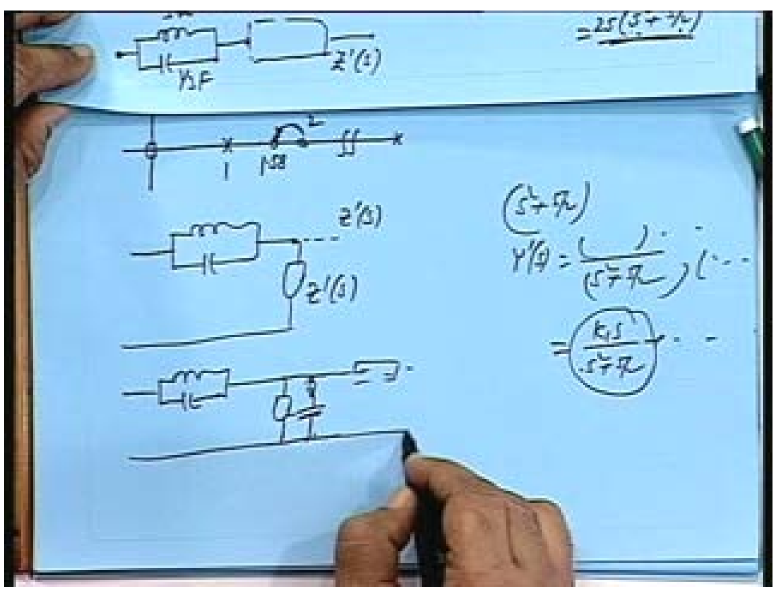

I am not bothered about the drift of all the 0s, I may be concentrating on one 0 specially then nearest one, the one which is nearest to this preferably to locate it in a new position and this is not the only pole there could have been other poles and my desire would have been to shift this to this side then I would have weakened this one, is it all right. So it all depends on what is the 0 wanted that means in Z_{12} in the transmission I am creating artificial 0s all right by removing the poles partly this is known as partial removal of poles and shifting of 0s okay. So you can weaken either the pole corresponding to this or this that means you weaken the pole here that also you could have seen. Let us see now, instead of 2S if I take S what happens I remove only one hand inductor and rest of it, I retain.

Now here I have created a 0 in Z_1 dash into S which is at root 5 by 2 so this element is having a 0 let me complete this then will go to partial removal of this pole at infinity. So I have got a network like this and then Z dash into S is a function which is having a 0 at S square plus that is at root 5 by 2 it is having a 0, how do I realize this any 0 you convert it to a pole. So I correspondingly I will take Y dash into S which will be giving me S square plus 5 by 2 in the denominator okay then there are other factors okay and this one I will write as K_1(s) by s square plus 5 by 2 plus other factors that means Y dash the admittance is taken as some Y_1, Y_2, Y_3.

So this entire Z dashed which was appearing here is now taken as some Y_1, Y_1 which will be corresponding to this all right that means it will be having, this is an LC series element, is it not if Y corresponds to K(s) by S square plus omega square that will give you a series LC element okay and then rest of it I will again switch over to Z(s). I am not bothered about this part, I am interested only rem in removing that 0, so convert it to a pole that is go to an admittance function then realize that and then again the balance you again invert you will get rest of the elements 1 by 1 you take out the 0s in the shunt elements okay.

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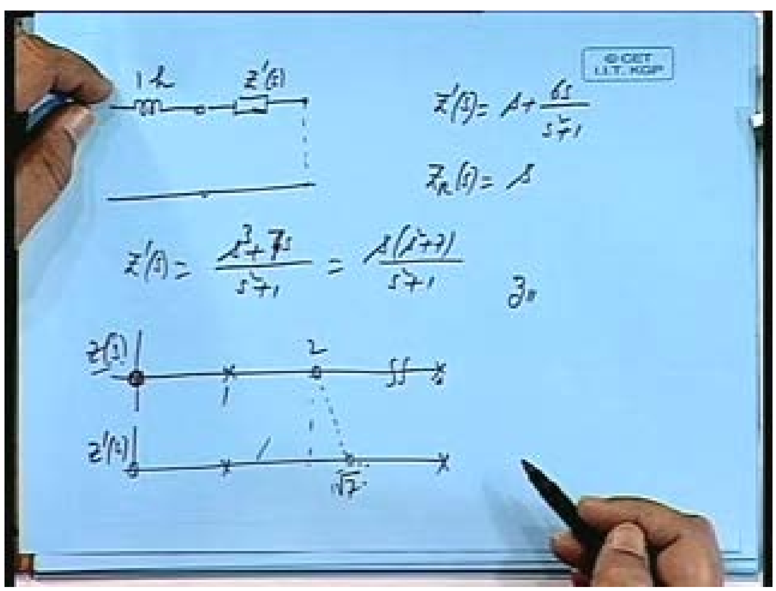

So this will be resulting at root 5 by 2 so that will be creating the 0 all right by partially removing the pole we can realize this. Let us see if we partially remove the pole at infinity that is let us take again 50 percent removal just take S. So we have got 1 henry it is Z dash into S now, is s plus 6 S by S square plus 1 and Z removed is just s okay. Now what is Z dash into S, s cubed plus S plus 6S, so s cubed plus 6S divided by s squared plus 1 sorry 7S. So s into s squared plus 7 by s squared plus1 now what will be the pole 0 location, earlier it was a pole sorry a 0, a pole a 0 and a pole.

Now I have weakened this so under the new situation Z dash into S this was for Z(s), Z dash into S is having 0 once again at the same point, pole at the same point, this pole is also at infinity but here it is earlier it was 2, now it is root 7, now it has drifted to this side. So 0 has drifted towards this pole which has been weakened, so wherever you are weakening the pole the 0s will drift towards that okay. Earlier it was in the other side so the 0 drifted to that side, so depending on the 0 that is given in the transmission, this is corresponding to the transmission 0.

So the 0s are transmission where they are located and what is given in z_{11} if you know that z_{11} is at 2 and the 0s required 0 of transmission is at root 7 then I will be weakening this pole first, so that this drift safe all right. If I weakened this pole then it will be drifting to this side I cannot realize this 0 all right. So this is the technique that we follow for ladder development and we want 0 at that particular frequency so what should be the value of that residue that is we have seen only 50 percent removal, what should be the percentage removal? What should be the percentage removal of the residue, percentage removal of the residue. So that we can get the desired location of the 0 okay.

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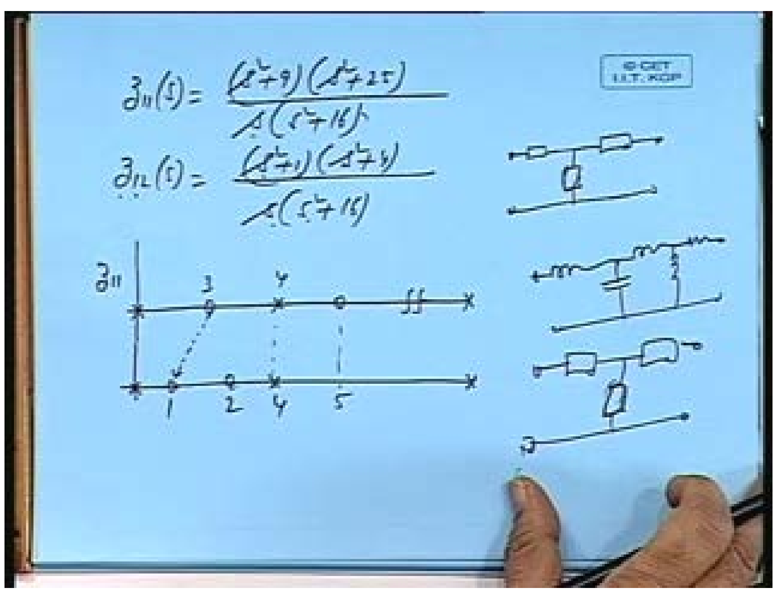

So that is a next question, let us take up an example it will be clear say $z_{11}(S)$ is given as s squared plus 9 into s squared plus 25 divided by s into s squared plus 16 okay. Let us make a plot of this poles and 0s first and then decide about the shifts, so there are no private poles z_{12} and z_{11} are having the same poles all right, if there are private poles remove that first. So poles and 0s are for z_{11} is it has is the original 0 pole then 3, 4 okay then not to the scale any way 5 and then at infinity there is a pole, this is z_{11}, z_{22}, z_{12} pole, poles are same because there is no private pole. So the poles are identical and then this one is at 1 and then this is at 2.

Now in the transfer admittance, transfer admittance or transfer impedance, there is no hard and fast rule that poles and 0s should alternate, transfer admittances z_{12}, how much is z_{12} in terms of z_1, z_2, they need not be okay we will discuss about it later on, see the poles and 0s need not come alternatively, so they need not be positive real functions, they need not be positive real function, is that clear. Any question? They need not be positive real functions how follows us z_{12} by repeated star delta conversions, you got z_{12} as this impedance but this need not be a positive real function, this need not be a positive real function.

Only thing is the numerator and denominator coefficient must be always positive when you go for repeated star delta conversions you must get all the coefficients as positive but repeated conversion of star and delta that need not guaranty positive real functions as elements that means the element may not be realizable, element may not be realizable. For example you are having an inductor and capacitor, an inductor, an inductor and resistor and so on. If I reduce it to star or a delta this may give you some Z(s) it may not be realizable it will be having a function all right with all positive coefficients $a_1(s)$, $a_2(s)$ squared and so on divided by $b_1(s)$ b_2 square but it may not be a positive real function they may not be able to realize it by RLC, okay it is a very interesting point.

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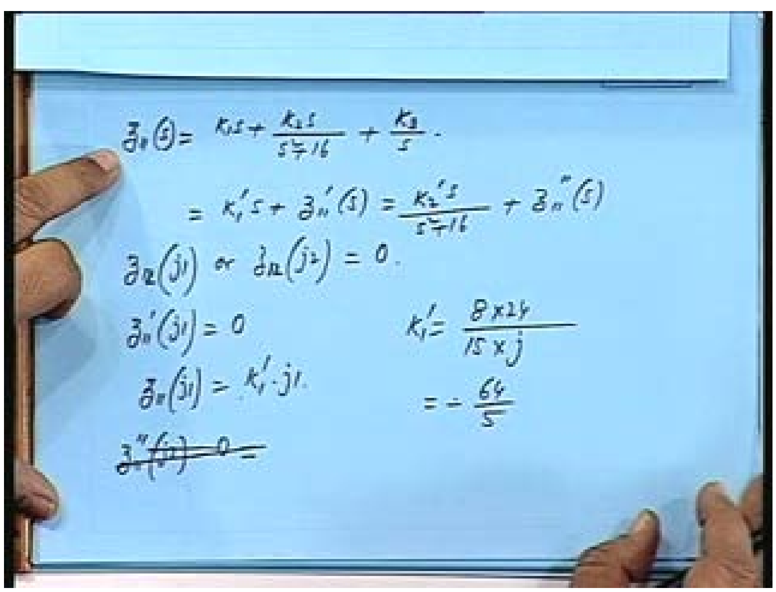

So this is very clear from z_{12} itself here s squared plus 1 into s squared plus 4 by s into S squared plus 16. So you can see the 0s are coming consecutively coming 2 consecutive 0s and then pole comes, so what should I do I can shift this 3 so these are the values 1, 2, 4 and 5 either I can shift this 0 here or this 0 here. So either you can weaken this or weaken this, is it not. So if you want to shift this here you have to weaken this that means a part of this residue has to be removed okay so $z_{11}(s)$ let us break up $z_{11}(s)$ if you write as $K_1(s)$ plus $K_2(s)$ by S squared plus 16 plus K_3 by S, I am not really concerned about all these, I am interested only in a partial removal of this.

Suppose I write this as some K_1 dashed S plus z_{11} dashed S that means this is the part removed, this is the part removed and this is what is remaining all right. Now $z_{11}(s)$, so z_{11} either at j equal to omega sorry omega equal to1 that is S equal to j_1 or s at j_2 this will be vanished all right sorry, the impedance at that point, impedance sorry impedance at that point j_1 or impedance at j_2 I want this to be 0 for z_1, z_{12} for z_{12} this will be equal to 0, is it not. So z_{11} dashed, z_{11} dashed at j_1 if I take it as 1 because I am shifting it here, should be equal to 0 is it not this must be 0. So what is this if I put on this side z_{11} at j_1 should be equal to K_1 dashed into j, S equal to j_1. So how much is K_1 dashed calculate from here, is it okay z_{11} that was given as s square plus 9 into this thing, so put S equal to j_1, so that is 9 minus 18 into 25 minus 1 into 24 divided by 15 into j okay and this j is there already.

So that gives me K_1 dashed into j_1 all right K_1 dashed into j_1, so calculate this how much is K_1 dash comes out as minus 64. Now check whether it should be minus z_{11} j omega plus K_1 dash into j_1, j_1 is equal to 8 into 24 divided by S into s squared plus 1 or 15, is it all right. We are trying to make it at 1 it is a 0, so 1 okay K_1 dash it is let us see whatever be that value minus 64 by now j into j will be so it will be plus minus 64 by 5 so that means we will have to add that seems to be some out funny this there made some slip okay.

Let us see the other one I come back to this you also, also think over it why we should get this or what we are trying as K_2 dashed S by S square plus 16 into some z_{11} double dashed S either remove this part partly, remove this partly then it will be shifting to 2 sorry, sorry, sorry, sorry 3 will be 2, thank you very much that is why this scaling was very very important. So in any case you can weaken this to shift it to either 1 or 2 okay, thank you.

So in that case what will be this one, z_{11} double dashed at j_2 should be equal to 0, this is a_0 which gives me yes, sorry we are weakening the pole at the origin that is what I am putting j_2, I am going to write that, that is what I am going to do, this will be z_{11}. Okay, let us write it here, okay we will continue with this in the next class. Now time is over we will continue with this in the next class, thank you.