Networks Signals and Systems

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Lecture - 31

Tutorial

Good morning friends, today we will have another tutorial session.

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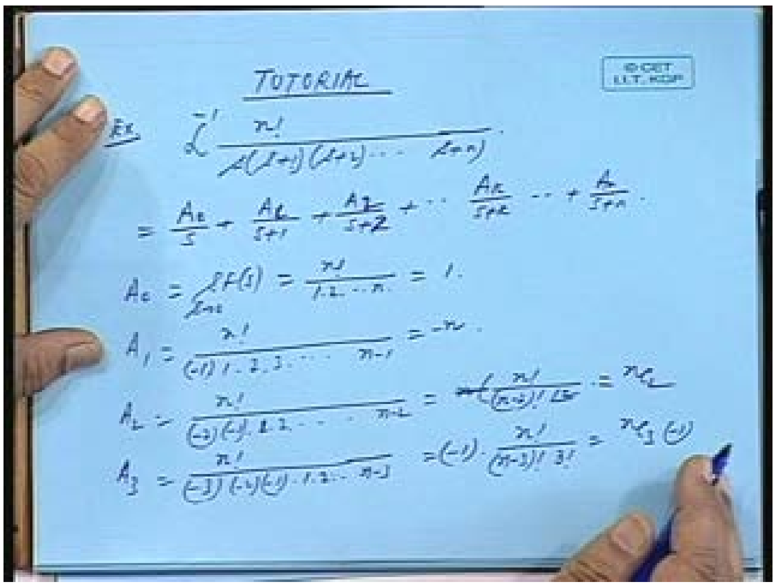

The first question is determine the Laplace inverse of factorial n divided by s into s plus 1 into s plus 2 into s plus n okay. So easiest method is to find out the partial fractions I will write A_0, A_1 A_2, S plus 2 and so on, A_k by S plus k general term okay. So how much will be A_0, A_0 multiply by s this F(s) make s stand into 0, so that gives me factorial n divided by 1, 2, 3, 4 up to n that is equal to 1, A_1 multiply by s plus 1 make s plus 1 equal to 0. So what we get A_1 as 1 into 1, 2, 3, 4 up to n minus 1.

So that gives me n with a negative sign because the first one will be minus 1, A_2 will be factorial n divided by multiply by s plus 2 then make s plus 2 equal to 0 should be minus 2 minus 1 minus 2 minus 1 then 1, 2, 3 up to n minus 2 and that will give me plus n into see I get minus 1, minus 2, n minus 2, so factorial n divided by factorial n minus 2 into factorial 2, A_3 and what is this you can identify this as nc_2, A_3 similarly will be factorial n divided by minus 3, minus 2, minus 1 then 1, 2, 3 up to n minus 3.

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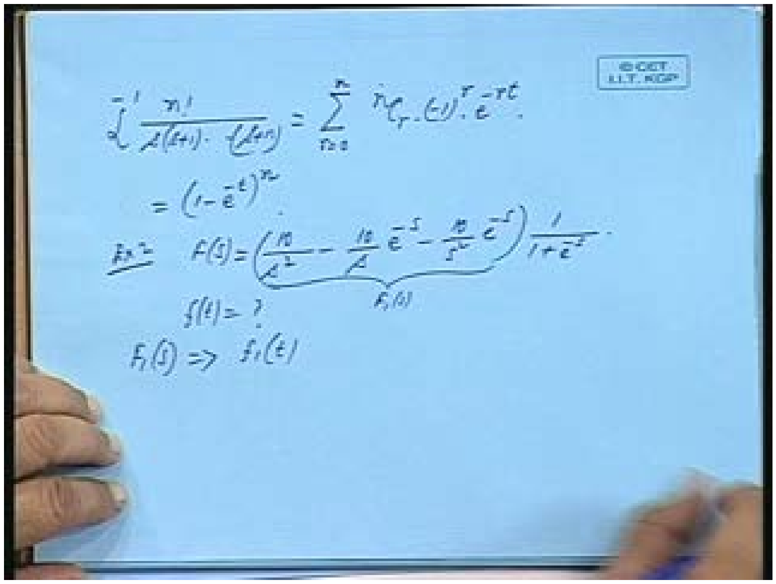

So I get minus 1 into factorial n by n minus 3 factorial into factorial 3 that is nc_3 with a minus 1. So alternately the signs are coming plus 1 and minus 1 and these are combination terms, so this can be written as therefore Laplace inverse of factorial n by s into s plus 1 etcetera up to s plus n as factorial n. So I can straight away write ncr minus 1 to the power r, e to the power of minus r t thus it is A_0 into A_t, A_1 into t or minus t, A_2 into t or minus 2 t and so on.

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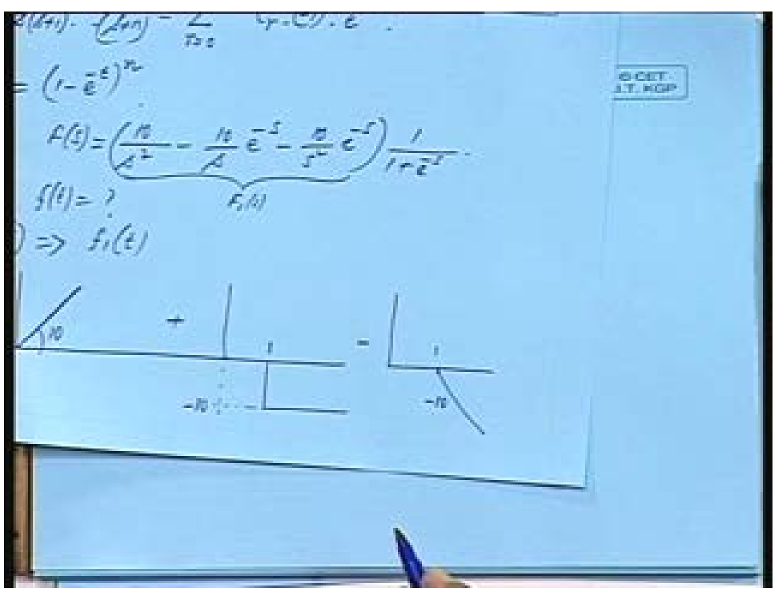

So it will be it is the minus rt, r varying from 0 to n, so this is nothing but 1 minus e to the power of minus t to the power of n okay. So this is the response there is a function given F(s) equal to 10 by s squared minus 10 by s into e to the power of minus s minus 10 by s squared into e to the power of minus s whole thing 1 plus e to the power of minus s, what to be f(t) like wherever you come across terms like this it signifies basically periodic function okay.

So let us first of all consider only this bracketed quantity. Let $F_1(s)$ be this bracketed quantity and let $f_1(t)$ be the corresponding Laplace inverse. So what is the function 10 by s square giving me $f_1(t)$ will consist of the inverse of this this and this what is 10 by s squared it is a ramp function okay of slope 10, I will write slope in terms of this minus 10 by s. So 10 by s minus 10 by s, so this is e to the power of minus s, so at 1 second we apply a step of 10 volts negative steps and then minus 10 by s squared e to the power of minus s. So again at 1 second we apply a slope of minus 10, what is the summation of these 3 summation of these 3 will be giving me net value $f_1(t)$ these are all the t axis and these are the 3 components of $f_1(t)$ whose Laplace transform is this.

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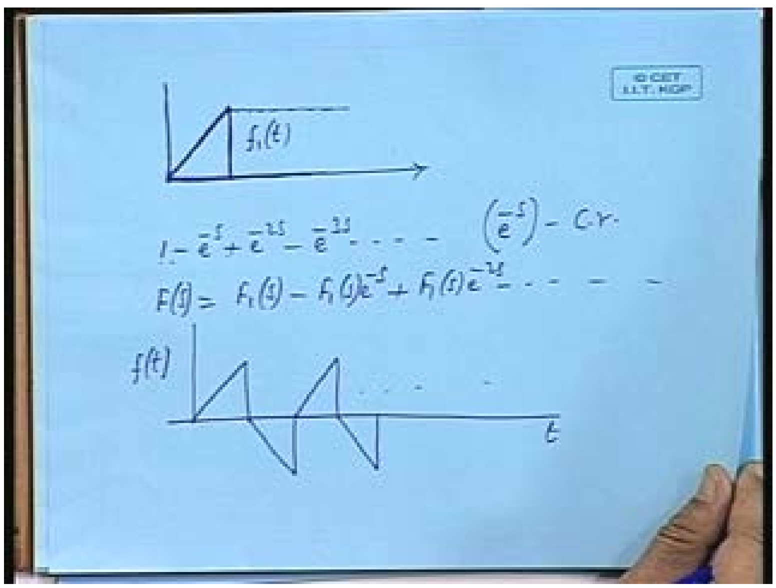

So $f_1(t)$ will be this one till it gets interrupted with the second part which is a negative step and again a negative ramp, the negative ramp will try to counter the increase. So it would have if I take this plus this it would have been a function like this but then there is negative step of the same magnitude see after 1 second this goes up to 10 volts, this goes up to a magnitude of 10. So then it would have remained constant if I apply just a negative ramp of slope 10, slope 10 but then there is a negative step also of minus 10, so it will be coming down to 0 and this will be the net value of $f_1(t)$ okay this is $f_1(t)$, this thick line.

Now let us see what is $f_1(s)$ finally giving me if you want to compute F(s) this $F_1(s)$ in what way it is being repeated 1 by 1 plus e to the power of minus s comes out of a series e to the power of minus s plus e to the power of minus 2s plus minus e to the power of minus 3s and so on, if you have alternately changing signs then only you get that is a common ratio of minus e to the power of minus s if this is the common ratio then the geometric series will give me a series like this that gives me $F_1(s)$ minus $F_1(s)$ into e to the power of minus s plus $F_1(s)$ e to the power of minus 2s and so on. Alternately $F_1(s)$ shift with a shift of 1 second, 2 second and so on, alternately plus and minus will give me F(s) that means correspondingly $f_1(t)$ if I take alternately plus and minus that will give me f(t).

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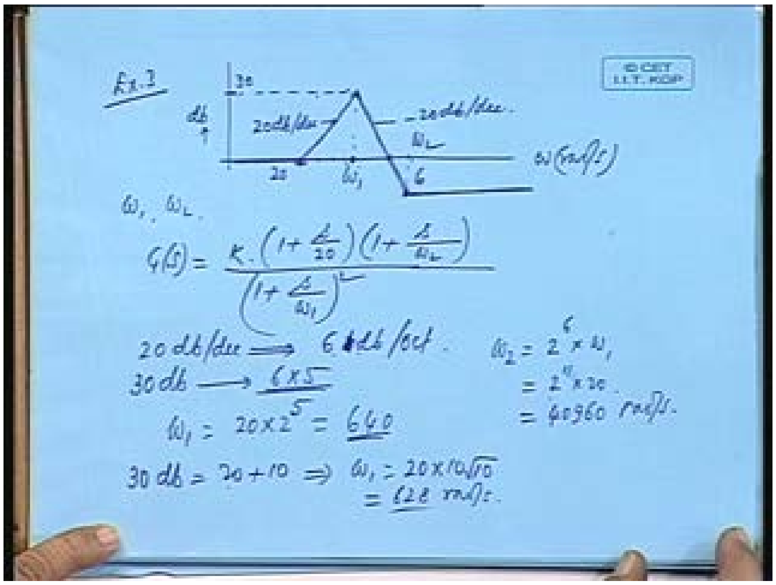

So f(t) will be this $f_1(t)$ then after 1 second it is negative, then positive, then negative and so on this will be the nature of f(t) okay. Next there is question there is a boarded plot where at 20 there is a break this is omega 2, this is omega 1, this is 6 db and this is 30 db, this is 20 db per decade and this is minus 20 db per decade. Calculate omega 1, omega 2 and the function G(s) and the function G(s), so what would be the boarded plot like G(s) what will be the form some constant times at omega equal to 20, it is going up so will be 1 plus s by 20 okay this is omega in radians per second, 20 radians per second.

So 1 plus s by 20 will be this then at omega equal to omega 1 there is a fall of 20 db because minus 20 db, so there is a fall of 40 db. So 1 by s plus omega 1 whole square the whole square term will give me a slope of 40 db per decade, is it all right and then at omega 2 again that is restored so 1 by 1 plus s by omega 2 this will be the sorry 1 plus s by omega 2 will be the other factor which will finally make it horizontal. Now what is a omega 1 and omega 2, now 20 db per decade is a slope, so 20 db per decade can be written as 6 db per octave that it is getting doubled every 6 db.

So 20, say 30 db means 5 into such 6 db's are there okay. So this can be written as 30 db, this 6 into 5 so there are 5 octaves from 20 which will take me to omega 1, so omega 1 will be 20 into

2 to the power 5, is it all right. So 20, 1 octave make it 40, next octave it is 80, next octave it is 160 and so on, so it will be 640 again. How much is this fall 30 and 36, 30 plus 6, 36 okay so this is an approximation because 6 db per octave is also an approximation of 20 db per decade, omega 2 should be 36 means 2 to the power 6 into omega 1 from omega 1 there should be 6 such octaves.

So that means 2 to the power 6 into omega 1 and 2 to the power 5 into omega 1 that is 20, so 2 to the power 11 into 20, so whatever be that value this should be the approximately 40960 radians per second okay. If one goes by the actual value then it is 30 db is 20 plus 10 db, so 20 db means 1 decade so that gives me in terms of frequency omega 1 is 20 into 10 and 10 db will correspond to root 10 okay. So this is approximately 628 radians per second instead of 640, we should get 628 and 2 to the power instead of 2 to the power 11 if you take this is 30 and 30, 60 db up to this and then 6 db.

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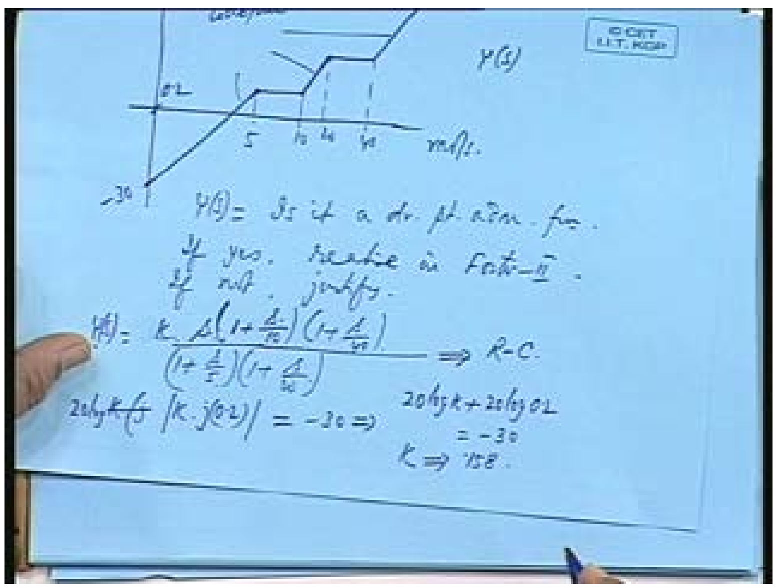

So if you take 66 db in that way it will come to about 40k, about 40k. Next we have another question the boarded plot of a network function Y(s) is shown like this for Y(s), some Y(s), this is minus 30 db these are all 20 db per decade slopes, this is 5, 10, 20, 40, this is 0.2 these are not to the scale exactly this is 0.2 minus 30, what will be the value of Y(s) and the next question is, is it defining a driving point impedance or admittance function, is it driving point admittance function if not justify, if yes state that is this and then realize 1 in faster 2 form. So if yes then realize in faster 2 form and if not justify, why it is not a driving point admittance function?

Let us write Y(s) the nature of Y(s) it will be k times there is a function that is going up so the factor corresponding to that is s then the 20 db per decade slope is made horizontal that means there is an addition of a term 1 plus s by 5 okay, this is in radians per second then 1 plus s by 5 numerator again to be 1 by s plus 10, 1 plus s by 20, 1 plus s by 40 okay. Before you go into the evaluation of k, let us see k is just a scaling factor is this an admittance function driving point admittance function yes because 0s and poles are coming alternately. If I simplify this it will be s plus 10, s plus 40, s plus 20, s plus 5.

So if poles and 0es are coming alternately and if the differentiate degree is restricted to 1 then it will be possible to realize this as RC or RL okay all the routes are in the negative on the negative real axis is it an RC or RL, it is an Y(s) function and s is coming in the numerator. So this is an RC that is route closes to the origin is a 0, if it is 0 for an admittance function it is an RC network. Now you can evaluate k very easily at .2 at a is equal to .2 when we are making a board assume to take plots and making calculations on that basis we neglect the effects of higher frequencies at such low frequency.

So at .2 basically these are all negligible terms, so these are all, this is all reduced to 1, so k into s equal to j .2k into j 0.2, 20 log of this 20 log of this magnitude is equal to minus 30. So that gives me 20 into log of k plus 20 log of .2, 20 log of .2 is equal to minus 30. so you can calculate k from here all right, so k turns out to be .158 approximately.

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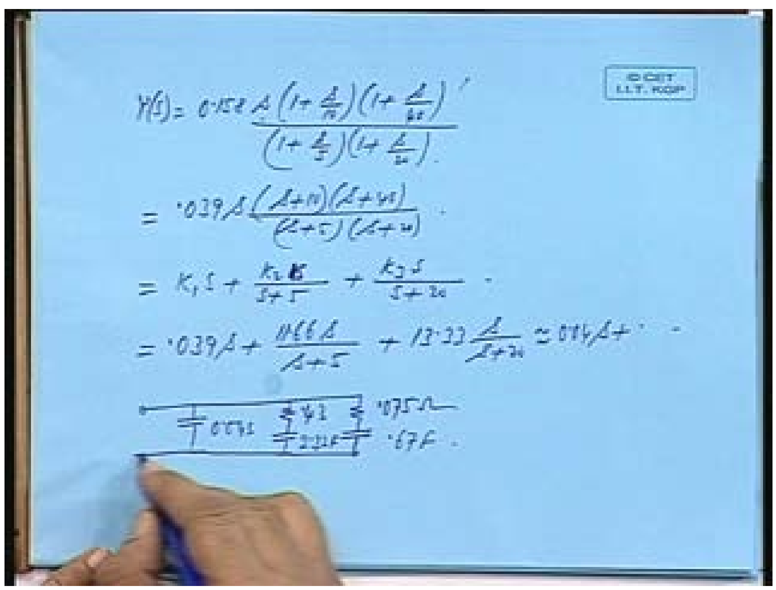

So Y(s) is .158 into s into 1 plus s by 10 into 1 plus s by 40 divided by 1 plus s by 5 into 1 plus s by 20, so that gives me if I multiply by these 10 and 40, 400, 20 and 500, so 4 times is 4 will come in the denominator so .039 s okay into s plus 10 into s plus 40 divided by s plus 5 into s plus 20, if I fast at 2 means it break up Y(s), so it is K_1 into S plus K_2 into S by S plus 5 plus K_3 into S, so db S okay K_2 into S by s plus 20, it is an admittance function. So that turns out to be .039s by the usual method of partial fractions 11.66s by s plus 5 plus 13.33 into s by s plus 20, correct me if I am wrong. So that gives me a capacitor of value .039 if you permit me I can write this is .04 an approximation plus 1.

So .04 s and then this is an RC, this is also an RC of corresponding values if we take s to be you just invert it and then put the values this comes out to be .43 that is 5 by 11 .66 and 1 by 11 .66, so that gives me 2.32 farads okay and then this one comes out to be .075 ohms and this one is .67 approximately farads, so this is the realization. Next example that we shall take is on a filter determine the values of L and C, for a low pass filter, low pass filter R_k is equal to 500 ohms and cut off frequency 60,000 radians per second okay. Here to calculate also alpha at 15 kilo hertz beta at 5 kilo hertz and thirdly if I have an amplitude filter the value of m for getting f infinity at 12 kilo hertz that is attenuation should go to infinity at this frequency. what should be the value of m? what should be the value of m? So the relationship is well known for a low pass filter, you know z_1, z_2 if you take the products.

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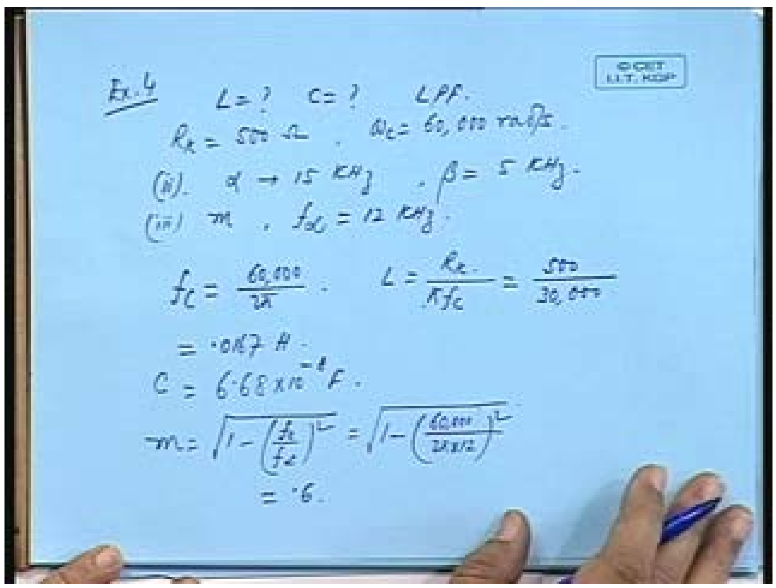

So f(c), f(c) is given as 60,000 divided by 2 phi okay L, how much is L? R_k by phi f(c) okay R_k is 500, R_k is 500 and phi into f(c), 2 phi f(c) is 60,000. So 30,000, correct me if I am wrong all right. So that gives me .0167 Henry, what about C? Similarly, C also you know the relationship for f(c), L and C, C comes out to be 6.68 into 6.66 or 67 are there 10 to the power of minus 8 farad, m in the usual way 1 minus how do you calculate m? What do you put for m? 1 minus f by f infinity squared okay 1 minus how much is f? or how much is f? sorry this is f(c), f(c) by f infinity f(c) is 60,000 divided by 2 phi and f infinity is 12 squared, so that turns out to be point6 approximately all right.

Now how alpha and beta alpha is 2 cos h inverse we have to calculate alpha for 15 kilo hertz, 15 kilo hertz all right mind you this is given in kilo hertz and this is given in radians per second. So divided by omega by omega c okay, so either you multiply this by 2 phi and 60,000 2 cos h inverse omega by omega c. So that gives me 2 cos h inverse 30 by 60, so phi by 2 mind you this is 15 kilo hertz, so into 10 to the power 3, so10 to the power 3 will get cancelled, so phi by 2.

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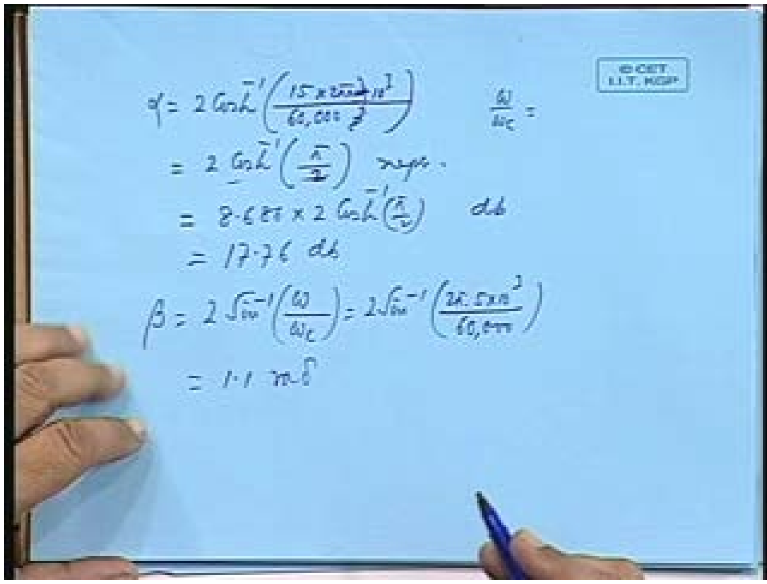

So that gives me a value of if I multiply by 8.68 this is in nepr 8.686 into this quantity that will be give you giving you the value in db that is 17.76 db this is approximately 2. Something 2.1 also. So approximately 17.76 db, beta 2 sin inverse omega by omega c, 2 sin inverse beta you have to calculate at 5 kilo hertz. So 2 phi into 5 into 10 to the power 3 divided by 60,000, so that gives me in radian 1.1 radian, this comes out to be 1. 1 radian all right. So these are the values of alpha and beta next we have question on a network graph, you are given a network this is example 5.

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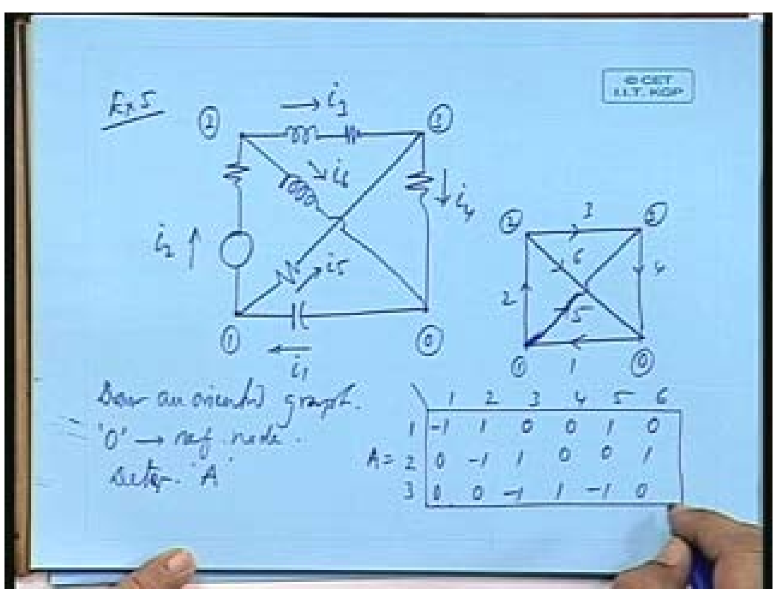

You are given a network like this, this is node number 0, 1, 2, 3, this is carrying a current I_1, I_2, I_3, I_4, I_5 and I_6, sorry, you are asked to calculate first of all draw an oriented graph, draw an oriented graph and considering 0 as the reference node. Determine matrix A okay considering 0 as the reference node determine A. So let us complete these parts first, so I will draw the network graph like this basically it represents it is not in the same plane, not coplanar 0, 1, 2, 3 and elements are numbered as the current element that this is as 1, this is as 2 and the direction is also the direction of the current which is 3, 4 this is easy to comprehend 5.

So this is a network graph then by choosing 0 as the reference you can very easily write matrix A as elements are 1, 2, 3, 4, 5, 6 and the nodes are 1, 2, 3, 0'th node is absent. So the bus will have the matrix A will have element 1 coming out from 0 entering at 1, so this will be minus 1, 0, 0, 0 element 2 is from 1 to 2, so plus 1 and minus 1 plus 1 minus 1, 0 okay then element number 3 is 2 to 3, 0, 1, minus 1, at 2 it is plus 1, at 3 minus 1 element 4, 3 to 0 so 0, 0, 1 at the 0'th node it is minus 1, so the 0'th node is not present then element number 5 is 1 to 3, so 1, 0, minus 1 element number 6 is 2 to 0, 0, 1, 0 okay and then the next question is for this network graph determine the total number of trees, total number of trees is equal to determinant of A, A transpose, if A, A transpose determinant of that is equal to the total number of trees, what will be the total number of trees for this particular example.

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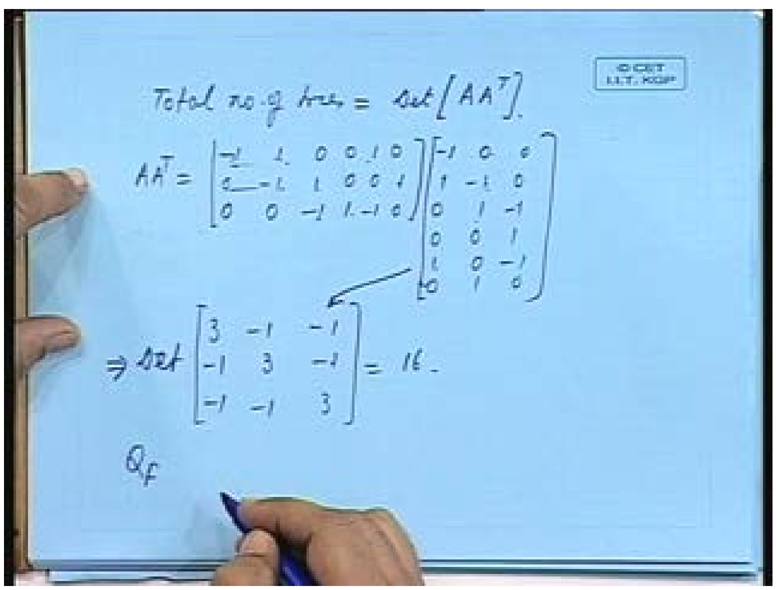

So it will be A, A transpose is minus 1, 1, 0, 0, 1, 0, 0, minus 1, 1, 0, 0, 1, 0, 0, minus 1, 1, minus 1, 0, minus 1, 0, 0, 1 minus 1, 0, 0, 1, minus 1, 0, 0, 1, 1, 0, minus 1, then 0, 1, 0. So that gives me minus 1 into minus 1, 1 into 1 and 1 into 1, 3 then minus 1 into 0, 1 into minus 1 and then 1 into 0, so minus 1. Similarly, this into this this means 0, 0, 0, 0, 1 and minus 1 okay and minus 1 this will also be 3 you can see this will be always a symmetric matrix.

So once you have this got these values you can club this similarly, if this is known then this will be known and I just completed just now, see there are only 3 elements, 3 non-zero elements when they are multiplied by the corresponding elements here they will be getting plus 1 plus 1 plus 1. So you have to count actually we have to count how many non-zero elements are there so so many plus 1s will be in the elements. So that gives me 3 into 9 minus 1, so 3 into 8 then minus 1 into minus 1 into minus 1 plus 1 into this, so that comes to 24 minus 4, so 16.

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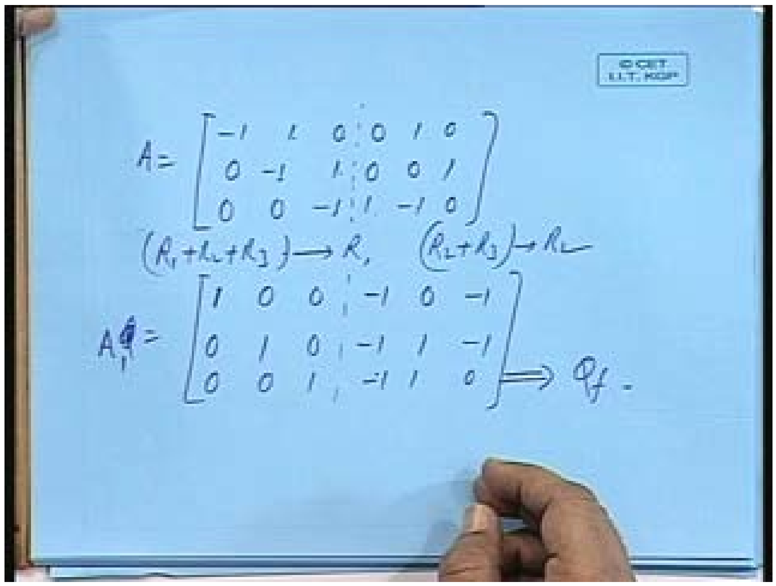

So total number of determinant this sorry, this into this, this matrix determinant of that is 16, so the total number of trees will be 16 okay next calculate QF fundamental cutres, cut set matrix converting A to a unity matrix all right, part of A by unitary transforms. So you have got A as minus 1, 1, 0, 0, 1, 0, 0, minus 1, 1, 0, 0, 1 and then 0, 0, minus 1, 1, minus 1,0. So very simple transformation I want Q, I want 1 here, so if I add this with this this with this that means row 1 plus row 2 plus row 3, I will call it some A_1, row 1 plus row 2 plus row 3 okay replacing row 1 and then negate the sign, so I get minus 1, 0, 0 and if I change the sign it become 0, 1, 0, 0 all right. This is the true element and then here it is 1, so that because minus 1, this becomes 0, this becomes 1, so minus 1 then this and this if I add this become 0 this becomes minus 1 and if I again change the sign so 1, 0 then this becomes minus 1 then this and this becomes plus 1 this becomes minus 1 and then this one I just change the sign I get 0, 0, 1 then minus 1, 1, 0 okay and if I find I get a unity matrix here, this itself is Q, Q_f.

So the transformation is very simple. So I have got from A directly Q_f by a simple manipulation one may question, one may question, whether this is representing a tree this part of A is representing a tree yes, that has to be verified because the diagram is very simple. So you take these are the nodes 1, 2, 3, 1, 2, 3 okay from A matrix if I take 1, 2, 3 elements 1, 2, 3 that forms a tree actually there are 16 possible trees, you take any 3 any 3 combination which will not make a loop that will be a tree because there are only 4 nodes, there are only 4 nodes, so 3 elements will require 4 nodes.

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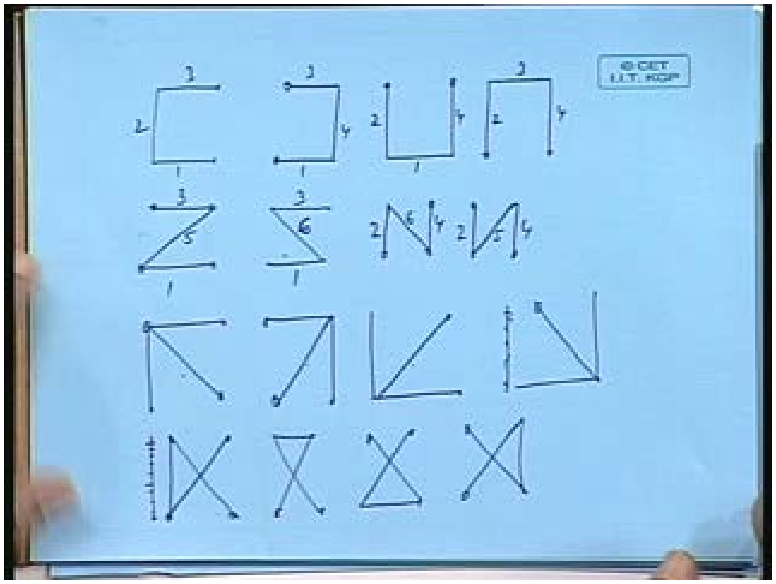

So you make any combination of 3 elements any combination of 3 elements you find this is an interesting figure any combination of 3 elements without forming a loop, without forming a triangle that will be a tree and there are 16 such possibilities. Could you draw the trees, all these possible trees. Let us see for this example what are the possible trees, we can have 1, 2, 3, this is 1, 2, 3 I will not number them then its counterpart is 1, 3, 4, it is another tree then 4, 2, 1, 2, 3, 4. So if one write 1, 2, 3, 1, 4, 3, 1, 2, 4, 2, 3, 4 okay then 1, 5, 3, 1, 6, 3 okay 1, 5, 3, 1, 6, 3, then 2, 6, 4, then 2, 5, 4 and so on.

You can make a few more could you suggest 1, 3, 5, 1, 3, 6 and so on and I leave it as an exercise all of you can try all other possible trees okay see the other possibilities 1 or 2 could some are suggest yes, this one, next this one, next this one, okay this one then is any other possibility 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 4 more are there, 4 more possibilities are there if I take this one and this one then I can have only this one, I cannot fit in the other one that will make it a triangle all right. If I take 1, 2 there are there seems to be 12 only, what is the other possibilities 1, 2 and 3, 1, 2 and 3, 1, 2 and 3, 1, 2 and 4.

So these are the 16 trees okay you cannot have anything more than this just 15 trees, 16 trees all right, it is better if you few keep on drawing for for any graph the possible number of trees and verify the relationship if the number is too big of course it is very difficult but we have to imagine what are different ways you can form a tree by connecting different nodes without forming a loop mind you there are chances of making errors here. There is no node here all right it is the lines are one over the other, they are not connected here.

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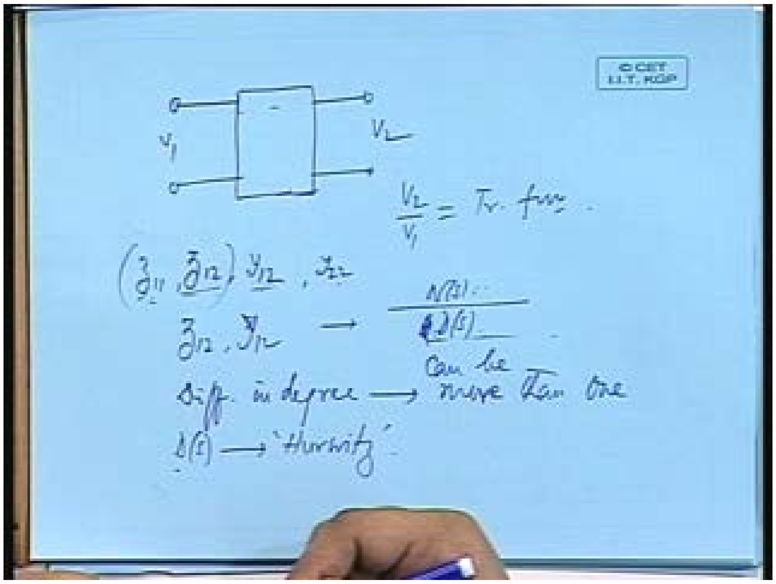

Next time you shall be taking up a 2 port synthesis that is if you are given network function in terms of in terms of say V_2 by V_1, V_2 by V_1 will be a transfer impedance or transfer function. I It may be V_2 by V_1 or V_2 by I_1 or I_1 by V_2, I_1 by I_2 and so on that means you may be given the specifications either in terms of Z_{12}, Y_{12}, Y_{22} and so on and Z_{11}. So specifications can be given in terms of Z_{11}, Z_{12} or Y_{12}, Y_{22} or V_2 by V_1 and so on. You have to determine a possible network.

So far we have studied the driving point synthesis, now we shall be going for 2 port synthesis with transfer impedances okay. Some of the basic conditions for these impedance and admittance functions that is a transfer impedances and transfer admittances a very interesting and so for transfer function also, we will find the numerator and the denominator polynomials N(S) by D(S) they need not have all the properties that were listed in the driving point synthesis that is the difference in degree can be more than 1, difference and degree can be more than 1, can be may or may not be but D(S) must be a Hurwitz polynomial routes must be always in the left up line.

So D(S) must be Hurwitz there can be multiple route, multiple routes, multiple 0s and N(S), there can be all possible combinations. So we shall study in details what are the conditions to be satisfied for 2 port synthesis what are the conditions for Y_{11}, Y_{12} or Z_{11}, Z_{12} this pair of admittance or impedance functions, thank you very much.

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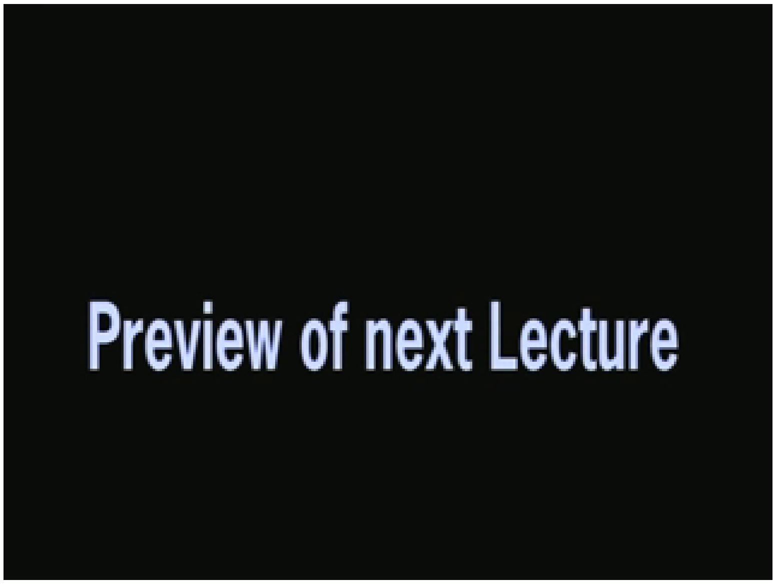

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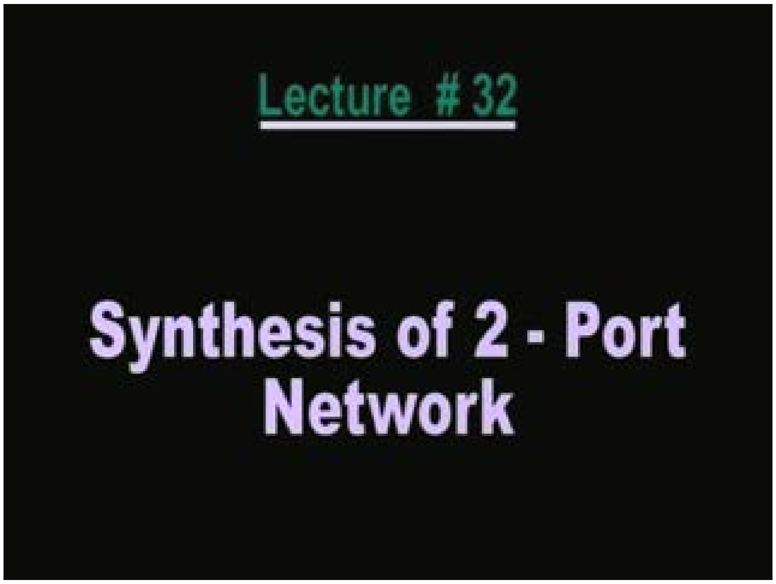

Good morning friends, today we will be discussing about 2 port network syntheses. You know in case of a 2 port you have various specifications now, for a 2 port network Y 11, Y 12, Y 22 these are the parameters or may be A, B, C, D or Z parameters and so on. These are the 3 commonly used sets sometimes you may be given only a gain function G 12 which you know is given by this or minus Y 12 by Y 22, so if this voltage by this voltage this is the gain function only the gain function is given then you are asked to realize a network to give that particular gain function all right.

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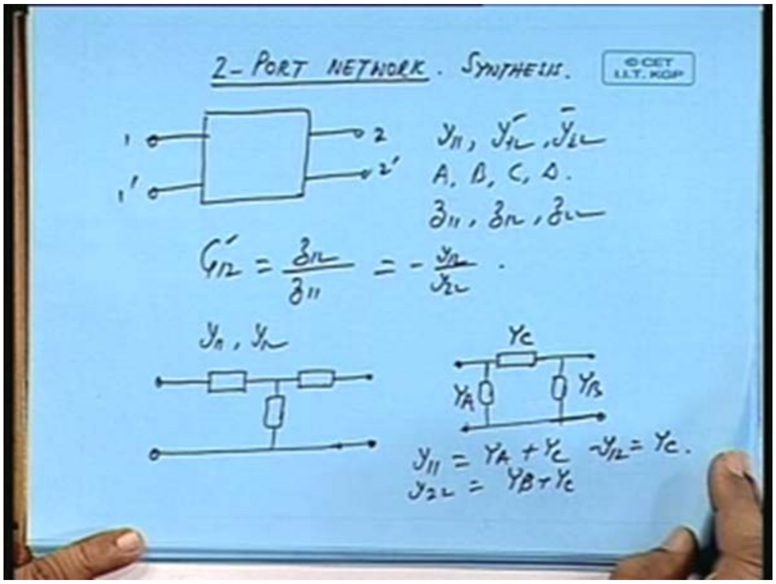

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So these specifications can be many of many types unlike 1 port network where we are given either Z or Y(s), Y(s) is just inverse of Z(s). So you have 1 specification here the specifications can be in terms of say Y 11 and Y 12 or Y 22 and Y 12, any 2 can be given or may be just the gain function all right. So how to realize a network to meet these specifications? So before we go into that we just see the network functions how they are related to this 2 port parameters network elements.

Let us take a t network okay or a phi network from a t, you can always find a phi, this is start star to delta or delta to star conversion all right. Normally if you are given the specifications in terms of Z parameters you go for an equivalent t this is easier. Similarly, for Y parameters you go for phi network suppose this is given as Y A, Y B and Y C. What do you mean by zeros of transmission, transmission zeros means what? I given a signal at the other side nothing is received, is it not? Now in how many ways that can be ensured, I give a signal you take a water pipeline alright. In water pipeline what are the possible ways say the delivery of water at the other end can be disrupted, there may be a leakage, very good there is a leakage. So water is grounded basically the water pipe is grounded, the entire pressure is grounded, is any other way of blocking, so if you have some blocking in the series path or the short circuit in the short path then you have the zeros of transmission that means how do you create the zeros of transmission by short circuiting at some frequency, suppose at some frequency I find s squared plus 9 in the numerator that means at s equal to J 3, this s square plus 9 will be vanishing that is a zero, zero at that particular frequency.

So at that frequency I want there should be no transmission of signal and I want short circuit at that particular frequency, no short circuit means what, it should be a series resonance, if I have a series resonance s equal to J 3 and that element is put in shunt okay so you are having an element like this, there are many such elements in the ladder I create, this element is Lc combination, thank you very much that is why this scaling was very very important in case you can weaken this to shift it to either 1 or 2 okay, thank you. So in that case what will be this one Z 11 double dashed at J 2 should be equal to 0, this is a 0 which gives me yes, sorry we are weakening the whole at the origin that is what I am putting J 2, at J, this 11 dash at J, I am going to write that, that is what I am going to do, this will be Z 11 double dash, okay let us write it here, okay we will continue this in next class, the time is over, we will continue this in next class. Thank you.