Networks, Signals and Systems

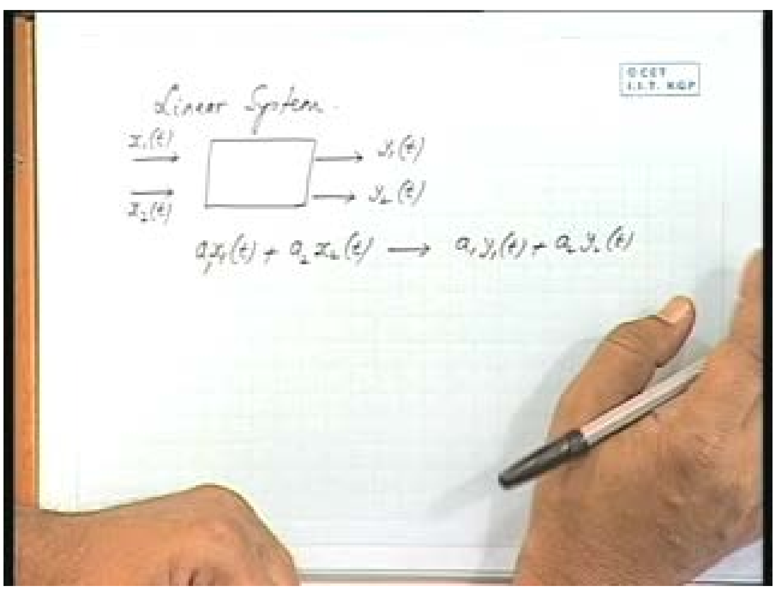
Prof. T. K. Basu
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 02

Introduction to Linearity and Non-linearity of Systems - Dot Convention

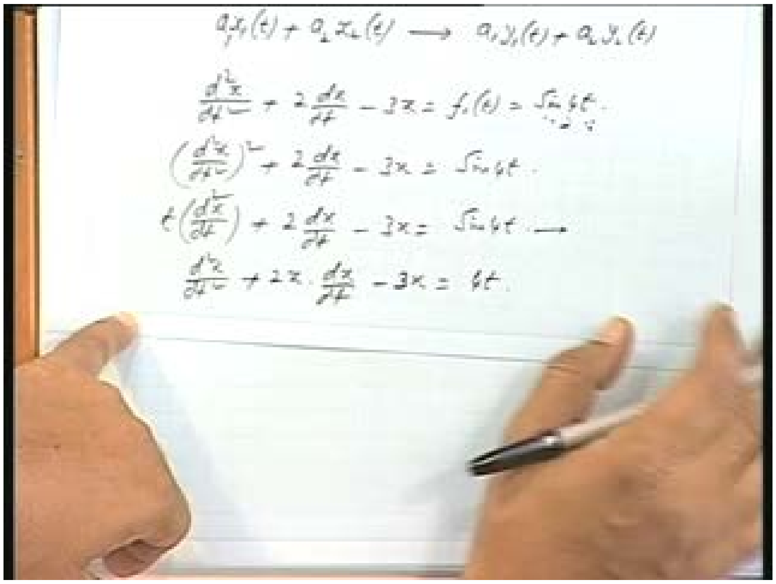
Okay, good morning friends will continue with our earlier discussions on linearity of systems. As we had mentioned a linear system will have the property like this mathematically we can write the property like this if we excite a system by an input x 1(t), it can be either a voltage or current and the corresponding output is Y 1(t). Similarly, if we excited by an input x 2(t) and the corresponding output is y 2(t) then any combination of x 1 and x 2, so a 1 x 1 plus a 2 x 2(t) will result in to a 1 y 1(t) plus a 2 y 2(t), if the system is linear. This is the principle of super position.

(Refer Slide Time: 01:18)



Now, sometimes you are given the system equation in the form of differential equations, how do you determine whether a differential equation is a linear differential equation or not. Suppose you are given I will give you a simple example here, d square x by dt square plus 2 into dx by dt minus 3 in to x is equal to some force in function f 1(t) may be sin t, sin 4 t, another equation d square x by dt square whole square plus 2 into dx by dt minus 3 x is equal to sin 4 t. The third equation is dx by dt, sorry d square x by dt in to t plus 2 in to dx by dt minus 3 x is equal to sin 4 t, d square x by dt square plus 2 in to x in to dx by dt minus 3 x is equal to 4 t.

(Refer Slide Time: 06:09)



Now out of these which one will be linear, which one will be non-linear, which one will be replacing the non-linear system yes, first one let us examine one by one first one is it a linear differential equation, linear differential equation, the right hand side is sin 4 t, is it a equation, is it a linear function, no this is a forcing function, what about this side, they are all linear elements, they are is it a differential equation, it is the forcing function whether it is sin 4 t log t root t does not determine the behavior of the system, forcing function is external to the system, it is this input is external to the system. So the right hand side the forcing function here can be anything.

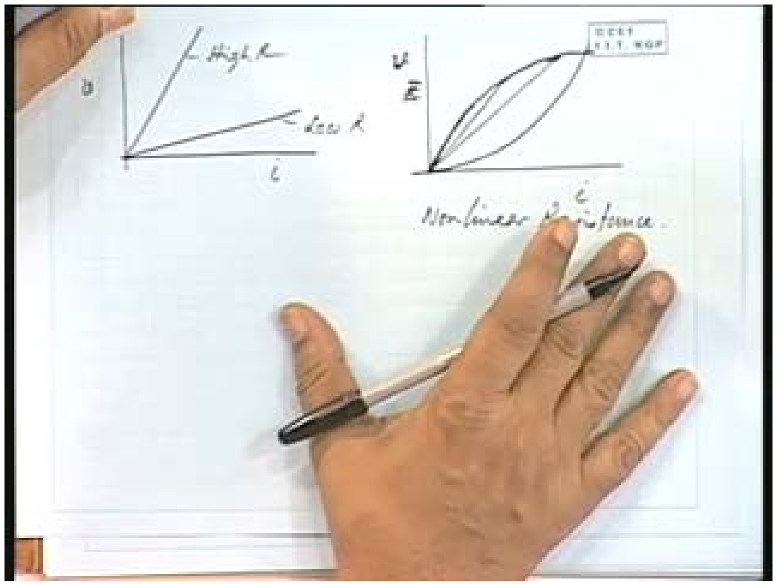
Now this side all these elements are linear when we say linear that means for a variable, the dependent variable x or its derivatives they all appear in a linear form okay. Here d square x by dt square is squared. So it is in a non-linear form similarly in this equation the 4 th one it is 2 x in to dx by dt, so dx by dt gets multiplied by x all right whether it is x in to dx by dt or x squared or d square x by dt whole square okay like this second equation these are all non-linear equation, non-linear form of x and its derivative, what about this one the third equation here it is multiplied by the independent variable, so it is a linear time varying system, it is linear time varying system but it is not non-linear okay. So you must be in a position to identify a linear system from a non-linear system, this also linear system but it is having a time varying coefficient, a very nice example of a linear time varying system will be say a furnish a furnish, you have the breek lining okay if you start an experiment now with so much of input of fuels.

So much of charge kept in this identical charge, if you perform the experiment say today and tomorrow, there is not much of a change in the furnish behavior furnish characteristics. So if you want to write say the heat flow equation if find the equations are identical but after 8 or 10 months, say what the use of the furnish the breek lining, the fair breek lining will gradually get damaged all right there will be aging. So the tower property of the breek lining will be changing periodically you have to go for maintenance you have to change the breek lining and after 10 months also if you perform the same experiment will find the heat flow equation will be different because the resistance the thermal resistance has changed over time.

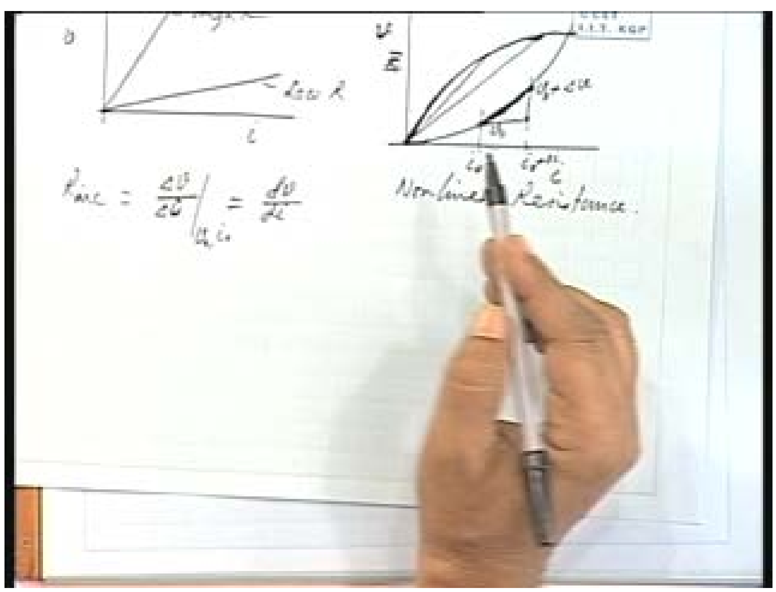
So it is an example of time varying coefficients the system differential equations may be identical but with time varying coefficients so sorry whenever the the elements like RLC etcetera are suffering a time varying change like aging you have time varying differential equations of this type even though this system is linear, it is not non-linear but there can be coefficients which may change. Now a resistor, pure resistance it is linear timing variant system, it is an over simplification all right people had conducted some simple experiments over a certain range of temperature, the resistance virtual elements constant.

So we simplify the model by a simple linear relationship V is equal to R into I but actual resistance may be varying in a non-linear fashion all right as the current increases the resistances may also change, there are many non-linear resistances in day today use for example, a bulb, the filament of a bulb, if you change the voltage current also changes, if you take the voltage by current ratio it is not fixed, it changes. For example, this resistance characteristics the ideal linear resistances say, this is a low value of resistance if the characteristics is like this, is the high value of resistance and you may have sometimes okay I draw on this side if you have performed any experiment on the bulb, filament bulb, incandescent bulbs depending on the type of filament, it can have a characteristic like this or this here the resistance sorry, this is the voltage say, the slope represents the resistance value.

(Refer Slide Time: 09:13)



(Refer Slide Time: 10:49)



So resistance keeps on falling in this case okay voltage by current ratio does not remain same, in the other case it may increase okay so this is the example of a non-linear resistor. Best example is incandescent lamp. So we approximate them to be linear for small values of current or will linearise around an operating point, say if this is an operating point, surround this point, it is the slope is taken as the resistance or we define as incremental resistance suppose this is v plus delta V, this is V and this is I, this is I naught plus delta I, this is say V naught and I naught, V naught and I naught, V naught plus delta V, I naught plus delta I.

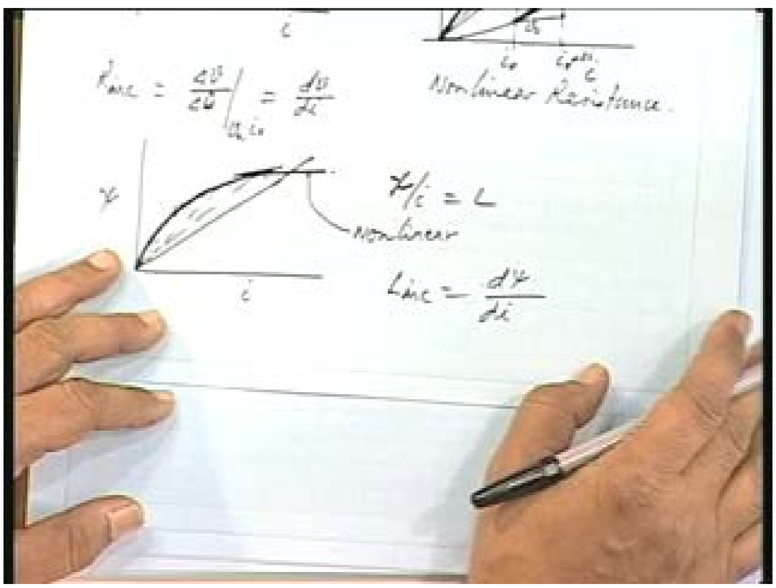
So the slope approximately is delta V by delta I at the working value V naught I naught. So this is dv by di, so the resistance incremental resistance is basically the slope of this vi characteristics at an operating point. Similarly, for an inductance how do you define an inductance by the way how do you define an inductance, inductance is it can be defined in 2 ways time, student: time derivative of the current, time derivative of current and the corresponding voltage induced a question that I very often ask and I would ask you also does the straight conductor carrying a DC have an inductance, does a coil carrying a DC have an inductance, does a straight conductor carrying an AC have an inductance yes, can you tell me a straight conductor carrying a DC will it have an inductance.

The first thing that comes to your mind inductance means Ldr by dt will be the voltage you were used to think like that. So the voltage since it is carrying DC. So the voltage induced will be 0 because there is no change in current, so I give an example you have a mass you apply a force there is an acceleration, so what is the mass yes, mass is force by acceleration. Now, you do not apply a force, so there is no acceleration does it mean the mass is not there mass is there it is irrespective with the force applied. So inductance is the property by which the conductor tries to establish the flux lines whenever there is a current flowing through it, so whenever you are have a current there will be a flux established by the very property of inductance.

So it is the flux linkage flux number of flux lines if you can count them from the conductor body to infinity so number of flux lines divided by the current will be the inductance. Now obviously it is not so easily measurable, I mean it is difficult to measure this but if by sum means you can measure the flux lines established then that will be giving you the inductance. So it is a material whether you are passing at DC or an AC, only advantage is for AC is this that whenever there is an AC you get a voltage induced which will be proportional to L dr by dt and you can compute the inductance. So it is only for our computational convenience that we try to measure the inductance with the help of AC but when there is a DC inductance is there because flux lines are established.

So any state conductor because current is always associated in the magnitude field, so any state conductor the moment you pass a current will always have flux linkage and hence there will be an inductance however small it may be, so for all practical purposes a straight conductor does not have a perceptible inductance especially when we deal with a very low frequencies, when we go to very very high induct, very very high frequency then the inductance will give rise to quieter substantial voltage drop if you pass a current of a very high frequency.

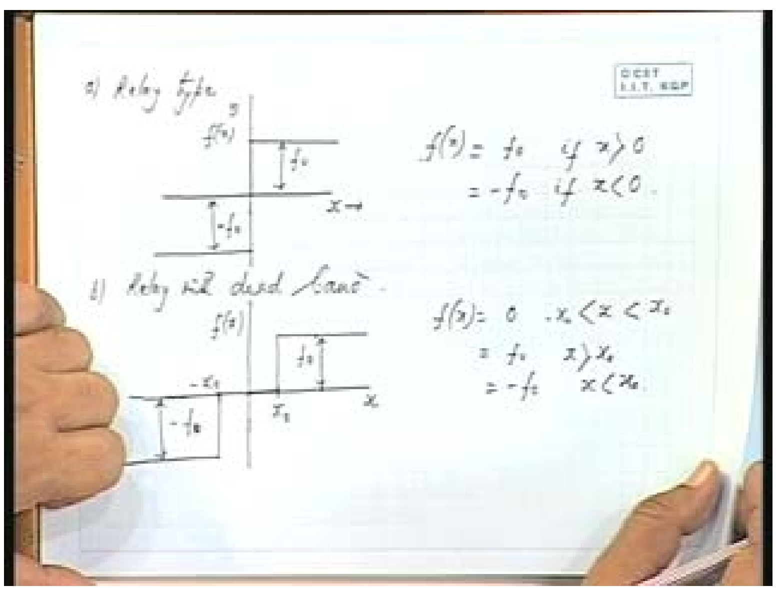
(Refer Slide Time: 16:16)



So an inductance will be present whether it is a straight conductor or a coil whether it is carrying AC or DC okay for example, when you go to the market to buy a an inductance, you do not say I will be using it for AC please give me an inductance to be used for an AC, you do not specify this is 2 milli Henry, 3 milli Henry, you specify only the value in Henry, I want so much current rating and so much inductance value, a coil. So an inductance will be given by this ratio, if it is a linear one that is flux linkage per unit ampere. Now it is not necessary it is not necessary that it will be always a characteristic like this there can be inductances were the variation of the flux linkage may be like this, can you say an example this is a non-linear characteristics that means once again here also the flux linkage per unit ampere keeps on changing with the value of I, with the value of I.

So can you say an example for example, the flux linkage in a coil where you have a ferromagnetic core which gets saturated after sometime okay, you do not get an additional flux per unit change of current at the same rate after sometime okay. So here the inductance value keeps on changing once again the slope as you have seen earlier dv by di, so it will be incremental inductance which will be d psi by di okay. So flux linkage and the current instead of pure ratio we have that the derivative d psi by di okay.

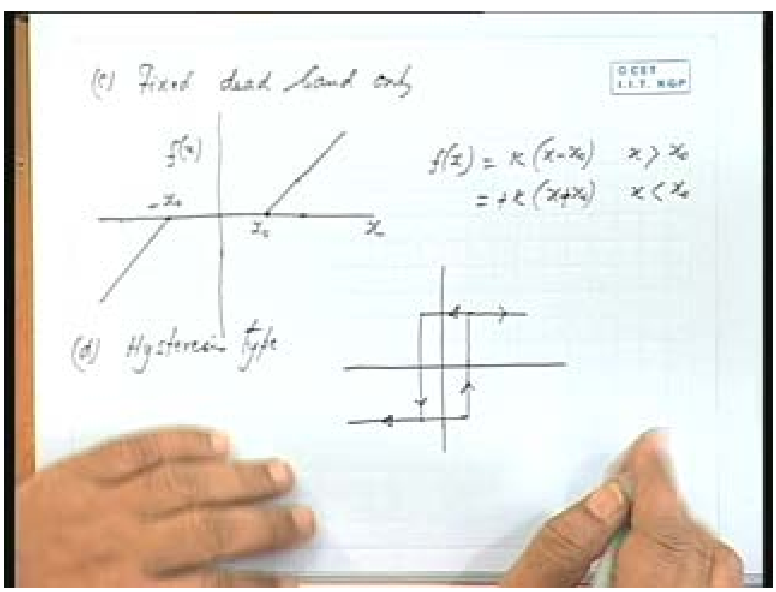
(Refer Slide Time: 19:05)



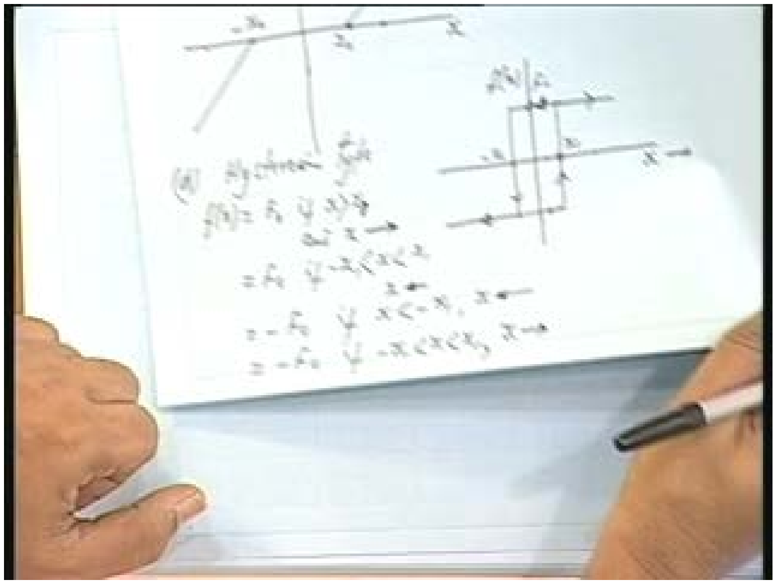
So that will be the incremental value of the inductance that is at any operating point what is the slope, capacitances normally are linear, normally capacitances are linear, it can be imperfect capacitances then of course have different frequencies we will have, we have to have a proper representation of the leaking resistance otherwise, capacity element is linear, there can be different types of non-relatives that may be present in a physical system. So before we go to networks in general, we will be discussing about mathematical as well as physical non-relatives that are present in most of the physical systems, what are the most common type of non-relatives, a relay type, relay type of non-relative, relay type means where the input and the output variables are relative like this that is y or fx if they write fx is output is equal to f naught, this is f naught and this is minus f naught, if x is greater than 0 and is equal to minus f naught, if x is less than 0.

This is a relay type, there is another non-relative it is a relay with dead band relay with dead band that is there is a dead band after which only the system response. So you have x naught and minus x naught as threshold so below this threshold value of x there is no response it is 0. So fx will be equal to 0 okay if x is between x naught and minus x naught is equal to f naught this is again f naught, this is minus f naught when x is greater than x naught equal to minus f naught when x is less than x naught okay. So this is relay with dead band then you have fixed dead band only fixed dead band only, here beyond this point x naught a relay output is proportional to the input, the effective input is this much x minus x naught.

(Refer Slide Time: 21:50)



(Refer Slide Time: 28:05)



So this is giving you the output accordingly so this is fx similarly on this side, so fx equal to some k times x minus x naught when x is greater than x naught equal to minus k times x minus, x naught correct me if I am wrong if it is x minus x naught let be all right, it should be x plus x naught it is minus x naught mind you.

Student: slope is positive,

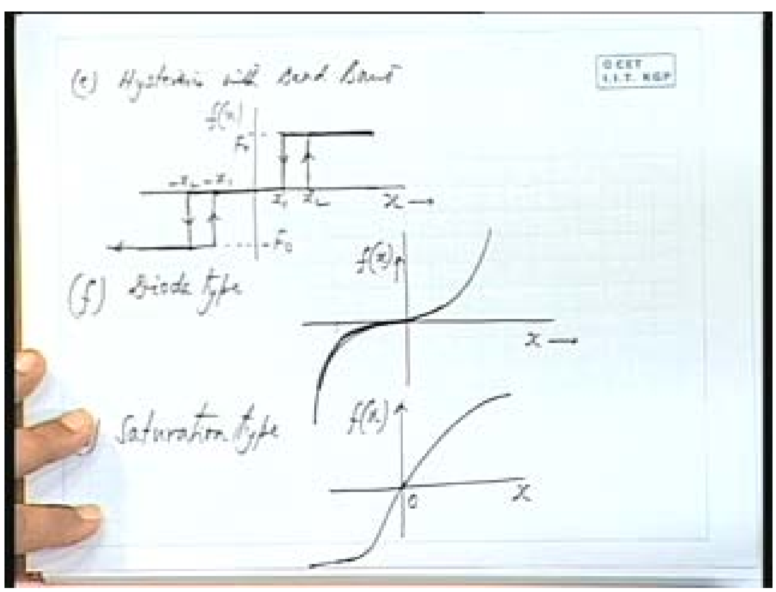
Slope is negative or positive, slope should be positive okay then x minus x naught is all right, is that all right when x is then there is no difference in there is no difference.

Student: but differing the value of x,

X naught was there now it is minus x naught, so I should replace it by plus know is that all right. So when x naught is in this position there will be an output okay next you have fixed dead band this one and then you can have dead band okay will take that later on let us see its hysteresis, hysteresis type of non-relative and ideal hysteresis is like this. So hysteresis, hysteresis type of non-relative is like this say this is x 1 minus 1 and this is fx, so fx I will write on this side fx equal to f naught this is f naught if x is greater than x naught and x is going these way okay, x is greater than x 1 x is greater than x 1 okay because we are using x naught, earlier. Now we are writing x 1 if x is greater than x 1 and x is increasing I will show this way this is the direction okay for x positive and then equal to f naught if x is lying between minus x 1 and plus x 1 okay, is that all right and x is moving in this direction, why is this direction so important, it is obvious at this value of x it is double valued it can have either this value or this value when is it having this value, when x is residing, x is going this way all right and when will it be in this position when x will be increasing. So the direction, it is a direction sensitive value it is a double valued function it can take either this value or that value, so that will be specified by the direction of variation of x okay.

So, similarly equal to minus f naught minus f naught if x is less than minus x 1 and x is going in this direction equal to minus f naught if minus x 1, x, x 1 when it is in this range but x is increasing and x is going in this direction all right. So these are some of the types of non-relatives that we have discussed you can also defined another non-relative hysteresis with dead band, hysteresis with dead band.

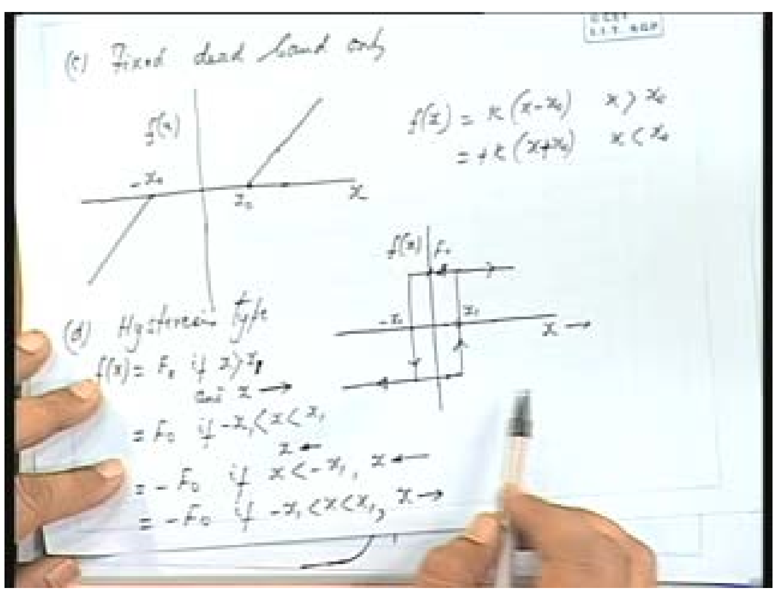
(Refer Slide Time: 28:11)



So I show you only the nature of variation I leave it to you as an exercise to define the function mathematically. So it follows a path like this as x varies as x varies it takes this path and then it gets saturated at f naught, when you are retracing back, when you are reducing x it follows this path and then it should have been yes, along this. So if I mark it as x 1 and x 2 if they are identical then this will be minus x 1 and minus x 2, these are the threshold values okay. So when again you are increasing x, it remains at f naught minus f naught then follows this path then from here, it goes to x 2.

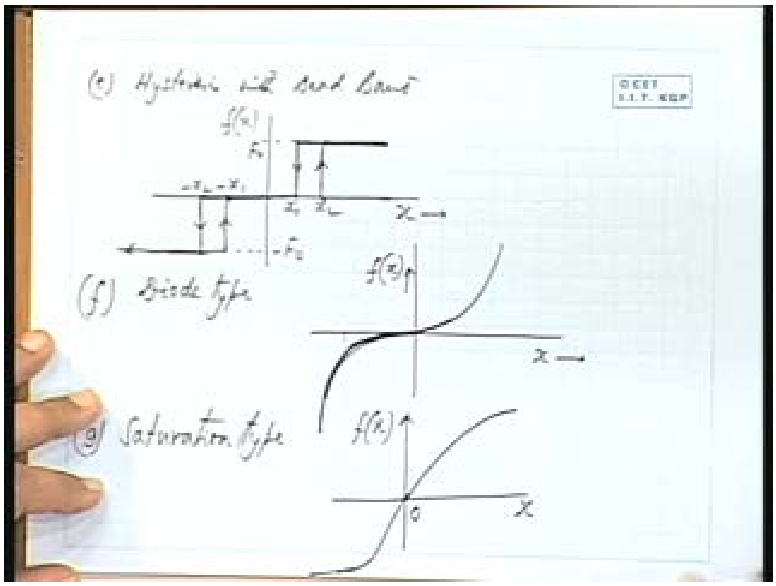
So from minus x 1 to x 2 it remains 0 in the return path from plus x 1 to minus x 2 it remains 0, okay. So you please try to define fx in terms of different ranges okay then there can be another type of non-relative diode type, where you define the diode characteristics like this, this side the break down is at a very high value whereas here it starts conducting. So this is diode type of non-relative, it is not to the scale as such and then you have saturation type of non-relative like this, they may be symmetrical.

(Refer Slide Time: 31:50)

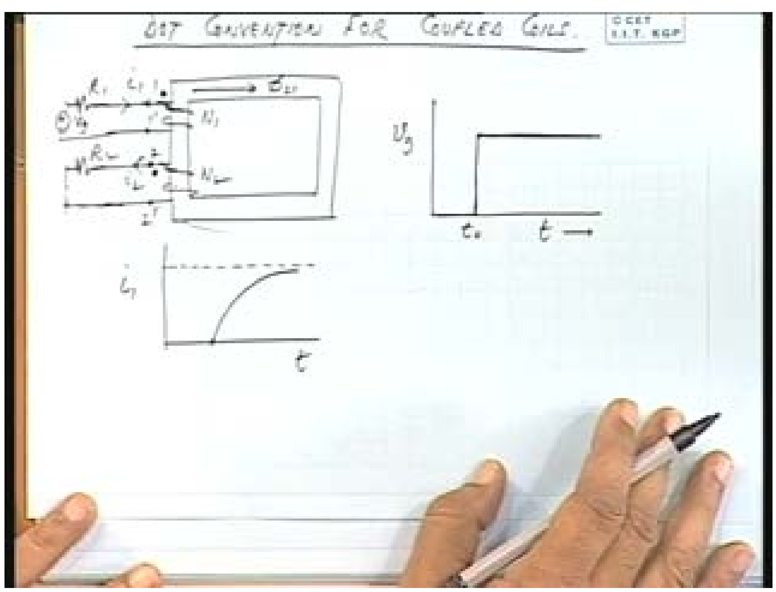


In all these non-linear functions that we have discussed, it is not necessary, it is not necessary for a physical system to have a symmetry in the non-relative, sometimes this side x naught, this side can be x naught dashed it can be something else, so there can be asymmetry in their should properties in their characteristics. Next, we come to dot convention of couple coils, what should be the dot convention for coupled coils, 2 coils are coupled.

(Refer Slide Time: 32:10)



(Refer Slide Time: 32:15)



Let us assume para magnetic core is used and 2 coils are around over it, to simplify the matter, let us see 2 coils in the same sense we have got 2 coils like this having number of turns N 1 and N 2 okay, there is a resistance connected here, R 1 resistance, R 2 and there is a current I 1 and I 2. The terminals are noted, we mark them as 1, 1 dashed and 2, 2 dashed okay. Now suppose from a source Vg, we try to send a current I 1 through this coil N 1, it should have been like this any way, I was thinking of putting another turn then anyway.

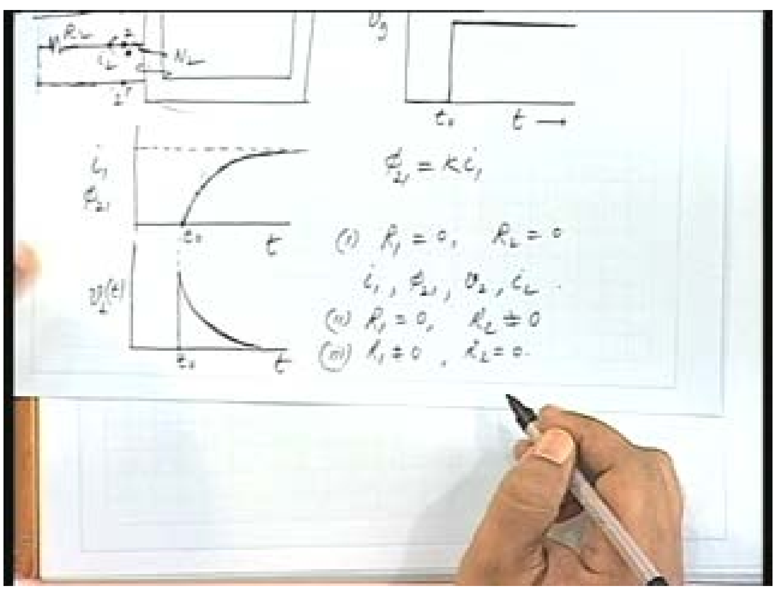
So current is going like this and suppose that establishes of flux phi 21 in this direction at some point of time then what should be the positive sense of the current if I short this, if I short this, if there is a current that is establishing of flux phi 21 which is established by I 1, then this will try to this coil if it is shorted, it will try to send a current in such way has to oppose this flux, is it not it is by law.

So any shorted coil will try to circulate a current so as to oppose the very establishment of the flux which is to oppose the cause therefore I will turn this as a positive current, when this is positive in coil 1 okay. So if coil 1, this terminal is noted with a dot that means current is entering from this terminal one living through one dash living through 1 dash then in the secondary coil current is flowing from 2 and going to the external circuit from 2 to 2 dashed, is that clear that means when a coil is shown 1,1 dashed then I put a dot here that means if I excite this coil, if I excite this coil 1 will be trying to send a flux phi 21 and correspondingly a shorted coil will try to send a current from 2 to 2 dash, if this is also shown by a dot okay. Alternatively, if I excite this coil by sending a current from here and if I remove the source, if I short circuit if I short circuit this coil externally then a current through this will try to force in an induced current which is being going from 1 to 1 dashed in the external circuit, if I inject a current from 2 inside the coil 2 to 2 dash then current from 1 will go to 1 dashed through the external circuit if this is shorted, okay.

So this is the convention that we follow for the coils. Now I pour a few questions before you and see what would be, what would be the nature of variation of different quantities. Suppose I have R 1 and R 2 some fix resistances, what would be the variation of the currents flux voltages induced and so on. We have applied Vg okay and time t, we have just switched it on at time t equal to t 0. So it is a step input of voltage that means we are giving a dc supply here, we are just switching on the supply dc supply across this coil, it is having some finite resistances okay, what would be the nature of addition of the current I 1, I 1 will be how much, suppose this is kept open, this coil is not their then at t equal to 0 will be go on like this just like an any inductor RL circuit okay, this you have studied in first year class, “principles of electrical engineering” you have studied simple RL and RC transitions.

So it is bearing like this is what will be the nature of variation of the flux phi 21, it is also identical it is proportional to the current I 1. So it will be like this, what will be the open circuit voltage if I keep it open, what will be the open circuit voltage, voltage across this secondary coil N 1 by N 2 voltage across this coil, what should be its nature N 1 by N 2 etcetera, I understand that will be only giving you a ratio but what about the voltage, how will it change, this is t 0, so before t 0 nothing happens after that as soon as you switch it on the current keeps on increasing.

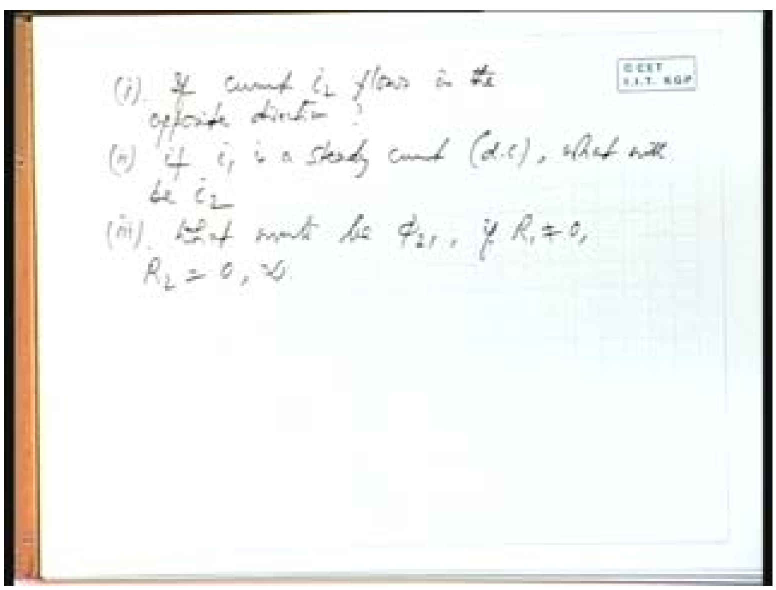
(Refer Slide Time: 40:00)



So flux is also increasing like this, so what is the voltage induced, it will be N 2 times D phi 21 by dt. So it will be depending on the slope, is it not, so how is a slope changing. Initially the slope is this much and gradually it is becoming 0 so if you apply dc, dc supply on to a transform primary and you measure the secondary voltage and finally it will come down to 0, 0 it will start with a maximum value and then it will gradually come down to 0 because the sloping here will be 0 is that all right. So this is v 2 with respect to time.

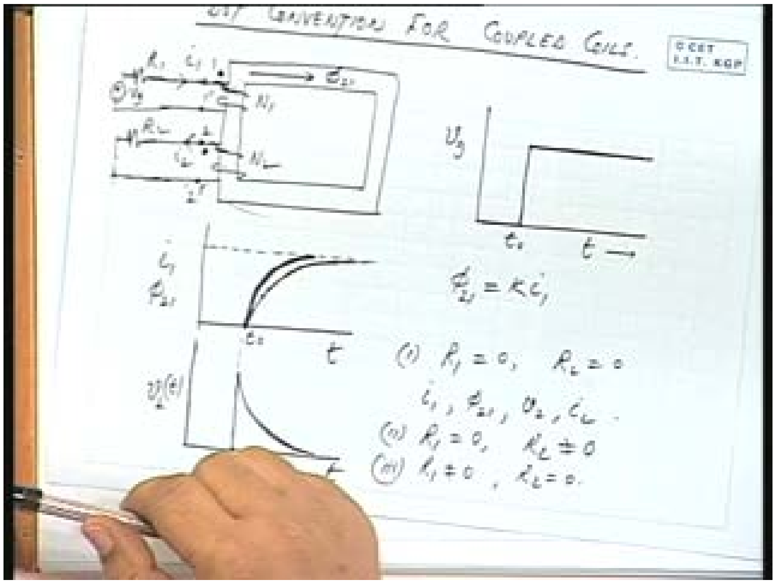
Now I pose few questions if R 1 is 0 sketch the 3 quantities I 1 phi 21, V 2 and also I 2 if I short it. So R 1 is 0 and R 2 is 0 that is you have to compute or you have to sketch I 1 phi 21, V 2 and I 2. Next case when R1 is 0 but R 2 is not equal to 0, R 2 is finite and thirdly, when R 1 is not equal to 0 but R 2 is 0 okay, another set of interesting questions I want to pose before you hence what if the current I 2 would flow in the opposite direction if the current flows in the opposite direction what happens, why can it not flow in the opposite direction. Next, what would be the value of I 2 in case I 1 is a dc that is the study value, if I 1 is a study value what will be I 2 and so the questions are if current I 2 flows in the opposite direction, can it ever flow in the opposite direction.

(Refer Slide Time: 43:40)



Next if I 1 is a study dc study current that is dc, what should be I 2 and what would be the flux phi 21 when R 30 and R 2 is infinity and R 1 is not equal to 0 what would be phi 21 if R 1 is not equal to 0 and R 2 equal to 0, next infinity, infinity means open circuited, what would be the values okay, one or two will discuss here because they were relevant sometimes by mistake he may put a dc supply on to a transform all right, transform is open circuited.

(Refer Slide Time: 45:18)



Suppose the transform is open circuited, so what happens to the transformer or the transformer you are applying you say, you are going for a short circuit test transformer secondary terminals had been shorted and you are supposed to apply a very small ac voltage on the primary instead of that you apply a dc, what happens. So this is the situation when, say when this side the secondary side is open, if you apply a dc, if say simple RL circuit.

So after sometime the current will be coming to a study value like this, so this current is decided totally by the resistance and the voltage supply. So voltage by resistance will be the current if you apply a very large amount of a large value of voltage, say 220 volts dc and the current resistance value small then it will be damaging the winding, it will burn out. So the voltage applied besides the level of, the level of the danger that is the chances of the winding getting damaged with very much controlled only by the resistance value which is normally very low and what happens to the flux, flux also goes on increasing like this. So just now we have studied, we have discussed the voltage of the secondary side will be varying like this.

Now suppose this is shorted the resistance is not there or even if there is a resistance, if there is a resistance here that will circulate a current that will try to circulate a current and that current will be trying to oppose the very cause, so it will try to oppose the establishing, establishment of the flux, sorry. So what happens to the primary, what happens to this side current I 1 will it be more or less, will it be more or less. If I short it through a resistance, why a phi 21 it will try to reestablish this flux, why should it be reestablished?

Students: try to increase that the i2 try to,

it will i2 will try to oppose the flux,

Students: try to reduce the flux,

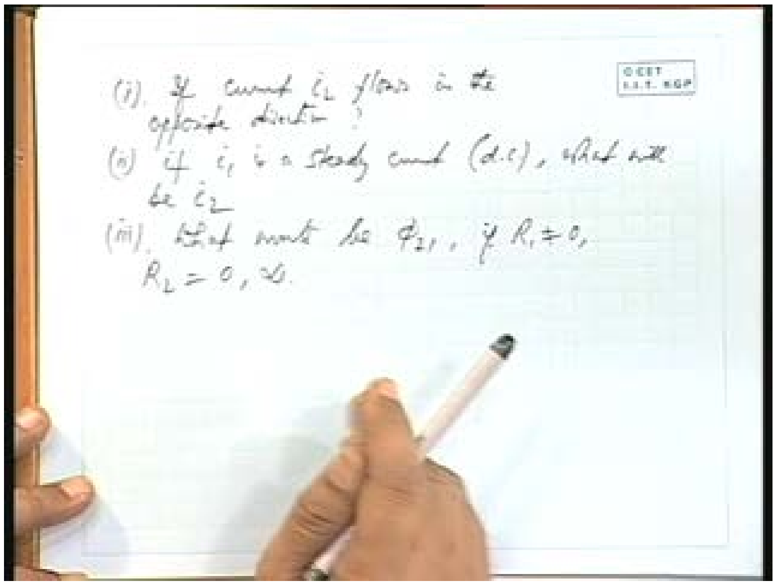
it will try to oppose,

Students: the flux it try to re-establish,

So we will that modify this characteristics like this this is what you anticipate that means it will be going like this is it all right you think of this situation need not transformer equivalent circuit if you have studied already. The equivalent circuit that is on the load side now you are having resistance R 2, earlier it was kept open, now you are putting a resistance R 2 that means in the equivalent circuit you are trying to provide a path already path through that secondary coil. So the total current increases earlier it was only the magnetizing current, now you are offering another path. So the total current will be increase okay.

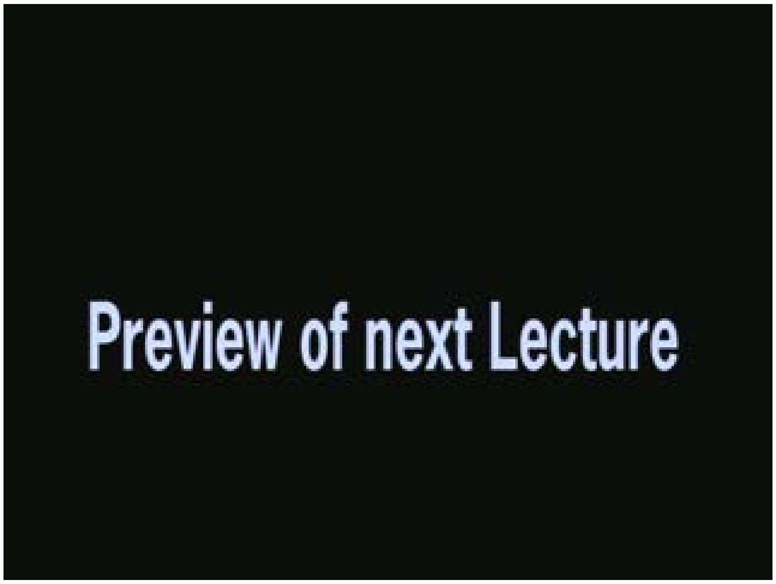
So the current I 2, I 1 will be having a higher slope like this. So it will be reaching that value very fast okay, if you have a resistance here instead of shorting it directly, if you have a resistance here were of considerable value. So it will be somewhere in between if R 1 is 0 and R 2 is 0, if R 1 is 0 and R 2 is 0, what happens? There is across an inductance across an inductance if I switch on a supply I said dc voltage, if I apply across an inductance, pure inductance there is no resistance what happens to the current, it is L di by dt which is constant V. So if it is L di by dt L di by dt which is constant that means I will be constantly increasing, it is integral v dt okay it is proportional to integral v dt.

(Refer Slide Time: 51:08)



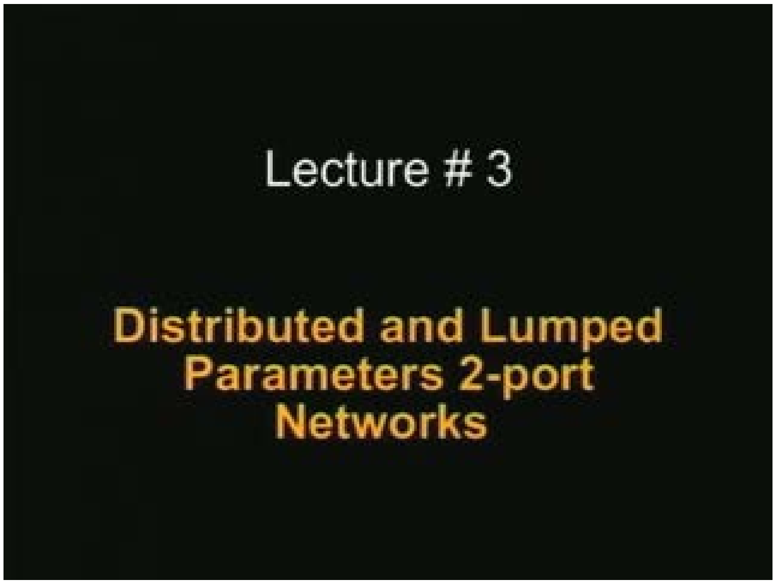
So I will be constantly increasing that means you will get a current that will be continuously increasing and that will cause a secondary current also in the reverse direction that will also be constant, constantly increasing. So both primary and secondary currents will be increasing continuously okay. So this is this a very interesting situation for example, sometimes when you apply a sudden change in the voltages and in pulse a pulse voltage, a pulse transformer, you get sudden changes and then though the voltage is there, there is no change, all right there is no change so on the secondary side there is no output whenever there is a change there is an output of the secondary side. So this principle is used in a first transformer, okay thank you very much, we will continue with this in the next class.

(Refer Slide Time: 51:47)

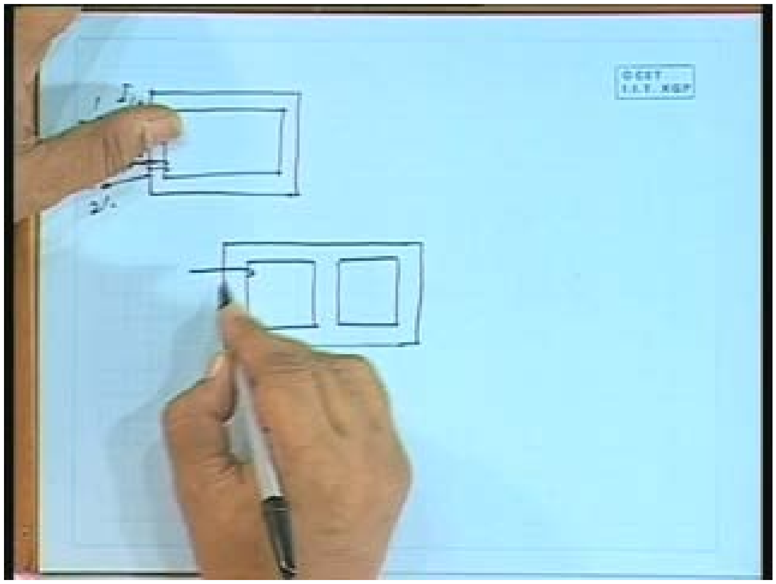


Good morning friends, yesterday we discussed about different connections of voltage and current sources and then we discuss something about non-relatives, different types of non-relatives then dot convention and will continue with that.

(Refer Slide Time: 51:50)



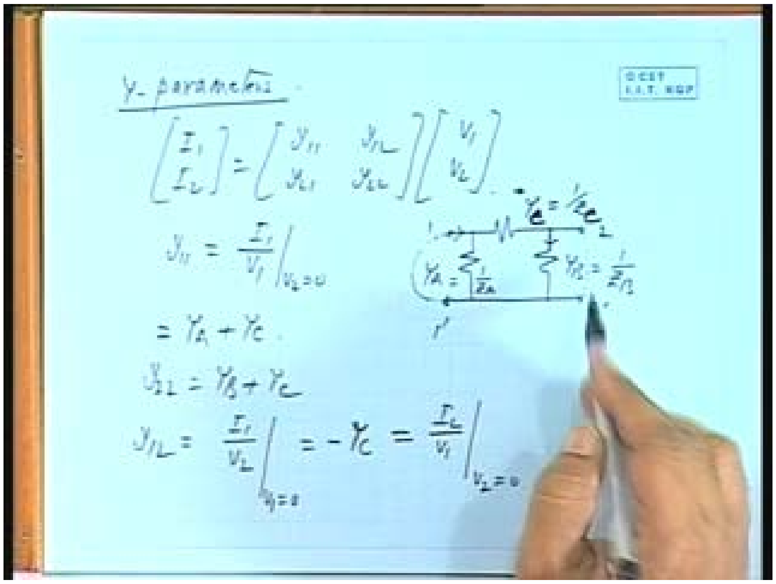
(Refer Slide Time: 52:29)



The dot conventions that we adopted yesterday, I will just repeat if you have a core like this, if you are having a coil, say coil A or coil 1, 1 dashed and another coil having the terminals 2, 2 dashed you see the coils are around in the same sense the coils surround in the same sense then if the current if I call1 as the starting terminal, 1 dashed is the finishing terminal. Similarly, 2 as the starting terminal, 2 dashed as the finishing terminal if current through terminal 1 enters into the coil, okay like this and current flow 2 also enters through this coil then both of them give rise to a flux in the same direction, all right or in other words, if we connect this with the source and if we short circuit this one externally, the current here will be flowing out of this terminal and entering through 2 dashed all right because it will be having by transformer action.

Okay, if we are having and alternating voltage say applied here then the current here will be going in this direction okay that means what? It means that if the coils, if the coils produce identical fluxes, when they are excited separately then those terminals the starting terminals will be shown by dots that means an a current entering here corresponds to current entering here from an external source. They are identical that means they give rise to the flux in the same direction, this is easy to remember all right.

(Refer Slide Time: 55:18)



Now I give you an example, you try to find out whether this is this particular coil 1, just 1 minute this particular coil arrangement and the dot conventions are correct, you are having a steroid with a central link, okay now you are having volt supply voltages and measure the current I 1 after shorting this. So how much is it, it is the admittance seen from 1, 1 dashed under this condition.

So it is 2 parallel elements Y A and Y C okay, is it not Z A and Z C are in parallel. So you add their admittances Y A plus Y C, Y A plus Y C very good similarly, Y 22 will be YB plus YC when I am shorting this side what about Y 12, what about Y 12, how do measure Y 12 is I 1 by V 2 when V 1 is when V 1 is 0. So I short circuit this and then measure the current here when I am applying a voltage V 2, so R, Y 12 is I 1 by V 2, I 1 by V 2 when V 1 is equal to V 1 is equal to 0 and how much is that by convention I 1 is positive, when it is going in this direction. Now when I am shorting this and applying a voltage here there will be a current that will be flowing like this. This current does not affect this, does it? This current is independent of this side, I am measuring only this current and since this is short, so this is redundant there is no current flowing through this all the currents will be flowing through this.

So the current flowing through this side will be through Y C and then through this 0 and a 0 impedance, so how much is the total admittance Y C, only Y C what about sign because the current is flowing in the opposite direction, so it will be minus Y C, okay. Now if you are given this Y parameters, there is a black box given to you I have given you 2 ports, 4 terminals and I ask you to perform this short circuit test that is you short circuit 1 side 2, 2 dashed make a measurement from this side, make the measurement of the current on the other side voltage from this side, voltage and current at this end and so on that is you measured V, V 1, I 1, I 2 again excite the other side short circuit this side 1, 1 dash, apply a voltage V 2, V 2, I 2 and I 1 okay.

So you will get all the 4 quantities, this is also I 2 by V 1 by the same logic when V 2 is 0 when I short circuiting this applying a voltage and measuring this current is that all right. If you are given these parameters but inside I do not know what it is, there can be number of elements that interconnected okay.