

Networks, Signals and Systems
Prof. T.K. Basu
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur
Lecture - 10
Frequency Response – Bode Plot

Before we start our next topic I would like to correct a small tutorial problem that we did last time, if you remember.

(Refer Slide Time: 00:54 min)

$$t^{1/2}$$

$$\Gamma(n) = \int_0^{\infty} \frac{e^{-t} \cdot t^{n-1}}{e^{-t}} dt$$

$$\Gamma\left(\frac{1}{2}\right) = \int_0^{\infty} t^{\frac{1}{2}-1} e^{-st} dt$$

$$= \int_0^{\infty} t^{\frac{3}{2}-1} e^{-st} dt$$

$$= \frac{1}{s^{3/2}} \int_0^{\infty} t_1^{\frac{3}{2}-1} e^{-t_1} dt_1 = \frac{\Gamma(3/2)}{s^{3/2}}$$

$$\Gamma(n) = n-1 \Gamma(n-1)$$

$$st = t_1$$

$$dt = \frac{dt_1}{s}$$

$$t^{1/2} = \frac{t_1^{1/2}}{s}$$

There was a function t to the power half for which we are computing the Laplace transform. So $\Gamma(n)$ is this so $\Gamma(n)$ is basically $n-1 \Gamma(n-1)$. So here it should be $\Gamma(3/2)$ which is half there was a slip here it should be $n-1 \Gamma(n-1)$ so it should be half, $\Gamma(1/2)$. So please make this correction I am extremely sorry will be half root s to the power 3 by 2 okay.

Today, we shall be starting frequency response of systems frequency response of various types of networks what do you mean by frequency response when you excite a system by say we have

a sinusoidal input $V_m \sin \omega t$ applied to a network whose impedance function is $Z(s)$ all right in the Laplace domain. In the time domain we are applying a function $V_m \sin \omega t$.

(Refer Slide Time: 01:18)

$$\Gamma(n) = 2\sqrt{n-1} \Gamma(n-1)$$

$$= \frac{2}{2} \cdot \sqrt{n} \cdot \frac{1}{s^{3/2}}$$

$$= \frac{1}{2} \frac{\sqrt{\pi}}{s^{3/2}}$$

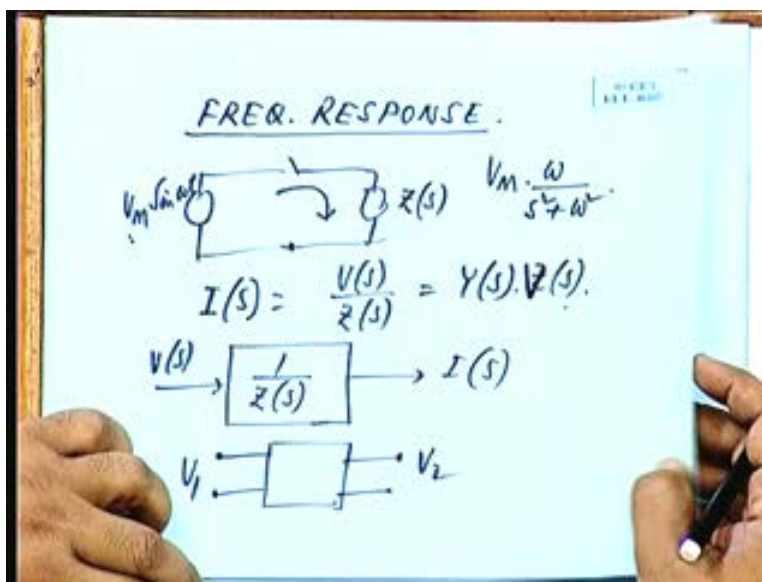
Ex 4 $\int e^{-at} dt \Rightarrow \int_0^{\infty} e^{-at} \cdot e^{-st} dt.$

$$= \int_0^{\infty} e^{-(at+st)} dt. \quad \int e^{-x} dx.$$

$$e^{-at} = 1 - at + \frac{a^2 t^2}{2!} \dots$$

$$\int e^{-at} = \frac{1}{s} - \frac{a \cdot 2!}{s^3} + \frac{a^2 \cdot 4!}{2! \cdot s^5}$$

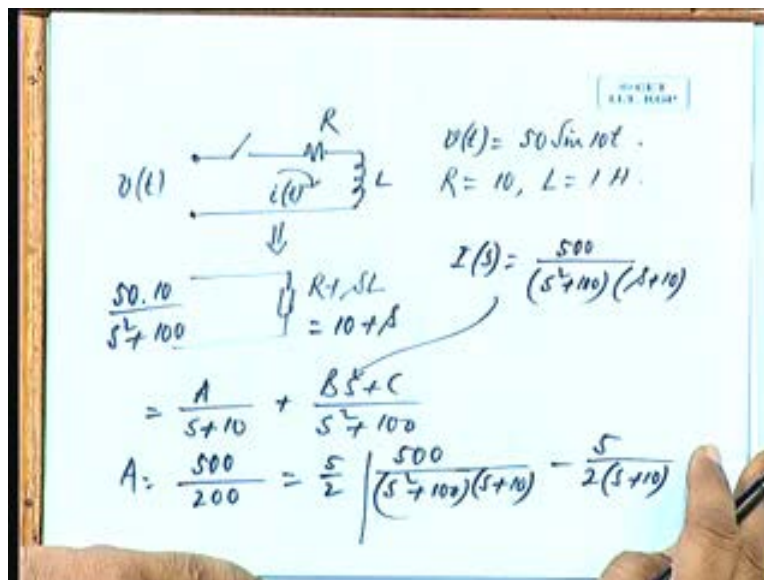
(Refer Slide Time: 01:49)



So in the Laplace domain this will appear as V_m into omega by s square plus omega square, so this is our input function, $Z(s)$ is the impedance function what will be the output. Now in this we may treat the current as the output okay. So $I(s)$ we will get as $V(s)$ by $Z(s)$ you can write $Y(s)$ into $Z(s)$, so for any system we define $Y(s)$ in to $V(s)$ thank you, $Y(s)$ in to $V(s)$. So if $V(s)$ is the input and $I(s)$ is the output 1 by $Z(s)$ is the multiplier we call it a transfer function okay. So input multiplied by a transfer function is the output in this particular case it happens to be 1 by $Z(s)$ the transfer function depends on how you define the input and the output, for a 2 port network we may define output as V_2 and input as V_1 , so $V_2(s)$ by $V_1(s)$ will be the transfer function somebody may be interested in the current that is flowing under open circuit condition or may be under short circuit condition.

So it is not necessary that it should be always V_2 by V_1 the transfer function depends on how you define the output okay. Now we will evaluate right from the start what would be the output in the time domain and at steady state, at steady state what will be the response like. So let us take a simple example, sorry.

(Refer Slide Time: 04:55)



We have a resistance and an inductance we may be interested in finding the current $i(t)$ when we are exciting this by a voltage $v(t)$ suppose $v(t)$ is given as, let us take an example, $50 \sin 10t$, R is say 10 ohms, inductance is 1 Henry, what will be the current $i(t)$? So I will replace this by the transformed equivalent which will be R plus sL which is 10 plus s and this side it is 50 in to omega is 10 divided by s squared plus 10 squared that is 100, is that all right? So what will be $I(s)$ it will be 500 by s squared plus 100 into 1 by $Z(s)$ that is s plus 10 if I make partial fractions. I can write this as A by s plus 10 plus B by s squared plus 100, so multiplied by s plus 10 then put s equal to minus 10. So that gives me 500 minus 10 squared is 100 plus 100, 200 is that

all right so it will be 5 by 2 okay. If I subtract 5 by 2 s plus 10 by s plus 10 from the original 1 I will straight away get B (s) plus C is it not otherwise by residues calculation by any method you can calculate s square plus 100 into s plus 10 minus 5 by 2 into s plus 10 is that all right.

(Refer Slide Time: 07:41)

$$\begin{aligned}
 &= \frac{A}{s+10} + \frac{Bs+C}{s^2+100} \\
 A: \frac{500}{200} &= \frac{5}{2} \left[\frac{500}{(s^2+100)(s+10)} - \frac{5}{2(s+10)} \right] \\
 &= \frac{1000 - 5(s^2+100)}{2(s^2+100)(s+10)} = \frac{500 - 5s^2}{2(s^2+100)(s+10)} \\
 &= \frac{5(10-s)(10+s)}{2(s^2+100)(s+10)} = -\frac{5}{2} \cdot \frac{s-10}{s^2+100}
 \end{aligned}$$

(Refer Slide Time: 09:02)

$$\begin{aligned}
 &= \frac{1000 - 5(s^2+100)}{2(s^2+100)(s+10)} = \frac{500 - 5s^2}{2(s^2+100)(s+10)} \\
 &= \frac{5(10-s)(10+s)}{2(s^2+100)(s+10)} = -\frac{5}{2} \cdot \frac{s-10}{s^2+100} \\
 I(s) &= \frac{5}{2(s+10)} - \frac{5}{2} \cdot \frac{s}{s^2+100} + \frac{5}{2} \cdot \frac{10}{s^2+100} \\
 i(t) &= \frac{5}{2} e^{-10t} - \frac{5}{2} \cos 10t + \frac{5}{2} \sin 10t \\
 &= \frac{5}{2} e^{-10t} + \frac{5}{\sqrt{2}} \sin(10t - 45^\circ)
 \end{aligned}$$

So that comes out to be 2 in to s square plus 10 in to s plus 10 divided by sorry multiplied by here that is 500 . So it to be 1000 minus 5 in to how much s square plus 100 okay is that all right so that gives me 1000 minus $5 s$ square divided by 2 in to s square plus 100 in to s plus 10 .

So that gives me if I take 5 common it will be 10 minus s in to 10 plus s divided by 2 in to s squared plus 100 in to s plus 10 . So that gives me 5 in to 10 minus s , so 5 by 2 minus 5 by 2 in to s minus 10 divided by s square plus 100 , is that all right? 10 plus s goes so it is like this. Therefore, the given function $I(s)$ is 5 by $2 s$ plus 10 minus 5 by 2 in to s by s square plus 100 minus and minus plus 5 by 2 in to 10 by s square plus 100 , is that all right? So what will be $i(t)$ it is 5 by 2 , e to the power minus $10 t$ minus 5 by $2 s$ by s square plus 100 inverse will be cosine $10 t$ plus 5 by 2 this one will be sin of $10 t$.

So 5 by $2 e$ to the power minus $10 t$ minus 5 by 2 sin I can write plus sin $10 t$ okay minus 45 degrees cosine $10 t$, sin $10 t$ both are having the amplitude 5 by 2 , so it will be 45 degrees 1 is plus the other one is minus anything else there will be a magnitude will be root 2 times this, is it not so 5 by 2 in to root 2 so I can put 5 by root 2 is that all right?

(Refer Slide Time: 11:03)

$$i(t) = \frac{5}{2}e^{-10t} - \frac{5}{2}\cos(10t) + \frac{5}{2}\sin(10t)$$

$$= \frac{5}{2}e^{-10t} + \frac{5}{\sqrt{2}}\sin(10t - 45^\circ)$$

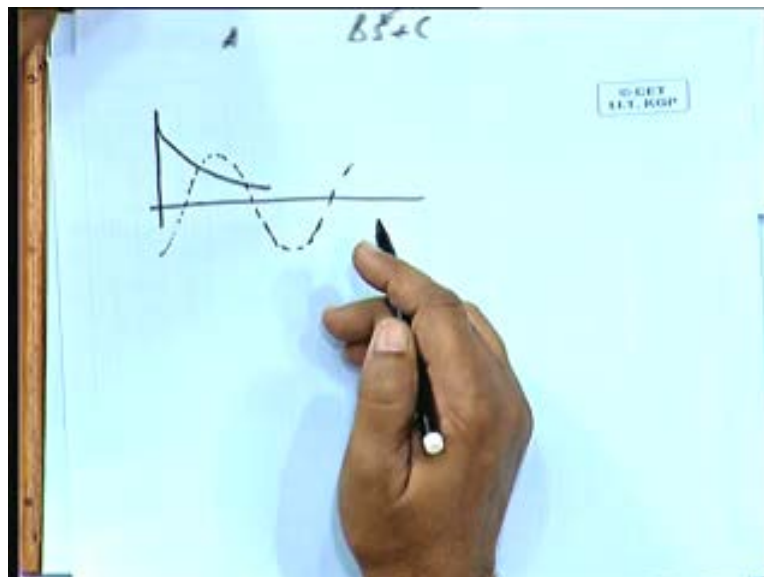
Now as time passes this will be dying down the response is the current is having a component which is dying down like this plus another component which is 5 by root root 2 sin $10 t$ minus 45 degrees. So like this okay, so the sum total of this will be the total response, this component will be vanishing after sometime it will die down. So if you are if you are interested only in the study state value then it is this component which will be continuing, so when we say steady state

response of the system, when you switch on a circuit this part will be vanishing within a few seconds, may be milli seconds depends on the time constants involved okay. So most of the time we are interested only in the steady value, what is that steady value?

(Refer Slide Time: 12:08)

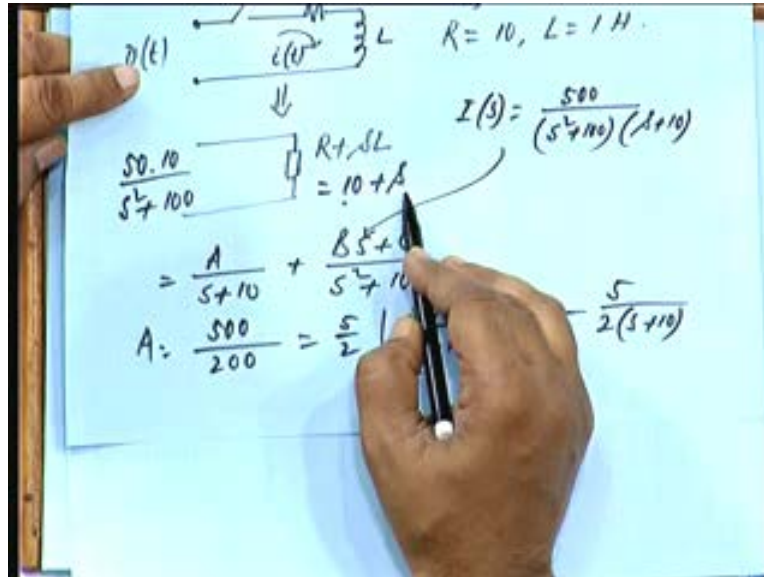
$$\begin{aligned}
 &= \frac{1000 - 5(s^2 + 10s)}{2(s^2 + 10s)(s + 10)} = \frac{500 - 5s^2}{2(s^2 + 10s)(s + 10)} \\
 &= \frac{5(10 - s)(10 + s)}{2(s^2 + 10s)(s + 10)} = -\frac{5}{2} \cdot \frac{s - 10}{s^2 + 10s} \\
 I(s) &= \frac{5}{2(s + 10)} - \frac{5}{2} \cdot \frac{s}{s^2 + 10s} + \frac{5}{2} \cdot \frac{10}{s^2 + 10s} \\
 i(t) &= \frac{5}{2} e^{-10t} - \frac{5}{2} \cos 10t + \frac{5}{2} \sin 10t \\
 &= \frac{5}{2} e^{-10t} + \boxed{\frac{5}{\sqrt{2}} \sin(10t - 45^\circ)}
 \end{aligned}$$

(Refer Slide Time: 13:09)



So it is this quantity, now when we say we are interested in the frequency response means basically we are interested in the steady state response of the system as a frequency changes as the frequency changes, this amplitude will be modified all right and also the phase this 45 degrees will not remain 45 degrees if I have a frequencies of 10 radians per second, if I go for 20 or 30 radians per second both this magnitude and this phase will be affected.

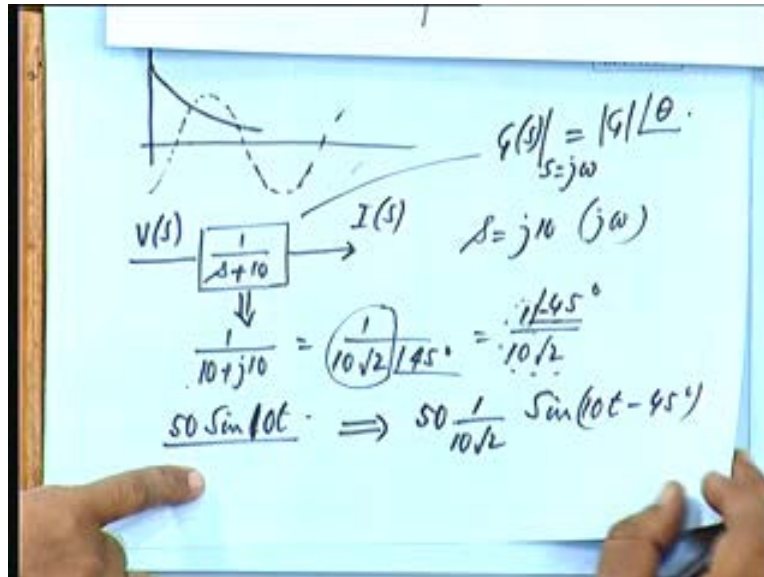
(Refer Slide Time: 13:18)



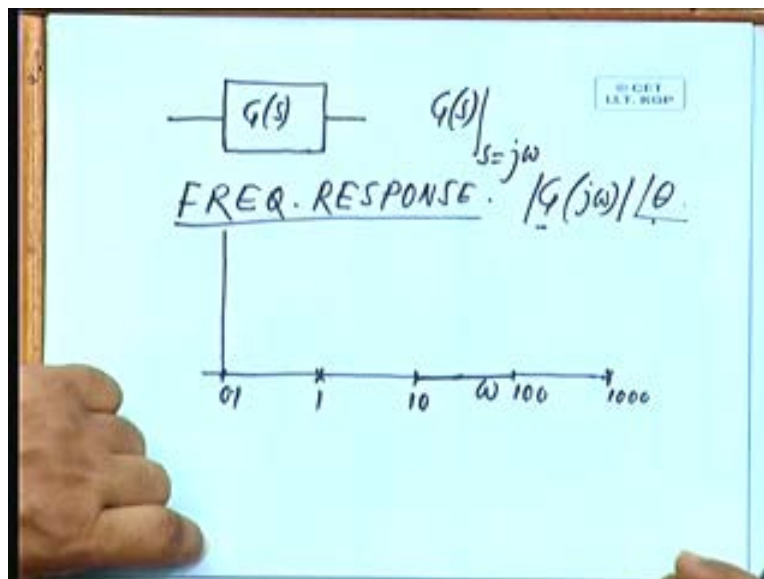
So how is this and this, how are these 2 quantities going to change with frequency if we can predict that if we can predict that then we need not go through all these computations okay. So you can see for yourself for this example, our function was the transfer function was 1 by s plus 10 that was 1 by $Z(s)$, is it not? So we have put $V(s)$ as the input 1 by s plus 10 as the transfer function and $I(s)$ was the output.

So if you are interested in knowing the steady value, the steady state value of the current then just put s equal to let us see if I put s equal to $j10$ means $j\omega$, where ω is the frequency in this case it was 10 so ω is 10 . If I put s equal to $j\omega$ then what would be this equal to 1 by 10 plus $j10$ which means 1 by 10 roots 2 and an angle of 45 degrees is that all right that means 1 by 10 root 2 minus 45 degrees. So now you see what was our input voltage $50 \sin 10 t$ this was our input here, what is the gain? We call this term gain that is the magnitude of $G(s)$ at s equal to $j\omega$ okay this is our $G(s)$ and we are evaluating $G(s)$ at s equal to $j\omega$ whatever be that ω .

(Refer Slide Time: 13:24)



(Refer Slide Time: 16:07)



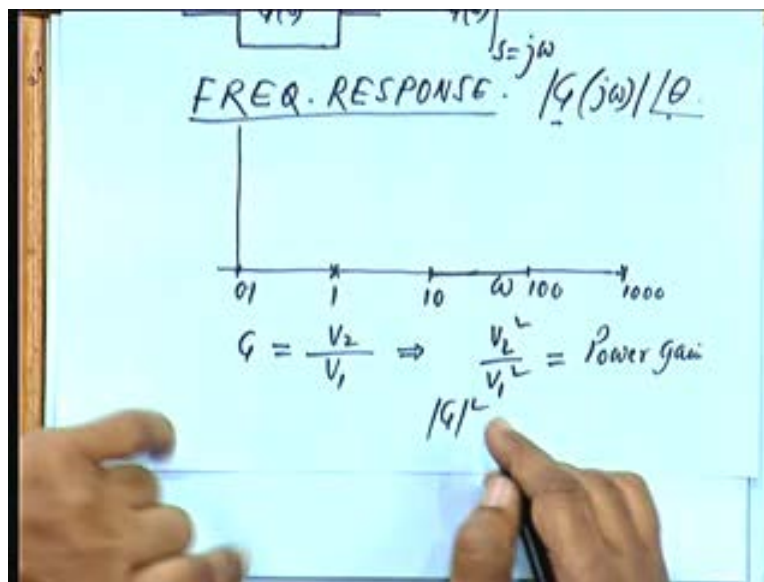
So this will be having an magnitude and a phase what is the magnitude? 1 by 10 root 2 , what is the phase? minus 45 degrees at this frequency ω will be 10 . So 50 was the amplitude of the input that gets multiplied by that magnitude of $G(j\omega)$ is that all right. So it will be 50 by 10 root 2 this will be the amplitude of the output $\sin 10t$ the same frequency $\sin 10t$ with the phase shift of θ here θ is minus 45 degrees, this will be the output. So the current you can

straight away write from the given input voltage provided you know the value of this G and theta from the given function all right.

So our primary task therefore we will be to we will be to evaluate Gs at different frequencies s equal to j omega, evaluate the magnitude and the phase then for any input I can always calculate current without going through this entire exercise of Laplace transforming taking inverse and so on. Once you know G (s) is that all right? So frequency response means calculation of G j omega and theta so we will try to find out a method of approximately computing these 2 quantities G and theta with respect to frequency, how it varies how G varies with frequency, let us see that.

Now since we will be varying the frequency over a wide range, we take the frequency scale in a logarithmic scale. So in logarithmic scale you do not have a₀₀ means log of 0 is minus infinity, so you can always have say we can choose arbitrarily any scale 1, 10 then the next equivalent length will represent a decade 10 to 100 next equivalent length will be 100 to 1000 and so on. So how much will be this this is 1 decade, so how much will this 0.1 had I had 1 starting from here then 1, .1, .01, so you can never approach approach 0 you can go to .1, .01, .001 depending on the scale that you want to choose the range that you want to choose you can have 3 decades 4 decades 5 decades in the market you will get you you can get the graph sheets graph sheets 4 decades or 5 decades or 6 decades or you can prepare 1 if you want so this will be depending on the range of the frequency that you want to work on.

(Refer Slide Time: 18:55)



This side instead of taking the gain as it is gain we define by say in a 2 port network, we write in terms of say voltage ratio of voltages or currents and so on. We are quite often interested in

knowing the power gain, how do you define power gain then over the same resistance say across the same resistance if I apply a voltage V_2 or V_1 how much will be the ratio of the power will be V_2 square by R and V_1 square by R . So if I take the ratio it will be V_2 square by V_1 squared that will be the power gain okay if G is the voltage gain then power gain will be G squared okay we are only interested in the magnitude at this time. Now human perception, human perception of power gain will be always in a logarithmic scale. When you see an image or when you hear a sound if the energy of the sound or the light is just doubled, if you just double the intensity the perceived intensity will be log of 2 in some logarithmic scale that means if I change the brightness, if I give the input energy to the light double the previous value then I will not be receiving I will not be feeling that the light has been doubled, the intensity has been doubled it will be in the logarithmic scale. Similarly, when you hear a sound if the energy of the sound has been doubled then I will be receiving it as log of 2 the perceived intensity will be varying logarithmically okay.

(Refer Slide Time: 20:58)

$$\log_{10} \left(\frac{P_2}{P_1} \right) = \log_{10} \left(\frac{V_2}{V_1} \right)^2$$

$$= 2 \log_{10} \left(\frac{V_2}{V_1} \right) \text{ bel.}$$

So it is better to define new gain function which will be log of the original gain function P_2 by P_1 we choose a base 10, base is not so much important we could have selected the natural log also that will be giving just a constant, how much is that constant log of 10 to the base e 2.303 okay. So log of say V_2 by V_1 square that means basically twice log of original G okay. We call this unit as a bel okay. Now in some countries you know the currencies are of such a denomination people get salaries in 1000 of rupees, 50000 an average man getting 50000 rupees salary will be somewhat unthinkable know but with 5000 rupees if he goes to the market he will be hardly able to buy anything.

So depends on the strength of the currency in Japan in Japan for example you have yen or in Italy you have lira people get say 50000 lira they go to market and buy something you do not go with say paisa I cannot say my salary is 50000 paisa, is it not? So or 5 lakhs paisa we have a little relatively hardware currency so instead of 100 rupees you may also have 1000 paisa I am going to the market with 10000 paisa is it not so or sometimes if you have the reverse situation, if you have the reverse situation that is I get salaries in lakhs of rupees so may be .05 lakhs all right or .005 lakhs with that rupee I will go to the market so that will be too small, is it not?

(Refer Slide Time: 23:28)

$$\log_n\left(\frac{P_2}{P_1}\right) = \log\left(\frac{V_2}{V_1}\right)^2$$

$$= 2 \log |G| \text{ bel.}$$

$$10 \times 2 \log |G| \rightarrow \text{db.}$$

$$20 \log |G| \rightarrow \text{db.}$$

So the units for any such quantity will have to be something which will be very easily which can be handled very easily so bel is such a unit that you will be getting a gain of say .03, .04 or may be .5 like that so we multiply it by 10 okay and define a decibel as the unit for gain that is in terms of rupees you are defining in terms of paisa the cost of anything you are defining in terms of paisa so you multiply by 100 similarly, in terms of decibel it will be 2 into 10 in to log G, is it not? 2 rupees means if I want to write in terms of paisa it will be 200 paisa similarly, 2 log G so much decibel will be 20 log of G so many, so much decibel.

So 20 log of G is mostly commonly used unit for gain in the in the logarithmic scale okay. So we will be interested in finding the variation of this quantity 20 log of G given a function G (s) variation with respect to frequency. So let us take let us take 1 simple example G (s) equal to say 10 by s plus 10 okay, what will be the frequency response for this? that means how is this quantity 20 log of G varying with frequency omega how is it varying, if I keep on changing s the value of s so this will be 10 by 10 plus j omega okay, 10 by 10 plus j omega I might as well write this as 10 by 10 in to 1 plus j omega by 10 okay. So that gives me 1 by 1 plus j omega by 10 what will be the magnitude G magnitude will be 1 by 1 plus omega squared by 100 okay.

(Refer Slide Time: 24:41)

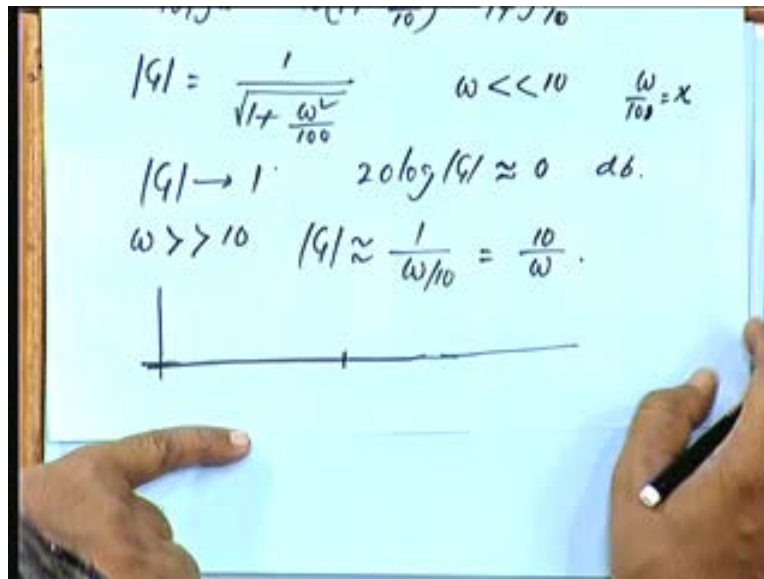
$= 2 \log |G| \text{ bel.}$
 $10 \times 2 \log |G| \rightarrow \text{db.}$
 $20 \log |G| \rightarrow \text{db.}$
Ex 1 $G(s) = \frac{10}{s+10} = \frac{10}{10+j\omega}$
 $20 \log |G|$ vs ω

(Refer Slide Time: 25:27)

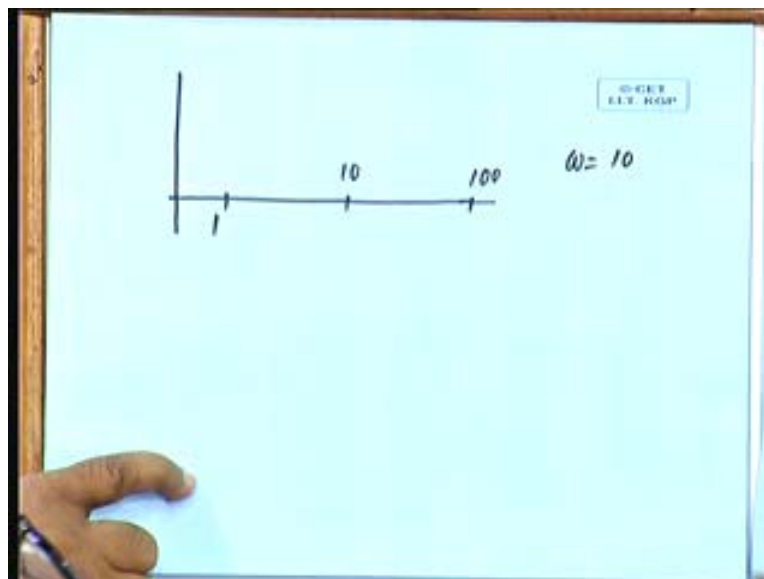
$\frac{10}{10+j\omega} = \frac{10}{10(1 + \frac{j\omega}{10})} = \frac{1}{1 + j\frac{\omega}{10}}$
 $|G| = \frac{1}{\sqrt{1 + \frac{\omega^2}{100}}} \quad \omega \ll 10 \quad \frac{\omega}{10} = x$
 $|G| \rightarrow 1 \quad 20 \log |G| \approx 0 \text{ db.}$
 $\omega \gg 10 \quad |G| \approx \frac{1}{\omega/10}$

Now when omega is very very small that means omega is much less than 10 G will be tending to how much this is negligible, so it will be 1. So 20 log of G will be log of 1 is 0, so it will be approximately equal to 0 db okay. When omega is much less than 10 when omega is much greater than 10 if I call omega by 100, omega by 10 as x it is basically 1 plus x squared okay under root.

(Refer Slide Time: 27:11)



(Refer Slide Time: 27:41)



(Refer Slide Time: 28:03)

$$\frac{10}{10+j\omega} = \frac{10}{10\left(1 + \frac{j\omega}{10}\right)} = \frac{1}{1 + j\frac{\omega}{10}}$$

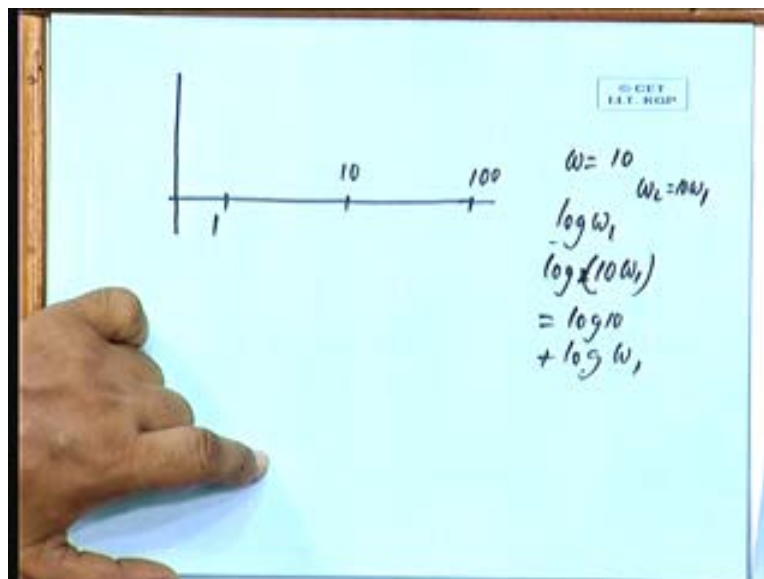
$$|G| = \frac{1}{\sqrt{1 + \frac{\omega^2}{100}}}$$

$$\omega \ll 10 \quad \frac{\omega}{10} = x$$

$$|G| \rightarrow 1 \quad 20 \log |G| \approx 0 \text{ db.}$$

$$\omega \gg 10 \quad |G| \approx \frac{1}{\omega/10} = \frac{10}{\omega}$$

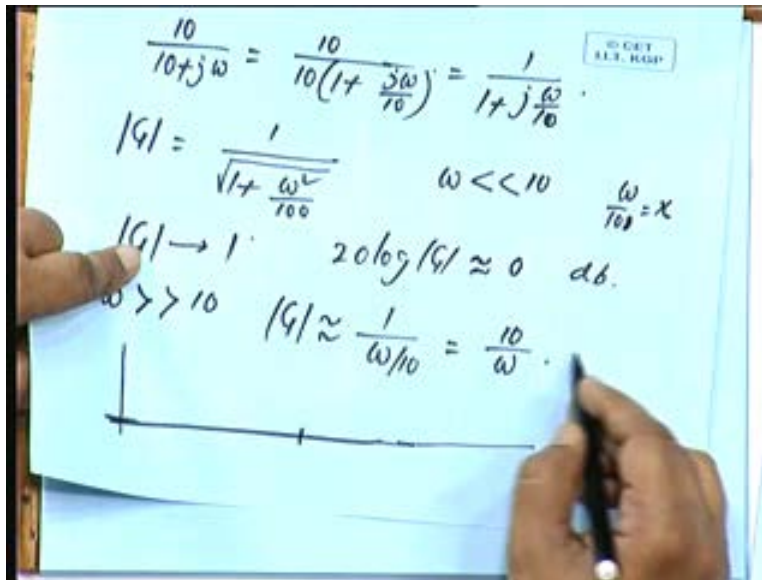
(Refer Slide Time: 28:15)



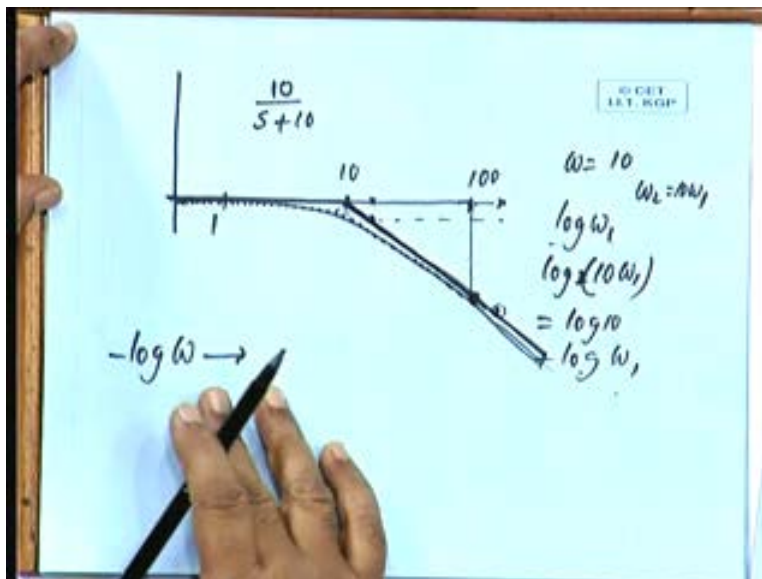
So this x squared will be dominating over this term, so I can approximate this to 1 by ω 1 by root over of ω squared by 10 , so ω by 10 okay and how much is this going to be this will be 10 by ω okay, so when ω is equal to 10 how much is it 1 . So I choose ω equal to this is ω okay let me take let me draw it here fresh let this be 1 this be 10 , this be 100 and so on when ω equal to 10 ω by 10 is equal to 10 by ω is 1 . So \log of 1 is 0 if I

increase the frequency 10 fold say you take any quantity log of omega, log of omega and at omega equal to omega1 say log of omega 1 and if I increase the frequency 10 fold at some other frequency omega 2 equal to 10 times omega 1, what would be the value log of 10 omega 1.

(Refer Slide Time: 29:20)



(Refer Slide Time: 29:26)

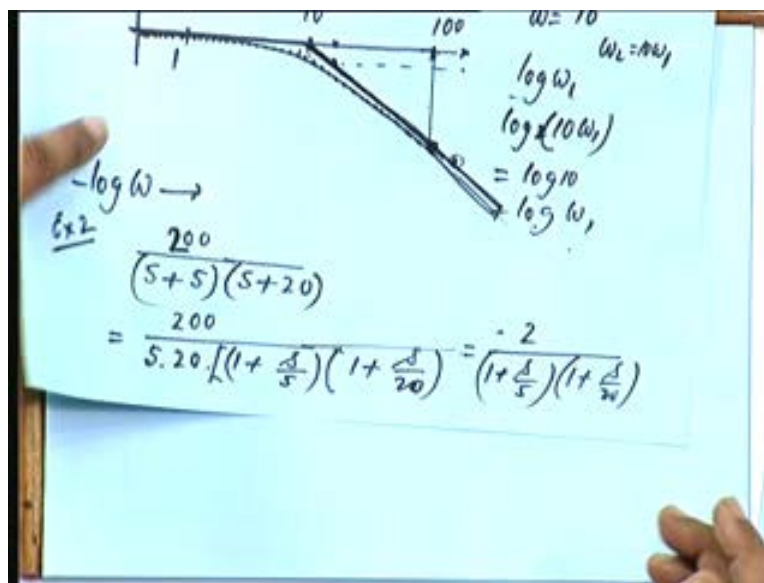


How much is this if increase the frequency 10 times from any value omega₁ is say 5 then from 5 to 50 if I go then this will be log of 10 plus log of omega₁ that means compared to the previous value log of omega₁ I am getting an additional term of log of 10 and log of 10 is. So if I multiply by the 20 log of G so it will be 20 in to 1. So I get an additional 20 db increase for a functional log of omega₁ and here omega is in the denominator if I take log it will be minus log of omega okay, so minus log of omega will have minus 20 db additional term for every decade increase in the frequency, is it not?

So whatever be the value at 10 omega equal to 10 omega equal to 100 will give me an additional value of 20 db it could have been any frequency I can take say 12 then at 120 that is 10 times that frequency whatever was the value I will go down by 20 db that will be the new value of the function at 120, so you take any frequency and 10 times that frequency the jump will be by 20 db. So if I draw a line at omega equal to 10 in this example it was 1 so at 100 it will be 20 db at 1000 it will be 40db, another additional 20 db.

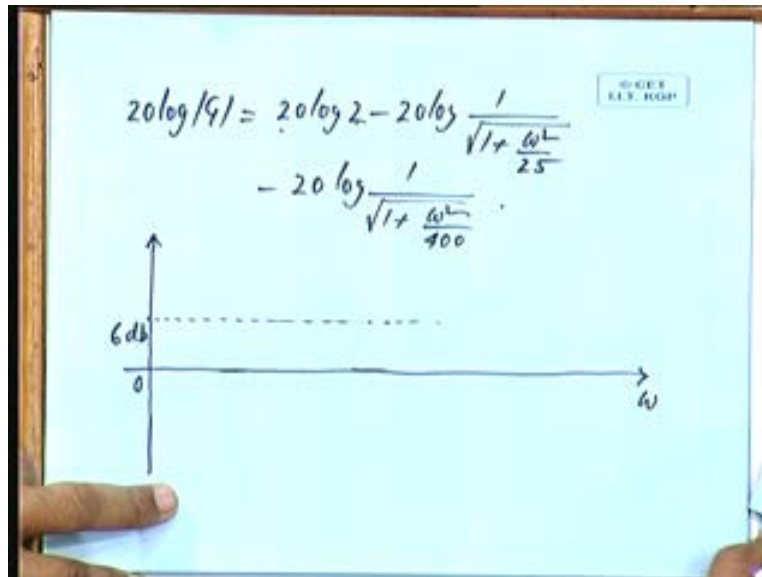
So at every decade all right if I increase the frequency 10 fold I will always get a 20 db additional term here it will be the negative region so I can join these points 20 db at 100, 40 db at 1000 and so on by a line. So it will be approaching either this 1 or this 1 as the frequency increases so these are the 2 asymptotes corresponding to a function 1 by s plus 10 okay it is 10 by s plus 10 okay. So the actual frequency response may be going close to this it will be meeting this asymptotically this 1 also asymptotically, so the actual curve will be somewhat like this okay.

(Refer Slide Time: 31:36)



Let us have another function 1 by s plus 5 in to s plus 20 say 100 divided by s plus 5 in to s plus 20 okay. Let it be 200 divided by s plus 5 in to s plus 20 I can always write this as 200 , I can take 5 outside, 20 outside and it will be 1 plus s by 5 in to 1 plus s by 20 , is that all right? I can write like this so that gives me 2 in to so 2 by 1 plus s by 5 in to 1 plus s by 20 . So what would be the logarithmic plot for this function.

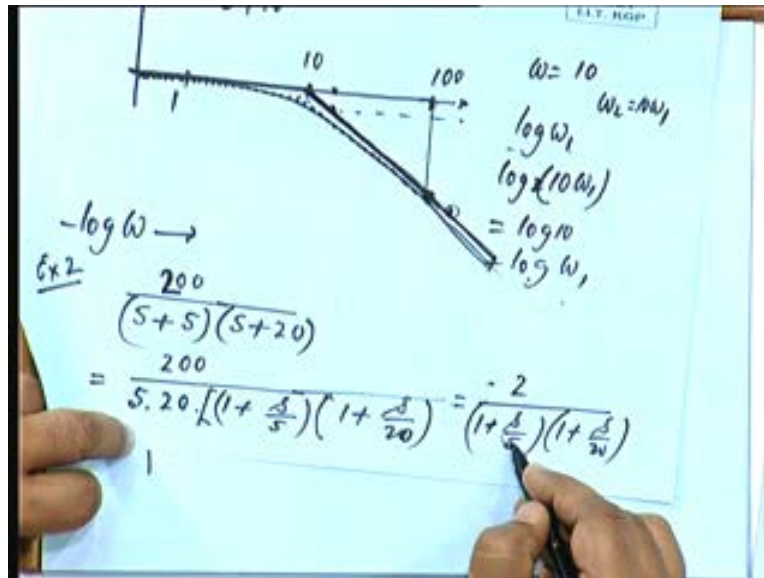
(Refer Slide Time: 32:54)



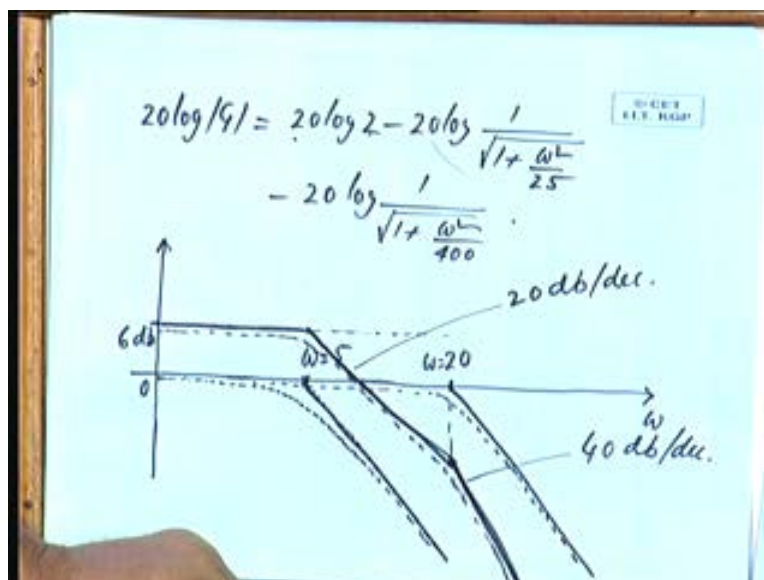
If I take $20 \log$ of G it will be $20 \log$ of 2 minus $20 \log$ of 1 by root over of 1 plus ω square by 25 minus $20 \log$ of 1 by root over of 1 plus ω squared by 400 , do you agree? So $20 \log$ of 2 log of 2 is $.3$, so $.3$ in to 20 , 6 db. So let us take each of this component separately plot them and then add them together that will be the resultant plot. So first component is 6 db that remains constant okay this 6 db all right. Next $20 \log$ of say the function 1 plus s by 5 what is that critical frequency where this component will be dominating over the other component it is at ω equal to 5 , is it not?

So ω equal to 5 is 1 critical frequency from where I can straight away show a 20 db per decade slope. So the function here $20 \log$ of 1 by root over of 1 plus ω square by 25 , 25 will be an asymptote like this, okay at ω equal to 4 sorry 20 similarly, there will be another 20 db per decade line so a curve like this will come is that all right.

(Refer Slide Time: 34:16)



(Refer Slide Time: 34:32)

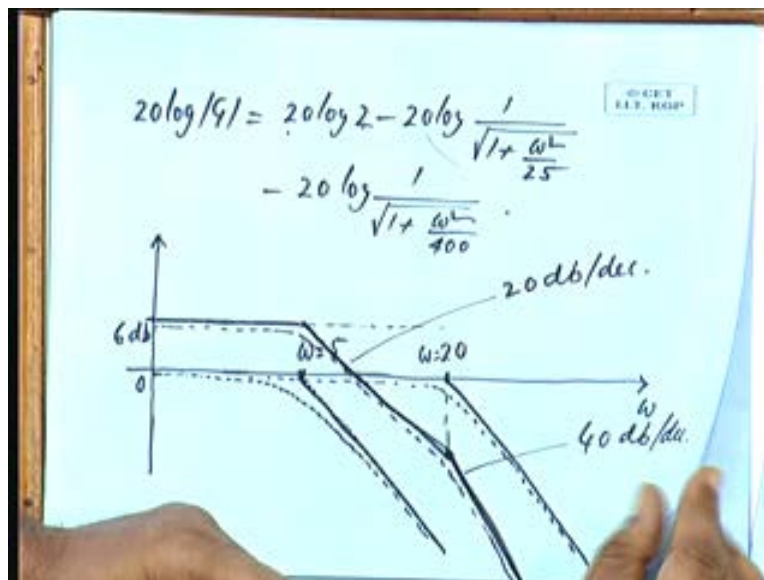


Now we are not really interested in the exact plot will be approximating that is good enough. So what will be the resultant of these 3 components one is a constant here then at this point there is a break 20 db per decade slope and at this point there is further 20 db fall. So there will be another

line of 40 db per decade so this will be the resultant one 6 db constant line then at omega equal to 5 it will be falling at the rate of 20 db per decade then here a from omega equal to 20 onward will be 40 db per decade, is that all right and then the actual plot will be somewhat like this. We are not interested in the exact plot so this is 20 db per decade this will be 40 db per decade okay.

Now instead of increasing the frequency 10 fold in an expression like log of omega if I increase the frequency 10 fold then I get an additional term of 20 log of 10 is it not plus 20 log of omega, if I increase the frequency 2 fold that is if I just double the frequency then what will be the additional term 20 log of 2, I have just double the frequencies so 20 log of 2 will be the additional term, how much is that 6 db. So 20 db per decade is same as 6 db per octave this octave is different from the octave that we use in music systems all right in sound octave here. In electrical engineering when you talk about frequency response octave means double the frequency that is the second harmonic so for any omega if I double the frequency they will be an increment or decrement of 6 db so sometimes these slopes are written in many books you will find 6 db per octave this as good as 20 db per decade.

(Refer Slide Time: 37:47)



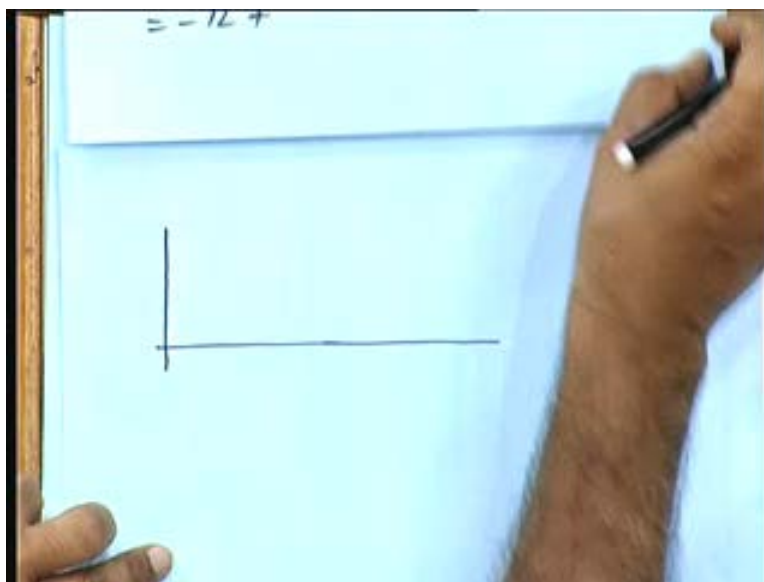
Now if you are having a factor of say instead of 1 by s plus 10 if you get a factor 1 by s plus 10 whole squared then what will be the rate of fall at that same frequency omega equal to 5 or omega equal to 10 or omega equal to 20, the line will be falling at double the rate. So it will be 40 db per decade okay. If you have a function in the numerator a factor in the numerator say G (s) equal to s plus 10 divided s plus 1 in to s plus 40 what will be the plot I can take 10 common 1 plus s by 10, s plus 1 in to 1 plus s by 40 and 40 common.

(Refer Slide Time: 38:28 min)

$$G(s) = \frac{(s+10)}{(s+1)(s+40)}$$
$$= \frac{10(1+s/10)}{40(s+1)(1+s/40)}$$
$$20 \log_{10} |G| = 20 \log_{10} \left(\frac{1}{4} \right) + 20 \log_{10} \sqrt{1 + \frac{\omega^2}{100}}$$
$$- 20 \log_{10} \sqrt{1 + \omega^2} - 20 \log_{10} \sqrt{1 + \frac{\omega^2}{1600}}$$
$$= -12 +$$

So 1 by 4 in to this if I take log 20 log of G will be 20 log of 1 by 4 okay plus 20 log of the numerator term root over of 1 plus omega squared by 100 minus 20 log of root over 1 plus omega squared minus 20 log of I am not writing to the base 10 every time it is understood we are working with the base 10 one plus omega squared by 1600 okay.

(Refer Slide Time: 40:22)

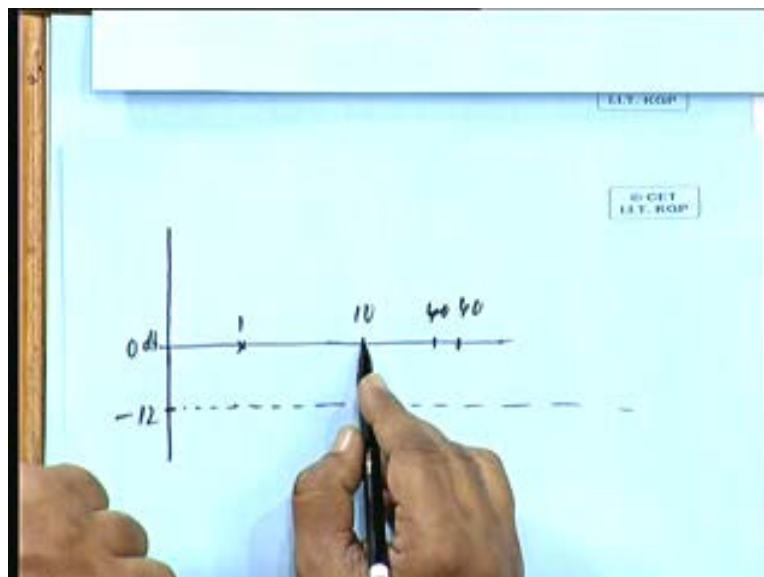


(Refer Slide Time: 40:32)

$$G(s) = \frac{(s+10)}{(s+1)(s+40)}$$
$$= \frac{10(1+s/10)}{40(s+1)(1+s/40)}$$
$$20 \log_{10} |G| = 20 \log_{10} \left(\frac{10}{40} \right) + 20 \log_{10} \sqrt{1 + \frac{\omega^2}{100}}$$
$$- 20 \log_{10} \sqrt{1 + \omega^2} - 20 \log_{10} \sqrt{1 + \frac{\omega^2}{1600}}$$
$$= -12 +$$

Once again you take term by term these quantities $20 \log$ of 1 by 4 how much is it \log of 1 by 4 means minus 1 \log of 4 \log of 4 is point 6, so is .6 in to 20 so it will be minus 12 db, this will be minus 12 db plus the other terms. So the plot will be now you see here we are having a numerator part numerator part which is giving me a plus term, which is giving me a plus term here and the denominators are giving me 2 minus terms.

(Refer Slide Time: 40:38)

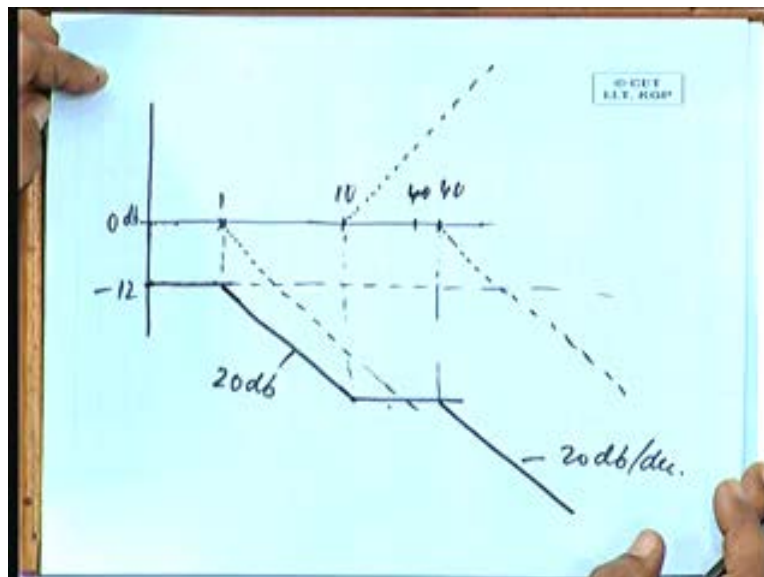


So you have minus 12 db, so if this is 0 if this is minus 12, this is the first part, the second part is at omega equal to 10 omega equal to say this is 1, this is 0 db, this is say 1, this is say 10, this is say 40, they are not really to the scale 40 may be somewhere here okay.

(Refer Slide Time: 41:22)

$$\begin{aligned}
 G(s) &= \frac{10}{(s+1)(s+40)} \\
 &= \frac{10(1+s/10)}{40(s+1)(1+s/40)} \\
 20\log_{10}|G| &= 20\log_{10}\left(\frac{1}{4}\right) + 20\log_{10}\sqrt{1+\omega^2/100} \\
 &\quad - 20\log_{10}\sqrt{1+\omega^2} - 20\log_{10}\sqrt{1+\omega^2/1600} \\
 &= -12 +
 \end{aligned}$$

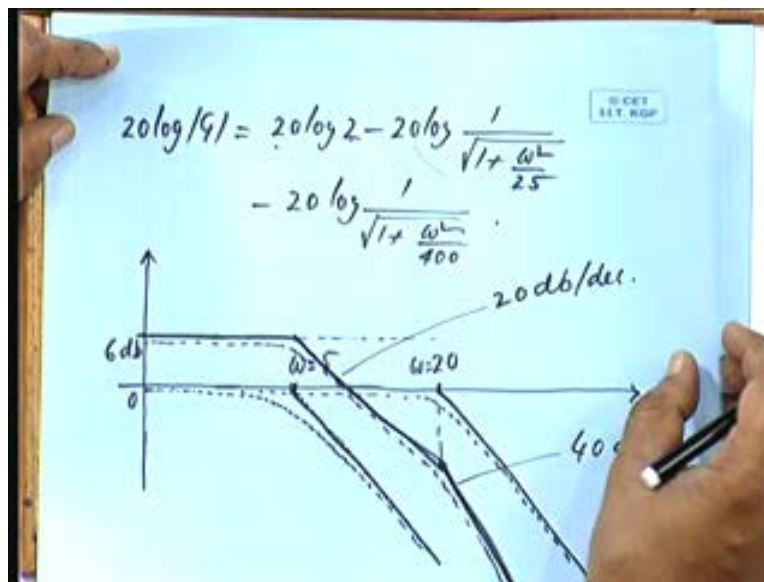
(Refer Slide Time: 41:24)



So at omega equal to 10 you are having a numerator function that means it will be increasing 20 db per decade line will be like this and at omega equal to 1 it is falling like this 20 db per decade at 20 again it is falling like this. So what is the net sum it is this plus, this plus, this plus, this, so minus 12 db the net value say in this range is minus 12. So it will be following this way minus 12 then at omega equal to 1 there is a slope of 20 db per decade the fall starts.

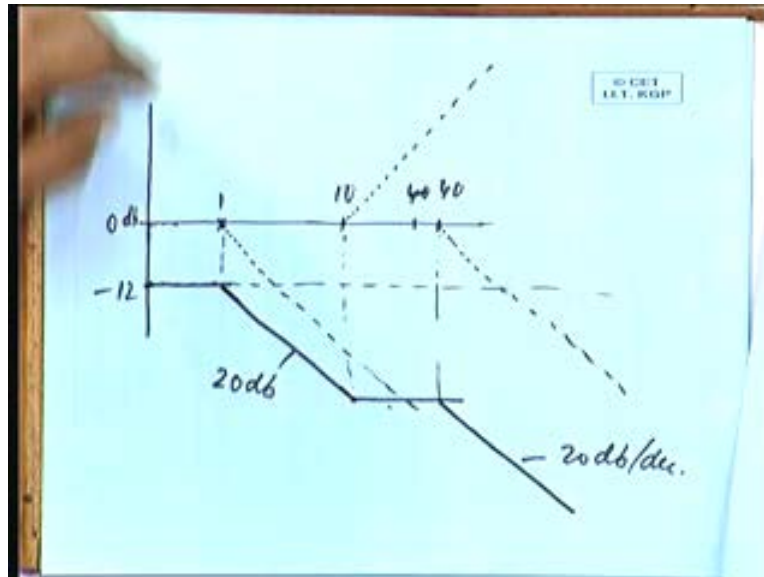
So it will start falling at 20 db per decade that means parallel line I will draw 20 db per decade like this at omega equal to 10 there is an increment of 20 db per decade so this will be offset by 20 db per decade increase, so it will be finally horizontal no increase or decrease this was falling at this rate this is increasing at the same rate. So it will be remaining constant at that rate at 40 again it will start falling, so this is 20 db per decade, 0 db per decade, again 20 db per decade, this will be the gain plot like is that all right 20 log of 2, these 2 becomes loss.

(Refer Slide Time: 43:10)

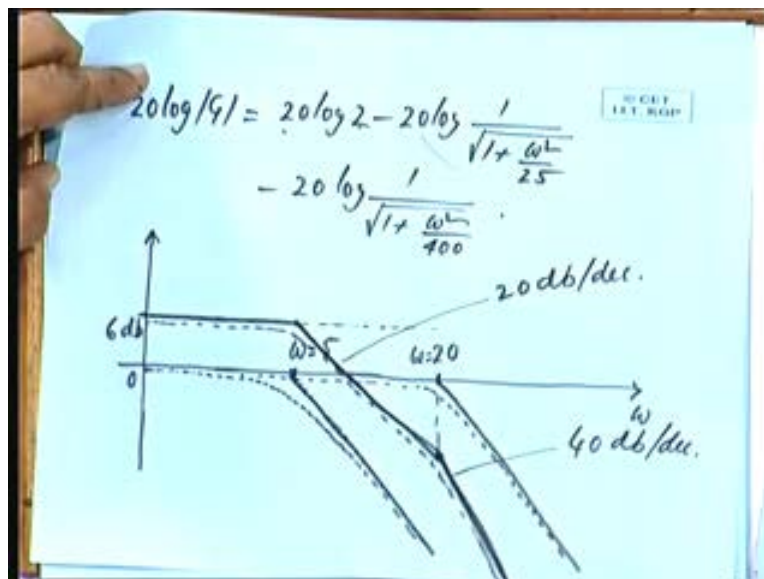


This problem you mean no no sir previous one, first example was this one, this one but the previous slide 20 log this one this becomes loss plus 20 log, Thank you very much, because all of them are actually thank you very much, these are plus signs sorry, there is a small slip earlier I was having this in mind that the denominator will come with a negative sign but instead of converting them I put negative signs it will be plus this I also be plus automatically when they come in the numerator it will be minus, thank you very much.

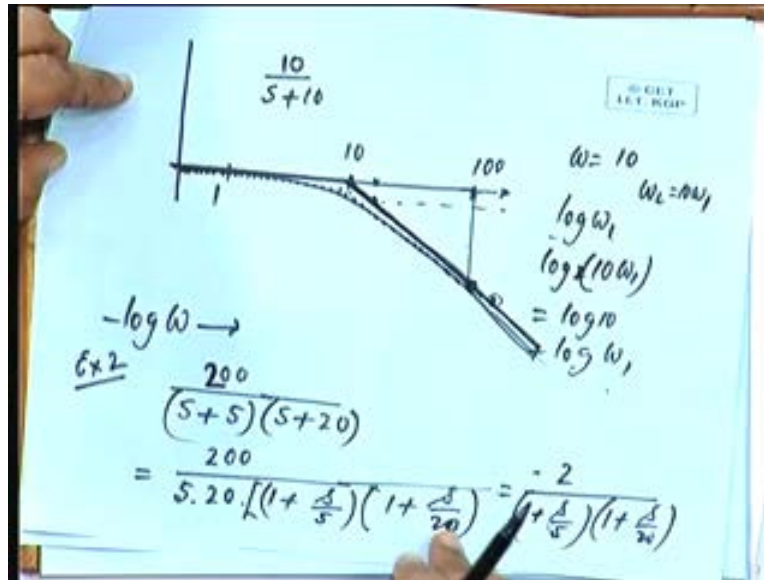
(Refer Slide Time: 43:21)



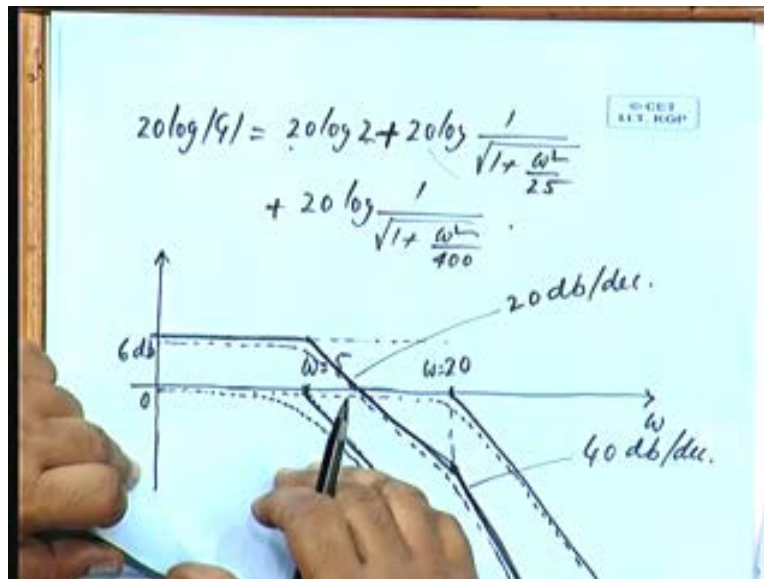
(Refer Slide Time: 43:22)



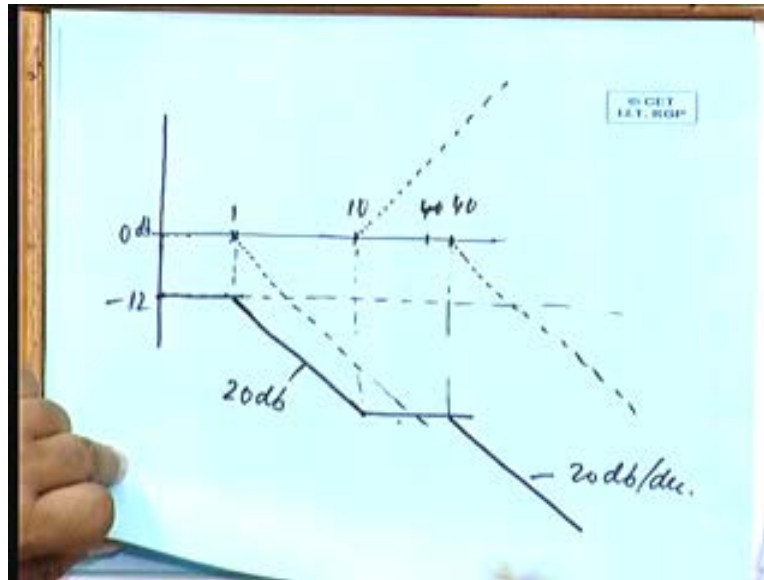
(Refer Slide Time: 43:26)



(Refer Slide Time: 43:33)



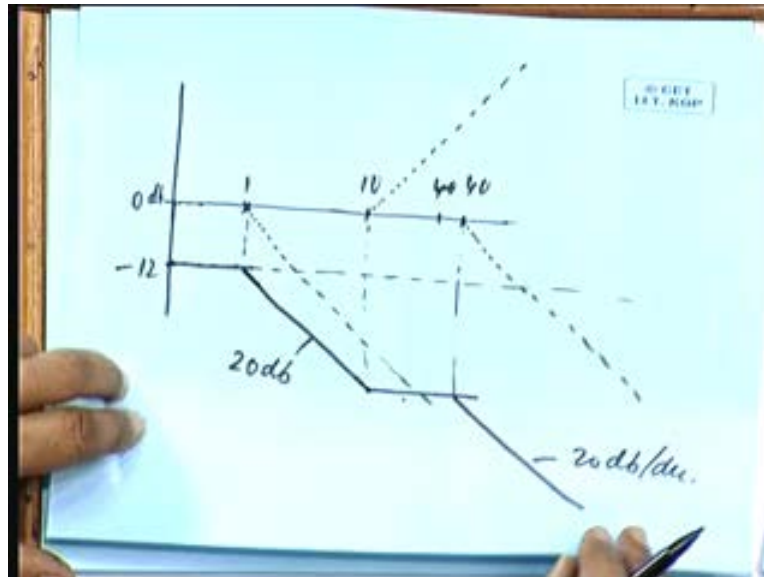
(Refer Slide Time: 44:17)



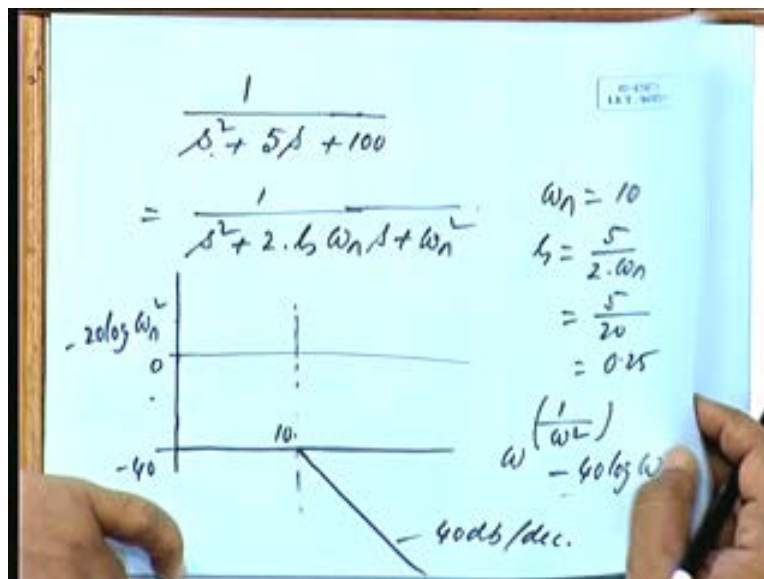
(Refer Slide Time: 44:22)

$$G(s) = \frac{(s+10)}{(s+1)(s+40)}$$
$$= \frac{10 \left(1 + \frac{s}{10}\right)}{40(s+1) \left(1 + \frac{s}{40}\right)}$$
$$20 \log_{10} |G| = 20 \log_{10} \left(\frac{1}{4}\right) + 20 \log_{10} \sqrt{1 + \frac{\omega^2}{100}}$$
$$- 20 \log_{10} \sqrt{1 + \omega^2} - 20 \log_{10} \sqrt{1 + \frac{\omega^2}{1600}}$$
$$= -12 +$$

(Refer Slide Time: 44:25)



(Refer Slide Time: 44:35)



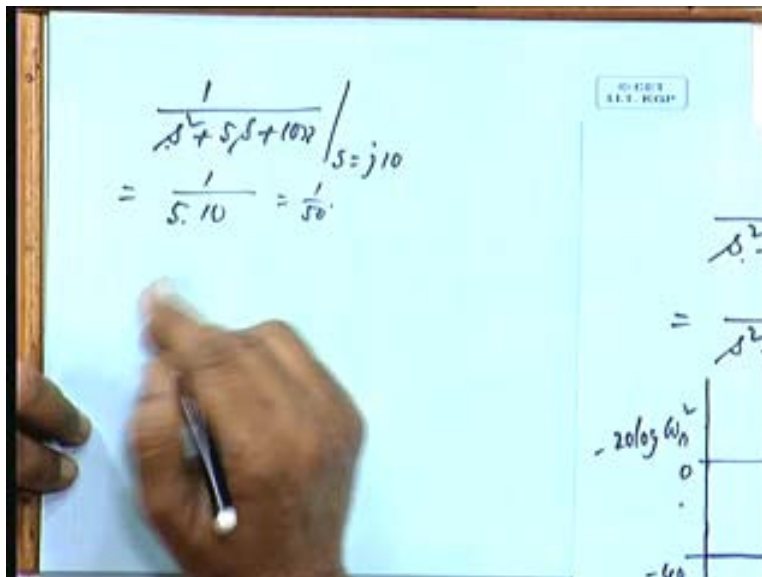
So this is the final gain plot sometimes you are having a function like 1 by s squared plus say how much should I write okay say, s squared plus this is 100 plus 10 may be $5s$ plus 100 you cannot factorize this in to real roots, V square is less than $4s$ c all right, so there will be complex roots you write this in the form of s squared plus twice zeta omega n s plus omega n squared

okay, ω_n is therefore 10 and ζ will be 5 by twice ω_n , is it not? means 5 by 20 so that is 0.25.

For such functions what would be the sketch like when ω is very very small when ω is very very small it will be 1 by ω_n^2 okay, s is $j\omega$ when we are evaluating this entire function for s equal to $j\omega$ and ω is very small. So this term will be 10 in to 0, this term will be 10 in to 0 because there is a multiplier s so it will be 1 by ω_n^2 , so this will be 20 log of ω_n^2 with a negative sign because it is 1 by whatever be that value depending on ω_n^2 in this particular case ω_n^2 is 100.

So 20 log of 100 so log of 100 is how much 2 so minus 40, so it will be minus 40 sorry if I take this as the 0 db line, so this is minus 40 okay. So this will be the value corresponding to $\omega = 0$ so this will be the asymptote like when ω_n is very ω is very very large this will be 10 db to 1 by s^2 squared all right that means 1 by ω^2 , it will be 10 in to this, when you take 2 extreme values s tending to 0 and s tending to infinity s equal to $j\omega$ ω tending to infinity then you get the 2 limits that means the asymptotes are estimated from this. The function will be approaching either this value or that value so, 1 by ω^2 if I take twenty log of that how much is it minus 40 log ω is that all right. For 1 by ω^2 square if I write 20 log of 1 by ω^2 that will be minus 40 log ω .

(Refer Slide Time: 48:49)



So how much will be the slope 40 db per decade is that all right so it will be from 10 onward at ω equal ω_n is equal to ω equal to 10 at ω equal to 10 or close to 10, we will see, we will see. So 40 db per decade will be the slope of this somewhere in between when

omega equal to omega n omega equal to 10 in this particular example omega equal to 10, how much is this? minus 100, plus 100 and minus 100 terms will get cancelled. So 1 by s squared plus 5 s plus 100 will tend to at s equal to j10, j10 means I am putting omega n value it will be this and this will cancel, it will be 1 by 5 in to 10. So that is 1 by 50 okay so it will be log of 1 by 50 is how much so 20 log of 1 by 50, how much is it?

(Refer Slide Time: 49:17)

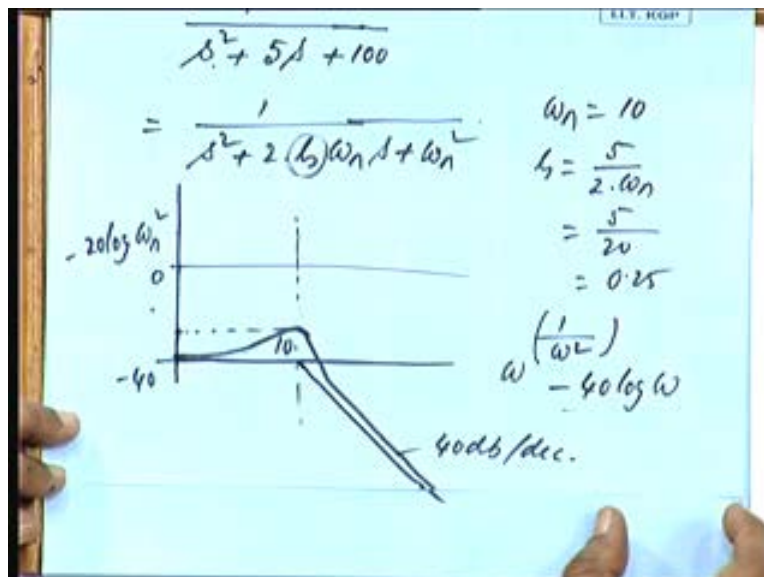
Handwritten mathematical derivation on a whiteboard:

$$\frac{1}{s^2 + 5s + 100} \Big|_{s=j10}$$

$$= \frac{1}{5 \cdot 10} = \frac{1}{50} \Rightarrow 20 \log \frac{1}{50}$$

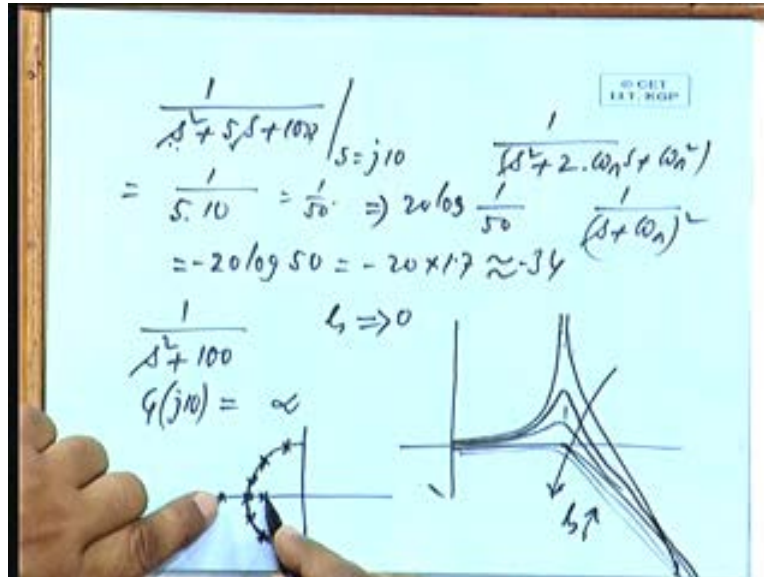
$$= -20 \log 50 = -20 \times 1.7 \approx -34$$

(Refer Slide Time: 49:54)



So I can always write 20 log of 50 negative sign so minus 20, so how much is log of 50, log of 5 is .7, log of 10 is 1, so 1.7 in to 20 34 is that all right, so 34 db minus, so earlier it was 40 now it will be minus 34 db. So it will be going like this going up to 34 and then falling like this, is that all right? it will be falling like this depending on the value of zeta, depending on the value of zeta this will be either very peaky or very flat. Now let us see how this changes with zeta.

(Refer Slide Time: 50:33)

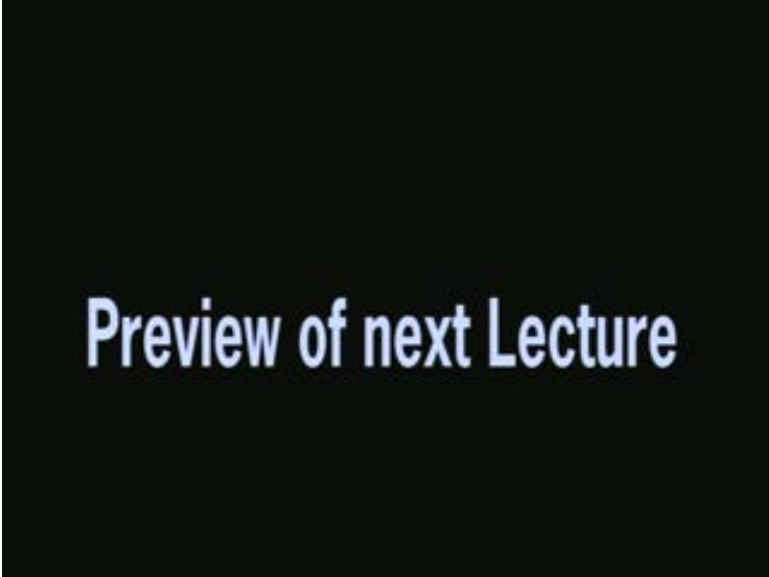


If $s^2 + 5s + 100$ if I reduce this make it approximately equal to 0 then what happens 1 by $s^2 + 100$ this term is a missing zeta is made 0, it will be 1 by $s^2 + 100$ and at s equal to $j10$. So how much is $G(j10)$ 10 in to infinity is that all right. So the magnitude if I take the log at ω equal to 10 , it will be approaching that 40 db per decade line okay slope will be finally 40 db per decade it will be tending to infinity and then again coming from there. So zeta as I keep on increasing zeta this will keep on falling like this okay.

So this is zeta increasing a simple way of giving the symbols this is the direction of the curve the curve tends like this as zeta increases, when zeta is more than more than say how much okay when zeta is equal to 1, when zeta is equal to 1 then it will be 1 by $s^2 + 2\zeta\omega_n s + \omega_n^2$ that means 1 by $s + \omega_n$ whole square. There are 2 real roots, 2 equal real roots so at ω_n straight away there will be a 40 db per decade fall all right, is that okay at ω_n that means 2 roots are coming if you remember for different values of zeta if you calculate the roots of this quadratic okay we will take it up in the next class how the roots travel, how the roots are changing for different values of zeta the complex pair of roots will be moving like this and then this is a time when they will become equal if you increase zeta beyond this the roots will be a real but they are separated now, 2 distinct roots will be getting okay. So


we will take it up in the next class and then will see how to make the phase plot from the gain plot.

(Refer Slide Time: 53:41)

A black rectangular slide with the text "Preview of next Lecture" in white, bold, sans-serif font, centered on the slide.

Preview of next Lecture

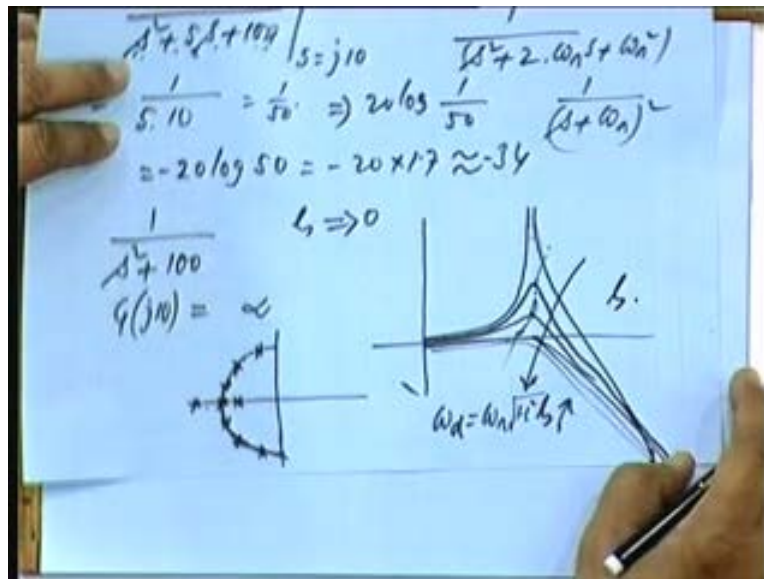
(Refer Slide Time: 53:44)

A black rectangular slide with the text "Lecture # 11" in white, sans-serif font, centered on the slide.

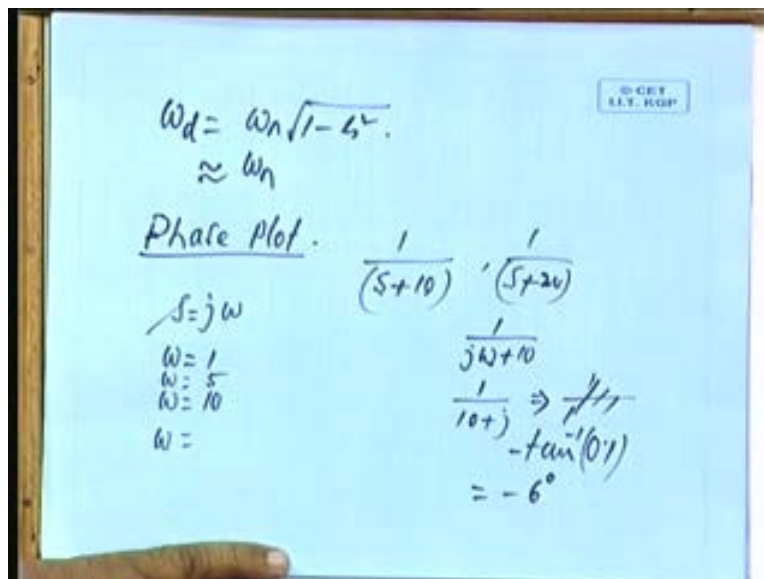
Lecture # 11

Bode Plot (contd.)

(Refer Slide Time: 54:10)



(Refer Slide Time: 54:49)



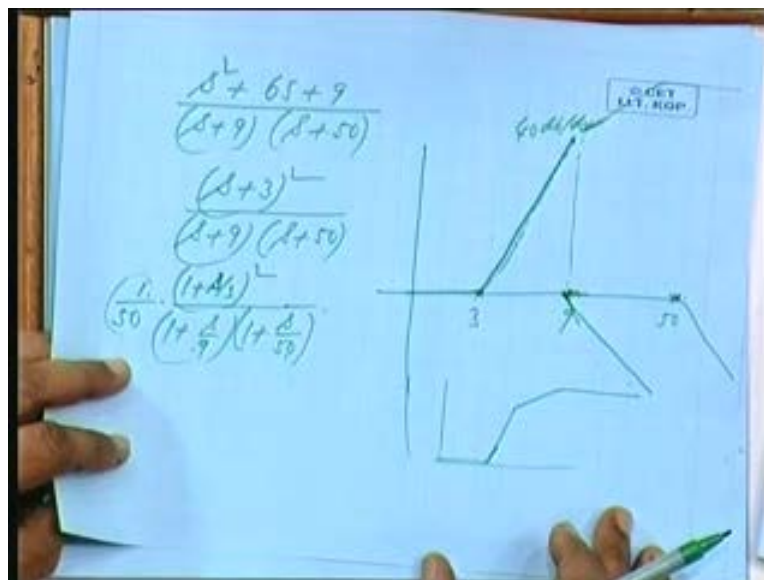
Okay friends, we will continue with some more examples on Bode plot. Last time we are discussing about the plot for a function of this form 1 by s square plus $5s$ plus 100 that is twice zeta omega n s plus omega n square for different values of omega n omega sorry for different values of zeta you will find this peak value occurs at a shifted value shifted point here that means the peak gradually shifts to the left at a frequency omega d which is equal to omega n in to root

over of $1 - \zeta^2$, ωd equal to ωn in $1 - \zeta^2$ when ζ is small we can approximate this to ωn . Now we come to a frequency plot a sorry phase plot.

Let us once again compute the phase for each of these factors like say $1 + s + 10$ by $s + 10$ and so on if you have a factor like this what will be the corresponding phase if I put s equal to $j\omega$, so ω equal to 1 , ω equal to 10 , ω equal to say 5 and so on. When it is $s + 10$ that means $1 + j\omega + 10$ when we take ω equal to 1 , ω equal to 10 , how much is this $1 + j10$ plus j okay. So $10 + j$ how much is the angle $\tan^{-1} \frac{1}{10}$, $\tan^{-1} \frac{1}{10}$ that is \tan^{-1} of $.1$ okay. So that gives me \tan^{-1} of $.1$ with a negative sign.

So that will be minus how much is it $\tan^{-1} .1$ is $.1$ $\tan \theta$ is equal to θ , when θ is small, so $.1$ radian approximately 6 degrees, 57 degrees will become 1 radian so $.1$ radian means 5.7 degrees approximately 6 degrees okay, if I take 10 times this frequency that is when ω is hundred then will be $10 + 100j$ all right so how much is $\tan^{-1} 10$, $\tan^{-1} .1$ is $.6$, so just $1 + .1$ that means $90 - 6$ degrees will be 84 degrees okay will be 20 db per decade minus 20 db per decade and again at for 50 minus 20 db per decade all right.

(Refer Slide Time: 57:31)



So what is the resultant of these 3 resultant will be if I am permitted to draw by a dotted line like this it will be actually this line and then at 9 okay, at 9 this will be minus 20 . So it will be plus twenty db per decade will go on like this and at 50 it becomes horizontal. So the net function will look like 40 db, 20 db and then horizontal and then depending on the constant $20 \log$ of 1 by 50 , it will be shifted up or down depending on its magnitude here it is 1 by 50 , so it will be negative

should be brought down by this factor twenty log of 50 is that all right. So we will stop here for today will take some examples in the next class. Thank you very much.