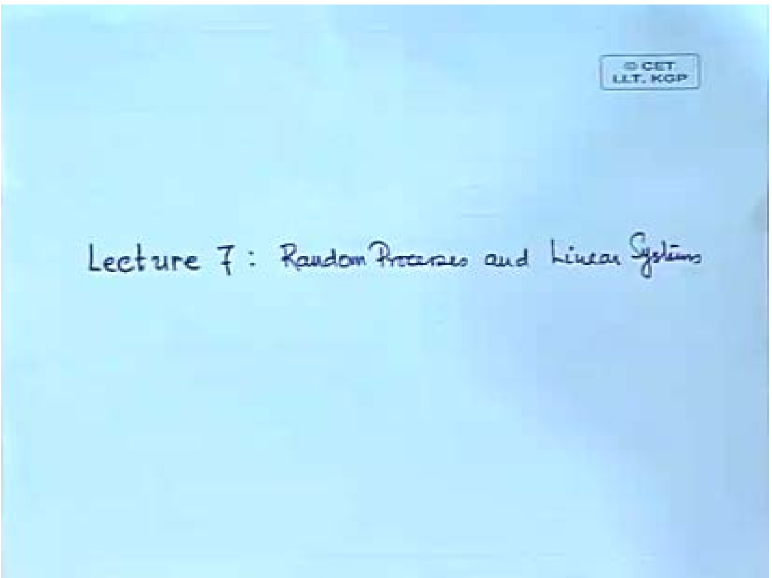
Estimation of Signals and Systems
Prof. S. Mukhopadhyay
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

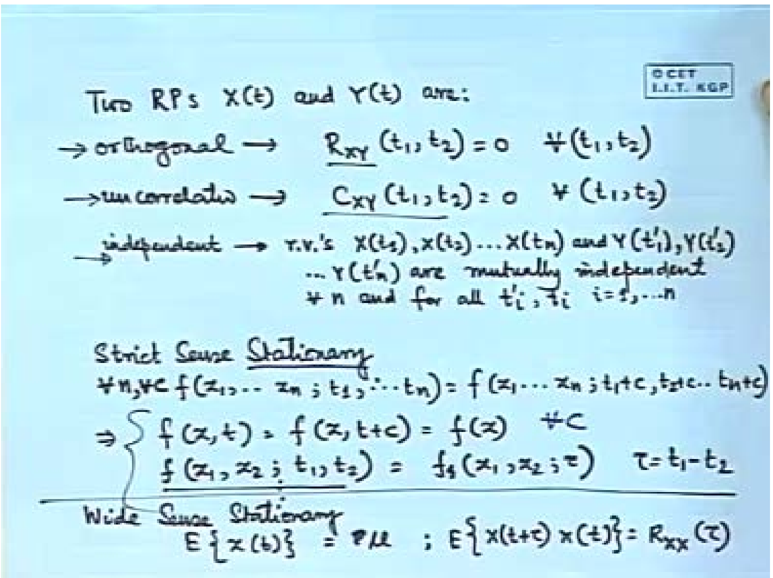
Lecture - 07
Random Process and Liner Systems

(Refer Slide Time: 0:55)



So today we will see some properties of I think we are interested in this, because we are in the course we are going to pump in signals through system. For simplicity we will consider linear systems. So and we will get some signal out, and we will be talking about the properties of the signals that are going in and that are going out. So, we need to know that,, if a random process excites a liner system and produces an output then, what are the properties of the output? Is a v we the properties of the input, right. So, that is a very important thing for us. So we are going to take a first look at that. Before that, remember that we had in the last class we had talked about the fined a random process, discussed.

(Refer Slide Time: 01:46)



We had defined a random process; is defined, two process is defined their cross co-relations. And finally we talked about stationary, because we said that we want to you know we do not want we do not the as such the problem is complex enough. And, if we have; if we have the, you know the statistical properties varying with time then we have a huge problem at hand. So, sometimes we will just you simplify the problem, and so that at least in you know there are I mean; if it so happens that that the stationary assumption cannot be satisfied, then we have to deal with it. Deal with such a complex problem, but there may be various situations where the stationary assumption is satisfied.

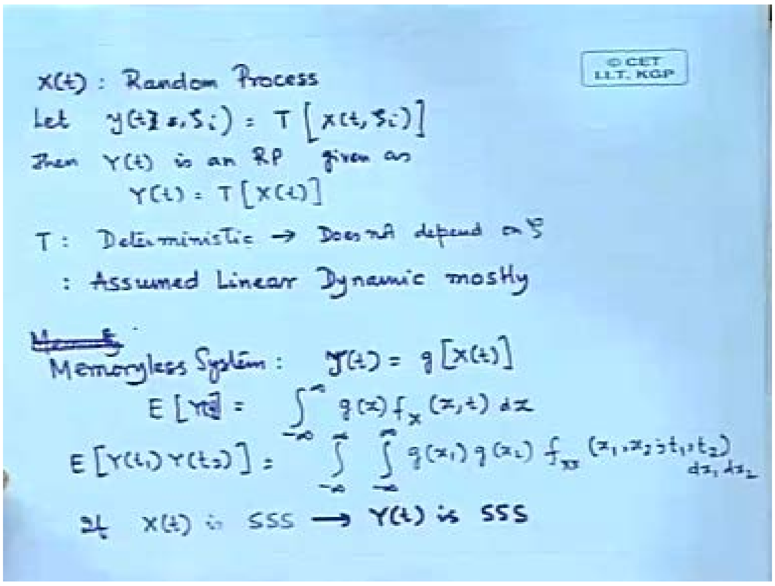
So it is better to see that, under that assumption what, what results we can obtain, right? It is very important to obtain result; otherwise we get nowhere. So we we said that, so we say that so we defined stationary; saying that the probability distribution function becomes invariant to a shift in the time axis. So, if u have effects T plus C; it is equal to effect T for all C right. And, and this happens for, I mean arbitrary degree of join probability distribution. That, you take a one anyone, you take two of them, you take three of them; and you take t1, t2, t3 up to tn times. Then you shift each of these times by C, it will.

So then we do not need to bother too much about, the time shifts that is that is an advantage. And now it is it is it is very difficult to for for processes, to to satisfy this definition; because the fact that you have to consider, nth degree join distributions. So that is even not only that, it is that is difficult to satisfy it is having difficult to check. I mean you cannott say, whether because something is really strict and stationary; it is very difficult to check. So people thought that; okay let me, can I do with only two degrees? So, came the concept of whites and stationary processes. Where the where the mean is a constant and the covariance which is, which will involve only two degrees of joint distribution; will only depend on the time difference, between the first time argument t1 and the second time time argument t2. It will only depend on the time difference, not the absolute time. So is so, if one thing is t1 plus 10, and other is t2 plus t10, it will still be same, and this will hold only up to the second degree. Here, I was telling that, it will that will hold for all n, so so now I do not want that it be.. that it be holding for all n. I would just want that, it.. holding for n equal to up to n is equal to two, okay? So if it holds that way, then I say that it is a white sense stationary process.

So, and then we had shown that, if it is if something is strict sense stationary; and naturally it is white sense stationary, that is a fairly obvious result. But now we are talking of random processes, and talking of functions of random not only function; the actually we had we are going to talk, of operators of dynamic of random process. What is the different between the function and operator? I mean generally a function is a is a memory less operator, that is if you give x you get y; but if you have an operator, then if you if you just give x you do not get y, to get y you need many x's right? So it depends on let us say, if it is a causal operator then then it will depend on all past values of x, only if you have given all past values of x you can determine the output y. Just give in one value of x, you cannot determine the value of y, right? As happens in any system, which stores energy or is or is supposed to have memory right? So now now we are going to talk about; such operators and but that immediately makes us makes the case much much much more complex, so we will restrict ourselves to linear operators.

So just once going back to the going back to our old definition, that is how do we finally, we are we should be able to relate everything from our probability space; that is from the from the experiments.

(Refer Slide Time: 06:12)



So, if we can find that given, x t’s I, x t’s i is a Random Process; so so corresponding to every experiments x i i, every experimental outcome you have a function x t. Now if if if you take an operator, if you apply an operator T on that function, and then you get another function y t. Then you can say that, now you can directly associate this function with this xi i. So now again you can you can that is for each y, you can associate it with an outcome; so it becomes a random process itself. So we say that y t is is a random process, and we briefly say that y t is, this T stands for an operator; so y t is the output when you operate it with T on x t, right? So so it is in this sense, that is remember the sense is that you are doing an experiment. Corresponding to every outcome, you are getting a function; on that that function you are passing through the operator and you are getting another function. And that function is the that function you are finally relating to the to the outcome, so so so that way it is going to be dilated to the to the probability space, right! It is also a random process right.

So, and remember that this function itself is is deterministic, that is it is it is argument is random, but the function is not random. Once you get an argument you are you are applying a deterministic function. It is not that for for one outcome; you are going to feed it through a low pass filter, for another outcome you are going to feed through high pass filter such the such is such as an event does not occur. You have a single low pass filter; let us see and every time you are getting a random signal x t, you are fitting that to the same low pass filter T and you are getting an output. So T is deterministic but its argument or its input is stochastic.

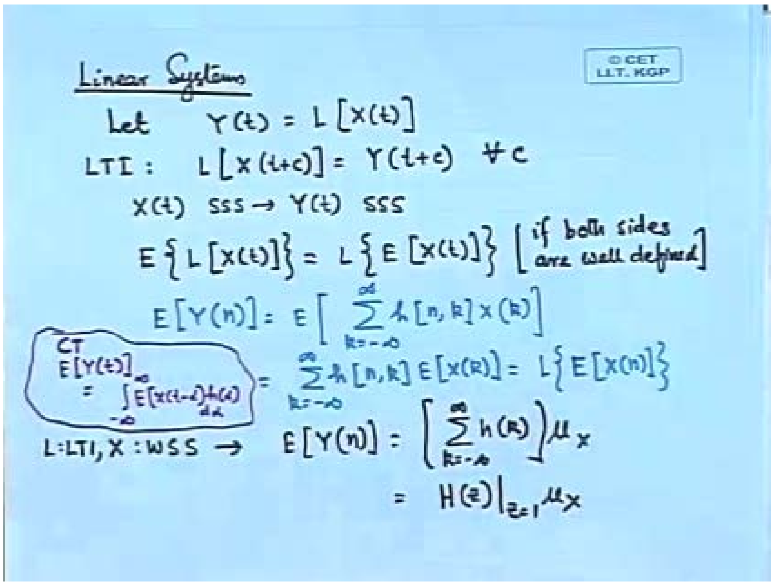
Student< sir T is the operator>

T is the operator, so in this case the whatever is the linear system operator; it could be a convolution operator with the impulse response, normally we get the output y, I cannot cover these things perhaps it is also known that, if you have a linear system then the corresponding operator is a convolution operator, with the impulse response. So if you convolve an input with the impulse response operator, then you get the output right? So so so so this is the sense, and and we will mainly generally assume, linear dynamic operator. See the case of static operators, or memory less system which had just functions is actually very simple. We have we have we have already derived, cases of how a random variable gets transformed to a function.

So now if you if you apply a memory less function, then you are simply taking each value of T, taking the value of x t passing it to the function. So it so it is nothing but a, I mean it is nothing but; transforming a random variable from time to time, I mean by instant by instant. So there is no no no additional complexity in that, so memory less operators are exactly to be treated in the same way, as you treat a function of a random variables. I am simplify every argument T; T into as a random variable because we know that, if you freeze T then a then, a random process becomes a random variable. So so simply applied through a function, we know how to apply functions of random variables, and you get the corresponding value of the process y at that point of T. And then you do it for all T. That is there is no complexity.

The whole complexity arises for memory systems vth memory; because now the now y t does not only depend on x t, it depends on and, in general it depends on an infinite number of past values. If we talk about causal operators; in the in the in the physical world, most operators are causal, but that that does not mean that in the signal processing world all operators are causal. I mean especially in image processing or even in problem, such as smoothing we we we will see I mean we will we we regularly regularly apply operators, which are non-causal. So, in any case causal or non-causal; they will they will depend on a number of values of x. That is what brings the additional degree of complexity here, okay. And it is it is also reasonably easy to prove that, if x t is strict sense stationary then y t is also strict sense stationary. That is that is that is very simple to prove just from this integral, because this remains this remains shifting Varian. This in any case does not contain t. So, this will also remain sufficient h shift in variable; that is very simple to prove, provided g is the function of x, only.

(Refer Slide Time: 11:15)



So now we come to linear systems, okay. So we we talk about, Y t is a now a now it is a linear operator of X t. By what what do we mean by a linear operator? That is, if you take an X one and you get a response Y one. And you get if you take X two, and you get a response Y two. Then A one, X one, plus A two, A two ,Y two, A two, X two, will produce a response, A one, Y one, plus A two, Y two, standard scuppered position, is the that is that that is what is I mean defines, a linear operator. And we have also talking going to talk about, we have, I am in simple case; I mean otherwise, the complexity is will I mean sort of blow up on our face. So we are thinking of a linear time in variance system, so which means that the characteristic; that is let us say the gain of the of the system, that you are talking of that does not change with time.

So if you if you apply if you apply a signal now and you get a response now, you will get the same response if you apply the input two seconds later, and you will get the same response two seconds later. So the gain does not change with time, so the so the system characteristic does not change with time, right. So we we will generally we talking about, linear time in variant operators. Now this is a very interesting result, first important result which shows the linearity of the expectation operator. That is if you take, the take the expectation of the output

Y, it is just like feeding the expectation of X, passing it through the filter. So if you know the expectation of the input, you can calculate you can directly calculate the the expectation of the output without actually feeding each one of them, and then calculating expectation. That is you do not have to calculate, I mean actually perform, how many Y's you got in that range and all that, you you you actually do not have to do it, if you know the properties of X just by feeding it to a linear operator directly you can get the expectation of Y. That is a that is a very significant result. I mean things really, I mean this is why I mean linear is so beautiful, and and and convenient and we spend so much time on linear, because if we did not have these simplicities, we will we will we will I mean it is very difficult to move head.

Similarly in in many cases, you know sometimes in this course we will try to restrict ourselves to discreet time systems, because of the fact that strictly speaking continues time systems poles some very tricky mathematical problems, like I mean how do you define the derivative of a random process, okay? These things become tricky and I mean you can have volumes; in fact there are two three different kinds of stochastic integrals. I mean very very very basic operators like, integrals, differential you have to you have to evolve their own definitions. I mean they have to be defined mathematically in some very complicated senses, I mean strictly speaking. While, if you have if you deal with discreet operators, then you are only only dealing with additions and I mean addition, subtraction, multiplication. So you are you are you can handle things with very simple operators, so that sometimes you know I mean reduces complexity.

So so we will often talk about, I mean present result with discreet time systems. So if you so in the discreet time sense; expectation of Y n is equal to what? I mean, this is a this is a kind of proof of these in the discreet time case. So what is the expectation of Y n? Y n is given by this impulse response operators because, we are taking about the linear systems. So it is h of n k of x k, if you had if you had linear time in variance system, then then this h of n k will be equal to h of n minus K, there will be there will there will be only one argument. So now what you have to do is you have to take the expectation inside linearity. So you can take the expectation inside; and this is the constant quantity because the out system is deterministic, so there is nothing nothing probabilistic about or impulses, that is a fixed function. So so we take it inside the summation and then it becomes this this is a very very simple proof.

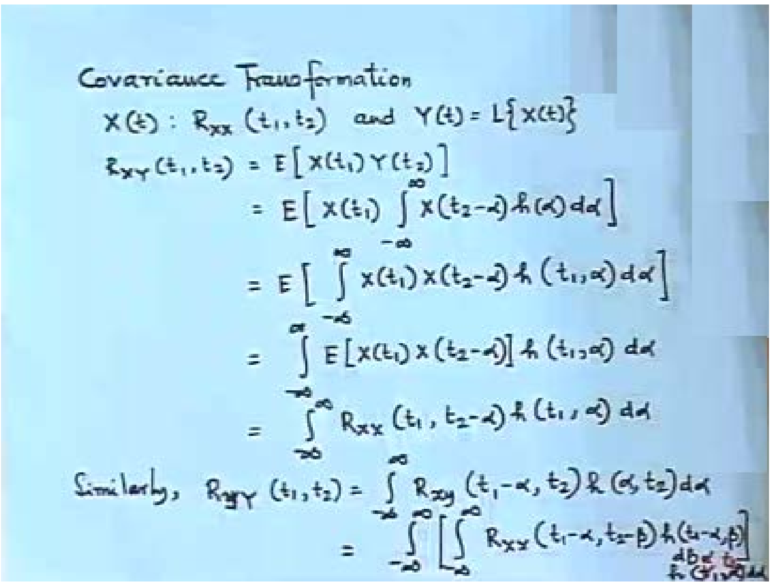
There are again mathematical; settle mathematical complexity especially when the system is unstable, because if the system is unstable some of this limits may not exists you know the sum will not exists and things like that. Suppose you have an integrator; then you suppose this x k has a non-zero mean, and this system this h n k stands for an unstable system, let us say an integrator. So what you are going to do is you are going to sum this x k every time. Now x k is non-zero and you are summing it over an infinite number of times, so it will so that limit will not exists at all. So such complexities may may exists, but for the time being we will we will keep our eye shut down on that, and only talk about you know debostable (16:32min) inputs, for which this this I mean actually the the impulse response sequence will be will be I mean convergent. That that is a result for which, this sum will actually converge. So we will only talk about those systems for the time being I mean, I am just mentioning that there are there are some pit falls mathematically speaking, which you might encounter in the textbooks. But I am for the time being, not not discussing them because then you will go to the depths of maths, okay. If you are interested, you can always read it. It is very interesting.

So it turns out that that that, expectation of Y n will be nothing but this for LTI systems. And now this is sometimes written as; you know if you have if you are, familiar with h Z- transfer functions. Then you can you can easily recognise that; if you get Z-transfer function and you can put Z is equal to one there, then you get what is this in fact it is the steady state gain of the operator, right. So so so if you are given a transfer function of a filter let us say; then and if you feed let us say a… random sequence, which are the mean of let us say one. And if you are given that filter; then you can using this formula, you can at one shot you can determine what is going to be the expected value of the output, right.

So so so that is simple remember remember one thing, that that does not mean that that we are going to feed a feed a DC to that filter right. If you if you feed an actual DC to that filter, you would have also got that output. But here we are not feeding a DC, we are feeding a signal whose mean value is something, signal is still AC and AC means varying, right. So do not make that mistake that, that that that we are going to feed a constant DC signals we are not going to feed that. We are feeding a random process only with a non-zero mean and then; we are and then we are trying to get the mean of the output, so output is also going to be a varying process.

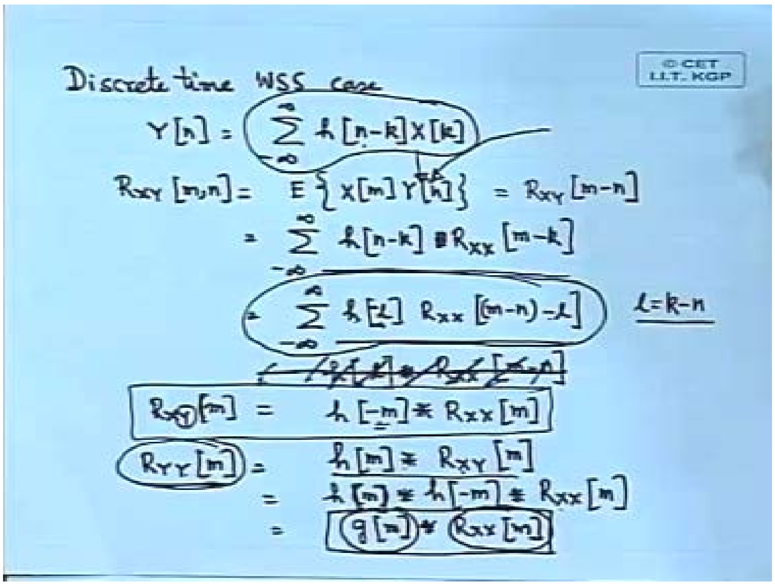
So so having talked about means, we need to talk about co variances. This is one of the bit frightening then, nothing I mean this is you know you if you if you if you really go continue as an let's not let's not probably I will skip this transparency, if it if it is so intimidating.

(Refer Slide Time: 18:46)



So let us come to a simpler case of discreet time white sense stationary process. So what is happening? This is this familiar this is the this is the standard impulse response, I mean convolution response.

(Refer Slide Time: 19:07)



So now if you want to calculate, what is the cross co-relation function? That is equal to nothing but expectation of X m and Y n. And other thing I would like to mention that in many books, you will find that these these random processes are are often assumed to be complex quantities. So so a complex quantity is nothing but, I mean it is something like a two dimensional vector. You know I mean you you you simultaneously deal with two different random processes, one is this real real component and another is imaginary component. And you I mean keep on dealing with them together; this is often done for especially for communication systems, but that again we will bring in more complexity.

So what I have decided in in the is there in our course, we will I mean unless we come across a situation, where it is really needed we will always talk about real variable. So all variables are real nothing is complex, okay. In the book you will find things which are complex; so all you have to do everything is similar is that, if anywhere you find a complex conjugate defined on a time function, simply simply ignore that complex conjugant, because for real variables complex conjugant is the same as itself. So wherever you find h star t, simply call it h t okay? That will give you the corresponding results for real variables; we are considering real variables here just to reduce complexity.

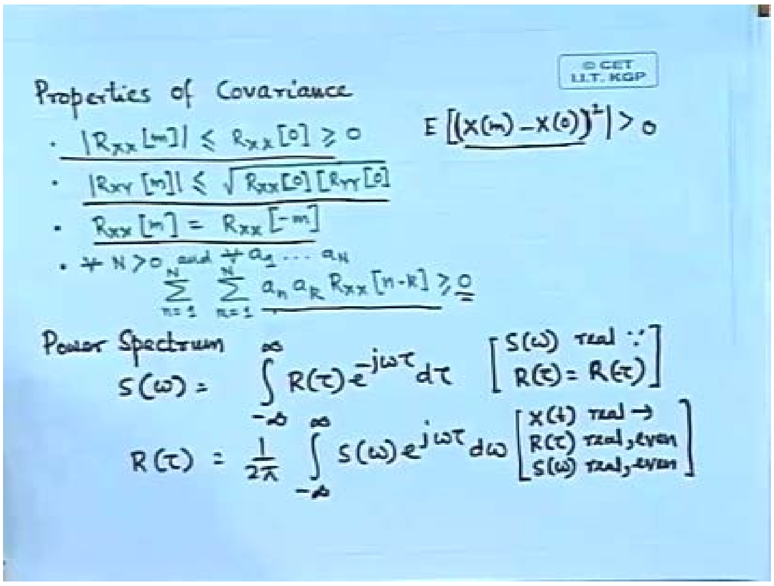
So that is why if you see complex; you will find that R x y, m n is actually defined as X m, Y star n that, is that is the definition of the cross co-relation function for complex processes X and Y. But since we are considering real processes, so it is X m into Y n Y star n is equal to Y n okay? So we need to remember that and obviously, if it is it turns out again these these are there are this is a very well developed subject mathematically. So I have written this; so it is it is very very legitimate to ask that that that how do you know that, if if I mean X is white sense stationary? How do you know that X and Y are going to be jointly white sense stationary? That is not yet been proved, but the fact is that it can be proved. So we will not go into those things and let us, because otherwise we will not be able to see any result. So it turns out that, if if X is jointly stationary and we have a linear system. Then X and Y are going to be jointly white sense stationary in which means; that their cross co-relation can be defined as n minus n, this is the result okay.

So so now now let us come to this, what is what is we know this so. It is nothing but what is going to be R x y, it is h if n minus k into R x x n minus k. This how to obtain this this is reasonably simple to obtain, only thing is that in place of y that is actually in place of y, that is what you have to do is what you have to do is just for this Y n, you you substitute this expression. Then you will get this, and then you put the expectation inside right. So you will get expectation of X m, into this people are feeling sleepy, so you put X m into in place of Y n, you can put this and then, you can bring the expectation inside. That is that is a reasonably simple thing, if you try. So so it turns out that this is going to be your R x y; and if you just play with the indices like for example, if you define a define l is equal to k minus n, then you will find that just by switching indexes from here you can. Suppose, l is equal to k minus n then you then here you have h of minus l. Here you have m minus l, m minus n, minus l, why am I doing it? I am doing it because; I want to get it into a beautiful form, which you will remember easily, okay. So now what is this? This itself is a convolution integral, if you if you realise. So it turns out so this is the beautiful result, that if by convolving X you get Y, then, by convolving R x x equal to R x y.

So as so even the so so the same convolution properties hold, even on the auto co-relation function that is what is the beautiful result okay. And now here why this minus m, here there is no minus m, this minus m because I have transformed the second quantity. If I would have transformed R y x, I would have got h m here okay. Now finally, we are interested in the R y y, that is what is the what is the property of the output with respect to itself. That is auto co-relation properties of the output. Then then this will be now; you see that if we as I said that, if you transform the second argument then you get minus m, if you transform the first argument you get this. That is also an exactly similar result. So if you have R y y, then you can take R x y, and then you will get h m star, so finally R y y can be expressed as these. So you see that the auto co-relation function of the output, you have obtained given the auto co-relation function of the input and the system properties. This is the importance of the result, without these results you can you cannot it is without these result it is nearly impossible to find out I mean, probabilistic properties of the output. Why? Simply because of the fact that; this is what complexity, this is why these results are so crucial.

Because just imagine that; if you really wanted to go from basic principles and tried to find out exact probabilistic descriptions of Y, then for each Y is now a function of an infinite number of X. So, if you really want to find out the the the probability density function of Y, then you have to consider an infinite order joint probability distribution of X. Just for one value of Y, because it is a function of an infinite number of points of X. So that way, if you go you will you will, I mean you are nowhere. So so so that is why essentially; we have to restrict ourselves to first and second order properties, and we will have to use these results. See this result, does not at all go through that path right. That is why it's so useful, and these results will be used very much.

(Refer Slide Time: 26:07)



It turns out that these are these are very interesting to prove, you know you can you can really try to proves of these. That is the auto co-relation function of with any value of m is going to be less than or equal to the auto co-relation function at zero. It is a very interesting to prove and it it is very to prove actually if if you realise that for this must be positive this has to be greater than zero because each each each term is each term is positive, and all probability density functions are positive. So if you take an expectation of an all positive quantity, then its expectation must be positive, right.

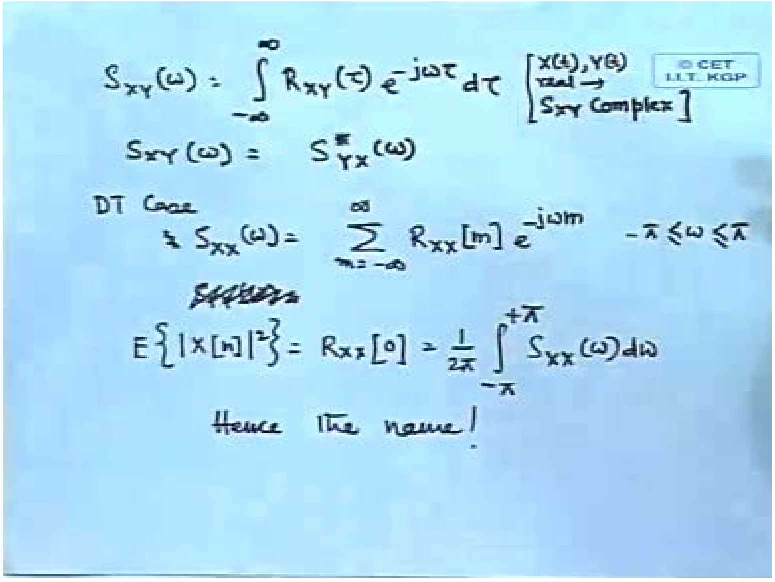
So if the given, this to prove this is simple just open the bracket, okay. In fact you do not need to put magnitude, you can just put simply square. Similarly you can prove that, the co-relation coefficient is always less than one. You can also show that, it is a symmetric function R x x m is equal to R x x minus m. And this is a property which is known, as the positive definiteness property of R x x. Actually if you, put it in matrix form it becomes simpler to understand. In this scalar form this is the relationship. It says that; if you choose any number of constant and then construct this function, it will always be greater than zero. That is the for all for all co-relation function; in fact I mean unless these properties are satisfied, a function cannot be called called as co-relation function. These are some well-known properties.

Now you see remember that; linear systems we are always think talking in terms of frequencies, hardly we really talk about time, why? Because of the fact that our algebra becomes so simple, one of the one of the you know abiding reasons. Because every time evaluating a convolution integral is is not a joke. So it turns out that if you, I mean furrier (28:26min) transform has has has such a nice property that is I mean; that is why out of all the transform, this transform is a popular is because for linear operators the convolution integral becomes a multiplication. So you know everything is simplified, you say you say Y s is equal to G s into U s, right. That is so convenient I mean, you just simply multiply right? Calculus becomes algebra. So so I mean that is why, we are so we are so obsessed with Furrier transforms and Laplace transforms. So so so it will be of natural interest to know that; can we define these these these time domain properties we are decide that we are deciding in this means and I mean, auto co-relation everything is so far we are describing in terms of time, right? Can we can we talk about it in the frequency domain also, right?

So so the first thing that is done is that, we define what is known as a power spectrum okay. We realise that, this R has not after you have taken the auto co-relation function; it has now become a function of tau. See if it is a function of tau, we can always take it as furrier transform. Any function f t under certain regularity conditions; under the regularity condition that this integral will actually converge, it will have a it will have a valid furrier transform. So now the question is that, what will happen if we define a furrier transform of an of an auto co-relation function? That function is given the name of power spectrum, why? We will see, why this particular name power spectrum?

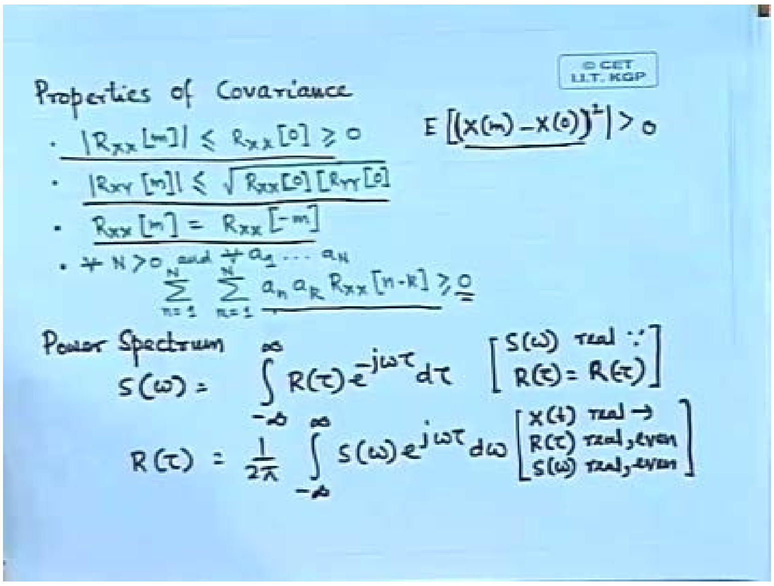
So at this point of time, power spectrum is nothing but the furrier transform of of auto co-relation for an LTI system, correct. And obviously; if you have an if you have a furrier transform, you can always have an inverse furrier transform, right? So so given the power spectrum, you can also get back the auto co-relation function with these integral. This is just furrier and inverse furrier; and incidentally this there are there are these these two are defined generally in all books in the same manner, except for this factor sometimes people define it, that is overall you have to have a one by two pi factor. Sometimes some people put, one by root two pi here and one by root two pi here and all. That that is that just matter of you know adjusting a factor, so sometimes you may have a slightly different definition of furrier transform. We will we will work with this this is more common.

(Refer Slide Time: 31:07)



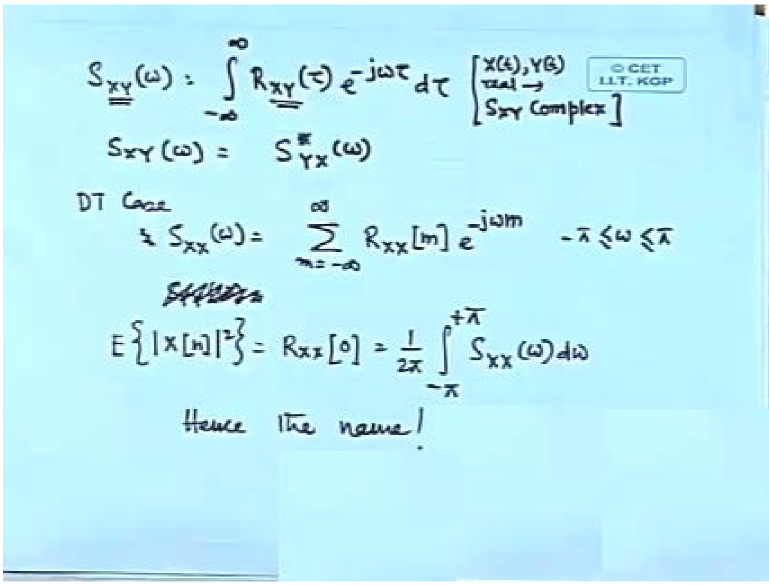
So now you have for example, similarly so here we have defined this is this is the general. Now, now the question is that this this auto co-relation function, this this co-relation function is between what and what?

(Refer Slide Time: 31:18)



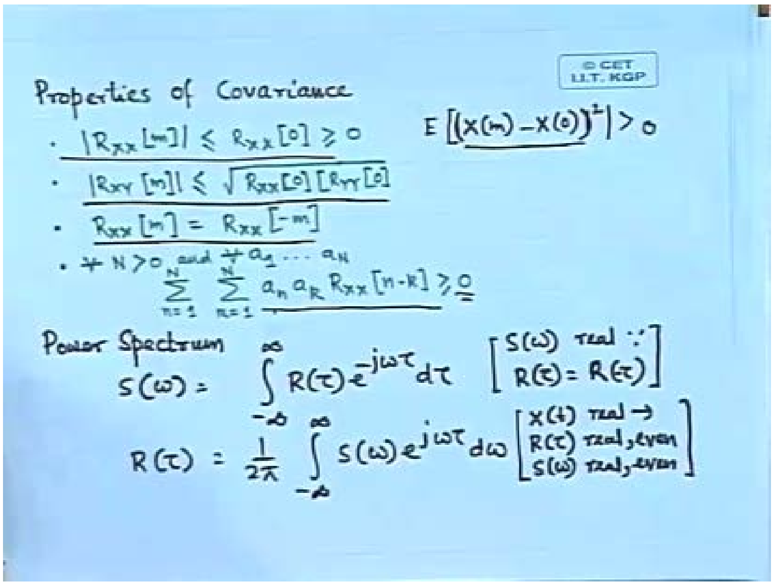
So you can define again between X and X, Y Y and Y, X and Y various kinds. And so you will get the auto co-relation, I mean cross power spectrums, auto power spectrums, etcetera various kinds of thing. So so you so you can define, this S x y omega; so if you define a cross power spectrum, then you take R x y tau.

(Refer Slide Time: 31:47)



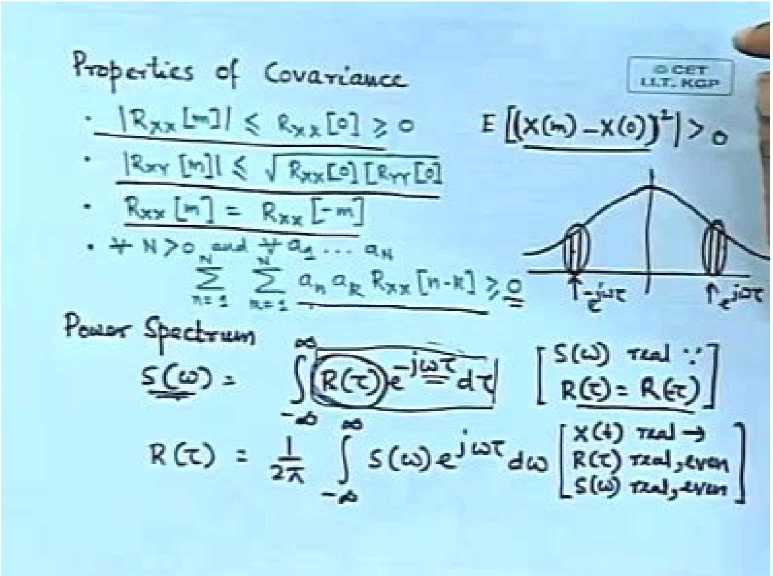
Now it is it is it is reasonably easy to find out that, S x y omega is S star y x omega. You know that is incidentally, if this S omega is a function of omega, what sort of function? Real? Complex?

(Refer Slide Time: 32:06)



It it will be a real function of omega. Why? See inside we are having complex integrals, furrier transforms are always give us give us I mean out of real things we get complex things right. X t is a real function, but x y omega will be complex. But here we are going to get a real function, why? because of the fact that, this R tau is symmetric, so R tau will be equal to R of minus tau.

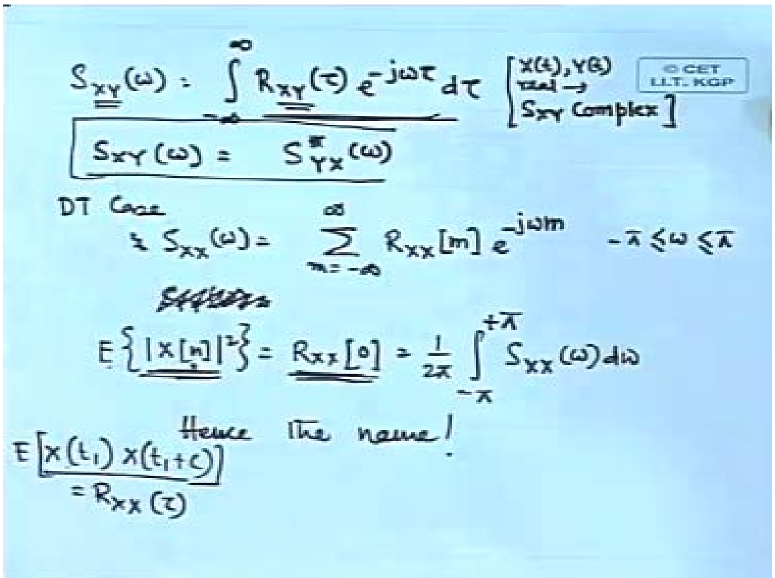
(Refer Slide Time: 32:36)



So for so if you are integrating; between x and x plus dx on the right hand side, positive side, take the integral on minus x and minus x minus dx, R tau is going to be the same. Now you add them, so so R tau is going to be the same but but but these two terms will now give you two complex conjugate quantities. So if you remember; do you understand, what am I saying? Suppose you are suppose R tau is a function like this, now what are you doing? You are actually taking this and this and then, multiplying this one by into the power j omega tau, multiply rather minus j omega tau and multiply this one by to the power j omega tau. This is tau and this is minus tau. So the so the integral term, that you will get here is going to be the complex conjugant of the term that you get here.

See if you add them up because; you are integrating some minus infinity plus infinity, so if you consider such pairs of terms then those complex then if you add two complex conjugant quantities get a real. So all this pairs will will add up, and keep on cancelling the imaginary term, so you will get a real integral. That is why S omega is real always, it is not a complex function, right. So that is because of this symmetry. So this is on the other hand, if x t is real it will be an it will be an even function. Even function definitely even function; because of the fact that, two the power minus j omega tau will be equal to cos omega tau plus g sin tau. Now now the sin omega tau tau will get concealed, so it so so what will remain in is only only cos omega tau, so cos omega tau is an even function, correct. So so this is fine.

(Refer Slide Time: 34:47)



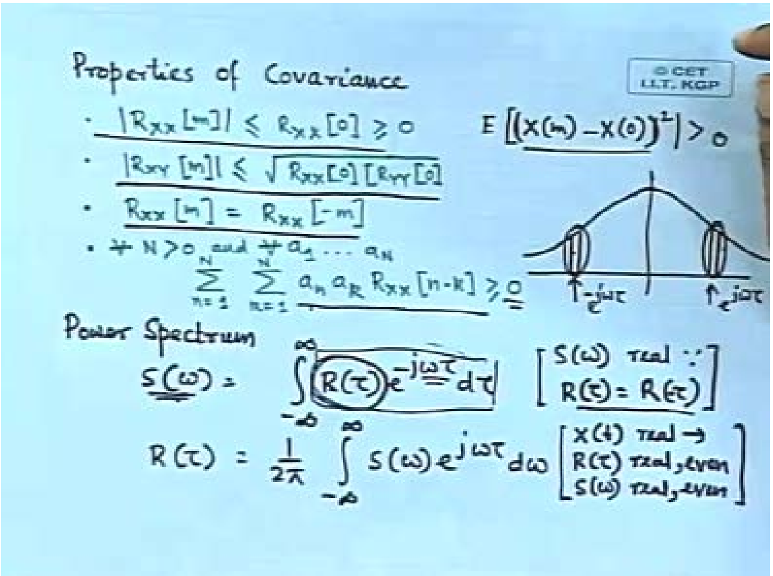
Now but but S x y omega may not be; S x y omega it is not guaranteed that that that, it will be real that may be complex. Very well because, why?

Conversation between Student and Professor – Not audible (35:11)

So so so simply because, of that fact so so you are not going to get real parts, imaginary parts will not cancel. And this you can this you can easily find out that, S x y omega is equal to S star y x omega, why again? because of the fact that, if you if you go back to this definition in case of y x this will become R y x ;and remember that that we had said that R y x you will get from R x x, if you then you will have to convolve with h m. If you do x y; you have you have convolve with h of minus m, so so because of that minus m here here you will get a complex, here you will get a complex conjugant quantity. Just just you can try writing this, I mean writing writing everything is time consuming, but simply writing the integral will give you that this will become a complex conjugate quantity of this.

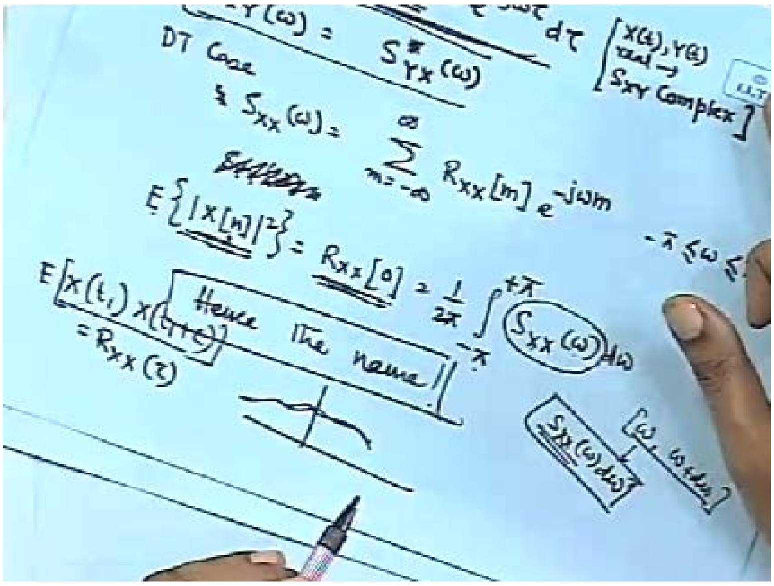
Now now let's come back to the old question that, why is it suddenly called the power spectrum? What has power got to do with it? It is called the power spectrum because; see what is the power, the power is the expected value of x n square. That is you take every sample you take, that is generally defined as power. I mean we are not talking of v i kind of power, we are talking of generally the power of a signal x t is defined, as especially for a random process you know power has a sense of square, is not it? So so if you want to have an expected value of power, then you have to take the expected value of x n square. Now what is the value of x n square? That easily you can find out that it is R x x zero, just put tau equal to zero in in R x x tau. Then you are doing x t one, see R x x tau is what? R x x tau; you are taking the expectation of x at t one and same x at t one plus tau, this is equal to Rxx tau. So you put tau equal to zero. So you will get expectation of X t one square, and this holds for any t one. So if so if you want to get the power of the signal, it is nothing but R x x zero. Now you, put R x x zero, in this that is here you put tau equal to zero. So rather rather rather in the inverse, so here you put Rx here you put tau equal to zero.

(Refer Slide Time: 37:56)



So if you put tau equal to zero; this will becomes one. So if you put so if this becomes one, then my R x x zero is nothing but, one by twice pi minus pi to plus pi S x is omega d omega.

(Refer Slide Time: 38:07)



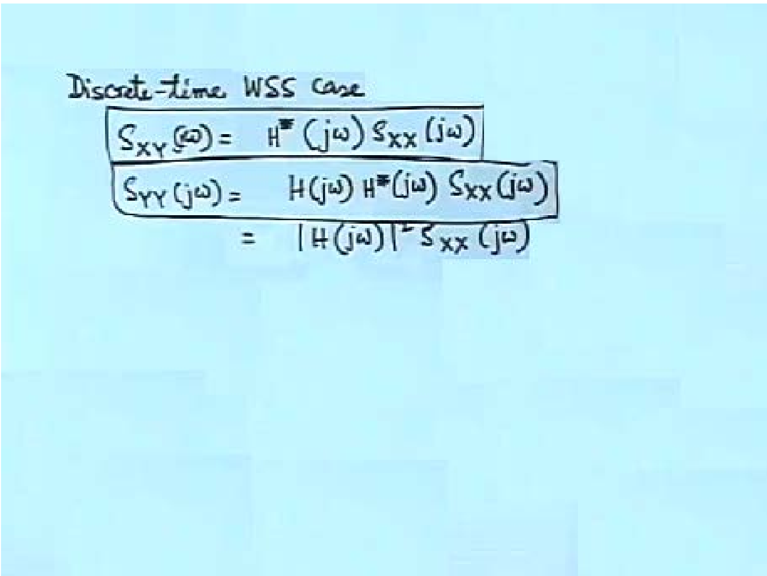
Which means that; this S x x omega function, if you are, if you if you see its area under the curve, then you get the total power, right. So so in that sense, the function stands for what? It says how the power is is distributed over frequencies. So if you integrate from one frequency to another; what the whole range of discreet frequency is minus pi to plus pi, then you get the total power, right. If you want to find out, what is the energy of the signal stored between omega and and omega plus d omega, that will be S x x omega d omega. This amount of energy is actually stored, between the frequencies omega and omega plus d omega. Well it is the continue of variable; so you cannot say that what is the power at this frequency that you cannot say, but you can always that what is the power between omega and omega plus d omega.

So you see that this stands for this just like, the probability density function says what is the probability that it it lies between x and x plus dx? Here you are asking; what is the of the total energy in the signal? How much of energy lies between five hertz and five point zero zero zero zero one hertz? So it is nothing but a density of a power. And and this density is now expressed over over it is spectrum; that it is it is a it is a function of frequency, that is why it is called the power spectral density. So hence the name, it is come from this. Now you it is very natural to ask that; if I have a signal which will have a certain power spectral density and then give it as a input, then what is going to be the power spectral density of the output, right.

You will ask that suppose I, I am giving a white noise, you have not yet defined what is what is what is white noise, we will. Suppose I, I am given that the that is the input random process, has some power spectrum density, okay. So it has some power spectrum density, so what is going to be power spectrum density output process?

Obviously; I mean common sense will tell us that, if you are sending it through a low pass filter, then the power spectrum density will will will probably be preserved around the low frequency zone. And it will be much more reduced around high frequency zone, because it is a it is a low pass filter. So the high pass so the high frequency signals will be attenuated, so their power will be reduced. So you see that the filter characteristic has has something to do with the power spectral density of the output; that is a filter characteristic will actually transform the power spectral density of the output, but why? But how, that is a question.

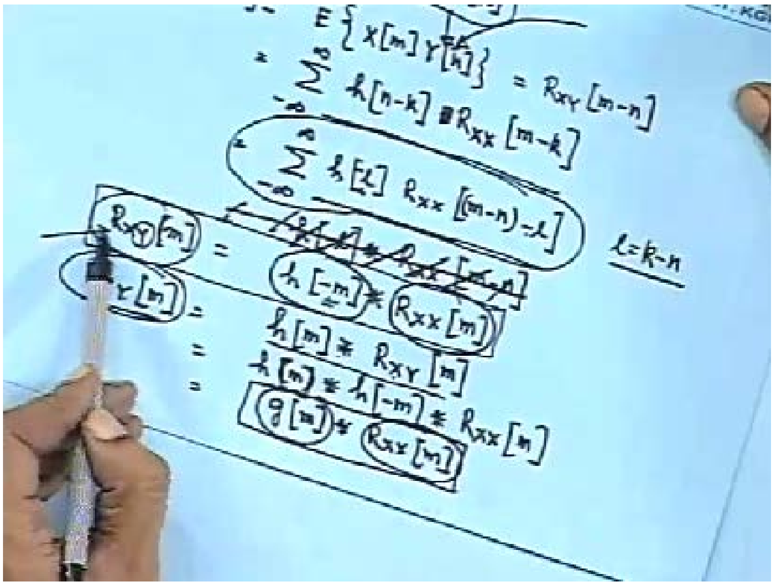
(Refer Slide Time: 41:20)



So so so that is very simple to obtain, these are these are these are very beautiful results. So it say is that, S x y j omega is H star of j omega into S x x j omega. That is if you, this is a cross power spectral density. And finally; S y y j omega again all these these relationships are can be obtain, just from that basic convolution integral, right. That is I will show you from where; from here.

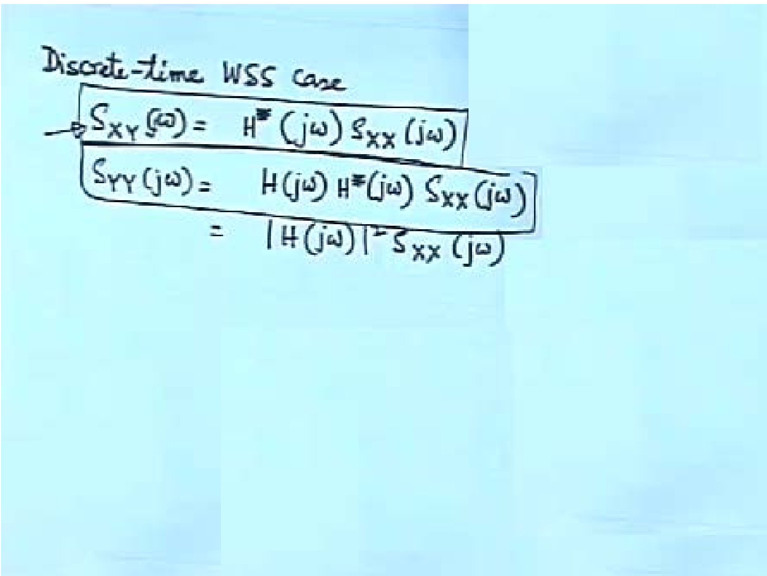
See R x y m is h of minus m into R x x of m; so we know that if two quantities are related by their, if if two quantities are related by the convolution integral, then if you take the furrier transform of those two quantities then then will be a product that we know.

(Refer Slide Time: 41:54)



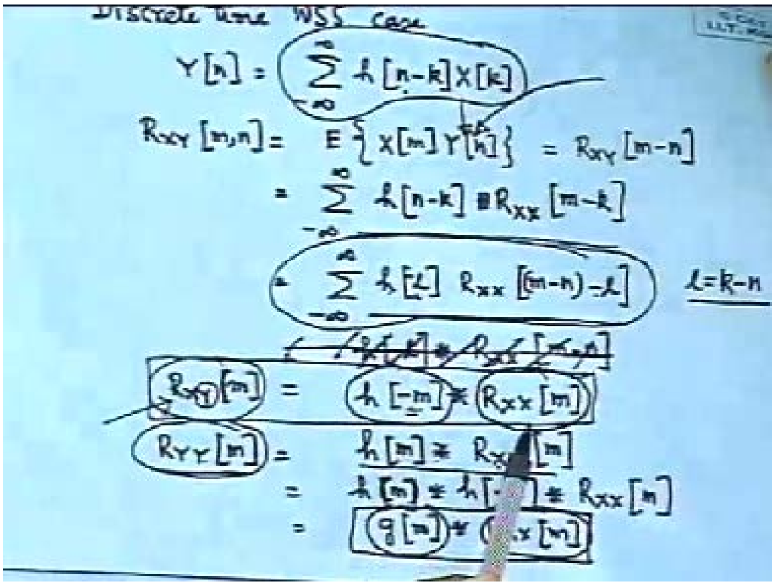
So the furrier transform of this; is is going to be the furrier transform of this, multiplied by the furrier transform of this. So what is the furrier transform of this?

(Refer Slide Time: 42:28)

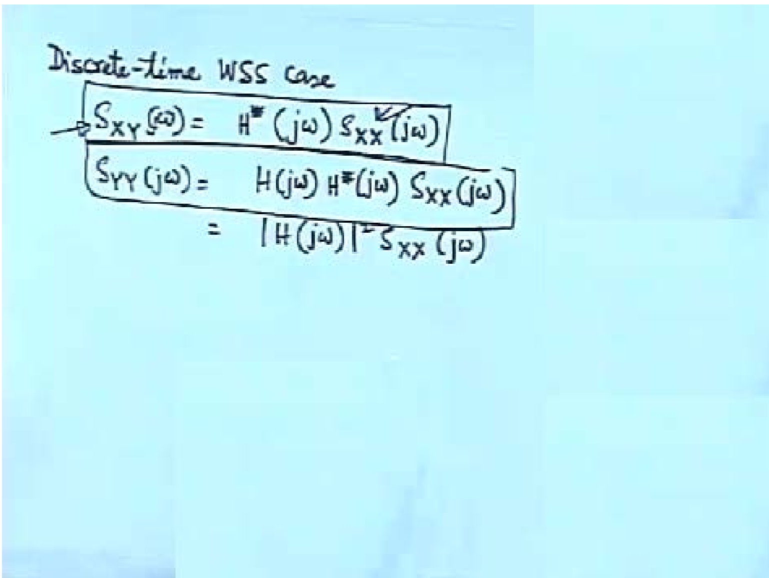


It is S x y j omega, by definition. And what is the furrier transform of this?

(Refer Slide Time: 42:34)

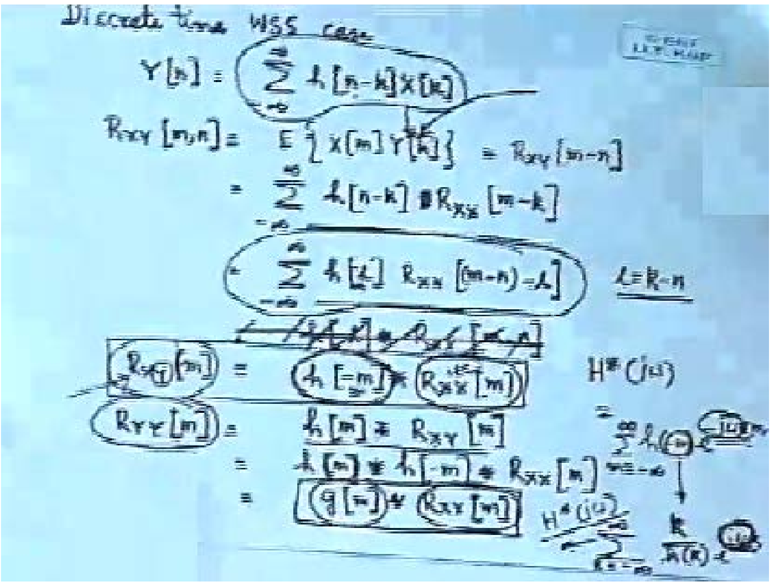


(Refer Slide Time: 42:36)



It is this, by definition. And what is the furrier transform of h of minus m? It is h star g omega, because of this minus m, right. Not following?

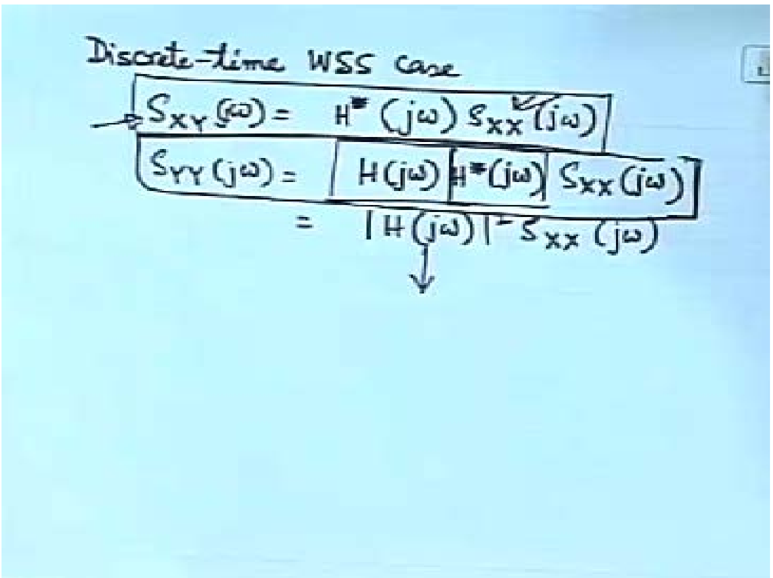
(Refer Slide Time: 42:38)



You can you can just put it, and then put put omega equal to minus omega. See if you want to take what is h star g omega? h star g omega is is nothing but, the the I mean in place of e to the power g omega tau, you have to put minus g omega tau. So you have to take the complex conjugant, right. So so it will be now h of minus m; e to the power minus g omega tau and then sum of because, it is the discreet thing so we did sum, so we will take m is equal to minus infinity plus infinity. Now you put, let let minus m be equal to k… Then again you will get k equal to minus infinity to plus infinity, h k e to the power now you will get j; no this this is this will be j omega m, so it will be plus j omega k, so it is like so you see in in place of minus j omega you have put j omega. So you can equivalently imagine that; in place of minus j omega you have put plus omega, so you have taken a complex conjugant. So then this will simply become h star j omega, just a complex conjugate. So you just manipulation okay, very simple mathematical manipulations.

So that is how you get S X y j omega is H star and then S y y j omega by the same formula, it will be now H j omega into S x x j omega.

(Refer Slide Time: 44:19)



Because now we are transforming the first argument; so it will become H it will not be H star, because now you do not have H of minus m, you have H of m. So finally if you multiply this; you will get these two you multiply together you get, magnitude of H j omega square and this is. So now you have you have a established a relationship that, the power spectrum of the input and the power spectrum of the output are related by this quantity. This is the magnitude of the H j omega function whole square. And this makes sense, why it makes sense? Because the H j omega function let us say, for a for a for a low pass filter at at at high frequency, this magnitude is going to be low. Remember, you know bode plots of low pass filters, the which is the first order filter will come down with six per octave, if, it is second order twelve per octave. So so it comes down as frequency increases.

So at high frequency; this this magnitude is going to be low, and at low frequency this magnitude will probably approach one, or if you have an amplifier it may even more than one. So the so so this means that, at so for a for a low pass filter intuition is satisfied by this formula. So at low pass, this is going to have a lot of power and a high pass it is going to have a less number of power, amount of power. So we will stop here today, we are we are we are gradually approaching our actual subjects and in a in a in a class or two we will we will probably landing filters. Actually I have to see a little whether; what to what to cover first, I have not yet fully decided whether to cover principles of estimations first, and then come to filters. Or but probably you are getting a bit impatient; you want to actually deal with voltages and currents or signals in that case, it may be better to start with filters. We are we are I understand that, we are dealing with abstract fearsome looking integrals for a bit too long. So maybe I will into filters that, I have to decide. So that is all today, thank you.