Estimation of Signals and Systems

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Lecture - 06

Random Vectors Random Processes

So now we will continue from where we left today. First of all let us... so we are today we are going to complete little bit was left on the random vectors and then we will go to random processes okay; that is a major concept because throughout the course we will be dealing with random processes. So at first recall first we had confusion.

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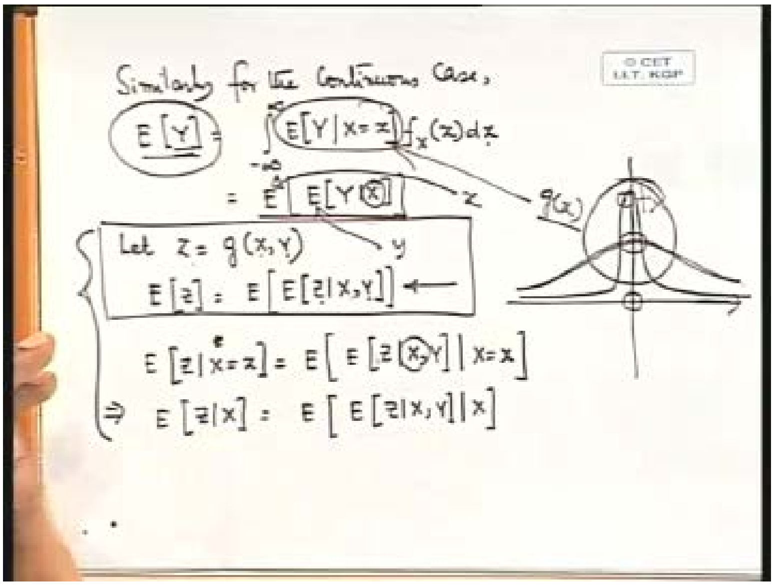

Actually that confusion was created today when I opened the book to see what was the confusion I got that I got the meaning in roughly 5 seconds; just looked at the book and I knew what I was where I was wrong; where I was is this that is if Z equal to g x what we were saying yesterday that if Z equal to g(x, y) and then once x and y are given Z is a deterministic function there is no there is no expectation about it. I mean if you take expectation you will get that function. So that was that was our major confusion. So actually this is not... what was written in the book is actually I misinterpreted, what was written in the book is that... I mean a sentence like this that expectation Z X given XY is a function of XY therefore it is a function of XY, I mean one same same clause was repeated twice in a sentence so I... the first one I interpreted like this that is not correct.

So if you forget this then then this is very clear that expectation of Z given XY is a function of XY okay. Z is not a not a Z is not a deterministic function of XY, Z is some random variable, X and Y are some other random variables alright, so then if X and Y are given then you will get a different probability function different statistics. So if you take the expectation of Z given XY remember that this expectation will be a function of X and Y and then if you take this expectation this expectation is what; this expectation is over X and Y.

Whenever you see these expectation operators you have to mentally determine that this expectation is being taken over what, over which random variables, which probability space; that is very important to think because in general in the books they will not write that, they will just write E okay. In this case the... so basically this is nothing but you can say that, what is this? You can write this as: integral z f z given X, Y f z f XY (x, y) dx dy dz this will be there will be there will be three integrals because finally what you are going to get is a number; look at this, finally what what you are going to get is this one. See the first expectation is z, what is the expectation of z given XY? It is z f of (z) given XY which is a standard formula; what is the expectation? X f X dx.

Now this... so the first one is if you put the dz here then you get the first one. Obviously it is going to be a function of XY. Now if you want to take expectation over both X and Y together then this thing will come because because this is the probability that X is between... capital X small x plus dx and capital Y is between small y and y plus dy so that you have to now take expectation that is not expectation you have to now... yes, so this so this thing now you have to vary over the probability of... that is you have to do it for all possible values of X and Y so that is why another set of integral will come, these, which will consist of these two terms (Refer Slide Time: 5:32) okay.

So that is what is being written here. There is no confusion about it, only confusion was that this Z equal g (x, y) that I wrote interpreting the book and I was in error okay. Once you do that then for example here here expectation of z given capital X equal to x. So one you are now freezing, what is the difference between these? Here I am saying that if some value of X is given what is the expectation of z, I am not saying what value; I am just saying that if some value of X is given. So then what value of what will be the expectation of z, obviously depends on what value of X is given that is why it is a function of X. but here you are telling that the value of X is given, already told, so if the value of X is told then what will be expectation of z given that value of X right.

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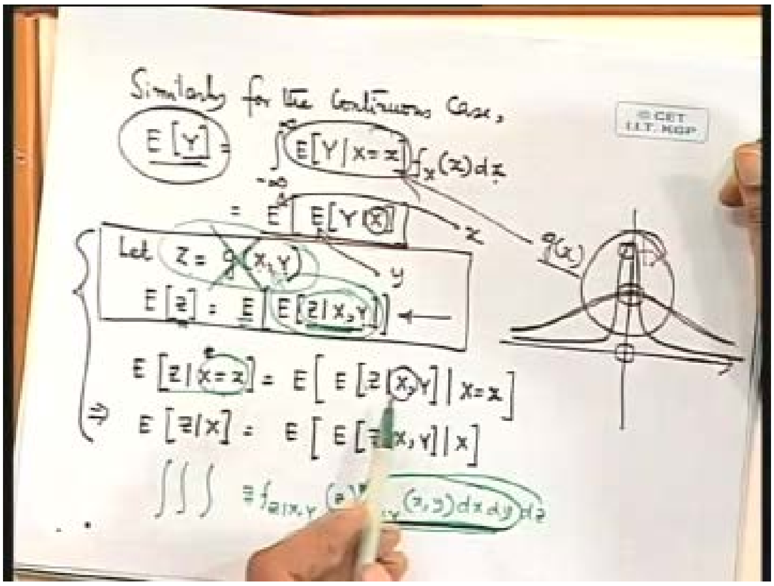

So now this will be this is a function of... what is this, this is a this is the expectation of z given some value of X and some value of Y. Further if you give that the that value of X is this then you give then you get then you get this outer bracket thing. So it is expectation of z given X Y in our function of X and Y given that X is equal to x further another piece of information is given. That is not only that function you are calculating, now you are putting the value of X in that function so then the then this becomes a function of Y.

Now if you take this expectation so then if you take this expectation that means you have to now consider all possible values of Y, so this expectation is now over Y.

[Conversation between Student and Professor – Not audible ((00:07:39 min))] then what you finally get, this it should be a number again. So so so this is the interpretation. So always remember that you have to know that where, over what you are taking the expectation. So I think this is clear now. So this point is clear.

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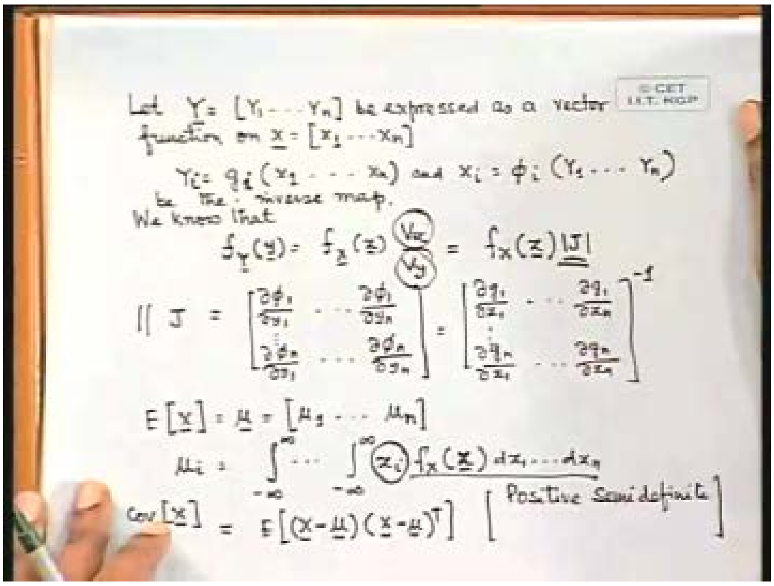

So now we now we continue from where we left okay. So we were talking about vectors and we said that as far as the probability is concerned; obviously vectors have certain algebra, vectors have certain kinds of certain kinds of objects you can add them in a certain way, you can multiply them in a certain way, you can invert them in a certain way so there are there are there are there are operators are associated with it, there are rules of working with such objects and they are not they are they are corrections of numbers but they are not exactly like sets which is like putting the numbers in your bag without any order but but which is like putting them in slots you know, this is X 1 this is in the first slot, that is X 2 that is in the second slot so the so the place is fixed, while in the set there is no place it is just all in a bag.

Okay but as far as the statistics is concerned they are pretty much like having N random variables together, so you have to always think of their joint statistics right so that is what we discovered and then we were trying to see that, first we saw that if we are given one random vector X and if we are given another random vector X both of which you have defined on the same probability space and they are also related by certain certain functions like phi and phi and G then how to... then then then given the given the sort of you know effects as that is given the probability distribution in the in in in the the space of X 1, X 2, X n how do we get the probability distribution of Y 1, Y 2, Y n that is what we saw and we saw that we have to we have to take the ratio of the volumes in the n dimensional space here and the rest,these things we discussed in the last class using what is known as Jacobian and the determinant of the Jacobian etc.

So we did roughly up to this, I think this then then you have to... how do you calculate the expectation of a vector X? It is nothing but the expectation of the elements. So this is the mu mu n where mu i is equal to expectation of x i. so you take element by element expectation nothing else. So the expectation of x i is again defined by this formula which is where you are keeping this x i and all others you are you are averaging on and okay.

Then then then then we defined what is the covariance of X, this is an this is an important quantity. Many times we are we will be evaluating this quantity for vectors covariance. So the covariance is defined as it is the... variance is square deviation from the mean as we know. Standard deviation is root mean square deviation from the mean. So this is deviation from the mean, this into this for for for scalar you do square and for vectors you do this into this transpose. So what happens? Actually... this is a this is written in vector notation but actually what are these quantities? It will be a matrix, this is a column vector. Actually we should not write it, actually what I have written is wrong. This (Refer Slide Time: 11:39) should be written as transposed because there are two kinds of vectors: one kind of vector is vertical they are called column vectors, the other kind of vectors are horizontal they are called row vectors.

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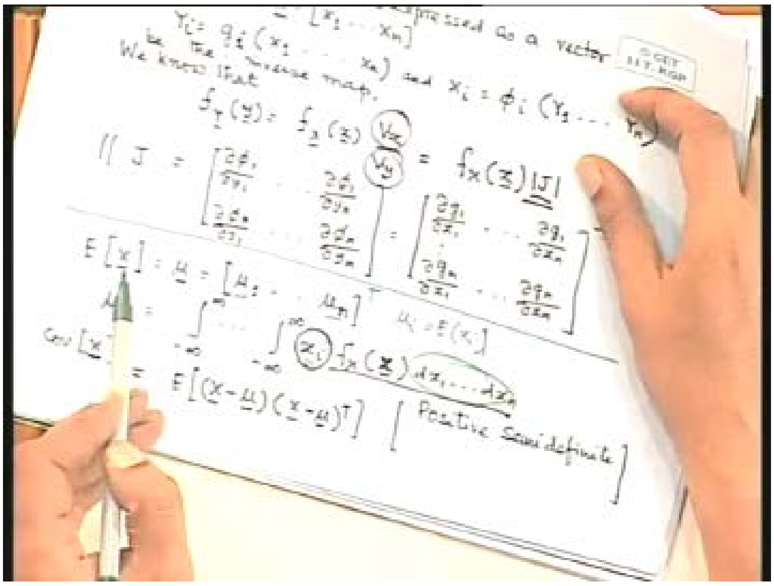

In this case I am assuming that X is a column vector. I have nearly written it like this because the page is like this otherwise it will take a lot of space to write that is why I put transposed. So now what do you get, if you if you multiply a vector like this into like this you get a number right this is called a scalar product; this into this plus this into this plus this into this you get a number because it is this is 1 into n, this is n into 1 so by matrix it will be 1 into 1 so it will be a scalar.

On the other hand, if you multiply this with this you will get a matrix because this is n into 1, this is 1 into n so you will get n into n right. So so this is a matrix. And what are its elements? So what you are what you are multiplying it is actually... can you see this okay, what I am writing you can see right; so what you are multiplying is x 1 let us say x 1 x 2 x 3 you are multiplying with y 1 y 2 y 3, so what you get is x 1 y 1 x 1 x 2 y 1 x 2 y 1 is here...

[Conversation between student and professor: 13:27] Sir, elements of those vectors will be same... yeah..... elements of those vectors should be same, means? No no I am just giving a general example why the elements of those vectors should be same.

[Conversation between student and professor: 13:34] Sir, in this case x minus b.... right, in this case it is defined like that, I am just saying that if you multiply a column vector by a row vector, in this case x 1 is x minus mu, in this case y 1 is also x minus mu it is a special case okay. So x 2 y 1 x 3 y 1 then x 2 y 2 x 2 y 3 x 1 y 2 x 1 y 3 and so on. So in general the the jth element is x i into y j alright?

So here also what am I getting; I am getting the expected value of the deviation of the ith component of the random vector x multiplied by the jth component of the random vectors same random vector x. So what am I trying to see in a way? And then I am taking expectation... so in a way what I am trying to see is that these these these various components of the random vectors they are they are they they... in general most signals you know... this is very important because in practice whenever you get... suppose you you are taking it from from from from circuit or something so you will... there will be a strong deterministic component which is which which arises from the circuit, the way you want to use it etc etc., on that you are going have some unpredictable thing happening which you do not know why they happen so your signal is at generally going to look like this right when you are when you are actually talking about signals.

So if you so if you have two such components; suppose you have another component like this which is which is the other component of the vector then what you are trying to see is that at what you are trying to see is that how are these increments on the mean, this is this will be in general the mean. That is if you see mean has again two interpretations. Often when we talk about a deterministic signal and say its mean... we general... we often mean that that we are talking about its time average. That is we take averages of time and then average it over time, that is a kind of mean. In this case what are we talking about? When we say mean we are talking about experiments, so we are talking about ensemble average. That is these are two different things. So when when we talk about means here and expectations here, expectations are also averages but they are ensemble averages. That is if you find if you create two hundred instances of the same thing by doing repeated experiments then you what you get is an ensemble; some people pronounce it as onsemble okay.

So you take an average over those okay. So what is means is that you first conduct the experiment one day you get, suppose you get these curves then you conduct the experiment on another day and you get maybe maybe the same curve but now the now these things are going to be different on another day, so you do these experiments and then...

Suppose you take at any point of time, suppose you take the increments...this pen does not write okay. So so what you are saying is that these variations about the mean in the component x 1 and the variations about the mean in the component x 2 are they correlated that is if one is positive does the other on an average is it positive also that is what you are trying to see when you are seeing covariance, this is very important right you are not talking about their means because means you have already subtracted. We are only talking about that random part which is beyond the mean and you are trying to see whether that random part is correlated or not right. So so so that is the physical meaning of this and we are we are in many cases interested in that quantity.

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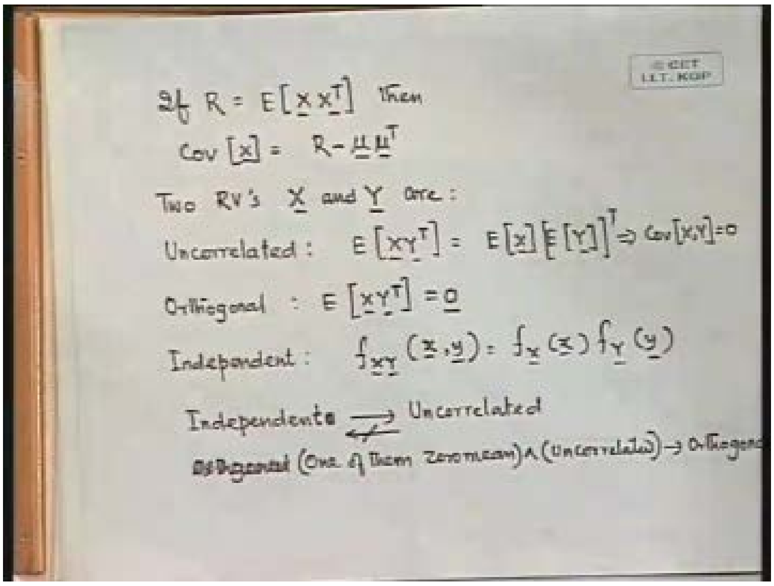

Similarly you can define what is known as correlation from R E say here we have not subtracted them. So when we take correlation we directly take XX transpose, we are trying to see the whole signals and you can easily just by simple algebra you can prove that covariance of X is is that is covariance is related to auto correlation like this. This is very simple algebra; just X minus mu into X minus mu transpose you open out by... by explicit multiplication you will get this.

And now there are three different concepts which are sometimes used. For example what is meant by two signals uncorrelated?
Two signals are uncorrelated means that expectation of XY transpose is equal to expectation of X into into expectation of Y transpose. That is this is equal to this (Refer Slide Time: 19:10). So that means that this equal to 0. If... what is this, what is this correlation actually this is this is sometimes called auto correlation. In general you can you can calculate this for different signals when both places you write X, you are finding the correlation of one signal with itself that is why you have written X in both places, that is why it is called auto correlation.

In general you can write X here and Y here or Y here and X here in which case it will be called cross correlation okay. So so so in general if you talk about... so if the if this cross correlation is equal to mu X into mu Y this is mu X into mu Y transpose that means this equal to this that means this equal to 0 and in that case it will be called covariance of XY. So then that equal to 0.

What does it mean is that the... covariance XY equal to 0 means that the increments of the... it is zero it is zero means what? It is zero means it is the zero matrix; covariance is a matrix. What is meant by a matrix is zero. It means that every element of it is zero. It means that if take if you take an arbitrary component in X and take its variation from its mean and take the take another arbitrary component in Y and take its variation from the mean, if you take their correlation and then take expectation it will be zero for all components any pair of ij's right.

If it is orthogonal then expectation of XY transpose itself is zero right? So then this itself is zero which generally means that it does not have a it will probably not have a mean because if these have a mean then their then their correlations will tend to be... at least at least one of them will have to be zero mean otherwise getting getting a correlation of, if they have a persisting mean then getting their correlation as zero is difficult.

So expectation of XY transpose is zero then it is called orthogonal. Actually two vectors are called orthogonal I mean two vectors are called orthogonal if then their projection becomes zero. And there is something called independent. Independent means that probability distribution-wise they are independent. That is the joint probability density of X and Y can be obtained as simply as the product of the two joint probability. That is... which means that that in the in in the in the underlined experiment space these two experiments have no relationship, have no bearing with each other, may be you are tossing a coin here and then you are casting a dye here and then you are combining the results to make your experiment.

Tossing a coin has has no relationship with casting the dye, so which face of the coin comes up so I mean saying that what is the probability of having a head and a 4 coming upon the dye is simply nothing but the probability of having a head multiplied by the probability of having a 4; they are disjoint events so their probabilities are multiplied right? So in such a case you call them as independent. These terms will sometimes be used so that is why this is the meaning.

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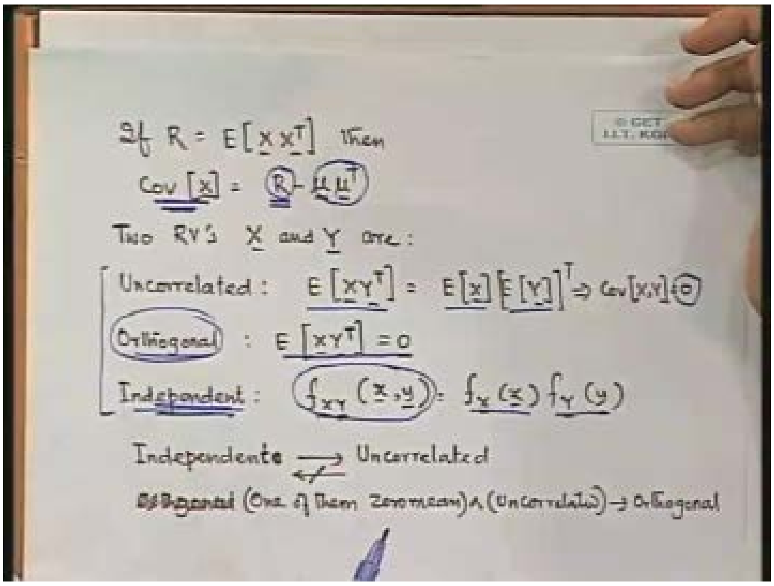

Now if you have... see this is the most strong condition, if this happens then... so so independent will mean that they are that they are uncorrelated. Obviously because because they are coming from totally different experiments which have no relationships and the experiments are random and you are subtracting the mean so then what is happening around the mean is they are not related. So if you take many cases and you average it will come to zero right? But but but uncorrelated will not mean that they are independent that may not happen. So these are... we will we will see them in much more detail but these are some terms which are used.

Now we come to a... one of one of our very important concept that is the concept of a random process. See first we said that the underlined space is always the same that you are performing some experiment and you are getting outcomes, those outcomes are x i's okay. Now now in the case of random variables what you did is for every for for every outcome you you picked a number X. So for every xi you picked a number X on the real variable and you said that therefore this this real variable is the random variable. So remember that for every outcome you are picking a number in the case of random variables.

In the case of random processes for every outcome I will I will pick a function of time right, so I will pick a different function of time every time. So what what is happening is suppose this is my t axis (Refer Slide Time: 24:56) okay function of time we are... because in engineering we are mostly concerned with functions of time, in statistics or in economics they are more concerned with they they they they are also sometimes concerned with functions of time but they are many very often not concerned with functions of time but but simply with random variables.

But in engineering especially in signal processing, what is a signal it is a function time. So our prime object of interest is random functions of time. So that is why these random processes are... that is the central concept that we all the time we have been climbing to come up to this concept. So here we have the t axis and here we have the... what is known as a zi axis and this is my... so I am not defining a function of time; for every zi I am defining a function of time. So wherever I am getting a zi I am mapping and getting a function of time.

So what are those functions of time?
Those functions of time are whatever... some function of time. So for zi one zi this is zi axis I am getting this function of time, for for another zi I will get another function of time, for another zi I will get another function of time, these together is the random process; you know there are there are circuits which you get which will generate random functions of time they are called noise sources, they are used to calibrate instruments in communication engineering, we have them in our labs.

So if you take a noise source and today come and switch it on and then see it see the function on the CRO you will get one function. If you come tomorrow and then again switch it on you get a different function of time, it is like that. So the output of that noise source is a random process okay. So basically this is a this is a first concept that it is a function of time. It is a mapping every time I am given an outcome of an experiment I have to pick a function of time, previously I was picking values. So now that means now my random process X becomes a two variable thing.

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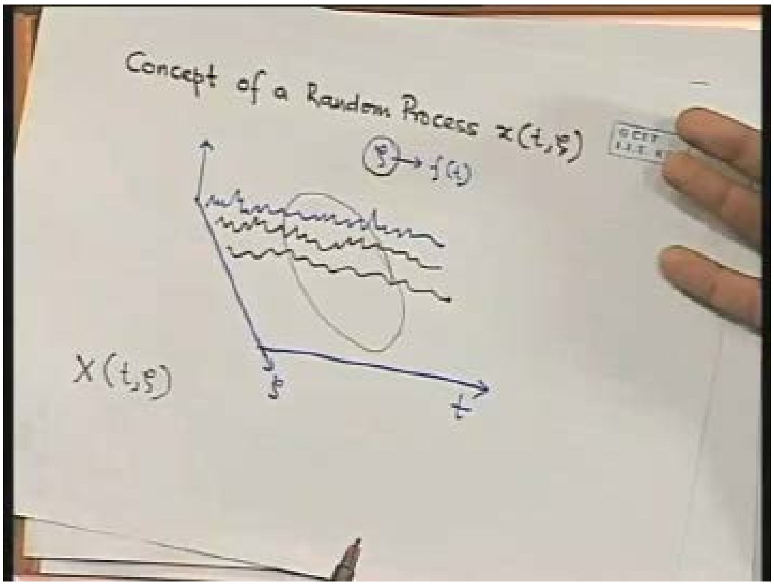

For every outcome I am getting a function of time so it is in the form of a function of X of t and zi right. So this is an ensemble of functions, it an ensemble of time functions. If you freeze this (Refer Slide Time: 27:51) that is if you see for a particular outcome what you get is a simple function of time, it becomes some X t if you freeze zi. If you freeze t what does it become, that is if you take, if you see the... if you switch on that noise source and every time just see the value at 2 seconds every day; you come, switch it on, you get a new function of time, just see the value at 2o seconds every day so you are freezing the time but you are doing an experiment over and over. So you get a value here (Refer Slide Time: 28:29) you get a value here, you get a value here, so if you take all these values what is it it is a random variable. When you freeze time it becomes a random variable. it is a random variable for time t is equal to 2 seconds, you will get another random variable at time 2 equal to 3 seconds another at 1 second.

So if you freeze t it becomes a random variable and if you freeze both it becomes a number. If you freeze t and zi both then it becomes then it becomes becomes a number 1.5 volts. So if you freeze both it becomes a number, if you freeze zi it becomes a function of time, if you if you freeze t it becomes a random variable, if you if you have both floating then you have an ensemble of time functions in general. It is a process which is capable of various time functions okay which are which are uncertain; that is a random process.

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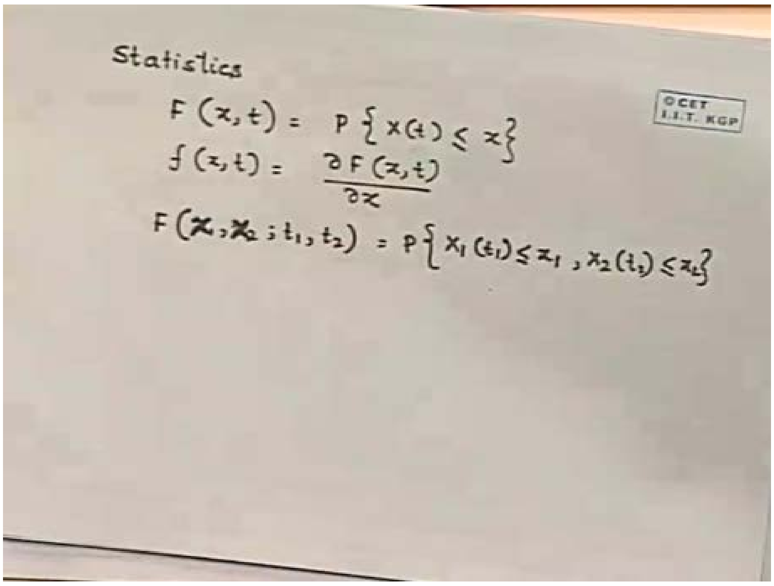

Now every time, now we will be talking of you know of our standard quantities of interest that we that talk about that is we will be defining F x t. So what is what what does F x t mean? it means the probability that X t becomes less than or equal to x, that is why now it is a function of t. So what is the... so as if you are saying that if you come if you switch on then suppose you are saying that what are the what is the probability that I will get a function whose value at 2 seconds is going to be less than 1.2 volts if you saying like this.

So at t you are saying that the value of the function will be less than or equal to a particular given number say 1.2 volts and if you perform repeated experiments what is the probability that this is going to happen at 2 seconds, at 3 seconds, at 4 seconds if you characterize that then that is the distribution function of the random processes right. And naturally its density function is going to be del F by del x that will be that this into dx is going to be the probability that at t the value is going stay within 1.2 to 1.21volts that is the probability.

Previously I was saying that it'll be less than 1.2 volts. Now I am saying that, what is the probability that the function at t equal to 2 second the function value is going be within 1.2 to 1.21 that is going to be this into dx, same interpretation, absolutely same interpretation; only since we are dealing with functions we have to always talk about t here but it is you know always you can nicely go back to the probability space. The moment you freeze t you get a random variables so you are back to your more comfortable region right.

[Conversation between Professor and Student: 31:59] so... in this function we have both x and t variable... that is right... why not tau xy tau t; yeah you can do that, but whether... yes you can do that that is that is if you want to differentiate this then then if if you if you differentiate this with respect to time what is the what is the physical statement you are making that how does this probability vary with time?

[Conversation between Student and Professor – Not audible ((00:32:26 min))] that is how is this going to probability going to change if you if if you measure it at 2 seconds and if you measure it at 2.01 seconds. If you are interested in that quantity you have to take tau x by tau t right? Correct? So we can in general we will we will get more than one value we will get one more than one random function you can have. In this case have so so if you have x 1 and X 2 as two different random processes then what is their joint distribution function that is going to be probability that X 1 at t 1 is less than or equal to small x 1 and X 2 at t 2 is less than or equal to small x 2 that is the interpretation physical interpretation and now it is going to be a function of X 1 X 2 and t 1 t 2 it will be a function of four four things correct?

We will have to see examples of this but we will see them in the tutorial.

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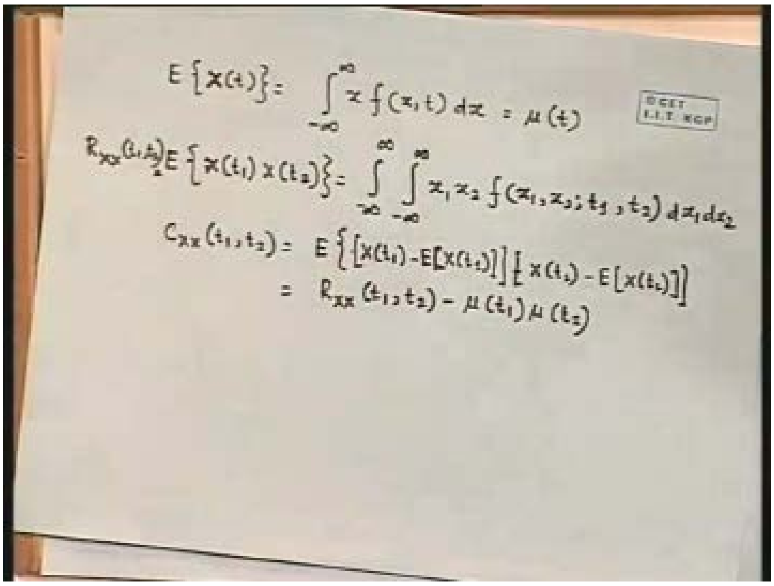

There are there are... if you get any other textbook there are large large number of examples. So you can go through you can go through them, we will see them in the tutorial. But now you have to quickly convert this so I am not going into examples.

Now we are in general we are more interested in you know expectation I mean expected value. We are always expectations is about you know finding likely things. So what is expectation of X t it is this (Refer Slide Time: 34:04) it is again a function of time. Now everything has changed to to now functions of time. Previously we were having a random variable so its expectation was becoming a number. Now we are having an ensemble we having a random process so its expectation is a function of time, so it is now mu t naturally because the expectation is over X so X will get eliminated and you will get t obviously. So so at each point you are going have a different mu naturally.

In many cases now... so you can define now just just previously we were defining correlation of vectors now we have vector functions of time that is X 1 and X 2 so you can now define correlation.

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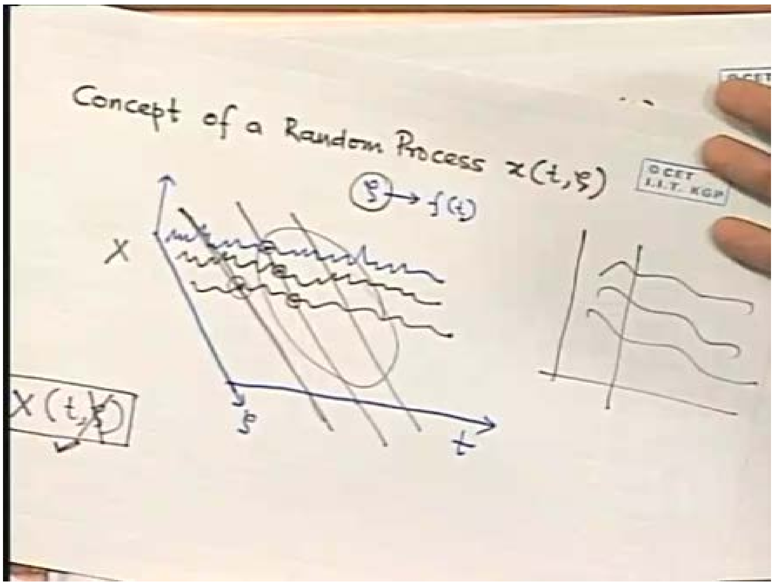

Now if you want to you see that if you have our previous drawing that suppose you have one random process which is this X okay. Now you have another random process which also is generating these things. So now if you say so now if you so if you freeze time t then you get a random variable X X 1, if you if you freeze time here you will get an another random variable Y 1, if you put them as components of a vector you will get a random vector right. So you can get vector functions of time also, no problem, which are basically... if you... in a vector if you put each component as the scalar random process then you get a vector random process.

Basically you are taking the signals and putting them in slots again. Just like you are putting variables in slots now you are putting functions of time in slots so you will get a vector function of time correct? So now here similarly if you if you are given two two two different random variables you can now say what is their correlation. But but now when you are saying what is their correlation you have to say that you are taking this correlation between which two points of time; you can take a correlation between Xt 1 and Xt 2 in general right, so if you take now expectation then it will be a function of t 1 and t 2. That is you you you can say... suppose suppose there is something that if this is positive now then after 2 seconds this is going to be positive. So if you take t 1 equal to something and t 2 equal to t 1 plus t 2 may be they will be strongly correlated in between maybe ((00:36:43 min)). Such things can happen.

So when you take correlation now because the because if now it is a function of time. So when you are taking expectation you have to say between which kinds you are taking, what is t 1 and what is t 2 because this correlation will change depending on your t 1 t 2 choice right. So therefore now when you define correlation you have to say R XX of t 1 t 2 this this times you have to mention which will be this thing (Refer Slide Time: 37:14) so I am multiplying them and I am taking their joint densities at t 1 and t 2 and taking expectation, same thing.

Similarly we can take... this is this is the correlation you can take, covariance where you are subtracting the mean, same thing. We know that for covariance calculation we always subtract the mean and for correlation we do not subtract the mean. So this is these are these are these are exactly same thing. And just like the previous thing you can also say that R XX t 1 t 2 minus mu t 1 mu t 2 is going to be covariance. This is again algebra, you just multiply it out you will get and you take expectation you will get it.

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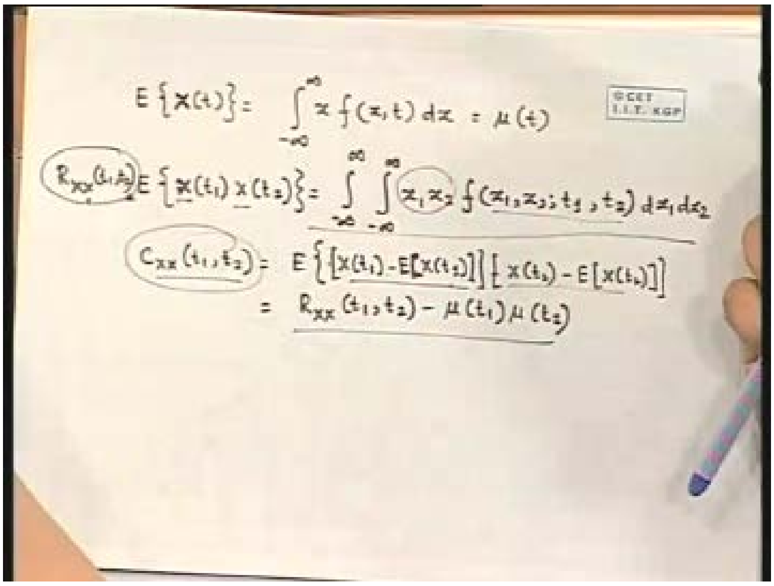

So these are... they are... what I am trying to say is that you have absolutely parallel definitions; only you have to keep track of the fact that now what you are dealing with are functions of time so t 1 and t 2 will come into play.

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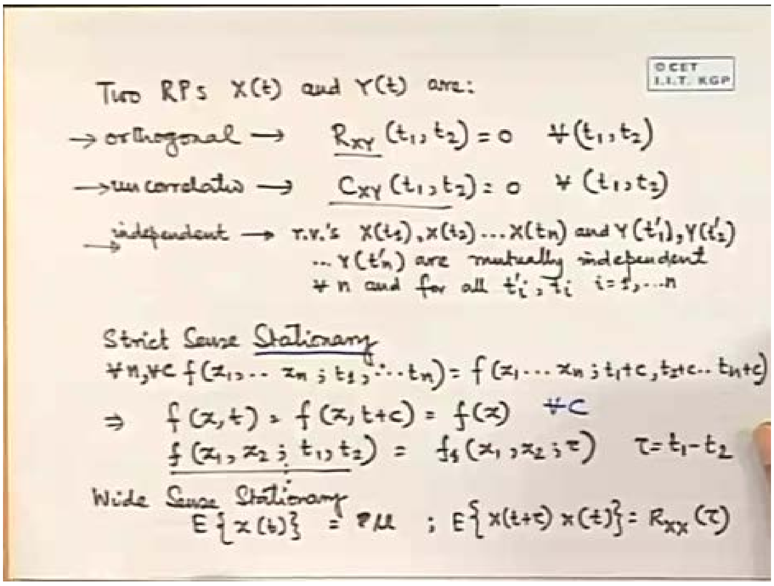

Again you can have definitions of orthogonal which says that this equal to 0; of uncorrelated which says that covariance is to be 0, they are they may both have 2 volt means but they are excursions about the mean that is going to be uncorrelated.

And similarly, if you have independent, now this brings in the major part of the complicacy of random processes. See in any random process, even if you have that... the whole problem is that even if you have only one random process what you have is an infinity of random variables immediately because it is a function of time and time, time extends from minus infinity to plus infinity and for each of those times you are getting a random variable. So immediately even if there is only one random processes in the picture the the number of random variables shoot to infinity straight so it is a much more it is like it is like increasing a increasing the dimension of the space.

I mean the moment you have you have a plane you have an infinity of line because you have increased the dimension of your space. So here also you are adding one dimension which is the time dimension which is which is infinite so obviously you will get an infinity of random variables. So now if you see independent... these are this is a very strict condition to be to be to be satisfied; says that you take, you choose any arbitrary number of time points t 1 t 2 t 3 any number, either 2, 3, 4, 20 and you take samples Xt 1 Xt 2 Xt n they are going to be random variables, the moment you freeze time it becomes random variable. So you choose an arbitrary number of time points so you get an you get an equal number of random variables in X. you choose another arbitrary set of time points and you get and you choose from Y so you so now you have t 1 to t n and t dash to 1 to t dash n number of random variables.

So so naturally now you have to study their properties means you have to study that how these random variables behave over experiments, you have to you have study you have frozen the time points now so you have got some variables. Now, when you are when you when you will try to understand their statistical properties then you will see how how these set of random variables are going to behave over experiments. So then that is characterized by their joint probability densities.

So if you if you calculate the joint probability density of this they are mutually independent means that they again that that f x y is equal to f x into f y so they are so their joint probability densities are going to be guided by their individual probability densities only. That means that that they come from unconnected experiments and and this is going to happen for arbitrary number n and for arbitrary choice of time points t dash t 1 to t n.

you take any number of you take choose any number n and after you have chosen n you choose as many number of time points arbitrary this is this must happen only then these two processes will be called statistically independent. This is rather a difficult condition to satisfy and check. But but mathematically that means. It means that that means they are they are variations at... all points of times are they are they are not only random in time they are also random in random amongst themselves so so that is a that is the definition.

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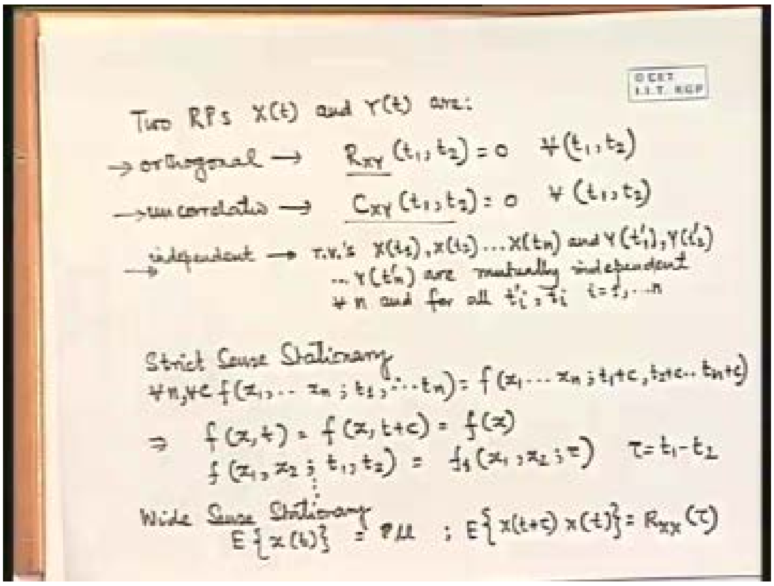

Now we do not we do not we do not... I mean these things are really mathematical concepts but they are in general difficult to satisfy, very difficult to satisfy. Only why we have considered is that once you can choose them. You know the whole problem is that if you want to characterize the behavior of the random processes in statistical terms then you have to always talk of their joint probability densities and their and their joint probability densities each one will be different. suppose you take t 1 equal to 1 and t 2 and t dash to 1 equal to 3 then they are then there are two two random variables they will have some some joint density.

The moment you change t 1 to 1.1 and t 2 to 1.9 they are not two different random variablesb so they so in general they can have different densities. So you will get an infinity into infinity number of density functions so that is that is that is a that is a hopeless case actually, you can you cannot even deal with it properly. So so that is why you know mathematicians make various kinds of assumptions to see how to how to how to simplify matters that is get some results making things very generalized usually does not help and that is why we all throughout your controls course you are you are so obsessed with linear systems why because by by making the assumption of linearity you can at least get some results, if you do not make an assumption of linearity you... I mean in many cases you cannot get any results, is is very difficult to proof stability, it is very you know I mean nonlinear system stability in general you have to find a Lyapunov function. How to find the Lyapunov function nobody knows. If you can find it you can prove it to be stable.

So complexity meaning that is why people have tried to simplify matters and assuming that two processes are independent simplifies a lot of Maths simply because now you have... their joint densities that is you can reduce one dimension, joint densities simply means this into this. So if you know each one’s individual densities the combinations are not going to different, you can you can generate the combinations from the individuals that is an that is an advantage. So for such reasons people make these assumptions okay.

So now we are talking of another thing. Again that this concept has come because we want to reduce complexity. we have two dimensions: in one on one dimension which is the value dimension that is the signal’s value that is that itself is uncertain and another dimension is time. Sso now what I am saying is that can I do something can I make some simplifications on the time dimension as well. Then I can deal with things. So that is why people talk about this this concept of stationary. So people say that on the time axis things are going to be regular. That is that is there is some some regularity that you can expect on the time axis at least that that simplifies matters for you. So they are saying that a function is in a strict sense stationary, stationary in the real strict sense if the density function f x t does not vary with time. So you you evaluate it at t evaluate it any t plus c for all c immediately you have you have made a big big simplification.

You you are saying that the property of this random variables at at at any point of time are the same, they are different instantiations as if the as if underline you are having only one experiment. So when you are generating the value you are casting a dye and then generating a value 1.1, when you are generating the next value at a different time you are again casting the same dye. So all the numbers are actually coming from the same experiments so they have the same probability distribution, it does not depend on time. Immediately you see the see the amount of experiments are are totally reduced.

Previously at each time you are doing a different experiment having is different density so so one infinity you have reduced. Previously you were having an infinity of experiments, now you are having one experiment that is a big advantage. So and not only that they are saying that they they are also saying that the joint density function for t 1 and t 2 will in general not... you see everything is invariant to shifts in time by a constant shift in time but that constant can be different that constant anything. that is if you take the this at t 1 t 2 and then take the take the same function at t 1 plus c and t 2 plus c they are going to be same. That is what they are saying.

So what does it mean?
It means that it I mean c can be anything so it essentially depends on what is t 1 and what is the difference between t 1 and t 2 because when you are shifting you must shift t 1 and t 2 both by the same amount so which means that when you when you are shifting them what are you keeping constant, you are keeping the you are keeping the difference between t 1 and t 2 constant, you are shifting t 1 and t 2 both on the axis.

So the function essentially depends on t 1 minus t 2, it does not depend on the absolute value of t 1 and t 2 that is what it says right? So we will talk about auto correlation functions which which previously we were saying R XX t 1 t 2 we will say that it it does not depend on t 1 t 2 we will will talk about R XX tau where tau is equal to t 1 minus t 2 that again simplifies one dimension and then you can have a you know this is also very difficult to check who is going to check all these joint distribution joint density functions, it is very difficult to check. So we will we have to continue this, so let us stop here.

Alright thank you very much.