Digital Signal Processing

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Lecture - 9
Relation between discrete time and continuous signals

Today, before we start our topic; that is the relationship between continuous and discrete time signals, we will just briefly go through some of the examples that we have taken up last time.

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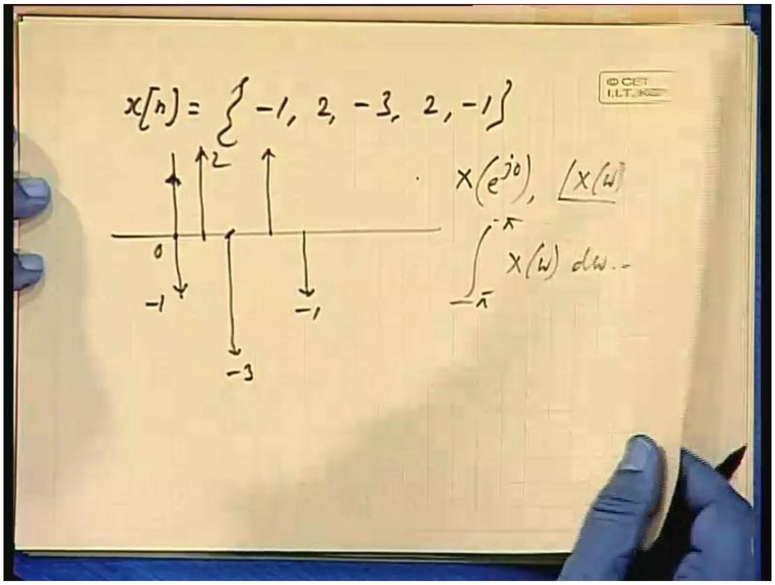

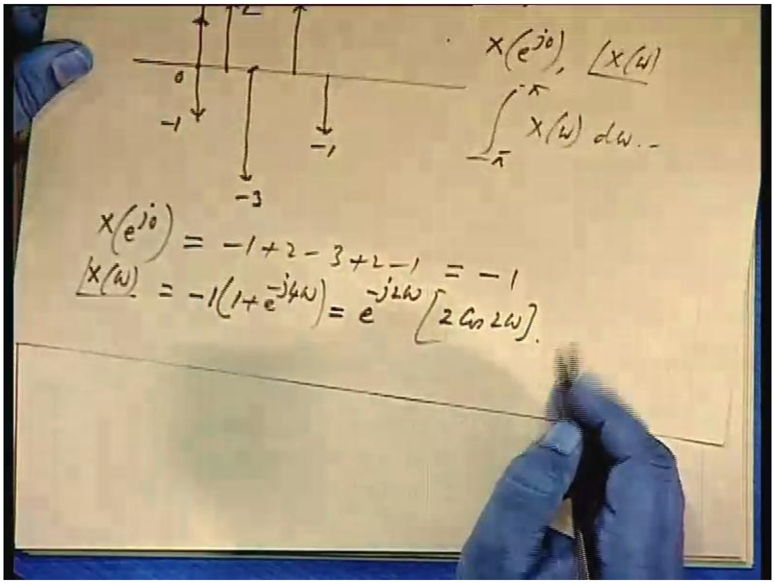

If you remember, the last problem that we took was a sequence which was symmetric, okay. It is like minus 1 plus 2 minus 3 again plus 2 and minus 1, it is symmetric about this central point, okay. So, if you consider the signal, from the origin then we are asked to compute some of these quantities; like this one and then angle of X omega, and so on, integral minus p i to plus p i X omega, d omega and so on.

Now, you can see first one; X e to the power j 0 by putting omega equal to 0 what we get is summation of these terms, minus 1 plus 2 minus 3 plus 2 minus 1 which gave us minus 1. Then angle of X omega while computing this, there was a little bit of confusion and I deliberately did not mention about the starting point.

See, the first one is minus 1, if I pair them minus 1 into 1 plus e to the power minus j 4 omega, okay; so that can be written as, if I take e to the power minus j 2 omega common, inside I will have e to the power plus j 2 omega and e to the power minus j 2 omega which will be twice cos of 2 omega. Next, if I pair this and this it will be, plus I will write these terms here.

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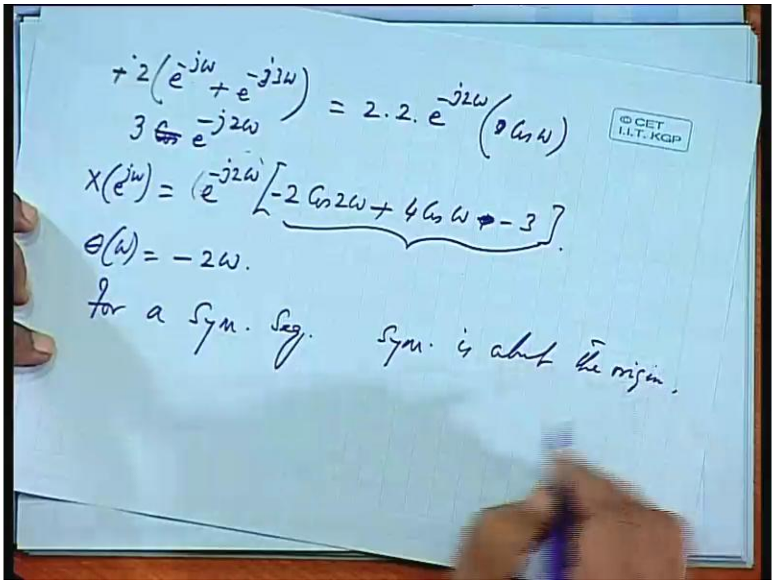

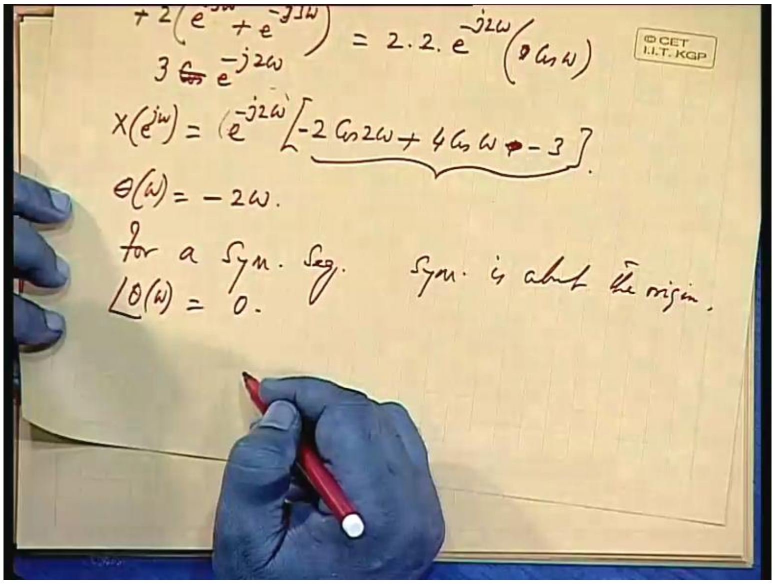

If I take two common, it will be e to the power; I will call this as X 1 okay, X 1 omega this is the first part, e to the power minus j omega plus e to the power minus j 3 omega that gives me, 2 into 2 into e to the power minus j 2 omega. Once again, I take e to the power minus j 2 omega common, so it will be e to the power plus j omega, e to the power minus j omega which will be twice cos of that 2 I have taken out, so cos of omega, okay.

And thirdly, it was 3 into cosine of sorry 3 into e to the power minus j 2 omega. So, if I add all the three terms, X e to the power j omega will be e to the power minus j 2 omega common and I get inside the bracket twice cosine 2 omega plus minus plus 4 cos omega plus or minus minus 3, that is all. Now, this is a real quantity, so this gives only me the phase, so the phase theta omega will be minus 2 omega, okay.

It is not 90 degrees, it is just minus 2 omega, last time there was little slip. Now, if I have the sequence instead of starting from here, sequence starting at this point; that is it is symmetric about the origin then this will be starting with 3 into e to the power 0 and this will be 2 into e to the power minus j omega, this will be 2 into e to the power plus j omega.

So this term, that is you are giving a shift of the origin. So, this term will be multiplied by e to the power plus j 2 omega and hence there will not be any angle associated with this. So, for a symmetric sequence, for a symmetric sequence if the symmetry is about the origin then we get angle theta is equal to 0, all right. Next the third part was, if you remember we have asked you to compute minus p i to plus p i X omega, d omega.

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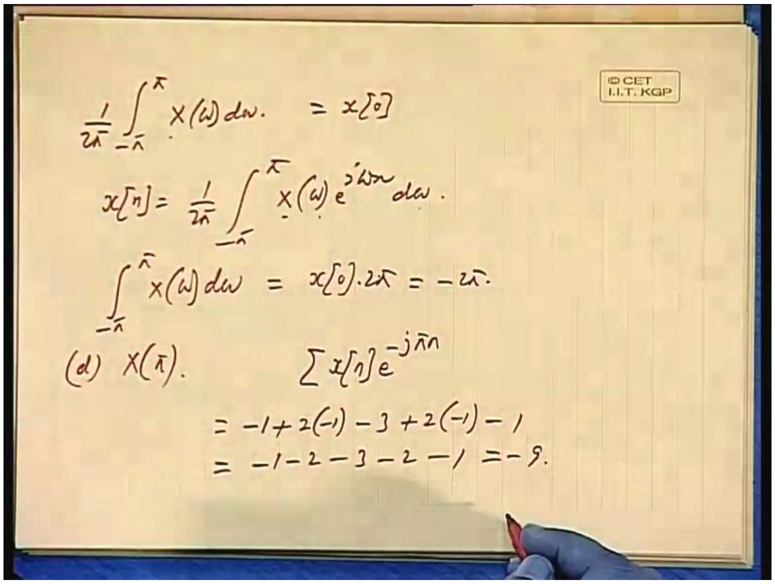

Now, what is this? 1 by 2 p i into this; if you remember, X n is equal to 1 by 2 p i minus p i to plus p i X omega e to the power j omega n d omega is that all right? Now, if I put n is equal to 0, this becomes just integral of X omega, d omega is that all right. So, that is equal to x 0. So, what is this integral equal to 2 p i into x 0?

And you have been asked to compute; sorry you have been asked to compute this integral, so that will be x 0. So, that is equal to into 2 p i, okay. So that is equal to minus 2 p i, is that all right, x 0, is minus 1. Next, it was what the fourth part? X at p i, all right. If I put here X at p i, yes? How much is it? If you put omega is equal to p i, what do you get from the series? What do you get from the series? X n e to the power minus j p i n, is it not?

And that is equal to; basically you are just alternately changing the sign, e to the power minus j p i n means, when n is odd it is p i, 3 p i, 5 p i when n is even it will be minus j 2 p i and that is equal to 1. So, you just alternately change the signs. So the original sequence that was given; minus 1 plus 2 into minus 1 minus 3 plus 2 into minus 1 minus 1, is that all right. So, that is equal to minus 1, minus 2, minus 3, minus 2, minus 1, so that is minus 9.

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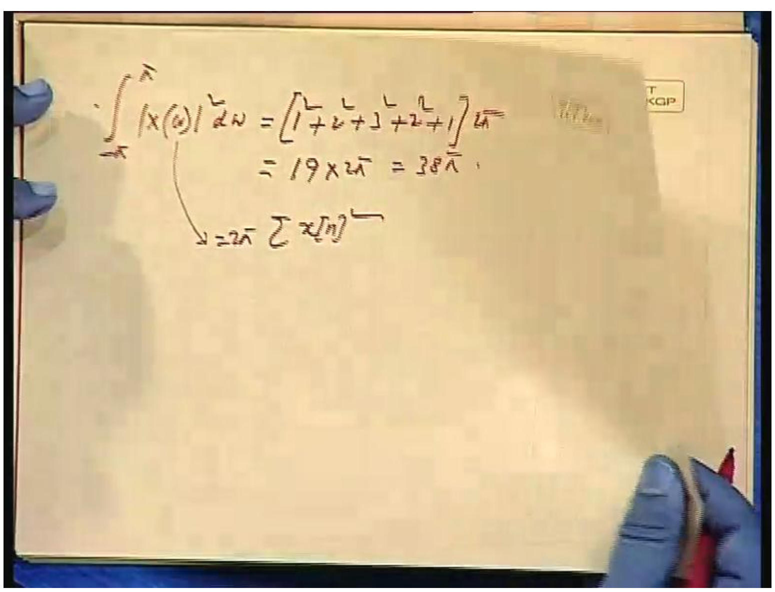

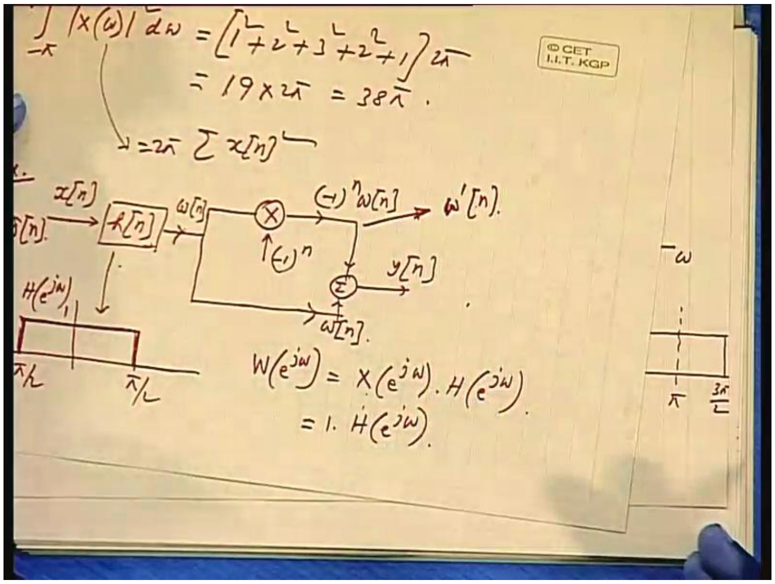

The last one, that we discussed, we did not discuss that was asked was X omega square d omega, okay. Now, from Parseval's theorem that is the energy content of the signal; whether you represent the signal in the time domain or the frequency domain will be same, and that is equal to square of the components, okay, summation of that, in the continuous domain it is integral. So that is will this one will be equal to minus 1 square plus 2 square plus 3 square plus 2 square plus 1, 4 plus 1, 5, plus 5, 10 plus 9, 19, okay.

Now this is actually, 1 by this is equal to this into 2 p i, okay. So, 19 into 2 p i; that is 38 p i. Parseval's theorem states that, this is equal to 2 p i into sigma x n square, all right. Now, let us take up another interesting problem, before we go over to the relationship. You are given, x n this is a low pass filter; you are having an adder here and this is y n. This low pass, low pass filter is having the characteristics like this, minus p i to minus p i by 2 to plus p i by 2, it is 1 and this is a characteristics of H e to the power j omega, okay, and ideal low pass filter.

What will be y n like; if I give say a delta input you are giving an impulse input, what will be the output, okay. So, let us take this as some w n. So this is minus 1 to the power n into w n and this is w n, okay. Now, what will be w e to the power j omega? It will be product of the two frequency transforms, frequency transform of delta n is 1, okay.

So, if I call that as input X into H e to the power and this is equal to 1; so, it will be H e to the power j omega okay. What will be the frequency transform of this? This is already known, same as h. What will be the frequency transform of this? If, I call that as say; some W dashed n then what is the transform of W dashed?

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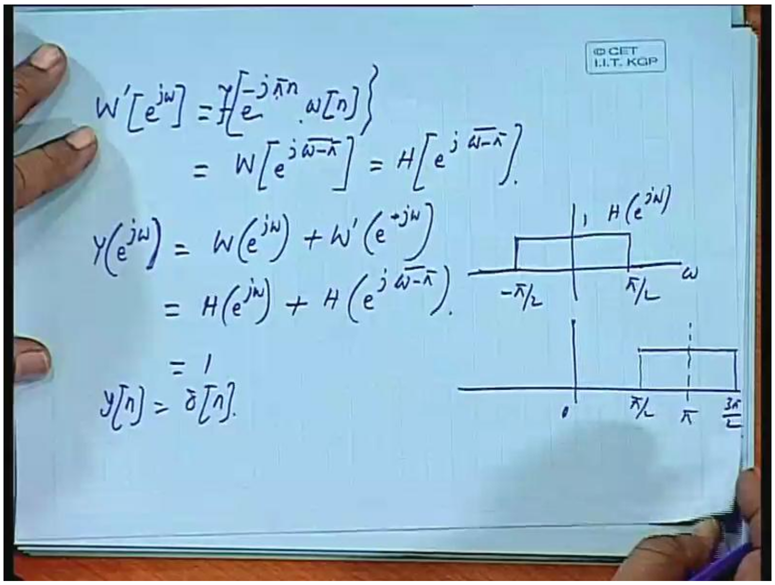

Now, minus 1 to the power n, I can write as e to the power minus j p i n, is it not? Into w n, the frequency transform of this, is it not? I am taking frequency transform of w dashed n which is nothing but w n into e to the power minus j p i, okay. So, in the frequency domain, if I multiply any function x or say f by e to the power minus some quantity j into something, then what is the transform of that product?

Will be the original function shifted by j p i, all right; so this is nothing but H because W is equal to H for a delta input, so H e to the power j omega minus p i. So, what will be Y e to the power j omega? It is nothing but W plus W dashed, okay. So, this is H plus H e to the power j omega minus p i, correct me if I am wrong, is that all right?

And, how much is this? If H omega is extending from minus p i by 2 plus p i by 2 and after that after that it is all 0. This is omega. This is 1, then just take any value. So, omega is equal to a little above p i by 2, all right. Then this will be zero and this will be one. You just see, what is w dashed, if this is H e to the power j omega, what will be the shape of e to the power j omega minus p i? It I will start from p i by 2, 2, 3 p i by 2, is that all right.

It will be like this. So, if I add them together, it will be always 1, okay. So, what will be Y n? This plus this, okay; if I keep on changing this periodic and it is coming continuously as 1, so y n is delta n. So, if I have an ideal low pass filter of ideal low pass filter of band minus p i by 2 to plus p i by 2 then this kind of arrangement will give me, an overall transfer function unity. I give delta n input, delta n output, okay.

Okay, before we take up any more examples, let us will take up some more interesting examples later on; let us get the relationship between the continuous domain and discrete domain signals. In real life, signals are signals are continuous, mostly all right; whether it is speech or it may be the temperature recorded or may be the voltage of a particular node. You record anything that is basically, analogue or continuous.

Now, you discretize it in the time domain and then you process it in a computer. So, for processing again the discrete signal depending on the memory available, we discretize in magnitude, okay, representing it by some bits. So, when we are doing that it requires some time.

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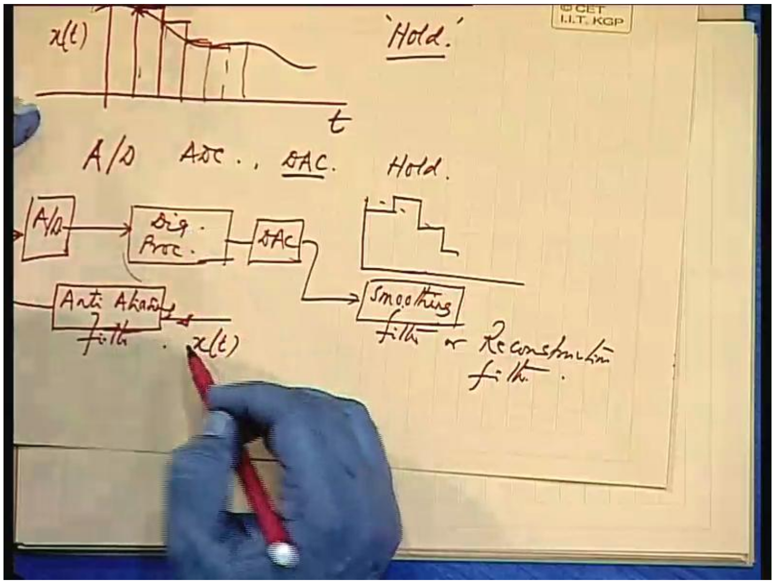

We discretize in magnitude that means, we measure them at discrete intervals of time then this value is to be represented in the digital form. So, it will take some time depending on the length of the memory. So, we allow that time; so till that time it is kept on hold at this value all right, so we also require sample and hold circuit. The device by which we discretize it is analogue 2 digital converter or we call it, ADC.

Then similarly from the discreet value; we translate them into continuous value we call it digital to analogue conversion and apart form that, we require whole circuits. These are not the only elements when we are getting a signal and we are using that hold element then we are getting the sampled value like this, okay.

These values are processed and then we are doing some kind of processing, depending on your requirement. And then you are getting the output converted into continuous domain signal; while doing that, what you are getting is a discrete even though it is a continuous function but it is having sharp bends like this, okay. You have to smoothen it all right. So, at the final stage you have a smoothing filter.

So, after processing digital processing, you have a DAC and then you have to have a smoothing filter, okay; smoothing filter, you also call it reconstruction filter all right or reconstruction filter, okay. Now before feeding it to teddy converter, you also take care of the signal; will very soon see what should be the condition for the signal for discretizing, before feeding it we also pre filter it or we call it anti-aliasing filter.

So, the signal is feed here pass through an anti-aliasing filter or pre-filter and then we discretize it, then we process it, then again we go for analogue conversion, then we have smoothing filter, okay. These two filters are in the analogue domain and this is the digital domain; this filter is in the digital domain. So, what is the most fundamental condition for processing a signal with the with the help of a computer? What should be the condition for anti-aliasing filter, because rest of them are very standard one; Anti-aliasing filter.

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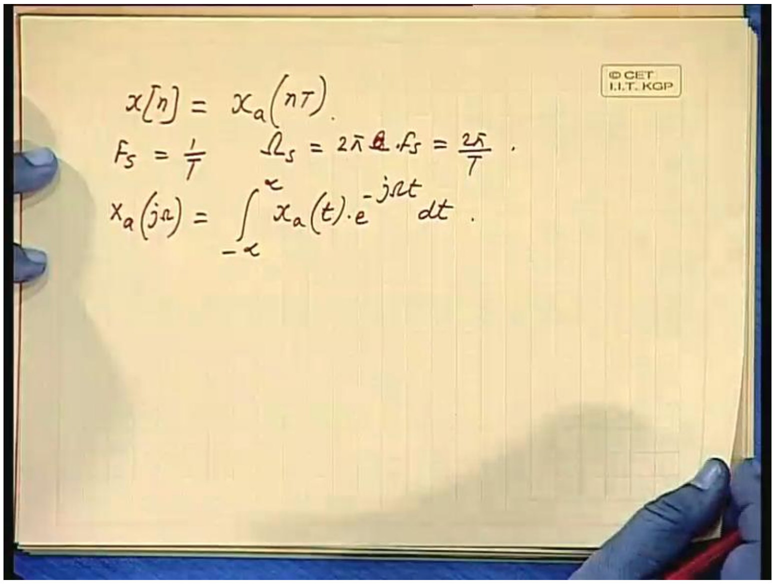

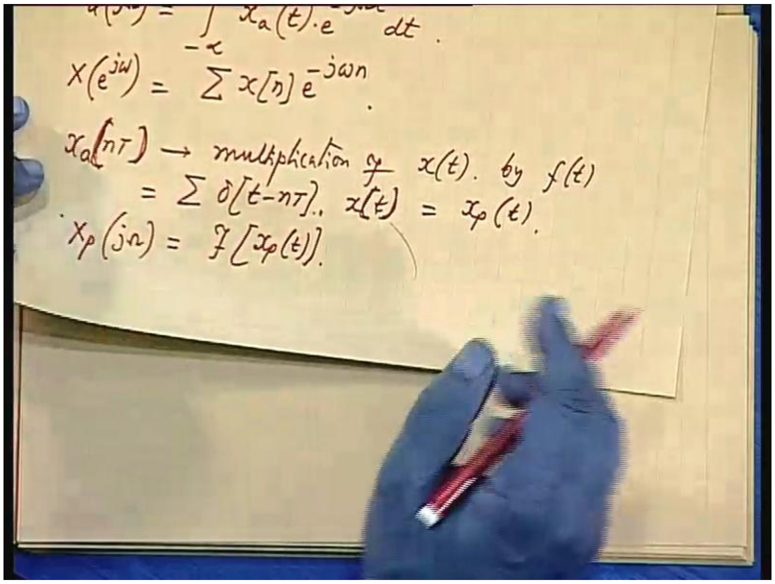

Now the signal discrete signal x n, is nothing but x analogue. We are computing the values at regular intervals of T, okay. So, I will write like this and the sampling frequency is 1 by T in the radian domain 2 p i into f s; that is 2 p i by T, so many radians per second. What will be its frequency transform? It is minus infinity to plus infinity, x at e to the power minus j omega t d t, okay.

And what is the discrete domain representation? It will be summation x n; this we are writing in the discrete domain, we want to establish a relationship between this and this, okay, you want to establish a relation between this and this. Now, x a n T, I can represent in two ways, I should put this bracket. Now, this can be shown as a multiplication of the function x t by a function f t by a function f t which is basically, a string of delta function with period T, is it not, that means okay.

If I call these as periodic, X p j omega, okay; this is a periodic function, so if I take the Fourier transform of this, I call this as so this multiplied by x t, I call this as x p t, all right. When we are looking at it as a product of this periodic function and the function x t, okay, I call it x p t. What is the Fourier transform of this? This is x p j omega, that is Fourier transform of x p t, okay.

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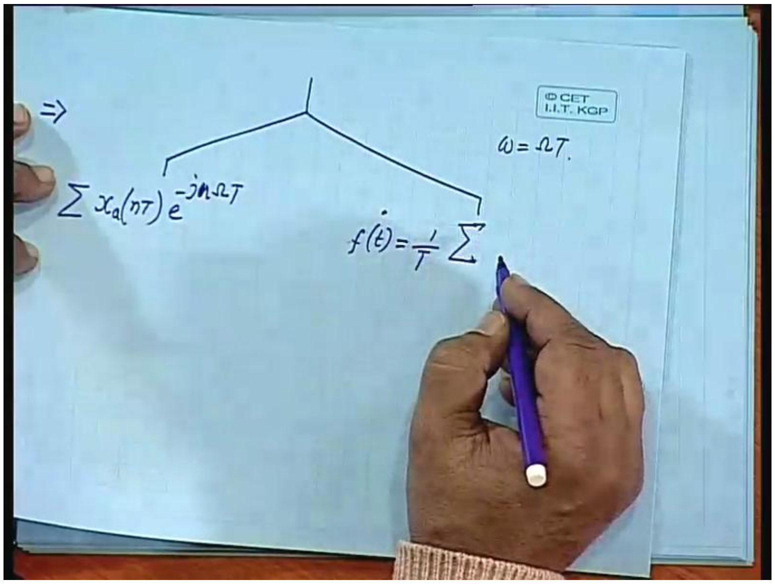

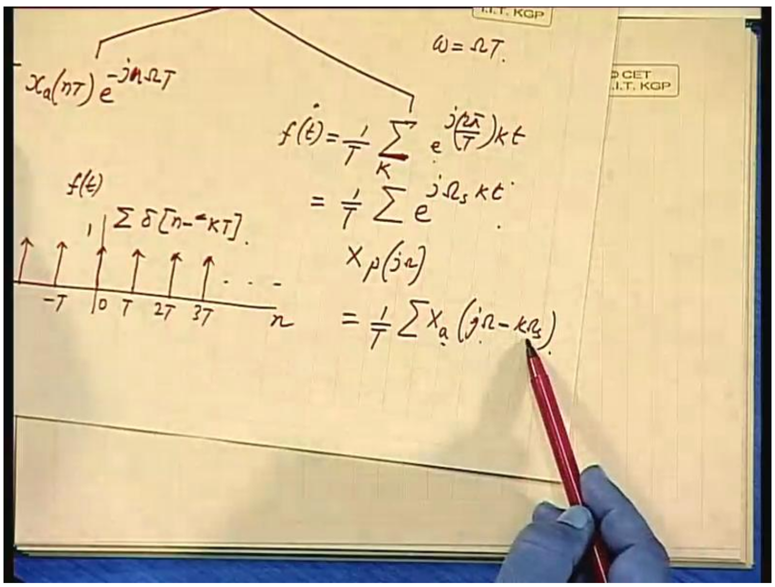

Now this I can look at it as, there are two ways of looking at it; one is a summation x a n T, x a n T is a magnitude, okay minus e to the power j n omega T, is it not. This small omega, I am writing as omega into T that is at only those times, if you are taking it, so it can be written like this. And the other way it will be; okay let me write for f t first, f t is this will discuss later on in details. See, if I have a delta function, a string of delta functions and so on.

This is summation delta n minus n minus, okay I can write this also in the discrete domain. So, the transform of this e to the power j, I am just writing this as K t okay, I can write f t like this, is it not? Whenever, T is equal to K t, I take only discrete values, okay. Whenever T becomes equal to this then it I will be giving me a pulse; so f t can be written like this, this is f t which is nothing but summation of this, okay.

So, 1 upon T, summation e to the power j omega s K t in terms of something frequency all right. So, correspondingly this is f t, so correspondingly if f t such a function; X p j omega will be 1 by T summation X a j omega minus K omega s, okay. Translation, any function is multiplied by e to the power some exponential function; e to the power j, this thing will be having in the frequency domain, a shift of K into omega s, okay, one of those properties of Fourier transform, shifting property.

So, it is nothing but a periodic function all right. A summation, summation of a function which is shifted by omega s, two omega s, three omega s and so on, is that all right. What does it mean?

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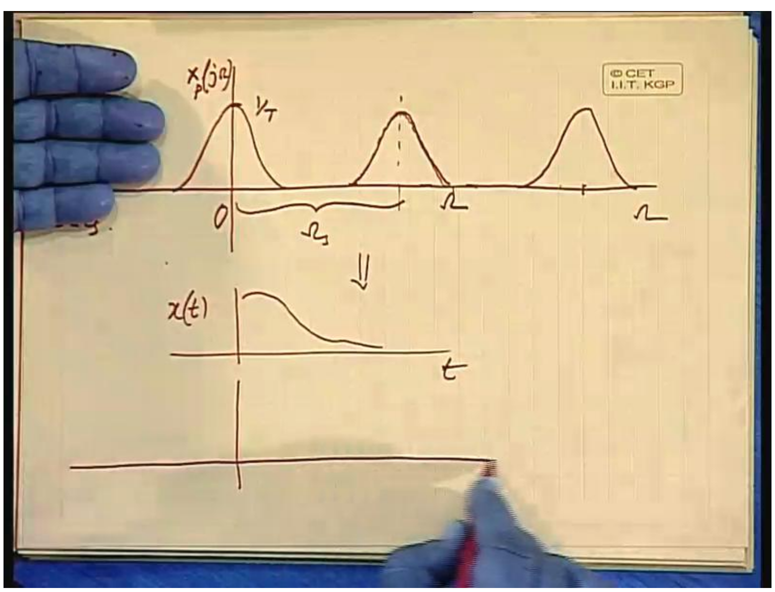

Suppose, X a is a function like this; in the null of domain, what is X a j omega minus omega s minus 2 omega s minus 3 omega s? So, this is omega s and again around that omega s, you are having a replica of this, is it not? Again, after 2 omega s we are having replica of this, similarly on this side okay, is it not?

So, if I have corresponding to this, say the time function is like this; this is the original time signal, all right who's Fourier transform will be this, okay. And the entire set corresponds to this one, if I scaled it down by 1 by T that represents basically X p j omega. So, if this was X then this whole thing is scaled by a factor 1 by T will be X p, this entire set is X p, is that all right?

So this, this one corresponds to the Fourier transform of this, and this one is 1 by T times; I have to multiply by a factor 1 by T, that I will give me X p, okay. How have you got it? If I consider, in the time domain you are multiplying X T by what? By a chain of delta function, all right? There is a train of pulses, impulses of magnitude 1 and you are multiplying by that, okay. So in the frequency domain you can take the products, all right. So, what is the product? One is this one to be multiplied, by no x omega?

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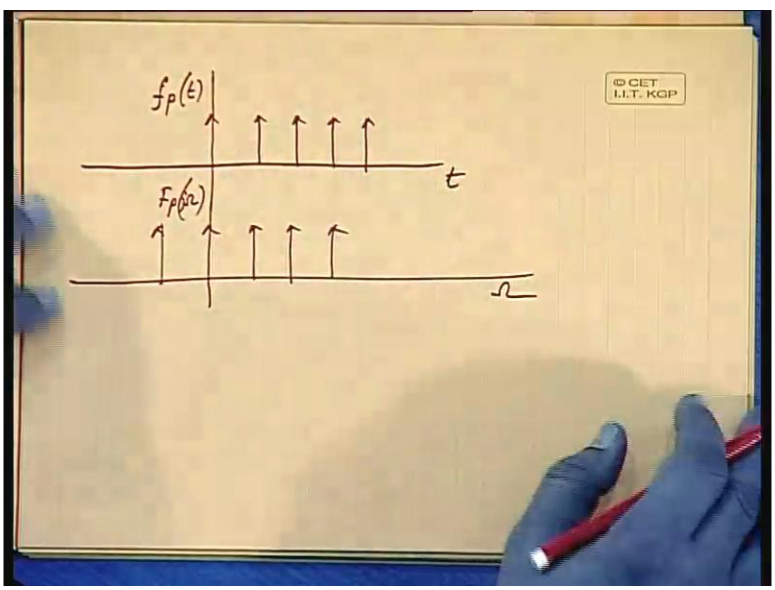

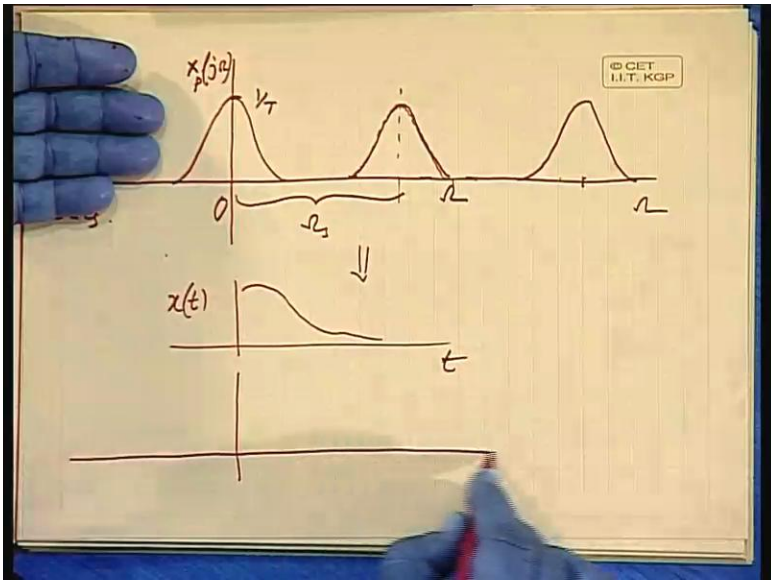

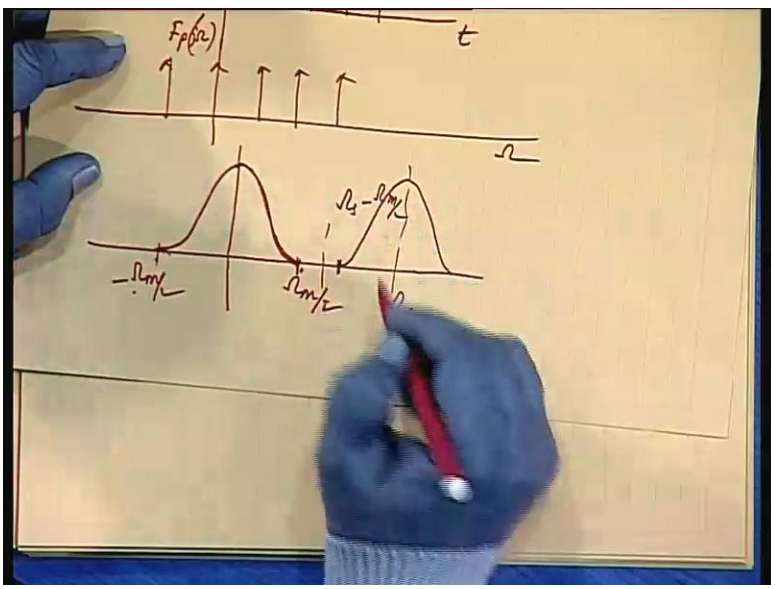

See, if I have a delta function; periodically appearing then what will be its Fourier transform? Also at chain of pulses in the frequency domain, if this is f p t, t then f omega F p omega will also be pulses, okay chain up pulses like this, sorry. So, this one I can put j omega, this one is convolved with this. You convolve, just X t, X j omega with these pulses, you will get this, is it not; convolution of one pulse with this one which will give me the same function.

And with a train of pulses, you will get repetition of this, is that clear? So, in the time domain when we are multiplying, in the frequency domain domain will be taking convolution of the two frequency domain representations. And that is how we have got it. And we have seen it also from the summation expression.

Now, question comes while taking convolution, this has got nothing to do with the with your sampling as such; it is depending on the type of signal that you are getting. So, this is a base band of the signal. Original signal, analogue signal is having this much of band width; minus omega m to plus omega m, okay you can call it omega m by 2 also, okay. Now, if this spacing is sufficient this is omega s, then how much is this?

Omega s minus omega m by 2, okay. So, if omega s minus omega; in many books, they write just minus omega m to plus omega m, okay, that is the normal convention, so let us also follow that.

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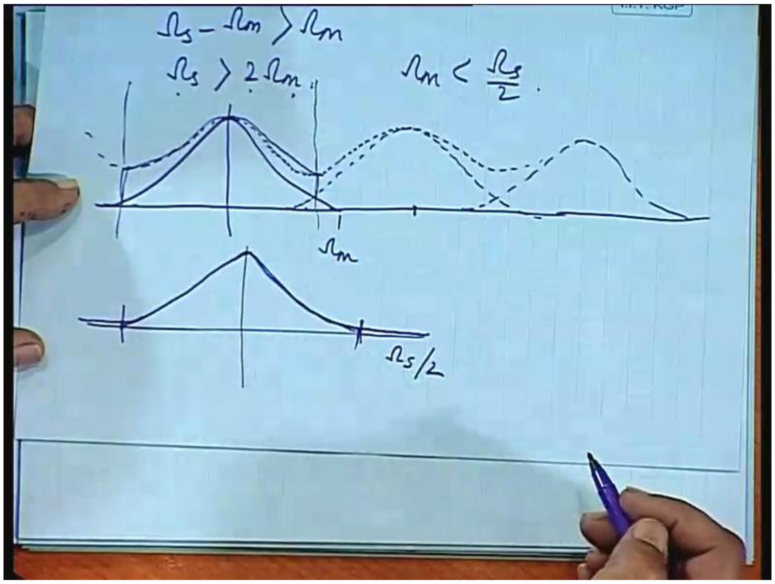

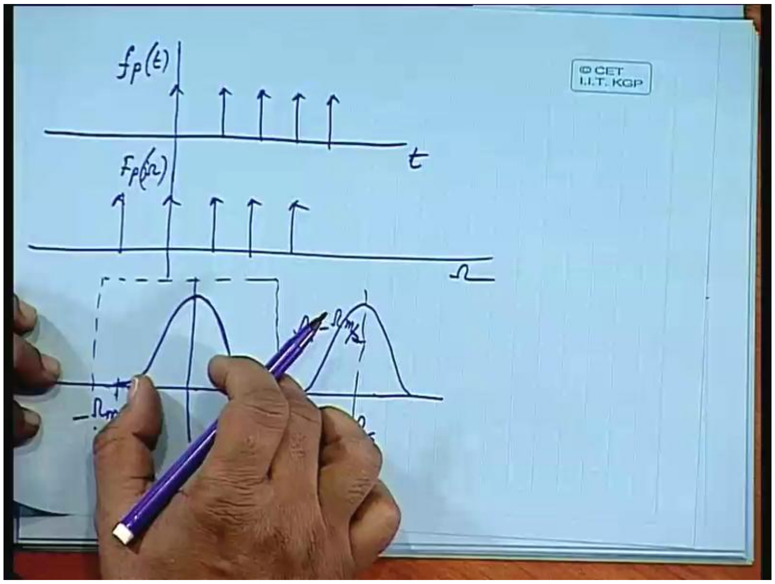

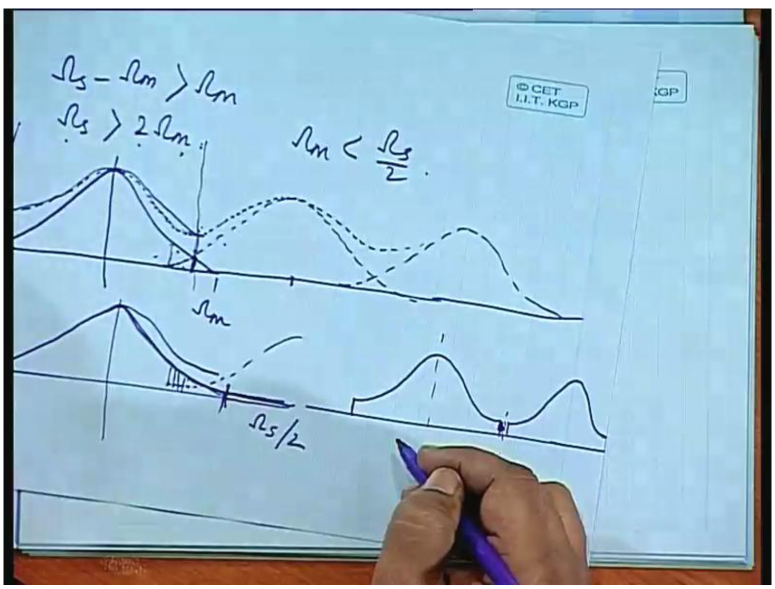

Omega s minus omega m should be greater than omega m. If, I want clearly distinguishable forms, okay that means omega s should be greater than twice omega m or omega m should be less than omega s by 2; that means sampling frequency should be more than twice the maximum frequency, that is present in the original continuous domain signal, this is the same theorem that we have already discussed, sampling theorem sampling theorem or Shannon's sampling theorem.

Now, if we have an overlap; if supposed this condition is violated then it is like this, this is omega m and omega s is omega s by 2 is coming inside and so on. So, now when you are taking the summation, the overall sum will be like this. So, this is not a clear picture from here; I cannot segregate the base band element all right, that is that segment. Earlier, when this was repeating with a distinct gap in between; I could have now used a low pass filter, I could have used a low pass filter and filter out this one.

If you take the inversion, you will get the original analog time domain signal; that means if you are given a discrete domain signal, if you take it is Fourier transform, all right, if you pass it through a low pass filter then again take inverse transform, you get the analog signal. But, now if I have a low pass filter, I will be having a signal which is not a replica of the original signal, you see here there will be lot of error, okay. So, to avoid this we impose this condition.

Now in reality, you may not have a signal with a definite value of sigma m or the magnitude goes on diminishing but at a very slow rate, it is like this. So somewhere we truncate it. This frequency is ensured to be less than omega s by 2, okay. Then you get this is the base band and then over this, you have another repetition and so on, but there is a separation. This is somewhat closer to the original signal then this distortion; because here because of the overlap even much before this here the distortion is only at, the tail only this portion is eliminated, okay.

But, if it is like this then the other tail also comes in, so the corruption starts from here because the resultant is somewhere here, you see somewhere here. So, the value gets corrupted even much before this value, omega s by 2, okay. So, that the distortion is less, if we do the pre filtering where we are restricting the band, okay.

Now, let us take an example; it will be very interesting, to demonstrate the effect of sampling on signals.

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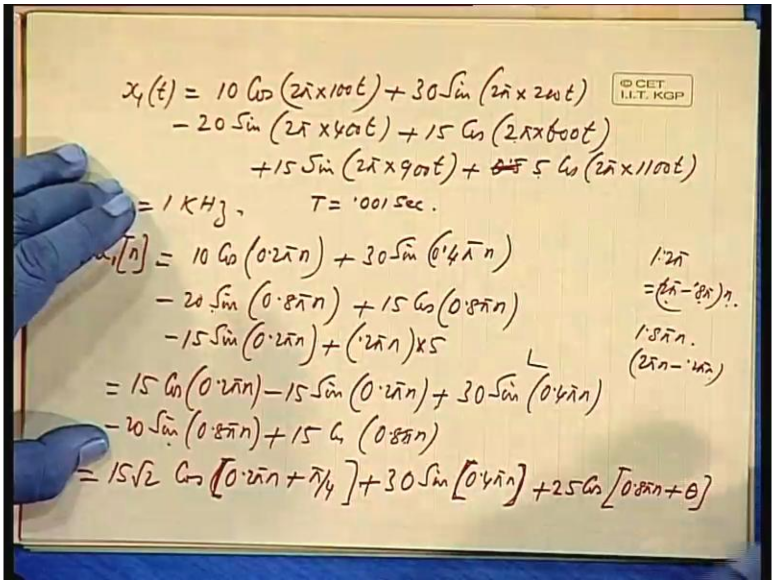

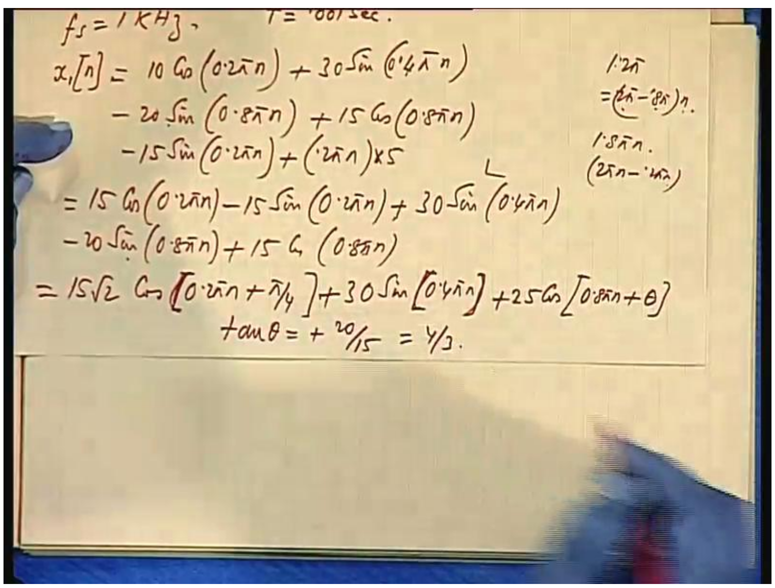

Suppose, you have X 1 t is equal to 10 cosine 2 p i into 100 t plus 30 sin 2 p i into 200 t minus 20 sin 2 p i into 400 t plus 15 cosine 2 p i into 600 t plus 15 sin 2 p i into 900 t plus 0.5, sorry, 5 cosine 2 p i into 1100 t. Suppose, we sample it at a frequency of 1 KHz, what I will be the discrete domain representation of this signal? Will be 10 cosine 2 p i into 100 into t, 1 kilo hertz is the sampling times, so sampling frequencies. So, T is equal to 0, 0, 1 second, okay.

So, you put n into capital T, so this will become 0.2 p i n plus 30 sin 0.4 p i n, correct me if I am wrong, minus 20 sin 0.8 p i n plus 15 cosine. Now, this will become 1.2 p i n. Now, 1.2 p i, I can always write as; 2 p i minus 0.8 p i into n, okay, so 2 p i n minus 0.8 p i n cos of 2 p i minus 0.8 p i n. So, it will become 0.8 p i n, if you permit me to write in one step.

And then, where it would be, this one and then plus 15 sin 1.8 p i n, 1.8 p i n. So, that can be written as 2 p i n minus 0.2 p i n. So, sin of this means, minus 15 sin 0.2 p i n okay plus 5 cos; this is 2.2, if I subtract 2 p i then it 2 will become 0.2 p i n, okay. So, check into 5,5 so 10 plus 5, 15 cosine 0.2 p i n, okay minus 15 sin 0.2 p i n then plus 30 sin 0.4 p i n. Again, minus 20 sin 0.8 p i n plus 15 cosine 0.8 p i n.

I can write this as; this is 15 and 15, so 15 root 2 cos and sin, so cosine 0.2 p i n then this plus 45 degrees or minus 45 degrees? Plus p i by 4, is that all right? Then this and this okay, 30 I write this first 30 sin 0.4 p i n and then minus 20 and 15 same frequency. So, this is 20 square plus 15 square under root 25 okay, plus 25 cosine, cos a, cos b minus sin a, sin b; so cosine 0.8 p i n plus theta where n theta is 20 by 15, okay. So, I can write like this where n theta is minus, sorry plus 20 by 15, okay. It is 4 by 3.

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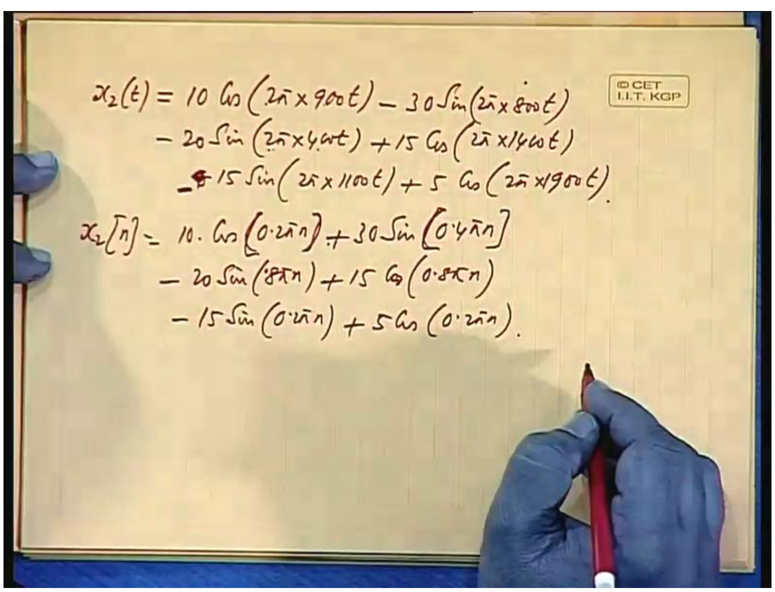

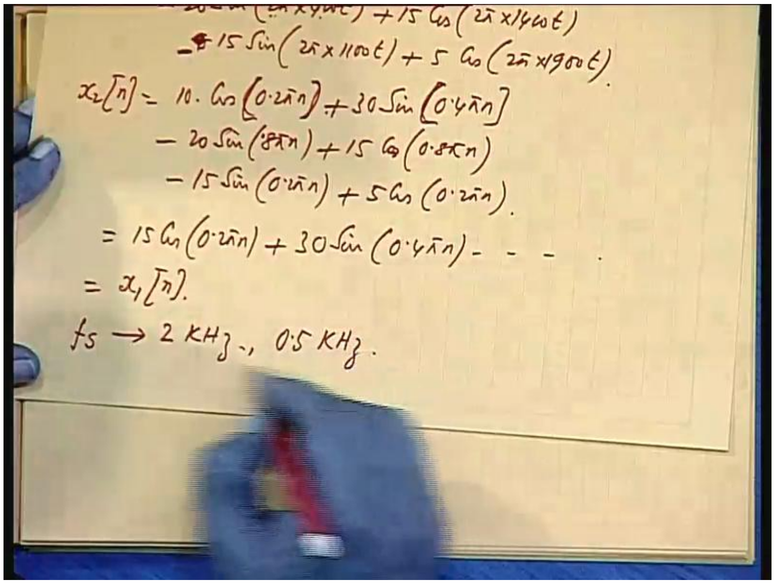

Now, one more signal; we take, x 2 t, x 2 t which will be 10 cosine, which is different from this minus 30 sin 2 p i into 800 t minus 20 sin 2 p i into 400 t plus 15 cosine 2 p i into 1400 t plus 15 sin of 2 p i into 1100 t plus 5 cosine of 2 p i into 1900 t. Now, if I have a function like this, what will be x 2 n? Once again by the similar conversions; we get 10 cosine, this is 800 into sorry 800 into p i, so, that give that will give you 1.8 p i. So, that is 0.2 p i, 2 p i minus 0.2 p i cos of 2 p i minus theta is cos theta itself. And then minus 30 into sin this is 1.6 p i, so that will be plus 30 sin 0. 4 p i n, okay.

Next, 20 sin this is 800, so 20 sin 0.8 p i, okay 0.8 p i, plus 15 cosine this is 2.8, that is equal to 0.8 p i n plus 15 into 2200. So, this is actually minus, this was minus I am sorry, so this will become minus 15 into sin 1, 2 p i n plus 5 cos, this one is 5 cos 0.2 p i n, is that all right. One is 900, other one is 1900 but both of them are generating 0.2 p i n. So, this is 10 plus 5, 15 cosine 0.2 p i n; you check with a previous example, is it coming same? Plus 30 sin 0.4 p i n. I leave it to you to, complete this, we will come as same as x 1 n.

Now, you change the sampling rate to 2 kilohertz; will find x 1 n is not equal to x 1 n or x 1 n can be equal to x 2 n, I ask you to verify for these two frequencies, what will be the sample sequences x 1 n and x 2 n? Are they equal for some of these frequencies? Will you guess or are they different? That means, two signals might appear to be identical under a certain condition of sampling, it might be different at some of the sampling rates. I want all of you to verify this by taking two different frequencies and once again translate the time domain description to continuous domain description, to discrete domain description like this, okay. Thank you very much. We shall take up some examples in the next class, using the frequency transforms.