Digital Signal Processing

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Lecture - 8
Tutorial on Discrete Time Signals & Their Transforms

We are taking up a few numerical problems on discrete time systems and their transforms.

So, let us start with the first question, the response of an LTI system, is given here for an input of 1, 0 minus 1; the output sequence is 1, 0 minus 1, 0, 1, 0 minus 1, 0, it continues.

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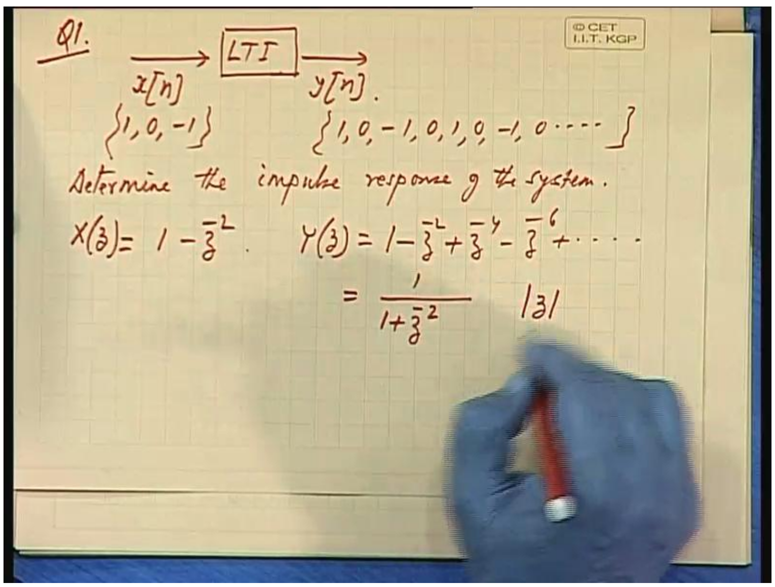

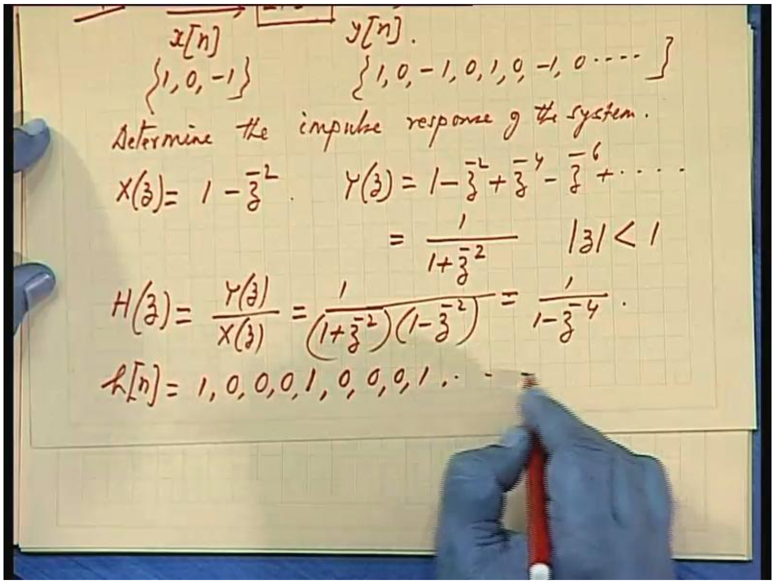

You determine the impulse response of the system; determine the impulse response of the system, okay so, let us see if we apply Z transform then what would be X z, what would be X z, 1 minus z to the power minus 2. What will be the corresponding Y z? 1 minus z to the power minus 2 plus z to the power minus 4 minus z z to the power minus 6 and so on, okay.

So, this is an infinite sequence, this will be giving me 1 plus z to the power minus 2, okay; z to the power z magnitude should be less than 1, that is our assumption. So, what will be H z? Y z by X z, there is a noise 1 plus z to the power minus 2 into 1 minus z to the power minus 2; that will be 1 minus z to the power minus 4.

So, what will be the inverse of this, very you are very familiar with this; it will be 1, 0, 0, 0 minus 1 or plus 1? If it is minus then in the series it is basically z to the power minus 4 with plus signs. So, it is a sequence like this, okay. This will be the impulse response.

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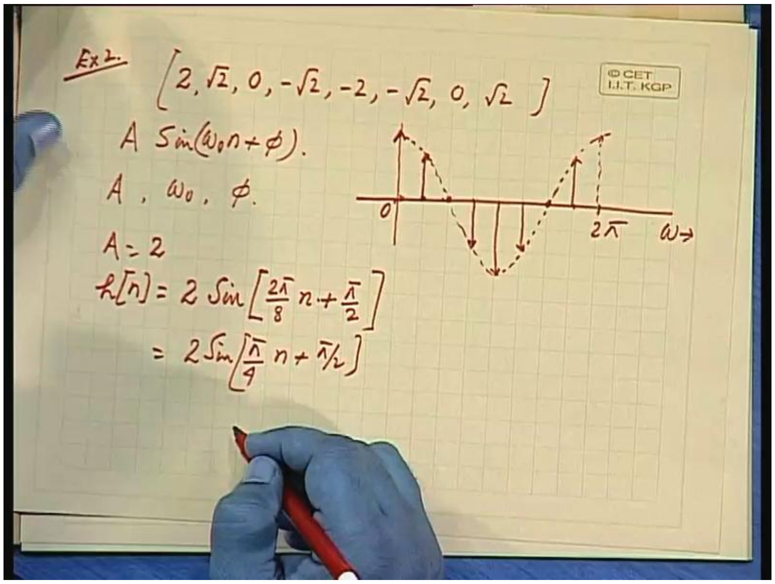

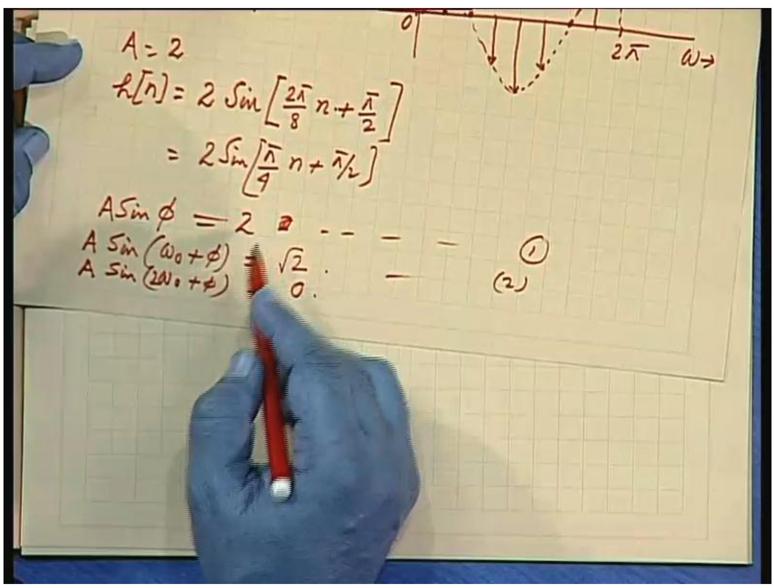

Now, next example is there is a sequence given 2, root 2, 0, minus root 2, minus 2, minus root 2, 0, root 2. This is just one period of a sequence; A sin omega naught n plus phi, can you determine A omega naught and phi? One may detected by inspection but we will do it, will solve it by rigorous mathematical technique.

Let me see, first of all these set of values first one is 2 next one is root 2, next one is 0, next one is root 2, next is minus root 2 of course then minus 2, then again minus root 2, then 0, then root 2, next it repeats again at this point. So this is a period, okay. You sketch it in the form of a sinusoidal function, okay. Now, tell me what will be A from here, what is the expression for this?

Okay, A is equal to 2 and then what will be the function h n, you may be able to write it, will solve it because the sequence could have been something else also, here it is very simple. So, this is 2 p i, okay, this is 0, this is the radian frequency omega. So 2, is it a sign function? Cos function, so in terms of sin, sin how much is omega naught? How much is omega naught, how much is this?

So, 2 p i by 8; this is 45 degrees, 90 degrees. So, 2 p i by 8 n all right and how much is the phase shift? Plus or minus, how much is the phase shift plus or minus? Plus p i by 2, is it all right? So, it will be 2 sin p i by 4 n plus p i by 2, okay. So, let us see how we solve this, analytically. Put n is equal to 0, so A sin phi that is equal to 2.

Next, A sin omega naught plus phi is equal to root 2, know this is equation 2. A sin twice omega naught plus phi is equal to 0, okay, is that all right, so substitute from here. In the second equation, the second equation is A sin, break it up omega naught, cosine phi plus A cosine omega naught, sin phi is equal to root 2 and A sin phi is 2, okay.

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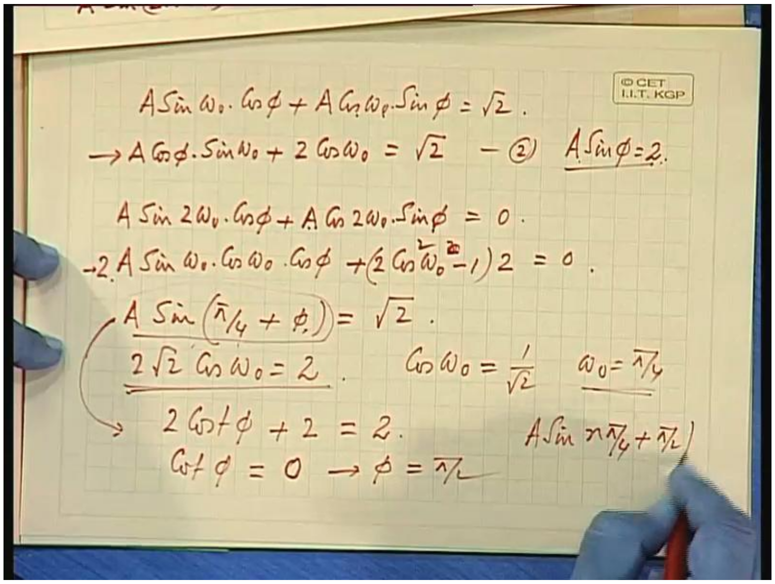

If I put A sin phi is equal to 2, this A sin phi will become 2. So, A cos phi sin omega naught plus 2 into cos omega naught is equal to root 2; so this is modified equation 2, what about the third equation? A sin 2 omega naught cos phi plus A cos 2 omega naught, sin phi is equal to 0, okay; put it to this side A will get cancelled, okay or okay retain A.

A sin omega naught, cosine omega naught, cos phi okay into 2 plus A sin phi is 2, 2 cos omega naught squared twice sorry, cos squared omega naught minus 1 into 2 is A sin phi is 2, is equal to 0. Now, from this 1 and this 1 we can eliminate A; if you try to substitute this here, will finally get A sin p i by 4 plus phi is equal to root 2. And from there another equation get; before this 2 root 2 cos omega naught equal to 2 from where we get cos omega naught is equal to 1 by root 2 and hence omega naught is equal to p i by 4, okay.

That has been substituted here, you will get this relation first by elimination okay and substitute it here, it will be root 2. So from there you can compute phi, okay. There is one relationship A sin phi is equal to 2 and this one, if you expand and then substitute this condition; will get 2 cot of phi plus 2 is equal to 2, you just expand it A sin p i by 4 is 1 by root 2 that 1 by root 2 I shift here, cos phi by 4 is also 1 by root 2.

So, the 1 by root 2 terms, if I put here that will become 2 and on the left hand side we will get this, friends. Cot phi will come out to be 0, so phi is p i by 2. So, it will be A sin n p i by 2 sorry, n p i by 4 plus p i by 2 which means of course cos sign, is that all right. Let us take up another example.

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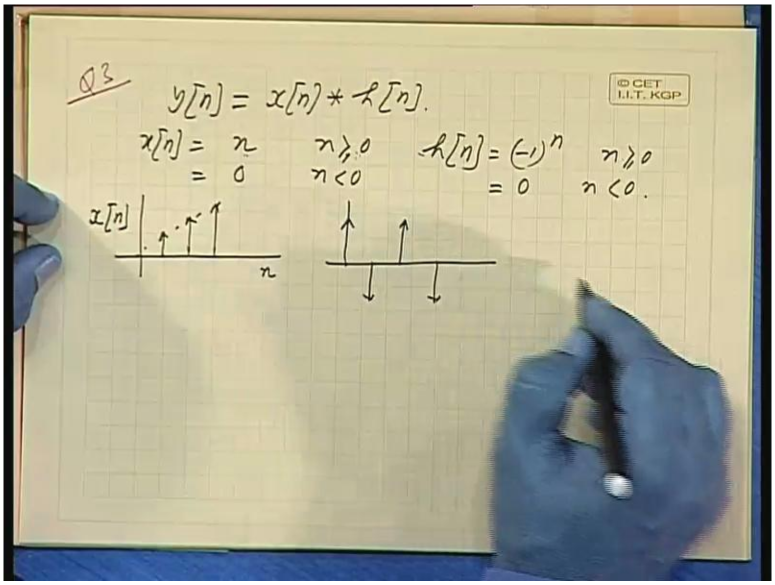

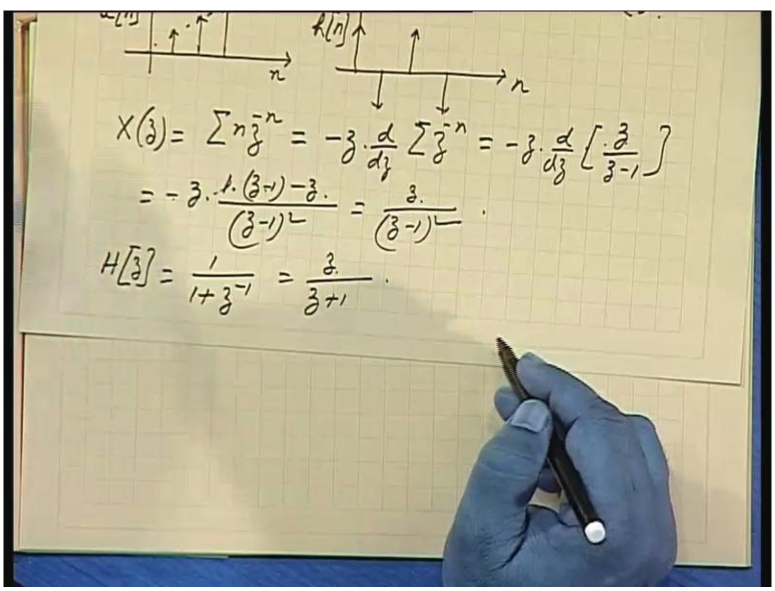

I think I should change the pen. You are asked to determine Y n which is the convolution of x n and h n, okay. x n is given as n, n greater than equal to 0, equal to 0, n less than 0, instead I can put greater than 0 because when n is equal to 0, it automatically become 0 at x is equal to 0. So, this equal to sign, in this particular case does not have any significance, it may be just greater than and h n, what is this function? It is a ram function; h n is minus n to the power n, n greater than equal to 0, equal to 0 for n less than 0.

The purpose of defining any any function in these two ranges, is to just a wide writing u n; otherwise you can write n into u n, we do not have to put all these, okay similarly this one. So, just just sketch the functions. One is like this 1, 2, 3 and so on, this is x n the other one is h n, all right which will be alternately plus 1, minus 1, this is start with plus 1 n is equal to 0, it will be plus 1, minus 1 and so on, okay.

So, let us see what will the response corresponding to this; this is h n. Now, using Z transform you are asked to find out response using Z transform, okay. So, X z is sigma n z to the power minus n, all right. So, if you remember this is z, d, d z of sigma z to the power minus n which is minus z, d d z of, for z to the power of minus n, what is the sum? n 1 by 1 minus z inverse? So, that you can reduce it to this form also, just multiplied throughout by z.

So, take the derivative that gives me z minus 1 into 1, minus z, so divided by z minus 1 whole square; z into derivative of this. So that will be, minus z will get cancelled, so minus 1 and minus 1 will get plus z by z minus 1 whole square, is it okay. What will be h n, H z, H z; will be 1 by 1 plus z inverse which means, z by z plus 1, okay.

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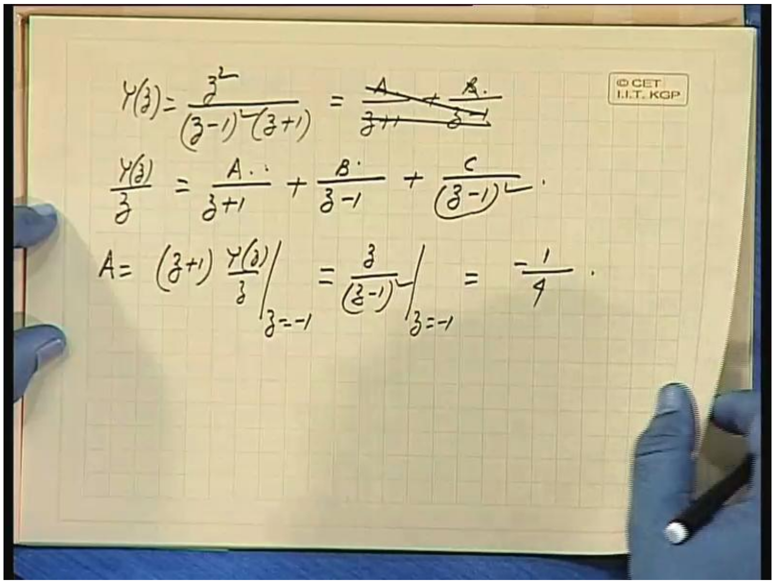

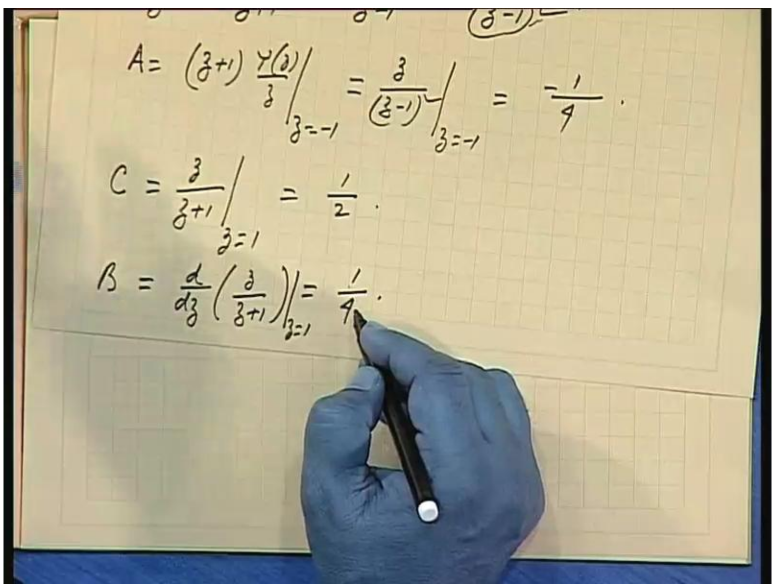

So, Y z will be the product of x n and x z; that will be z square by z minus 1 whole squared into z plus 1, is it so? So, take the product and make a partial fraction. So, z minus 1 whole squared into z plus 1, I can write this as A by z plus 1 plus B by z minus 1, okay sorry sorry; I take y z by z whenever you are having the function in terms of z, not z inverse in terms of z then it is better to take y z by z as a partial fraction, Why? After getting A B C, I will multiply z here then A z by z plus 1 is a standard form, B z by z minus 1, C z by z minus 1 whole squared, we know the corresponding values, okay values of variance. Similar thing you have done in the network synthesis also, remember RL circuit? You take z as p i s first, make the partial fractions and then get z s. So, what will be A if we multiplied by z plus 1, evaluated at z equal to minus 1?

So, if I multiply these by z plus 1 and divided by z; so it becomes z by z minus 1 whole squared at z is equal to minus 1, so that gives me minus 1 by 4, is it all right. Similarly, C let us evaluate C first; because that takes all the multiple roots. C, it will be Y z by z, z minus 1 whole squared, so that gives me z by z plus 1; to be evaluated at z is equal to plus 1.

So, this will be 1 by 2 and it is this function which is to be differentiated with respect to z, so for the evaluation of B and if there are more number of roots then you start with the highest term, term with the highest number of roots and then keep differentiating. So, B will be d d z of z by z plus 1 and evaluated at z is equal to 1, so that gives me again, 1 by 4. Is any other opinion about this for evaluation of B? Subtract, A by z plus one and C by z minus one whole square, from this side you can straight away get B by z plus 1, you do not have to take the derivative. Now, is it not all right?

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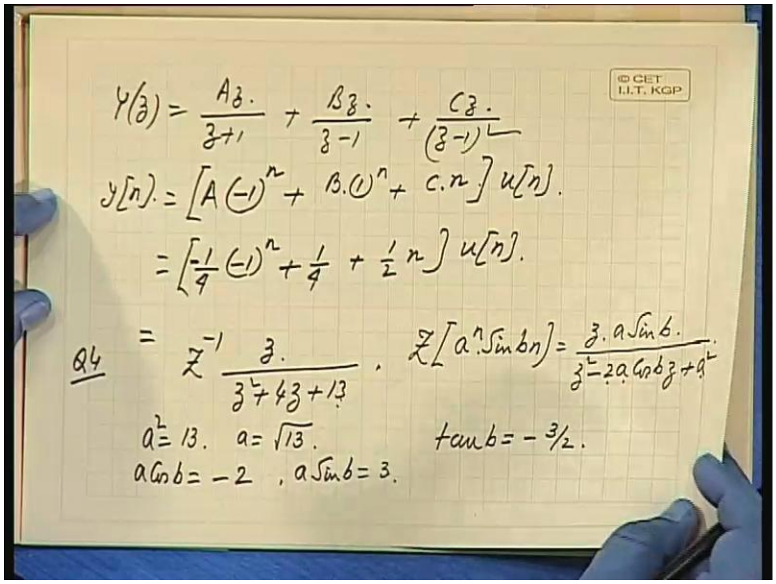

So, Y z will be A z by z plus 1 plus B z by z minus 1 plus C z, z minus 1 whole squared. Therefore y n, if you take the derivative or if you take the inverse of this; z by z plus 1, how much is it? A into minus 1 to the power n plus B into 1 to the power n, okay plus C into n just one or two problems earlier we have derived this, is it not; for that ram function. So, whole thing multiplied by u n. And now substitute the values A is 1 by 4 okay, into minus 1 to the power n; you can write 1 minus by 4 to the power n, can I write like that?

Should, should it be minus 1 by 4 to the power n, no this is all right? Okay plus 1 by 4 plus0.5 n whole thing into u n, is that okay? A was minus 1 by 4, so there will be a minus sign here, is that all right; this is the one we are talking about. Next, let us take another example on Z inverse, z by z square plus 4 z plus 13; this is a very familiar problem. We know Z transform of a to the power n sin b n, how much is that?

z a sin b divided by z square, no if I write in terms of z then it will be z square minus 2 a cos b into z plus a square, if you remember. So, straightaway matching the coefficients, a squared is 13, do you all agree? So, a is equal to root 13. How much is a cos b? Minus 2 a cos b is 4, so it will be minus 2. So, what will be a sin b? 13 minus 2 square 4, 9 under root; so 13 minus 4 under root so it will be 3, okay. a square, this square plus this square is 13 and that is equal to a square, a sin b is 3 is that all right? So tan b is minus 3 by 2, is anything wrong?

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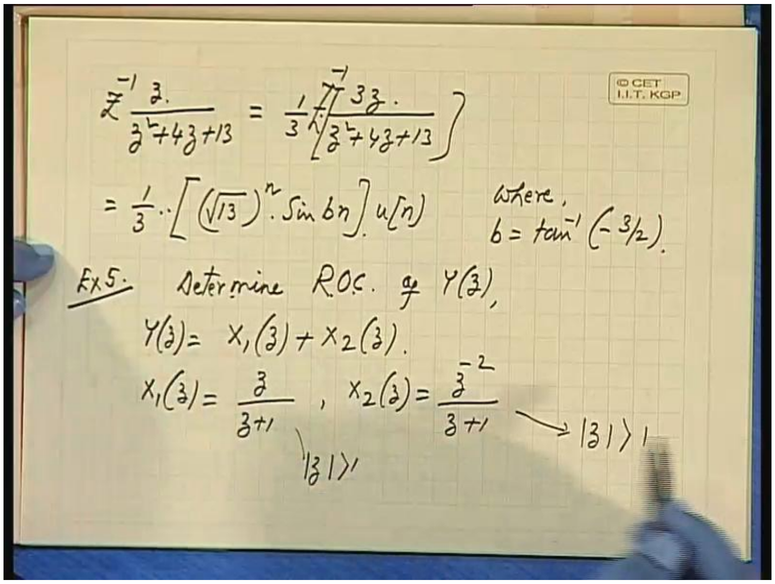

Therefore, Z inverse z by z squared plus 4 z plus 13 will be I can put; 1 by 3 here and 3 into z there, divided by z square plus 4 z plus 13. I can write this as z inverse of this and which is nothing but, I made it in the form of z a sin b, a sin b is 3, so I put 3 here and taken 1 by 3 outside. So, this is nothing but this term and the denominator is same, so what is the corresponding inverse a to the power n? Sin b, n.

So, root 13 to the power n and sin b n, is okay; I will write sin b n where into of course u n, where b is equal to tan inverse minus 3 by 2, is it okay? Another example would like to take; determine the region of convergence ROC of Y z where Y z is given as sum of X 1 z and X 2 z. X 1 z is given as z by z plus 1, X 2 z is equal to z to the power minus 2 by z plus 1; this will have a ROC, z greater than 1, this is also having an ROC z greater than 1, what will be the ROC for this function?

The function converges for this part outside the unit circle, for this part also there is outside the unit circle; so the total sum will have a region of convergence normally, normally let us see, okay. So, add them together, what do you get?

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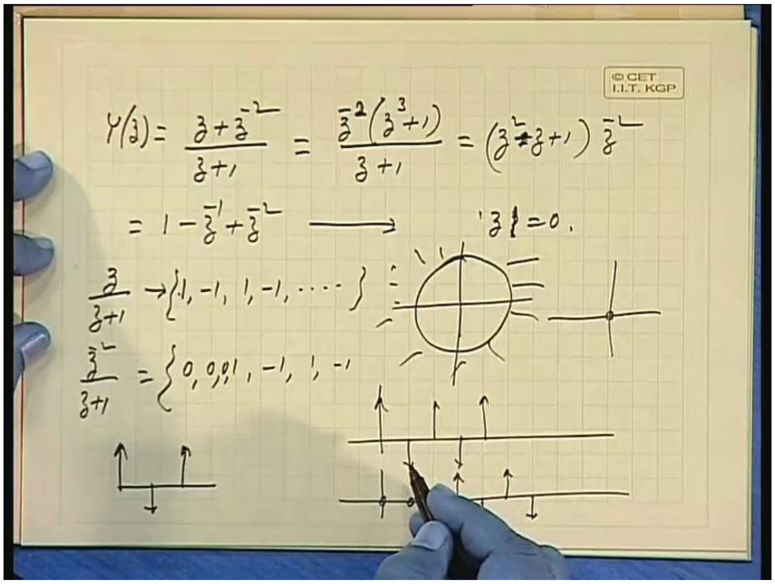

Y z, z plus z to the power minus 2 by z plus 1, okay. If, I take z to the power minus 3 outside minus 2 outside then I will get z to the power 3 plus 1 by z plus 1 which gives me; z cube plus 1 will have z plus 1 as a common, common factor, so z squared plus z sorry minus z plus 1, sorry this minus into z to the power minus 2 which gives me 1 okay, minus z to the power minus 1 plus z to the power minus 2. And what is ROC of this? When is it becoming infinitive? Only, at z is equal to 0.

So these convergence; except the origin, it converges in the entire space so, apparently the two function x 1 z and x 2 z, though they were having the region of convergence outside the unit circle, the function x 1 z was converging only in this region, so was x 2 z, but the total sum converges except at this point in the entire space, all right. Let us see, what will be the response, the impulse response of corresponding to i z? What will be x 1 n and x 2 n? If we add them together, what will be this?

z by z plus 1, what kind of sequence will it give? Alternately changing the sign, okay and z to the power minus 2 by z plus 1, what is it? 0, 0 and then 1 minus 1, 1 minus 1, is it 0, 0? Only one 0? 0, 0.

So, this was z to the power minus 2. This is z by z plus 1, thank you.

This is z by z plus 1 will be first one will be, second one; it will be 0, 0, 0 then 1 because I have to take one more 0. So, if you add them together; so it is a sequence first one is acceleratory, so is a second one with a shift but shift of 1, 2, 3, so now it is acceleratory. So, what will be the net result? It is a finite sequence. It is not an infinite sequence.

So, it will converge okay. Sometimes, it is a very good example; sometimes now a days to reduce noise of any system, we have anti-noise that means you create a signal which will be anti-phase with the noise, noise signal. So you create another noise, to cancel the parent noise. So, that you have silencing effect, you do not have to pad these, you do not have, to have a wall, barrier wall to prevent yourself from the disturbing noise. You can create it by having just some anti-noise generators, all right.

If you can know the source of the noise, actual noise, its behaviour; if you can sense it with the sensor and suppose the noise source is at a far off place and it takes so much time to propagate, all right, and you have a sensor there, it transmits a signal immediately the moment the signal comes here in that time, you have already started the anti-noise generator. So that, that will be cancelled with that; so you will be getting a signal free from noise. So, that is a very challenging task, scientists have working on it. So, this is a very good example of digital signal processing, in a very practical situation.

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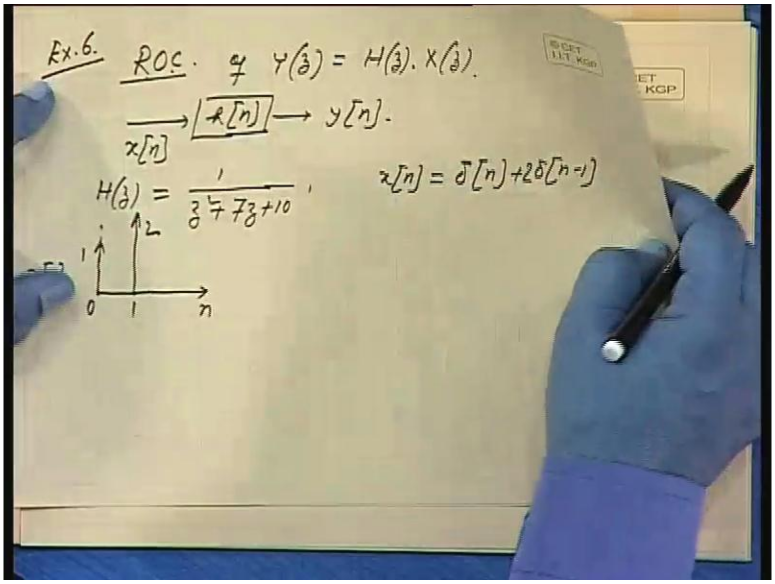

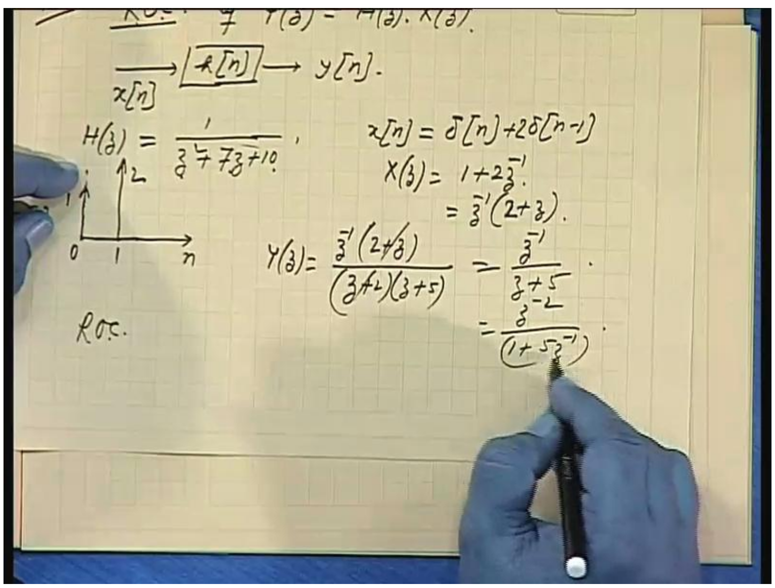

Let us take up another example, what will be the ROC? What will be the ROC of Y z equal to H z into X z? It is you are having x n and h n; convolved output is y n, so in the Z domain it will be like this where H z is given as 1 by z squared plus 7 z plus 10, okay. And x n is given as delta n plus delta sorry, 2 times delta n minus 1; that that means you are giving an input of 1 and 2. This is 0, 1, this is x n, what will be the region of convergence for Y z?

So, X z you can compute from here; it will be 1 plus 2 z inverse. I can write this as, Z inverse into 1 plus z, is that be, all right? So, Y z will be how much is it? Z inverse, it should be 2 plus z, 2 plus z divided by this and you can factorize this; z plus 2 into z plus 5, so that gives me Z inverse by z plus 5, okay. So, what is a ROC? One one may take 1 z common outside, so that will become z to the power minus 2 divided by 1 plus 5 z inverse, is that all right?

So, ROC should be 5 inverse, should be less than 1 or z should be greater than 5, okay. Let us come to another example; okay I will take another page.

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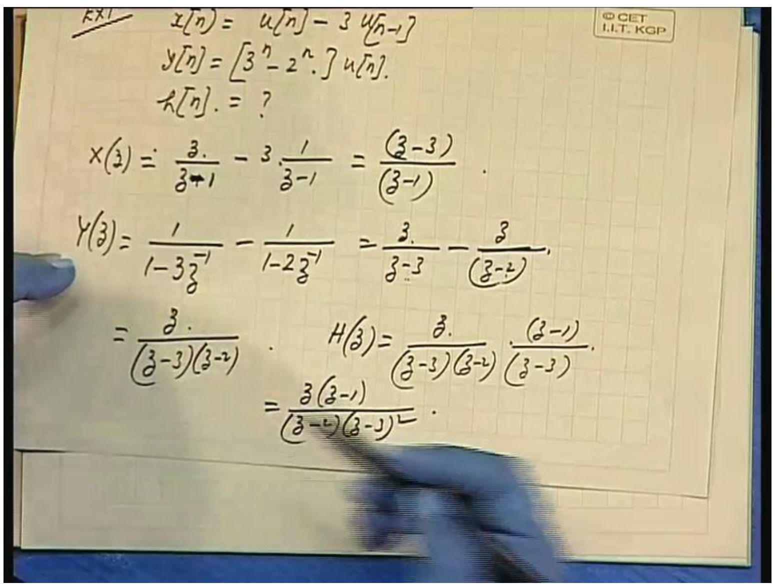

Consider the signal; x n is equal to u n minus 3 u n minus 1, okay, u n minus 1. y n is equal 3 to the power n minus 2 to the power n u n, what will be h n? So, let us compute X z, it will be okay, z by z plus 1. z by minus 1 all right, minus 3 into u n minus 1, it is z by z minus 1 shifted by one step; so z inverse multiplied by this, so z inverse will get cancelled with this z.

So, it will be 3 by z minus 1. So, we may take z minus 1, z minus 3, is that so? How much is 8 Y z? 3 to the power n. So, 1 by 1 minus 3 z inverse minus 1 by 1 minus 2 z inverse; which means z by z minus 3 minus z by z minus 2, correctly me if I am wrong, is it all right? So, that gives me z minus 2 minus z minus 3, so plus 1 by z minus 3 into z minus 2 and z can be taken out, okay. So, H z will be Y z by X z, is it all right? So, z by z minus 3 into z minus 2 into z minus 1 z minus 3, is this all right?

So, that equal to z into z minus 1 by z minus 2 into z minus 3 whole squared; make partial fractions and then take the inverse, can you make the partial fractions, let’s see.

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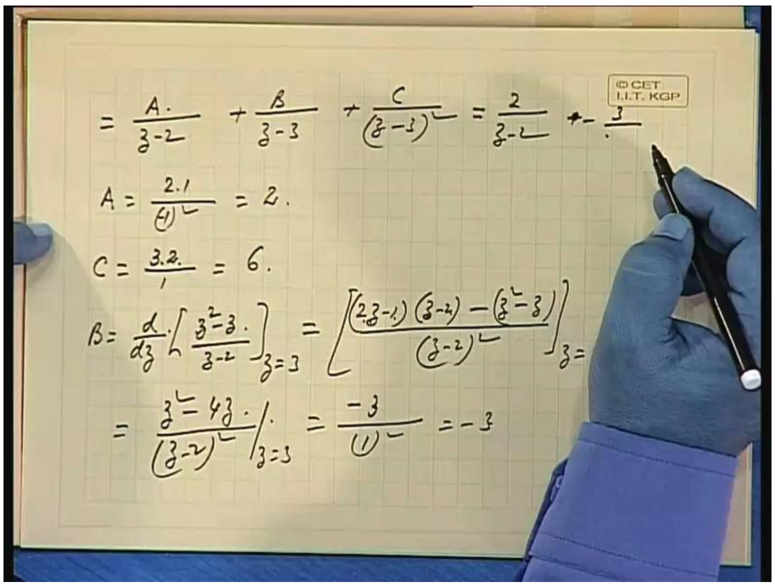

A by z minus 2 plus B by z minus 3 plus C by z minus 3 whole squared. So, A will be multiplied by z minus 2 and then put z is equal to 2; so it will be 2 into 1 divided by 2 minus 3 whole squared that is 1, so that gives me 2, you all getting that? C will be, I multiplied by z minus 3 whole squared then put z is equal to 3; so it will be 3 into 2 divided by 3 minus 2, that is 1, so that is 6, okay. And B will be, d d z of after multiplying by z minus 3 whole squared; that is z into z minus 1 by z minus 2, derivative of that, if I write z square minus z by z minus 2 and derivative of this at z is equal to 3, is that all right?

So, numerator will be z, 2 z minus 1 into z minus 2 minus z square minus z by z minus 2 whole square evaluated at z is equal to 3. So, 2 z square minus z square; that is z square, 2 z into 2, 4, 5 plus 1, so 4 is it so? z minus 2 whole square evaluated at 3; so that gives me 9 minus 12. So, minus 3 divided by 1 square, so minus 3, okay. So, you have got these three values. So, 2 by z minus 2 plus sorry, minus 3 by was any mistake? I should have factorised, I told you H z by z, okay. So, 1 z should have been reduced, okay.

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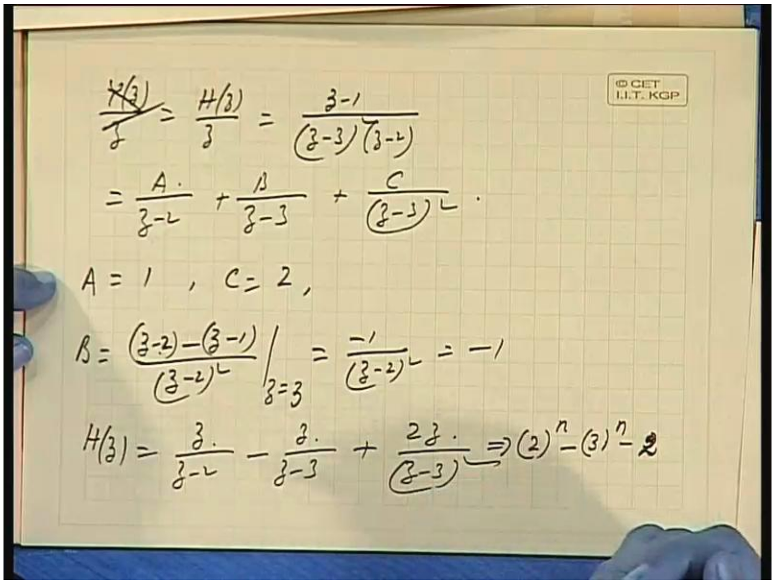

So, A z by a Y z by z; first of all we should compute H z by z, H z by z that is equal to z minus 1 by z minus 3 into z minus 2 remains 3 whole square into z minus 2 which will be A by z minus 2 plus B by z minus 3 plus C by z minus 3 whole square, okay. And you can compute A B C that, will be A is 1 all right multiplied by z minus 2; put z is equal to 1, z is equal to 2, so it will be 2 minus 1, 1 okay.

Then C, z minus 3 whole square, so it will be 2. And B will be, if I take the derivative of z minus 1 by z minus 2; so it will be z minus 1, z minus 2 into derivative of this is 1 minus z minus 1 divided z minus 2 whole square evaluate at z is equal to 3. So, z will go this is minus 1 by z minus 2 whole square, that is minus 1, is it so? So, H z after getting these you multiply by z, so it will be A z by z minus 2 which will be z by z minus 2 minus z by z minus 3 plus 2 z by z minus 3, whole square.

So, what will be the corresponding H n? This is 2 to the power n minus 3 to the power n minus 2, is it 2 into n? z by z minus 3 whole square, is it not a ram function? So, it is minus 1, 2 z by z minus 3 whole square, is it not same?

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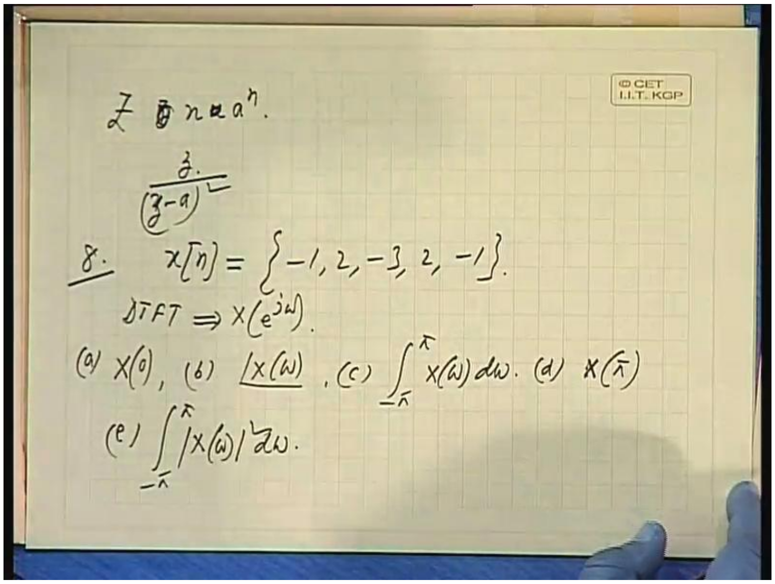

What is the Z transform of a ram function? Okay, n into a to the power n, how much is it? z by z minus a whole square. So, what is this? So, it will be minus 2 into it will be 3, it is 3, plus 3 it is minus. So, plus 3 into 3 to the power n into n and there is a multiplier 2; okay, whole thing into u n, is that all right?

The lase one is consider the signal, x n equal to minus 1, 2, minus 3, 2, minus 1 with DTFT, X e to the power j omega, determine a X at 0, b angle of X at any frequency, c minus p i to plus p i X omega, d omega; d X at p i. And lastly, minus p i to plus p i; X omega, magnitude square omega, okay. So, we will take up one by one at least some of them, rest of them I can leave it to you.

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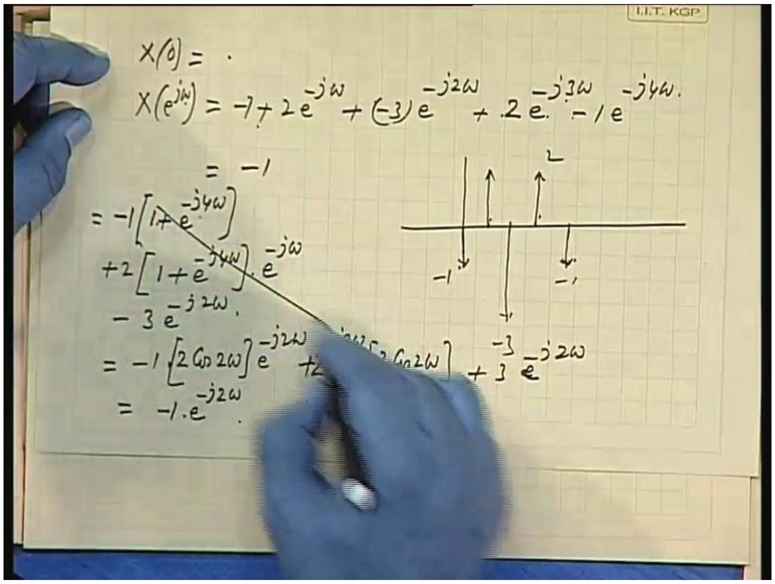

X at 0, how much is it? Okay, so all right let us see, what is X e to the power j omega? First, it will be minus 1 plus 2 into e to the power minus j omega plus, sorry minus 3 into e to the power minus j 2 omega and so on, all right. And I am putting omega equal to 0, except 0 means basically; I should have written e to the power j 0, okay omega 0.

So, it will be just sum of these terms. So, how much is it? If, you add up minus 1, 2, minus 3, 2, minus 1; it will be minus one, okay. What will be the angle of this? Okay, let me write this completely, minus 3 into e to the power minus then plus 2 into e to the power minus j, 3 omega minus 1 into e to the power j minus 4 omega. Now, the sequence that has been given to you, you see if you plot is there any symmetry? Minus 1, okay then plus 2 then minus 3, then again plus 2, then minus 1. So, it is a symmetric function, all right.

If I take this and this, this and this and this; if I pair them then it will be very simple; this is minus 1 and this one minus 1 into 1 minus e to the power minus j 4 omega, both are okay plus 2 into 1 plus e to the power minus j 4 omega into e to the power minus j omega and minus 3 into e to the power minus j 2 omega, okay. So, this one if I take e to the power minus j 2 omega common then that will give me, twice cos 2 omega into e to the power minus j 2 omega.

Similarly, here if I take e to the power minus j 2 omega out; so this will be e to the power minus j 3 omega into twice cos 2 omega inside and there is a 2 here plus 3 into e to the power minus j 2 omega, okay. So, it will be minus 1 into e to the power minus j 2 omega, 3 omega, sorry minus. Yes second term is; e to the power minus j, sorry, 1 and this I have taken 3 plus 1 minus 1, okay. Let me rewrite this, it will be clear; see minus 1 and this I have taken common, okay.

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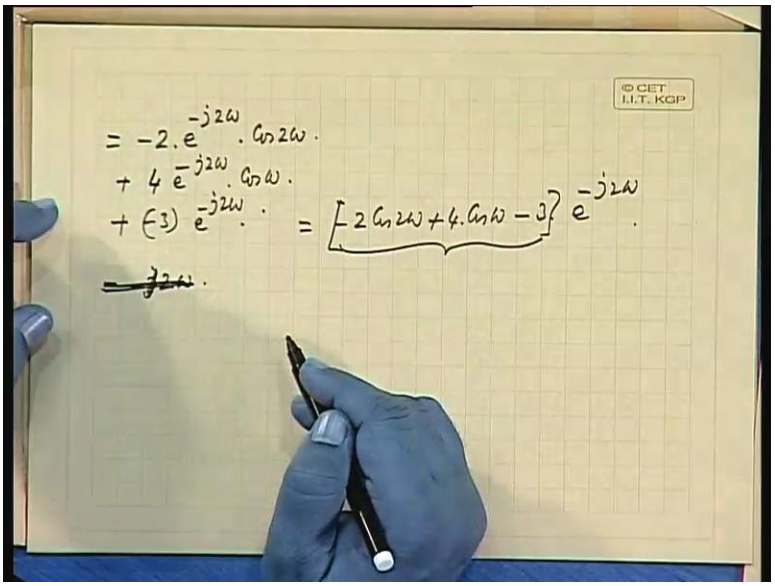

Actually it is 4 and 0, so it is minus 2 e to the power minus j 2 omega into cos 2 omega, is it all right, first one. Then next one; 2 into e to the power minus j omega, 2 into e to the power minus j 3 omega, if I take e to the power minus j 2 omega common. So, 4 into e to the power minus j 2 omega, all right into cos omega, is it all right.

And then the next one is minus 3 into e to the power minus j 2 omega all right; so if I add them together, it will be minus 2 cos 2 omega which is a real quantity plus 4 into cos omega which is a real quantity, minus 3 and multiplied by e to the power minus j 2 omega. e to the power minus j 2 omega for any omega, it is having an angle of 90 degrees minus 90 degrees and this quantity is real, all right.

So, this will have an angle of minus j, that is 2 omega; j will go, so minus 2 omega. Its sign is basically 180 degrees, no sorry e to the power minus j 2 omega; I made a I made a slip, e to the power minus j 2 omega, I this is real, angle is 2 omega, no. There is some slip here, minus 1 actually minus 1 plus, we are searching for the angle, okay it will have minus 2 omega. I think we will stop here for today, we will continue with the other two parts in the next class. Thank you very much.