Digital Signal Processing

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Lecture - 24
Lattice Filter (Contd…)

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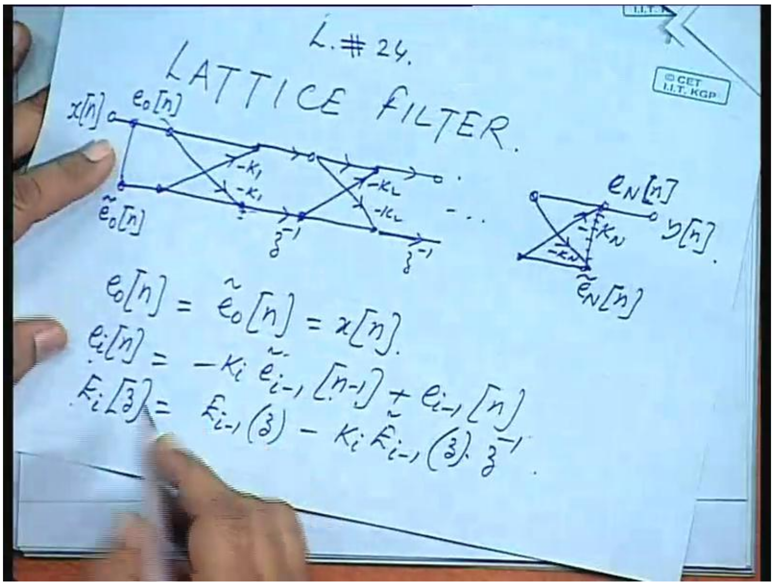

We shall be continuing with Lattice Filter. Last time we saw for a lattice structure for an FIR filter it was like this, the relationship between the input and the output was like this, this was e 0 n, this was e 0 tilde n. And then there are two constants, say minus K 1, minus K 1, in this path you are having a multiplier z to the power minus 1 at every stage and in this path it is a direct transmission. Again the second lattice comes minus K 2, minus K 2, and so on, lastly we had minus K N minus K N, this is e N tilde and this is e N n and this was our output y n, this is the input. So, in this structure we discuss last time, this an FIR structure, because all the signals are going forward even if there is a path like this or like this or like this, the signal is taking after sometime it will all die down; so for an impulse you will get a finite impulse response of this.

The relationship that we had was very briefly I will summarize, what we achieved last time and e i n was minus K i e i minus 1 tilde n minus 1 plus e i minus 1 n, that is at any stage e i, it was minus K i e i minus 1 tilde and that is delayed by one step. Z inverse means that is delayed by one step plus e i minus 1, the previous value of e i minus 1 that is transmitted directly. So, summation of these two quantities will be the output here, so at the i th stage this was the relationship, then E i z we obtain was E i minus 1 z minus K i E i minus 1 tilde z into z to the power minus 1, this relationship we established by induction if we remember.

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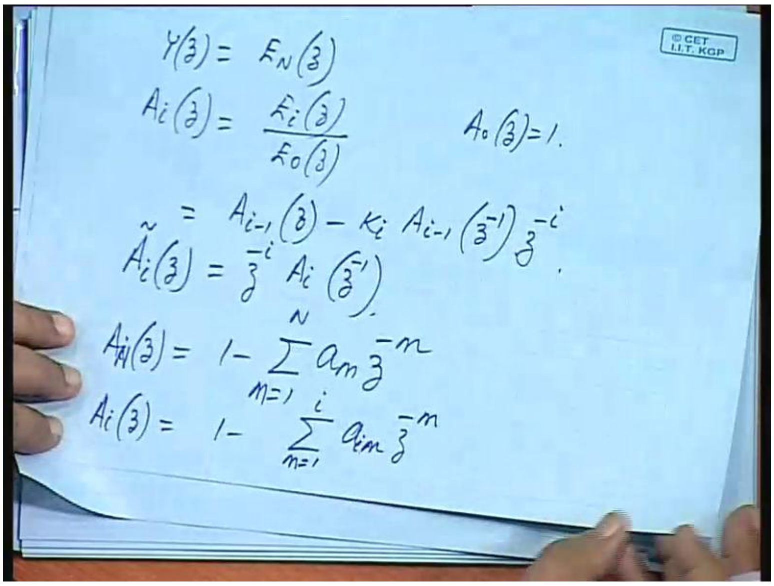

So, from there, so you got the transfer functions y z and the final output is E N z and at any stage A i z is E i z by E 0 z and A 0 z is equal to 1, so this was shown to be A i minus 1 z minus K i A i minus 1 z inverse into z to the power minus i. If you remember we derived this relationship and A i tilde z is z to the power minus i A i z replaced by z to the power minus 1, this was established last time.

So, today will see how to get the coefficients in a recursive form. See our equation is A i z is say 1 minus sigma a m z to the power minus m, m varying from 1 to n, this is A n z. And A i z at any intermediate step will be 1 minus sigma a m z to the power minus m this one is different m varying from I will call it a m, instead of that I write a i m, m is equal to 1 to i.

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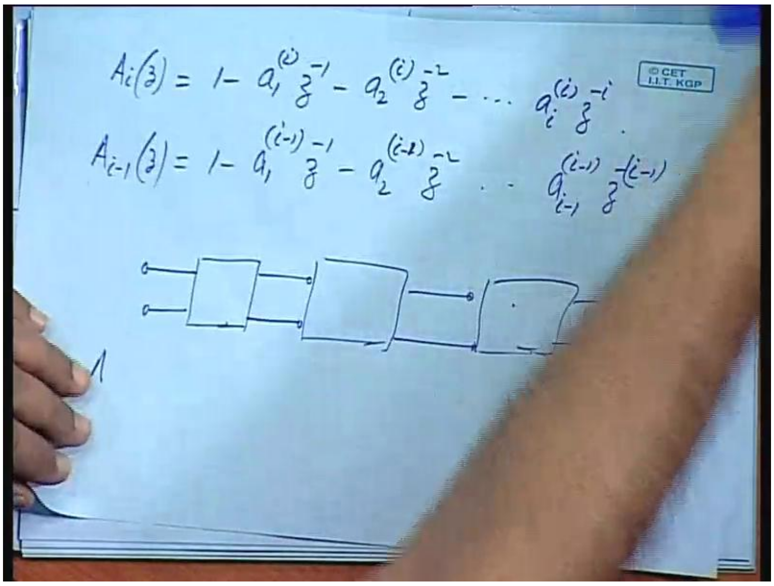

So let us see in the expanded form how they look like, any A i z I can write as 1 minus a 1. Let me put it as a superscript that is better z to the power minus 1 minus a 2 i z to the power minus 2, like that minus a i i z to the power minus I. Suppose that the i th stage we have got i th degree polynomial is like this. And at the previous step what will be the transfer function like, 1 minus a 1 i minus 1 z to the power minus 1 minus a 2 i minus 1 z to the power minus 2, a i minus 1 i minus 1 z to the power minus i minus 1 is that all right, see it is like this.

You are having say, let us replaces that does not represents the lattice by a block like this, so what is the transfer function at this stage, in terms of the transfer function at this stage, do you get my. So, the transfer function up to this you say A i minus 1, transfer function up to this is A i, what is the relation between those coefficients sets, this is what we are trying to establish. So, A i z we know the relation, ((Refer Time: 08:32)) A i z in terms of A i minus 1, is A i minus 1 z minus K i into z to the minus i into A i minus 1 z to the minus 1, I give an example that would be simpler.

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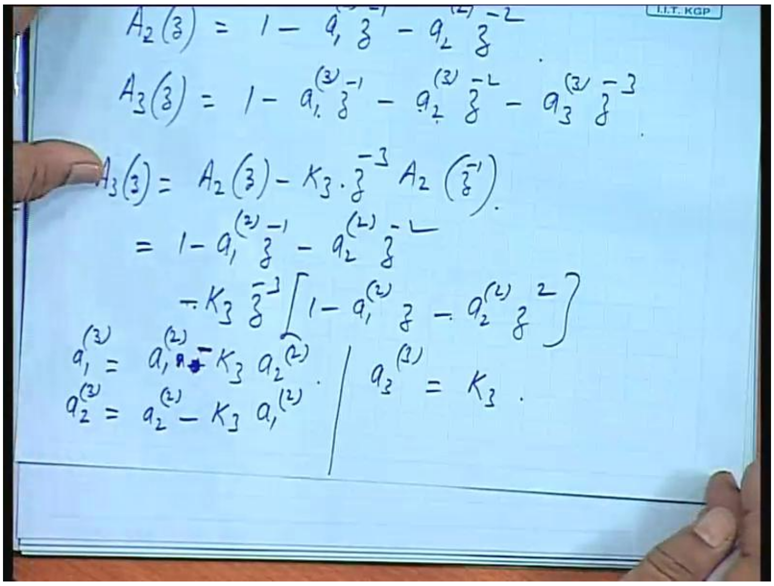

Suppose, we find A 2 z is equal to say 1 minus a 1 2 z inverse minus a 2 2 z to the power minus 2, this 2 stands for the polynomial order, we all to see the relation between the polynomial coefficients of A 3 z and A 2 z. And we know the relation A i z, ((Refer Time: 09:40)) that is A 3 z is equal to A 2 z minus K 3 z to the power minus 3 A 2 z minus 1, z to the power minus i, in case there was any slip last time was it minus 1.

Actually it should be z to the power minus i, we can check that relationship and there might have been some slip, so you put A 2 z that is and left hand side is this one A 3 z. So, this one will be 1 minus a 1 2 z to the power minus 1 minus a 2 2 z to the power minus 2 minus K 3 z to the power minus 3 into A 2 z inverse that is 1 minus a 1 2 z to the power plus 1 minus a 2 2 z to the power plus 2, z replace by z inverse.

So, this polynomial is replaced by this, now equate the coefficients, so how much will be a 1 3, it is the coefficient of z to the power minus 1, so here it will be a 1 2 minus and minus plus z to the power minus 3 and z to the power 2 will give me z to the power minus 1. So, K 3 a 2 2. A 2 3 minus and minus, is it total yes, that class is z to the power minus 3, so this should be minus, thank you; there z to the power minus 1 thank you, it should be minus. Then a 2 3, similarly z to the power minus 2, coefficients of z to the power minus 2 is a 2 2, then minus K 3 into a 1 2 and then a 3 3 is equal to K 3. Therefore now I have just given you an example of second order and third order, can you generalize it.

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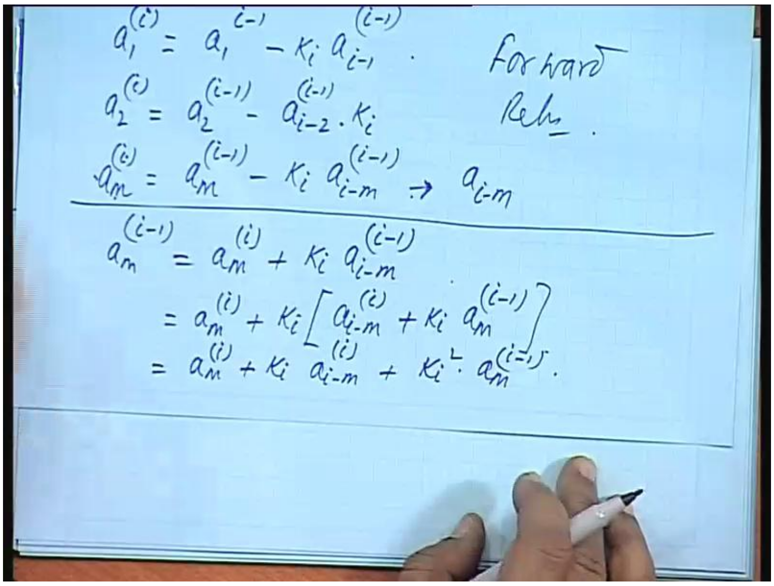

If you generalize it, I will get a i, i will be K i, like we have got a 3 3 is k 3, the last coefficient will be equal to the constant, then a 1 i is equal to a 1 i minus 1 ((Refer Time: 13:14)) see a 1 3 is same as a 1 2 minus K i a i minus 1 i minus 1. A 2 i is equal to a 2 i minus 1 minus a i minus 2 K i i minus 1 here, so in general a m i is equal to a m i minus 1 minus K i a i minus m i minus 1.

Now, this is the forward relation from second order polynomial. How to go to third order, from third order how to go to fourth order and so on, what will be the reverse relation; that means, if you are given a polynomial you can find out what will be the next polynomial. Now, what will be the reverse relation; that means, if you are given K 1, K 2, K 3 you can find out the polynomial coefficients, if the polynomial coefficients are given can you find out the lattice co-constants K 1, K 2, K 3.

So, let us try to find out what will be the reverse relation, so will be making use of this itself a m i minus 1. So this is the forward relation, a m i minus 1 can be written as a m i plus K i a i minus m i minus 1 equal to a m i plus K i, this I can write as a i minus m i plus K i a m i minus 1. See you have calculating a m i, can you calculate a i minus m from here, m replace by i minus m, so you can get this relation. Now, this will be transferred on this side, from here that will be plus, so finally we get a m i plus K i into a i minus m i plus K i squared a m i minus 1, so I can bring to this side.

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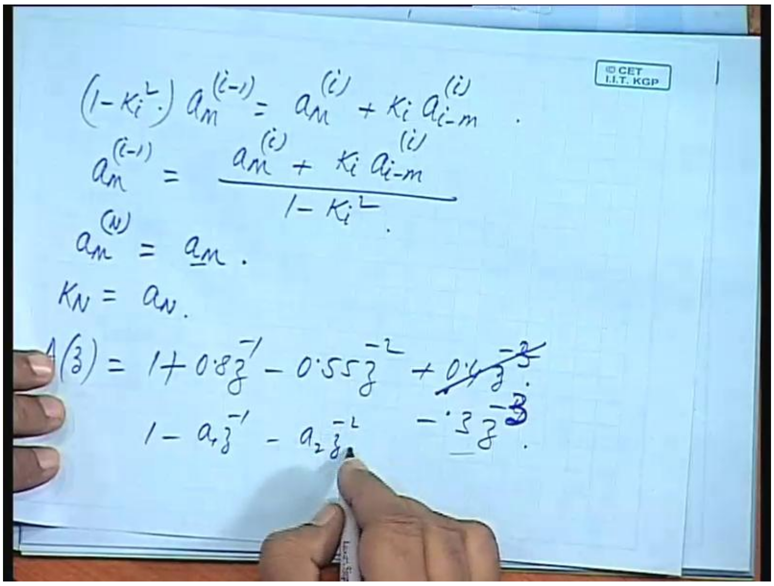

So, that will give me 1 minus K i squared into a m i minus 1 is equal to a m i plus K i a minus m i, therefore a m i minus 1, can be written as a m i plus K i a i minus m i divided by 1 minus K i squared. So, if you are given a polynomial you start with the last coefficient i th stage, take the entire polynomial that is the final stage, then you go to the previous step the coefficients corresponding to previous step. So, that will give me the solution for the constants K i, will see how this is to be computed.

A m n is a m itself and K N is a N, this is for the entire polynomial when the last term is given, if you are writing that as a N. Now, let us take up an example it would be then clear, you are given a polynomial A z equal to 1 plus 0.8 z inverse minus 0.55 z to the power minus 2 plus 0.4 z to the power minus 3 minus 1 minus 2 minus 3 minus 0.3 z to the power minus 4 I made a slip somewhere. Let me see my polynomial was of third degree, we are by mistake it has been made fourth anywhere, let me see the values K 3 probably it is not there, it is only 0.3, let us see.

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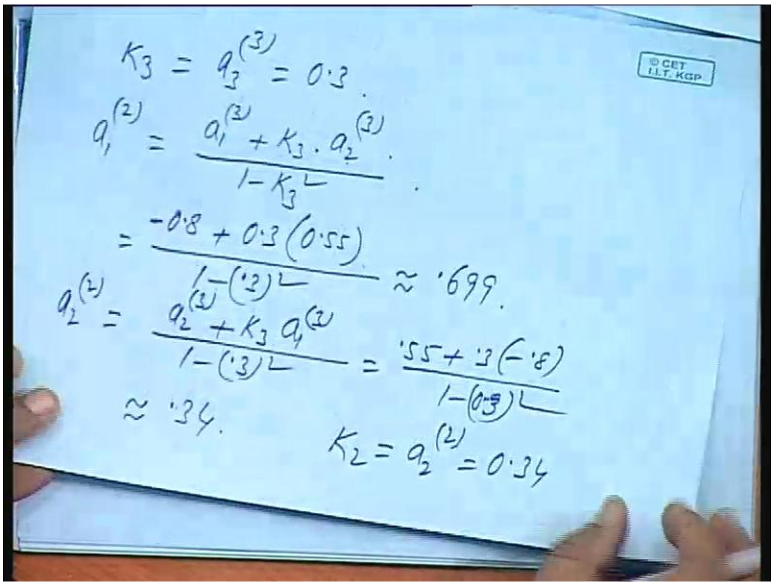

K 3 will be a 3 3, so the last coefficients is actually ((Refer Time: 19:57)) this is not there 0.3 with minus 3. So a 3 3 is 0.3, please correct the polynomial 0.3, a 1 2 is a 1 3 plus K 3 into a 2 3 divided by 1 minus K 3 squared, this is what we have got. And how much is a 1 3 minus 0.8, the polynomial is written as 1 minus a 1 z inverse 1 minus a 2 z to the power minus 2 and so on, so this is minus 0.8 plus K 3 is 0.3 into a 2 3 is 0.55.

And the third order polynomial the second term a 2 0.55 divided by 1 minus 0.3 squared, so that turns out to be approximately this is very rough calculation. Similarly, a 2 2 will be a 2 3 plus K 3 a 1 3 divided by 1 minus 0.3 squared. So, that gives me 0.55 plus 0.3 into minus 0.8 divided by 1 minus 0.3 square, so that gives me approximately 0.34. This is the second order polynomial and what will be K 2, therefore next lattice constant that is the 0.34.

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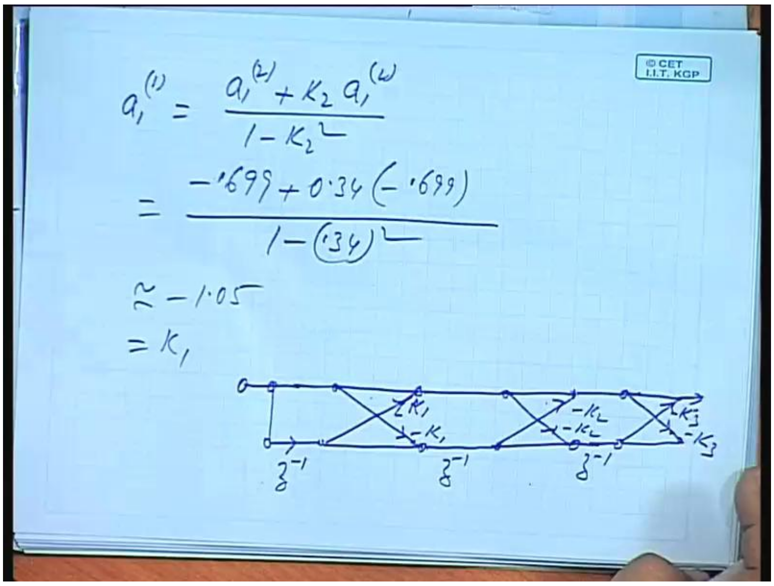

Then, a 1 1 will be a 1 2 plus K 2 a 1 2 divided by 1 minus K 2 squared, so that is minus 0.699, a 1 2 we have already computed 0.699, it was minus 0.699 ((Refer Time: 23:01)) this a minus 0.8. So, finally please correct it minus 0.699 plus 0.34 into a 1 is minus 0.699 divided by 1 minus 0.34 squared. This is very approximately coming out to be minus 1.05 and that will be equal to K 1, so the lattice structure for this polynomial will be it is like this.

((Refer Time: 24:28)) These are minus K 1 minus K 1, minus K 2 minus K 2, minus K 3 minus K 3, the values of K 1, K 2, K 3. We have got, so this one will be minus 0.3 minus 0.3, this is 0.34 0.34 and this is 1.05 1.05, these two will be with negative sign, this one will be positive, because K 1 is coming out to be negative. So, this one will be a plus 1.05, this one will be minus 0.34, this one will be minus 0.3. Now this is all about FIR structure, in FIR structure there is no problem of stability, let us see an IIR structure.

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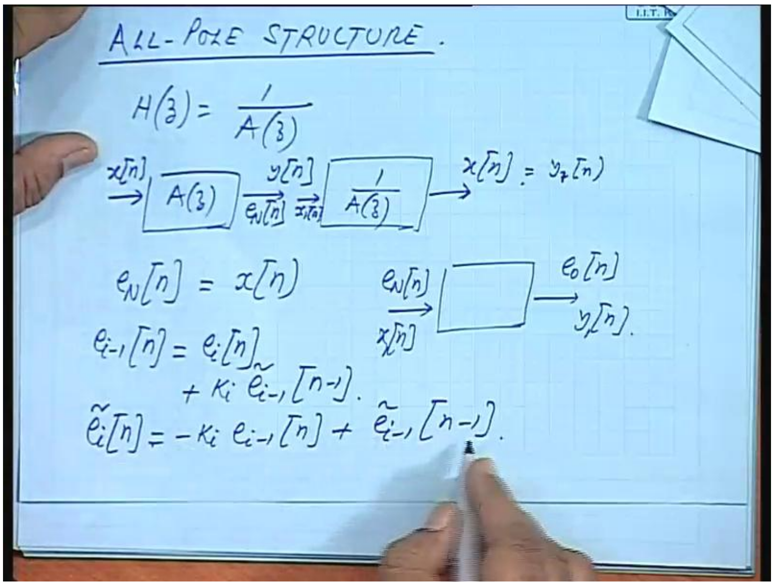

And rather on all pole structure, that is you have the transfer function as something like this, only the denominator is there, there is no 0, finite 0. Now, we can realize this earlier we are having x n and y n, this was say A z, suppose we interchange the position of x and y. If this is used as an input, if I have 1 up on A z, then I should get back x n provided this is a stable system, that means all the roots of A z should lie within the unit circle.

Then, I can see by interchanging the position of output and input, the same structure, rather same equations can be used with a little modifications for determining the all pole structure, let us see what it will be like. The same equations we shall be using, now e N n we are writing as x n, that is now we are giving the of the previous stage as the input. So we are taking this block where output has already come as e n from the previous step, that is going as input to the new block now.

And what should be the output, this is what you want in our new system, that means this y is now taken as some x 1 n and this x n will be the new output y 1 n, so it is this x 1 and y 1. So, e i minus 1 n I can modify if you remember the first equation that we had, ((Refer Time: 28:27)) E i n is equal to e i minus 1 n minus this. If I take it to this side, this one is put on this side, then e i minus 1 n is equal to e i n plus K i e i minus 1 tilde n minus 1.

And e i tilde n that we written I forget to write there, as it is minus K i e i minus 1 n plus e i minus 1 tilde n minus 1, this was our earlier relation that we are not disturbing.

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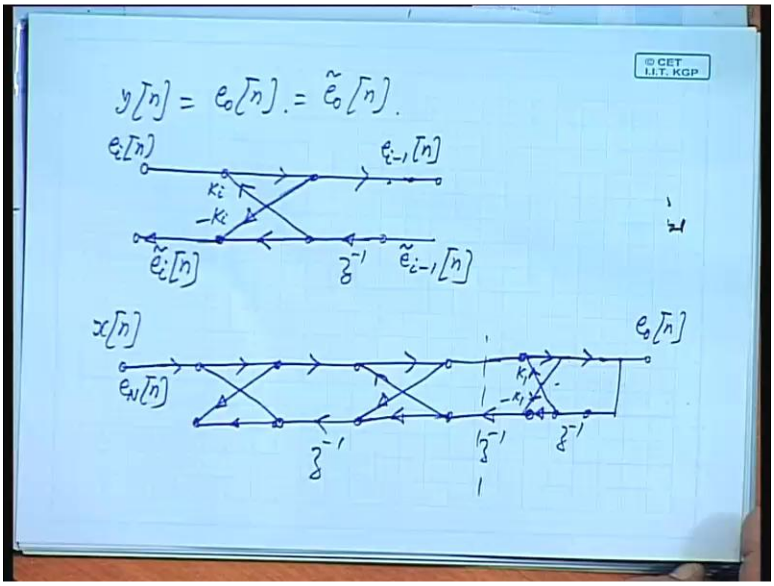

Then, y n is e 0 n and that is equal to e 0 tilde n, then what will be the lattice, you see the input is e i n and you are getting e i minus 1 the previous step as the output, like that we will come down to e 0, following this equation this one will be e i tilde. Now, it is plus K i on this side, see this is the input, this is the other input, this is multiplied by K i, they are added together and proceeding this way. And from here from the output there is this signal going out, so basically these are two outputs and these are the two inputs, we can concede this as if these are the two inputs and these are the two outputs.

So, this is just one stage the ith stage, this is the input output quantities, so x n in general this is E N n, this lattice filter is used in speech modeling. When you model the vocal tract or any acoustic system we use lattice filter it is very helpful the parameters are very robust. Quite often while we realize a digital filter will find that in the direct form the coefficients, because of the quantization and other in a sensitivities, the system may become unstable, the roots keep on changing their positions.

So, they are not very robust, in lattice filter this lattice constants if you take this K 1, K 2, K 3, that means in terms of K 1, K 2, K 3 you are representing a entire polynomial. So, this is much more robust and they are very useful in speech modeling, so like this the chain continues and this is one and at the end it is like this e 0 n, this is K 1 and minus K 1 this is the structure.

Now, you can see after evaluating all these K i is, just now we have shown by the same method that is in the FIR mode, you have calculated K 1, K 2, K 3, if you want to have an all pole structure. So, you first of all find out it is K 1, K 2, K 3 as if it is the FIR model, the same K 1, K 2, K 3 will be appearing here, because we are using the same equations. So, for an all pole model if you find the magnitudes of any of those K is greater than 1 it is an unstable system.

So, if you want to design, if you want to check whether an IIR filter is feasible or not you just take the polynomial of the denominator treated as an FIR filter, find out the K 1, K 2, K 3 by the same recursive relations. And if you find any of the case is greater than 1, then IIR realization is not possible.

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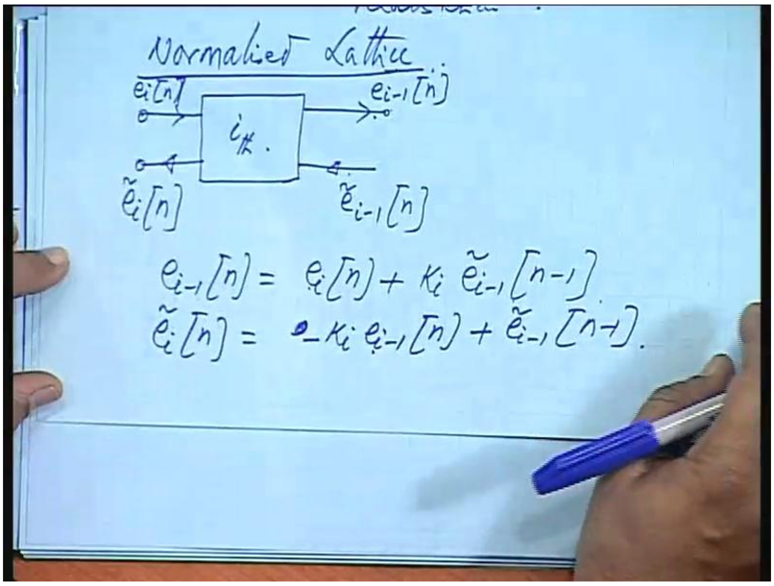

So, the magnitude of K i is must be less than 1 for an all pole realization, there are some interesting simplifications of this let us consider a normalized lattice, if you write this is the lattice block the inputs and outputs in pairs. So, this is e i n, e i n, this is e i tilde n, this is e i minus 1 tilde n and this is e i minus 1 n, so this is the i th block, e i minus 1 n is equal to e i n plus K i e i minus 1 n minus 1.

So, e i minus 1 is the output which is e i n input plus the second input this one. Similarly e i tilde n this output is equal to minus K i you refer to the earlier equations e i minus 1 n plus e i minus 1 n minus 1. Now, this is in terms of e i minus 1, that is in terms one output and one input, I want to write both in terms of the inputs, the outputs in terms of the inputs n minus 1 incorporates that, that is all inside.

So, we want to write output in terms of the inputs, what are the outputs this is one output, the other one is this one, so earlier the equation was like this e i tilde this output, in terms of this is an output actually e i minus 1 is an output. So, I want to write in terms of the input, both in terms of the inputs, so that is the normalized lattice, so that we can write here, so K i e i minus 1 I will e i minus 1 as this.

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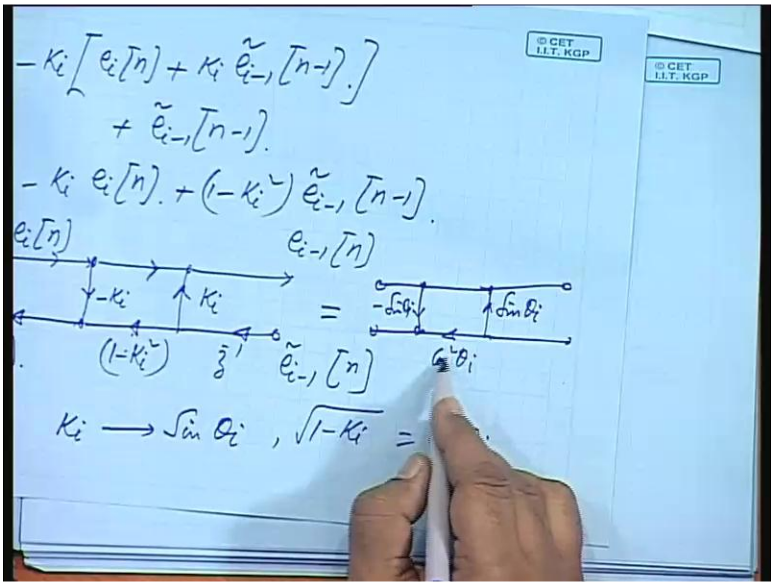

So, that gives me minus K i into e i n plus K i e i minus 1 tilde n minus 1 plus e i minus 1 tilde n minus 1 equal to minus K i e i n plus 1 minus K i square e i minus 1 tilde n minus 1. See this term was, in terms of this output, so I use this equation substituted here, so I have got both in terms of the inputs. Therefore, you can write e i n input e i minus 1 as the output K i minus K i 1 minus K i squared z to the power minus 1, now this is the new arrangement, this is the normalized lattice satisfying these two equations.

Since K i is less than 1, we can take K i as sin theta i, then this one will be 1 minus sin squared theta that is cos squared theta, so root over of that is cos theta i. Now in the loop gain will not change if I put one cos theta here and one cos theta here. So, for us the final output is concerned that will not get affected, if I put this constant in a square root of this here and square root of this here, so this structure is basically cos square theta, sin theta and this side minus sin theta i.

So, this cos square theta i, I am distributing in these two paths cos theta i and cos theta i, so that will not affect the overall poles distribution. It will be affecting only the gain it will be scaled by multiplication of cos theta 1, cos theta 2, cos theta 3 in the forward channel, so the overall transfer function for this normalize lattice.

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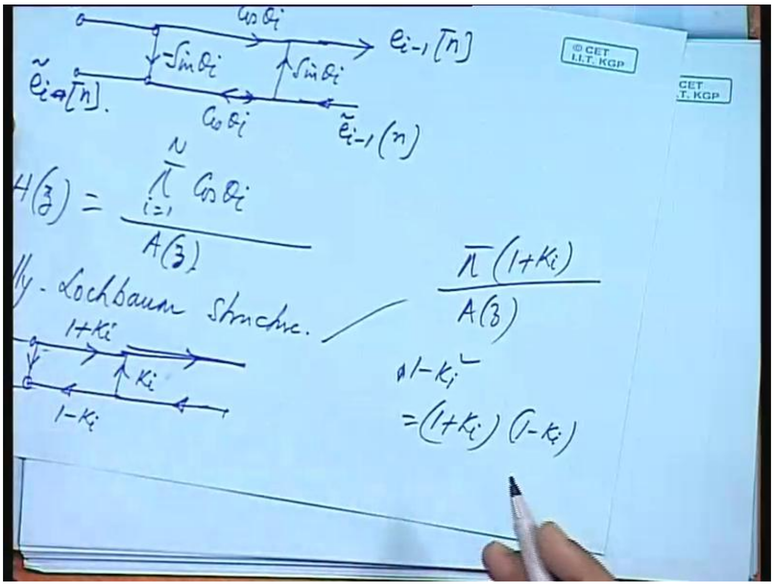

If I have the lattice structure like this, minus sin theta i cos theta i cos theta i, replace this way sin theta i, if I have a structure like this, then the overall transfer function will be just multiplied by product of cos theta i. So, H z will be product of cos theta i divided by the same A z, so it is only a scaling factor of this that will be change, so we can have this normalize lattice structure. There is also another structure Kelly-Lochbaum structure where that is the first one, where we took 1 minus K i squared here and 1 here.

So for that, for this structure that is the original one, the gain is actually it is 1 minus K i square can be factorized as 1 plus K i into 1 minus K i. So, instead of taking cos theta and cos theta, we can take 1 minus K 1 and 1 plus K 1 1 minus K i and 1 plus K i. So if you have 1 plus K i and 1 minus K I and then K i minus K i this type of structure, then the gain will be just 1 plus K i and same A z, this will be just product of these constants 1 plus K i.

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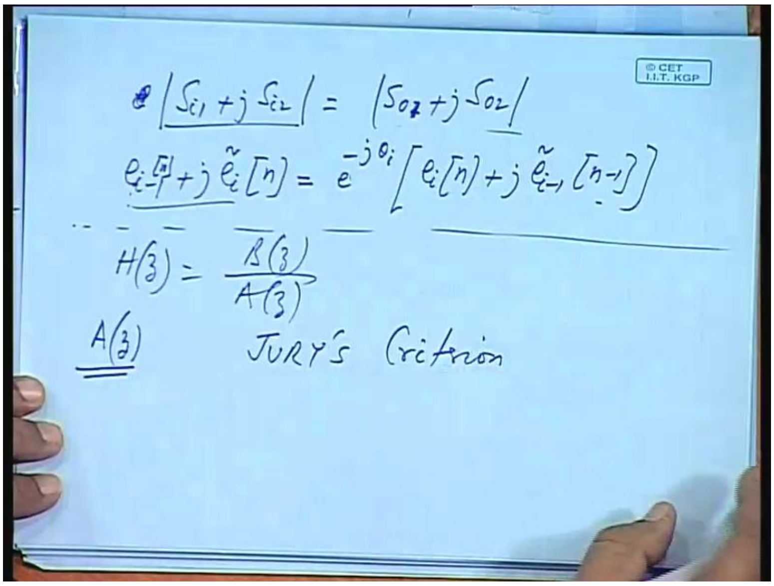

Another interesting feature is, suppose we take a complex pair of the inputs that is suppose I call it S i 1 plus j S i 2 input a complex pair, this magnitude is complex pair of output you will find. Let us see in magnitude they are identical it is a very interesting property gives you very useful property, so let us see e i minus 1 at any stage e i minus 1 plus j e i n this is a output.

Output terms are you see these are the input terms and these are the output terms, so e i minus 1 plus j this one these are the outputs, so e i minus 1 plus n plus j e i tilde n is how much e to the power minus j theta i e i n plus j e i minus 1 n minus 1 ((Refer Time: 47:18)) this one and this one. If you take the complex combination of the two and if you take the complex combinations of the outputs, you will find it is having a multiplier e to the power minus j theta, that is cos theta i and j sin theta i.

So, since this is having a magnitude of 1, so this magnitude is same as this magnitude, this you can see for yourself you just substitute the relations output and input, you will get this. Now, we are discussing about stability, I do not recollect whether we discussed about Jury's criteria for stability, now a polynomial given a polynomial that is H z is say B z by A z you have to test only the denominator polynomial for stability.

So, one is, calculate all the roots, there are standard programs. You can calculate the roots is any other method one is if you get the lattice structure the constants must be less than 1, the magnitude of the constants. There are many other methods for testing the roots of A z, whether they are within the unit circle or not, so Jury's criteria for A z, Jury's criteria is like this.

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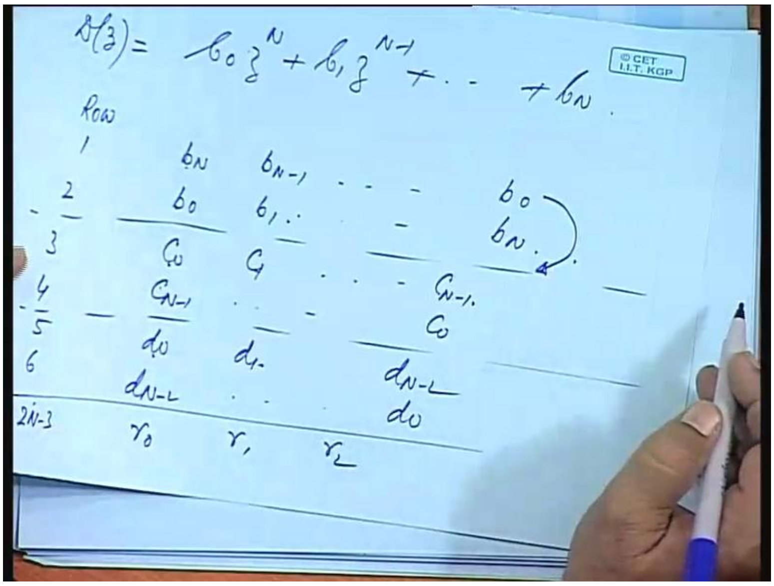

The denominator polynomial, we normally write n z by D z, so the denominator polynomial is written as b 0 z to the power n plus b 1 z to the power n minus 1 plus b n in this form. Then we form a row very much similar Routh Hurwitz criteria, row 1 you write the coefficients b N, b N minus 1 up to b 0, then row 2 you just reverse the order b 0, b 1 up to b N.

Then, row 3 you create from here c 0, c 1, one step less it will be c N minus 1, 4th c N minus 1 etcetera going up to c 0, where these synthetic rows are generated by various method very similar to Routh Hurwitz rows. In Routh Hurwitz criteria you create the rows once you have got the first set of coefficients, in a similar manner you have do it, I will show it here. Similarly, you calculate d 0, d 1, one step less, so it will be d N minus 2 d N minus 2 and d 0 and so on, lastly you stop at 2 N minus 3, row it is r 0, r 1 and r 2 when you are getting a quadratic that is three terms you stop there.

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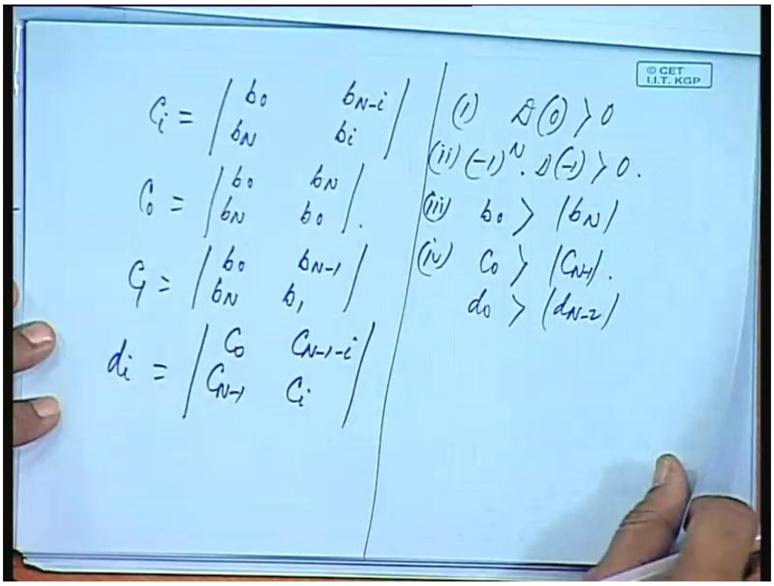

What is c 0, c 1, c 2 etcetera, c i will be b 0, b N, b N minus i and b i, c 0 will be b 0, b N, b N b 0, c 1 will be b 0, b N, b N minus 1 b 1, and so on. Similarly, d i will be c 0 c N minus 1 c N minus 1 minus i and c i like that, you get all the coefficients. Then the criteria says, first condition to be met is D 0 must be greater than 0, next minus 1 to the power N, D minus 1 should be greater than 0, third b 0 should be greater than b N. Similarly, c 0 should be greater than c N minus 1, d 0 should be greater than d N minus 2 and so on, so let us take an example, then it will be clear will use these conditions.

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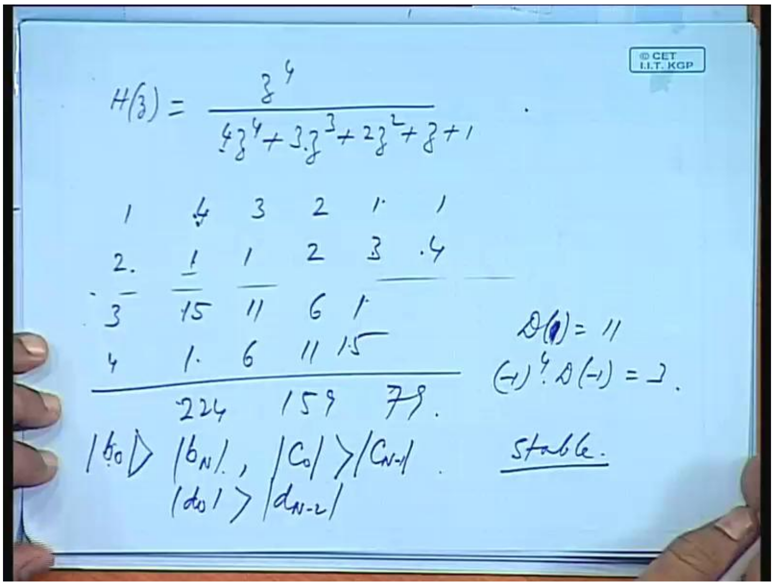

Say you are given H z as z to the power 4 divided by 4 z 4 plus 3 z to the power 3 plus 2 z squared plus z plus 1, so row 1 will be 4 3 2 1 1, check whether we are writing them correctly. Next row 1 1 2 3 4, next third it should be 15 check if I I am writing this ok, no there is some slip it should have been 4 4 z 16 minus 1 15, b 0; no. But it then it is b N it is somewhat b 0 square, it should have been the other way know it should have been the other way any way, I think I missed out the order.

Yes, first two rows are to be just interchanged, then it will be that is true, because if you see the arrangement is c 0 c 1 up to c N minus 1, then d 0, d 1, so it should have been b 0 b 1 up to b N and then this one, this should have come here. But, you have to interchange then only it will be, I mean I have written it 4 3 2 1, here I wrote b N from this end it should have been from this side.

So, it is like this. You write all in terms of z to the power plus 1, z to the power plus 2 and so on actually the confusion has come, because of that normally we write 1 plus z to the power minus 1 z to the power minus 2. So, those coefficients will be just reversed, so it is like this highest power of z the first coefficient and then you reverse the order, so 4 4 is a 16 minus 15, then 11 4 3 is are 12 minus 11, similarly 4 2 is are 8 minus 2 6 and 4 1 is are 4 minus 3 1, then reverse the order 1 6 11 15 and so on, you continue. So, I will write the next one 224 15 15 is are 225 minus 159 and 79, so all the conditions there are listed are satisfied here.

So, I will find b 0 is greater than b N, c 0 is greater than c N that is c N minus 1, and so on, D 1 is 11, 4 plus 3 plus 2 plus 1 plus 1 minus 1 to the power 4 into d at minus 1 is 3, so the system is stable, all the conditions are made. B 0, b 1, d 0 yes, that is also as I mentioned that condition is also satisfied, capital D 0 the first one is D 0 that is what 1, that is already there. And then D 1 the second condition was D 1 know, am I note where did I write ((Refer Time: 57:53)) this is a D 1, D 1 greater than 0, so will stop here for today.