Digital Signal Processing
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Lecture - 23
Computer Aided Design of Filters & Introduction to Lattice Filter

Shall we continue with Computer Aided Design, and then will take over Lattice Filter design.

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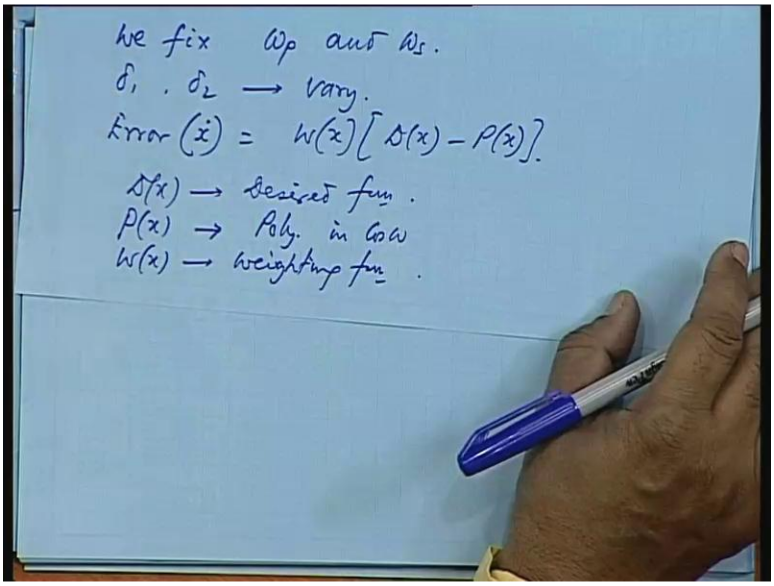

The Parks and McClellan algorithm that we discussed earlier for equi ripple filters was like this, we got say some function A e e to the power j omega as summation of a K x to the power K, so we called it P x where x was cosine omega. And then we fix the two frequencies omega p and omega s at their desired values and delta 1, delta 2 are allowed to vary, but the ratio is given it is normally kept constant. And then we took the error function if remember, if you write error function we took D x minus P x. Where D x was basically the desired function and p x was the polynomial approximation, polynomial in cos omega and W x was a weighting function and P x is not constrained in the transition region.

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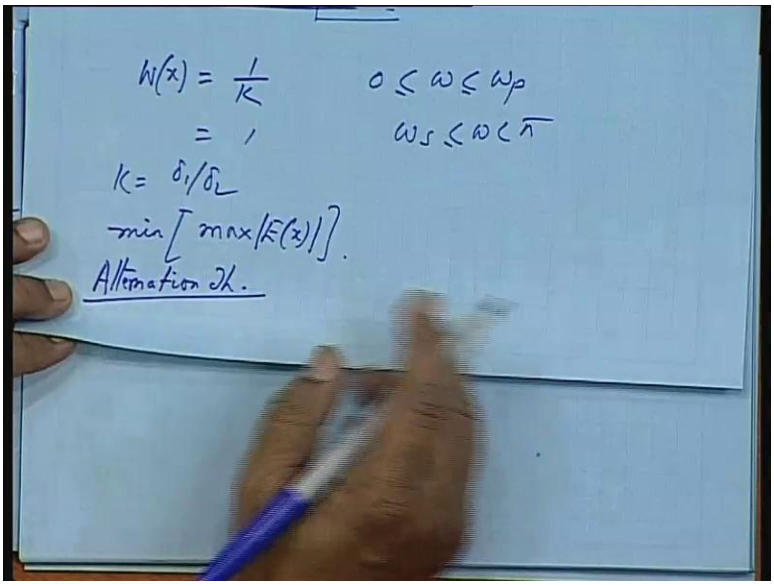

That is we compute the error only in these two zones, we compute the error only in these two zones. Here we do not constrict the function and W x we took as 1 by K and equal to 1 for omega s phi and K was delta 1 by delta 2. So, basically this is normalizing the error. So minimax criteria we had minimization of maximum value of E x modulus, so this we solved by alternation theorem.

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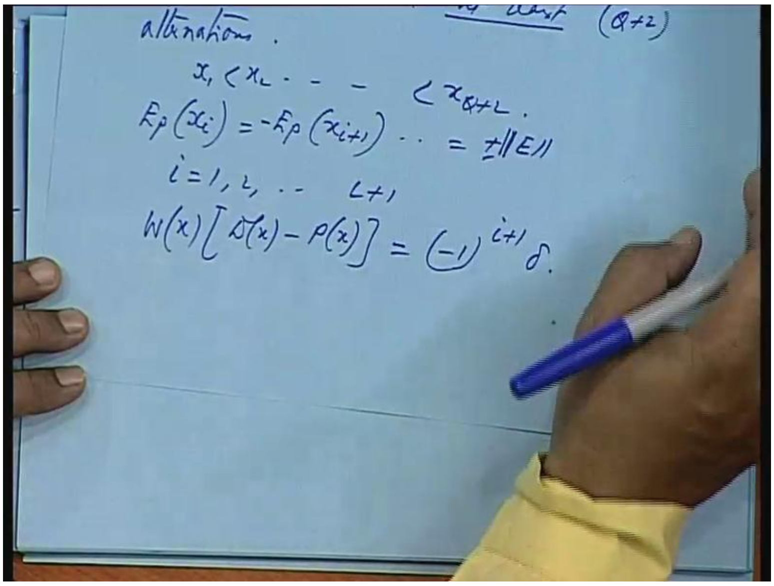

So, crux of that theorem was for P x to be unique, that minimizes the error is the maximum of the error we defined as E bar, so maximum of E we are writing as E bar.

So, P x to be unique that minimizes this E, E P x that is, that normalized error should exhibit at least, mind you at least Q plus 2, where Q was the order of the function, Q plus 2 alternations, that is x 1, x 2, x Q plus 2.

Such that, E P x i is equal to E P x i plus 1 with a negative sign and that is equal to plus minus this value, for all values of i this should be L plus 1 such points, the values of i. So, this was our alternation theorem, we found that W x into D x minus P x that is minus 1 to the power i plus 1 delta, so delta is to be optimized is optimum error.

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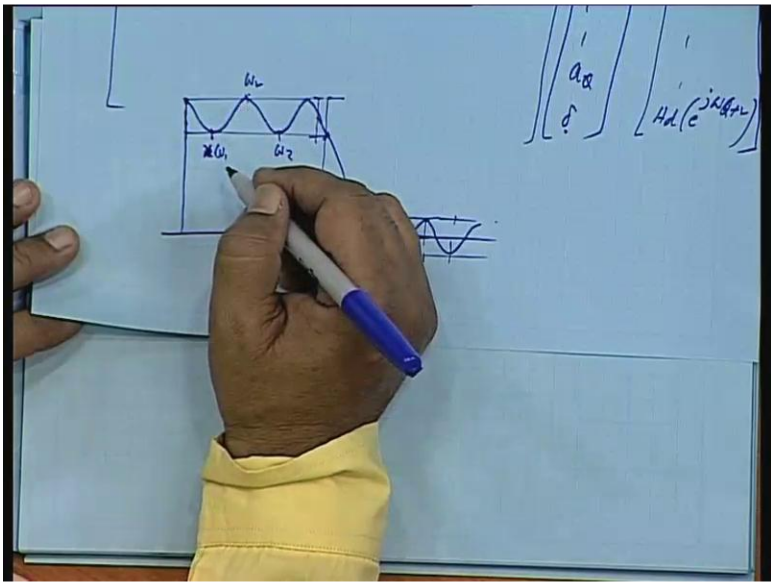

So, this we wrote as this center set of equations, we wrote as x 1, x 1 square, x 1 to the power Q and then W cos we wrote w 1, small w 1 into the coefficients a 0, a 1 up to how many such coefficients will come Q and then delta. And this will be equal to that function H d e to the power j omega 1 and so on, H d e to the power j omega Q plus 2, now obviously the value of delta is unknown, the omega is are also unknown.

So, there is alternative process involved, how do you solve this? So you make some initial guess I will just outline first how such a problem is solved, you are having within a bound, say may be from here, so this is fixed omega stop band, this is fixed omega pause band. So, these are the points of maxima or minima in the error function, if remember in the error function this will come as maxima and minima and these frequencies are not known, nor is delta known.

So, we first of all assume some values, so this is omega 1, omega 2, omega 3 and so on, I can write this also as one of those omega 0, so initially you are having an approximate values of these maximum. Then what you do try to fit it the polynomial, that polynomial may not have the maximum value here, that polynomial will be satisfying this.

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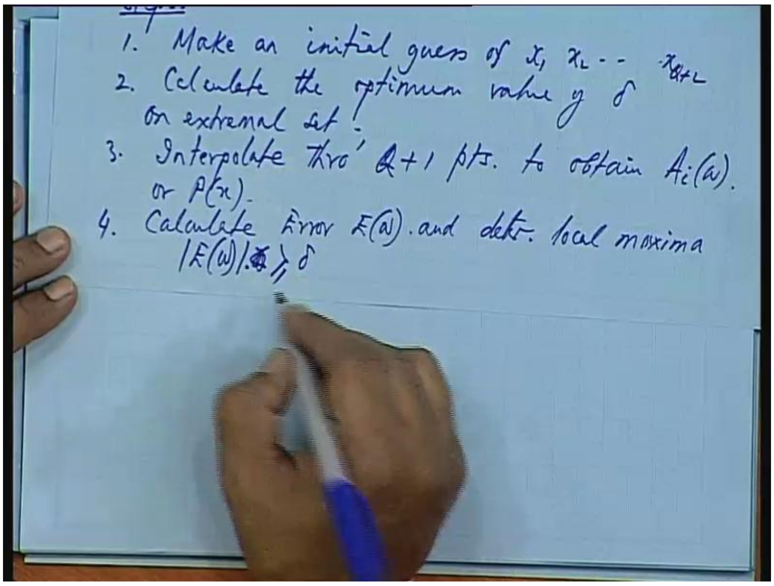

So, you may get a polynomial, where your earlier values where these. So they may not correspond to the maximum to get my point, so may be here, so your earlier points where these, whereas the new maxima are these. So, what you do, how do make an approximation for the polynomial, see it is passing through zero in between the maxima and the minima.

So, you have selected first some values at random for the frequencies omega 1, omega 2, omega 3, so between omega 1 and omega 2, there will be a zero processing, omega 2 and omega 3 there will be a another zero processing. So, you take even a linear interpolation you can make, so by interpolation technique you form the polynomial, then what you do take the derivative, find out the values of the maximum and the corresponding frequencies, you can do that from a polynomial of say order n there will be n minus 1 number of maxima and minima.

You can find out those frequencies and you can also find out the values of the function at those values, now find out all of them may not be equal, so take the maximum one then, take delta. Once again those frequencies you take up as the next approximation, find out once again the zero processing and it is an iterative process, when you find all of them are converging to a single delta that is delta is not modified much, then you stop.

So, I will write down the steps, make an initial guess of x 1, x 2, up to x Q plus 2, x means off course cosine omega 1, cosine omega 2 and so on, calculate the optimum value of delta on extremal set. Next interpolate through Q plus 1 points to obtain that polynomial A i omega or P x we can talk, calculate error E omega or E x whatever you call it and determine the local maxima, maxima and minima, that is local maxima such that...

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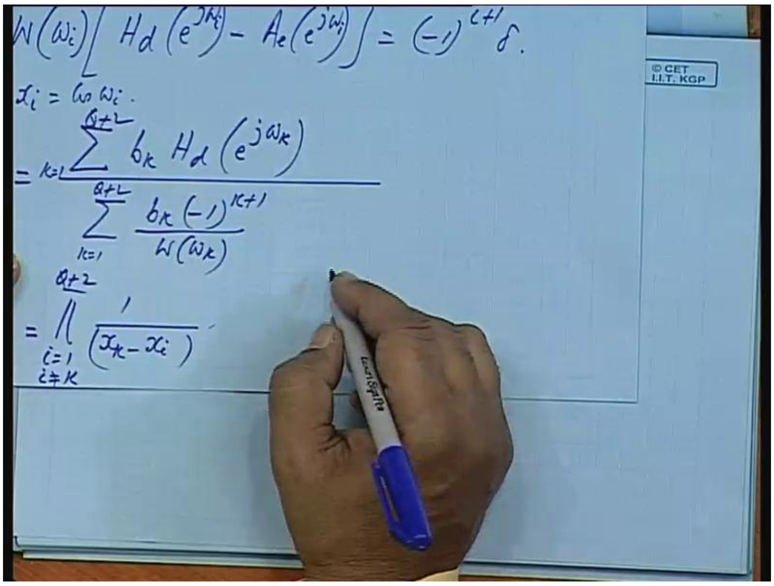

((Refer Time: 14:39)) Check whether the extremal points changed, if there is a change, then go back to step 2, step 2 is what calculate the optimum value of delta on extremal set. So, basically this is the desired function and this is the designed filter this is the equation, where x i was cosine omega i, so we are taking a different points basically this is i i.

So, after guessing, so you can put those iterative equations in this form, we write delta equal to sigma b k H d e to the power j omega k divided by sigma b k minus 1 to the power K plus 1 by W omega k, k varying from 1 to Q plus 2 again Q plus 2. S from here you compute delta where b k is 1 by x k minus x i phi i equal to 1 to Q plus 2 i not equal to K, this is Lagrange interpolation technique, mind you x i is cosine omega.

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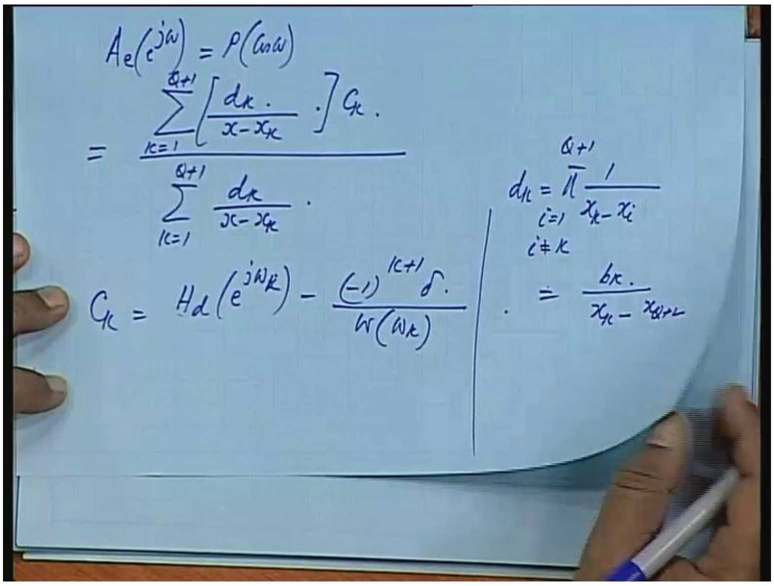

Therefore, A e e to the power j omega will be that is P of cosine omega is sigma d k by x minus x k into C k divided by sigma d k by x minus x k, again k varying from 1 to Q plus 1 and C k equal to H d e to the power j omega i minus, omega k. And d k is equal to phi 1 by x k minus x i, i varying from 1 to Q plus 1 i not equal to k, thus basically b k by x k minus x Q plus 2, because b k was this product where I was varying from 1 to Q plus 2.

And here for d k it is varying from 1 by Q plus 1, so in terms of b k this is one more x k minus x Q k plus 2 left here, so once you know C k and d k we can calculate these polynomial, so it is like this you fix the maxima minima frequencies to start with, you just assume some values. But, omega p and omega s will be one of the, I mean these two values are fixed and then try to find out the polynomial, again calculate the maxima and minimum points those frequencies and also the maximum value. And that should be greater than delta, that again change the polynomial and keep on trying this, next we shall see what will be the approximation of the filter order, how do calculate the approximate value of the filter order.

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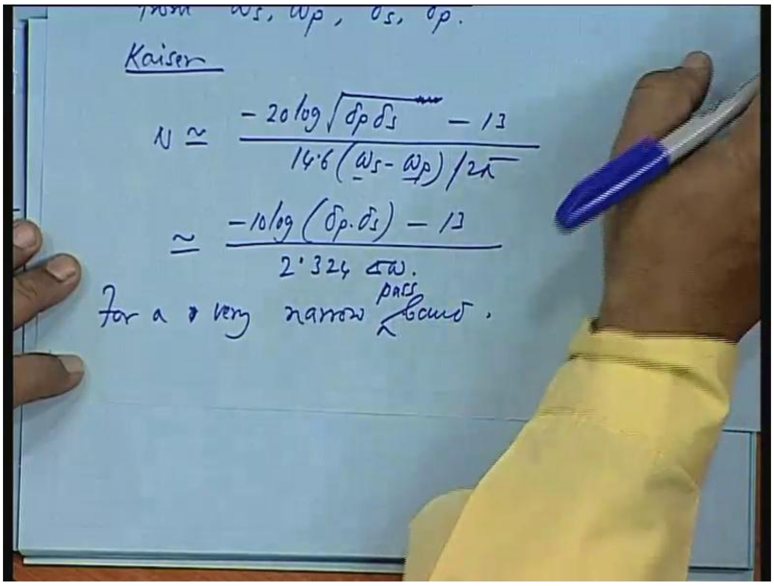

Specially, for FIR filters various suggestions have been made, so from these values omega s, omega p, delta s and delta p these are specified, what is the order of the filter. So, Kaiser suggested this relationship approximately N is equal to minus 20 log of root over of delta p delta s minus 13 divided by 16.6 omega s minus omega p divided by 2 phi, and that is 20 and root over of this.

So, that will give me minus 10 log product of delta p delta s minus 13 divided by 14.6 by 2 phi. So that gives me… and this I can write as delta omega delta omega means the stop band and pause band gap for a very narrow band, narrow pause band, this may not be very effective.

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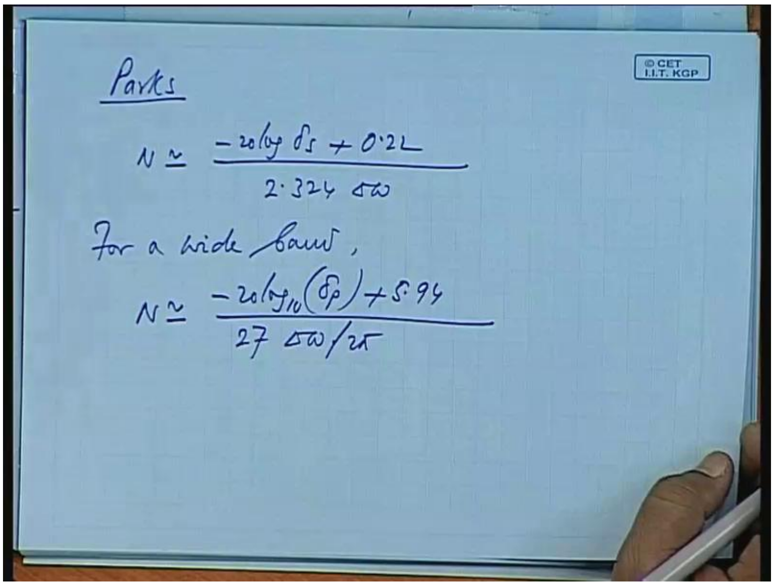

So, Parks has suggested N equal to minus 20 log of delta s plus 0.22 by 2.324 delta omega this is more effective, so instead of minus 20 log of delta p delta s, it is approximated as this. And for a wide band N is minus 20 log of delta p plus 5.94 divided by 27 into delta omega by 2 phi, so these two modifications were suggested by parks.

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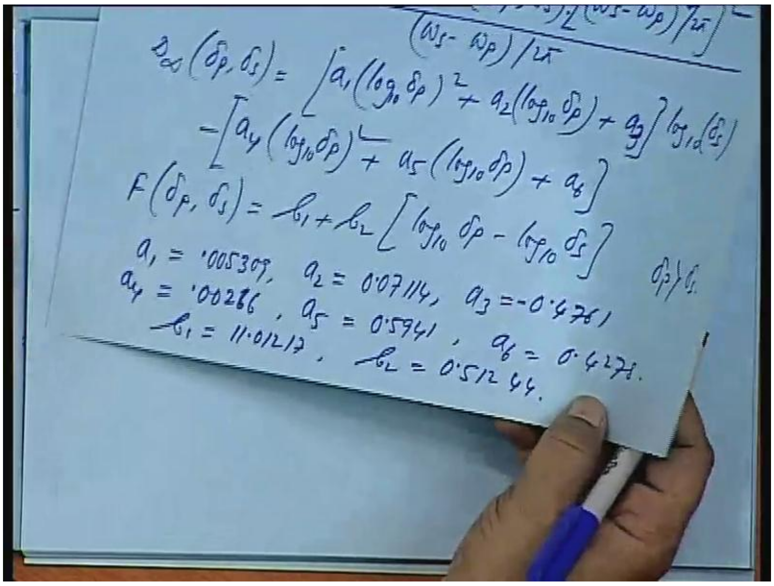

There are more accurate relationships, which is very commonly adopted and given by Herman it is like this, D infinity a function of delta p delta s minus F delta p delta s omega s minus omega p by 2 phi whole squared divided by omega s minus omega p by 2 phi where D infinity a 1 log of delta p to the base 10 of course, squared plus a 2 log of delta p plus a 3 into log of delta s to the base 10 minus a 4 log of delta p to the base squared plus a 5 log of delta p plus a 6, so it is like this.

D infinity delta p delta s is one quadratic involving log delta p to the base 10 multiplied by log delta s, another quadratic again in the same form without log of this and F delta p delta s is equal to b 1 plus b 2 log of delta p minus log of delta s. A 1is 0.005309, a 2 is equal to 0.07114, a 3 equal to minus 0.4761, a 4 is 0.00266, a 5 is 0.5941 and a 6 0.4278, b 1 equal to 11.01217, b 2 equal to 0.51244, this is valid for delta p greater than delta s.

If it is the other way round, if delta s is greater than delta p, then you just interchange delta p and delta s get this expression, so this is very commonly used in filter design. If time permits later on we take up one or two tutorial problems on this filter design. So, this is all about the computer aid design that we have studied so far. Now, I will take up lattice filter. Now, so far we have seen a direct form, which is taken from the difference equation or from the transfer function, lattice structure we had studied in signals and networks, networks synthesis.

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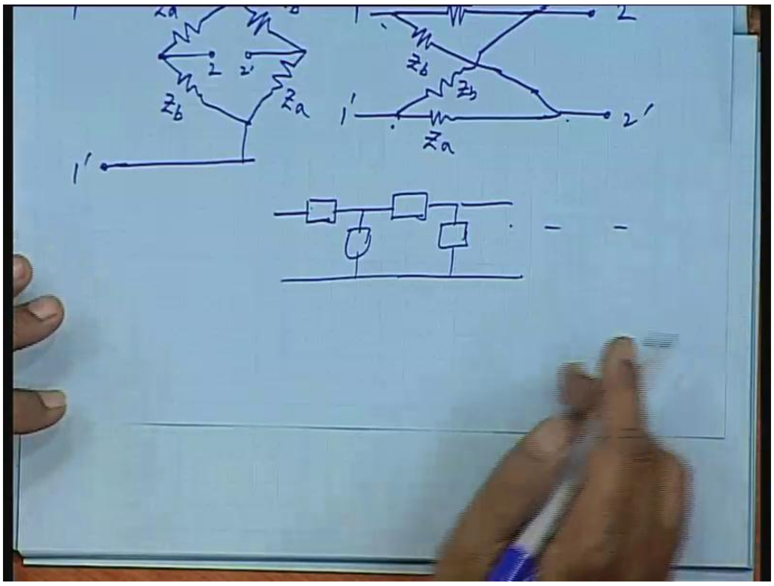

Lattice structure is like this, let me take a simple network. Suppose we given input here this is z a, z b, z b, z a and we take the output here, we will find this can be written as z a, z b, z b and z a, 1 1 dashed 2 2 dashed, this is a lattice structure. So, unlike a lattice structure, latter develops by alternatively putting series and shunt elements, this is a lattice structure and in a lattice you have cross connections. Now, in adaptive filtering speech processing modeling of the signals by filters will be very convenient with lattice structure, there many users.

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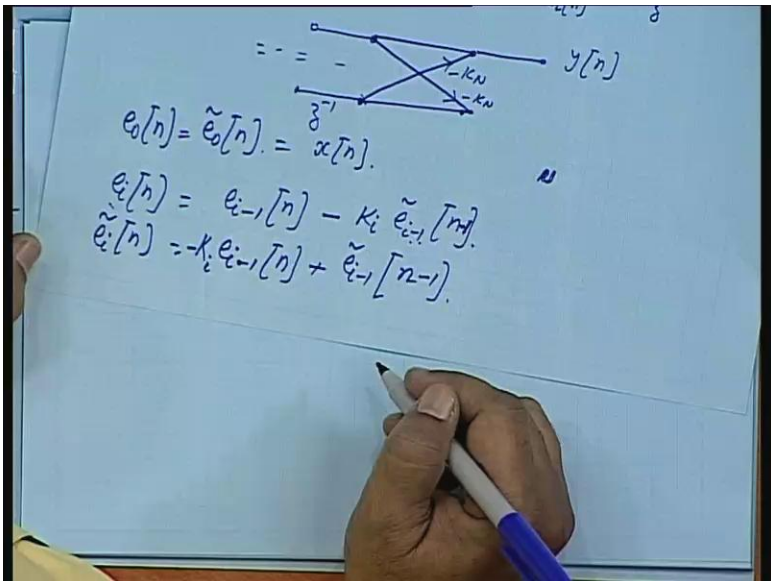

So, in a lattice structure we have like this. Hence forth I will not put z inverse as a single element I will show in the path itself have multiplied z inverse, this is in signal flow graph we use this symbol. So z inverse this is the input and so on, this is continuing from here. I will write these symbols, suppose this is e 0 n and this is e 0 n tilde, then in one path you are putting a delay elements.

And then this is minus K 1, minus K 1 they are multiplied these signals are multiplied by the lattice constants K 1, K 2 and so on, minus K 1 minus K 1, minus K 2 minus K 2. So, at their next stage it is minus K n minus K n, so at this point it is e 1 tilde n and e 1 n, similarly here it is e 2 n and e 2 tilde n and so on. The initial value of these two signals there are two channels through which you are sending the signals, so the initial values are same, so e 0 n is e 0 tilde n and that is equal to x n itself.

Now, will this be an FIR filter? What is this structure? Is it an FIR structure, of what order? if it is an FIR of what order, let us see whether it is an FIR or not. See there is no feedback, so the signal is transmitting only in the forward direction. At the end suppose I give an impulse input, I give an impulse input, then that impulse go straight here without any delay here it is delayed by one step, then again goes to this here it is through two delays, so like that there will be n such steps.So, there will be n delays and after that there is no response, so it is an FIR of degree n.

Now let us see what this relationship will be like at any stage e i n can you tell me what will be the relationship, I want to get a recursive relation e i n, in terms of these two signals in the previous steps. So, correct me if I am wrong and check whether you are also getting this, are you getting this say for example, e to n is minus K 2 times e 1 tilde n, check once again e to n, let us take e 2 n is equal to minus K 2 into e 1 tilde n no, there is a delay z inverse.

So, n minus 1 there is a delay, so in the second channel the signal is successively delayed by one step, whereas in the forward channel it is e 1 n, so e 1 n, so e 2 n will be e 1 n minus e 1 tilde n delayed by one steps. So, e i minus 1 tilde n minus 1 multiplied by the corresponding constant K i, so that will be e i n, check whether this is all right.

Now, you tell me what would be the relation for e i tilde n at the next stage what will be these value e 2 tilde n it is e i minus 1 into n into minus K i, this signal is multiplied by K i and then plus e i minus 1 tilde is it n. So, e 2 tilde n is e 1 tilde n delayed by one step minus K 2 into e 1 n, so this is the general form at the higher stage.

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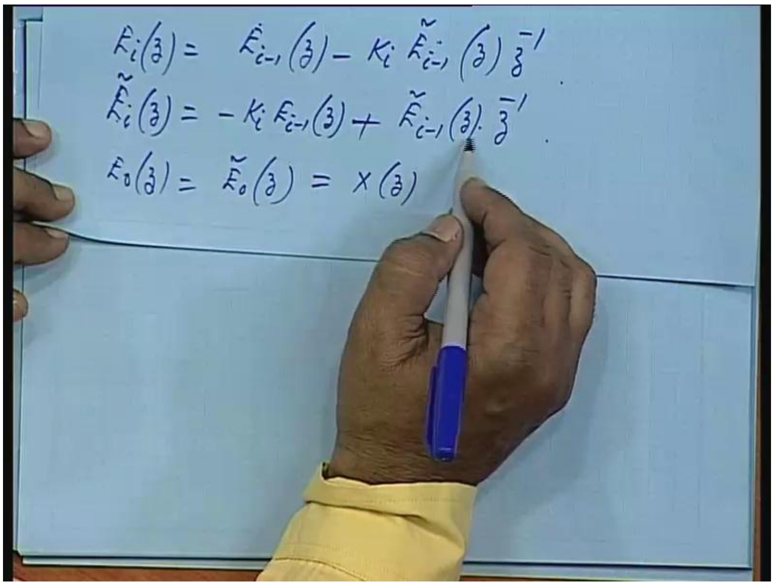

So what will be h at N th stage, what will be these value, if I write h 0, h 1, h 2, h n as the impulse response, what is the h n, this is what I am writing as h capital N, what is that h

N, after ((Refer Time: 38:55)) Nth interval what will be the output for an impulse input, this an impulse what will be the output after Nth interval it will be minus k n. The one of which suffers Nth delays, will be the one which will be coming through all these and lastly take up this.

So, if the signal is coming through these, there is only 1 delay, 2 delays, 3 delays and so on, Nth delay and then multiplied by minus k n that is the last signal, so h N is minus k n that convinced. So, we are interested in finding out H z, I will write Y z by X z, so this is A z a polynomial of this type, now what will be A i z, it is E i z by E 0 z, from here if I take the z transform, what do I get.

E i z here is equal to E i minus 1 z minus K i E i minus 1 tilde, then z into z inverse and from this 1 E i tilde z equal to minus K i E i minus 1 z, correct me if I am wrong, E i minus 1 tilde into z inverse. Now, there are two relationships, two recursive relationships involving i minus 1 and for both the channels and you have to find out what will be E i, it will be better to see for yourself some simple, starting from i is equal to 1, 2, 3, 4 by simple induction what these relationships will be like. So, E 0 z is same as E 0 tilde z is equal to x z is it not, ((Refer Time: 42:44)) this is equal to this equal to x, z inverse and E i z inverse is minus K i E i minus 1 z in I think this is, so let us see with E 1.

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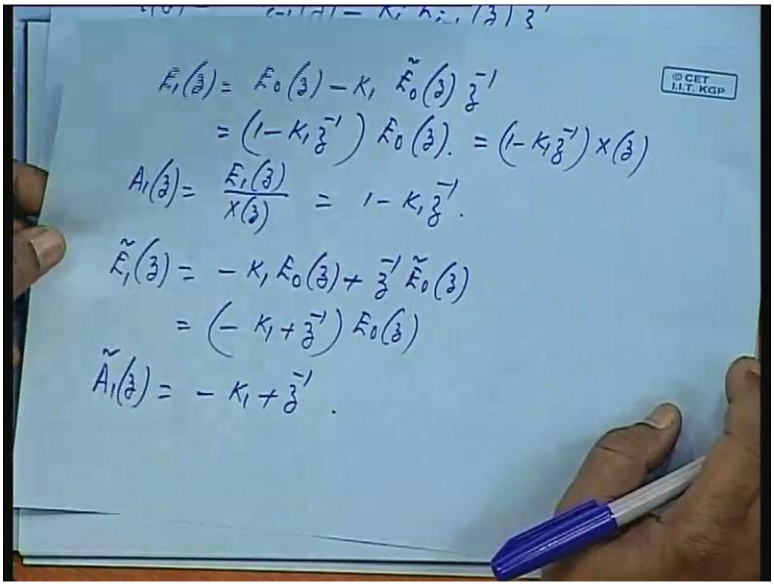

What would be E 1 z, E 1 z is equal to E 0 z minus K 1 I will use ((Refer Time: 43:36)) this. So let me put the paper here minus K 1 E 0 tilde, is same as E 0 into z inverse. So that is equal to 1 minus K 1 z inverse into E 0 z, that is 1 minus K 1 z inverse in to x z, is it not E 0 z is x z. So, A 1 z which is the transfer function E 1 z by x z is 1 minus K 1 z inverse, what will be E 1 tilde z, E 1 tilde tilde z is minus K i minus K 1 E 0 z plus z inverse E 0 tilde z, which is same as E 0 z, so it will be minus K 1 plus z inverse into E 0 z, because E 0 tilde is same as E 0. Therefore, what will be the corresponding transfer function A 1 tilde z minus K 1 plus z inverse, can you write A 2 and A 2 tilde z, then will generalize something from here.

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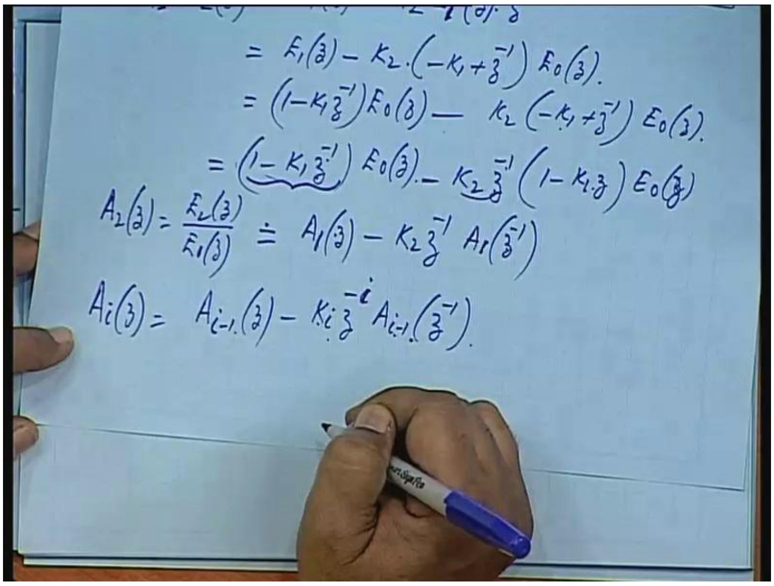

A 2 z it would be better if you write first E 2 and then derive A 2 z, so this would be E 1 z minus K 2. Now, once again taking this E 2 z is equal to E 1 z minus K 2 into E 1 tilde z into z inverse. So, E 1 z minus K 2 into what was E 1 tilde z earlier, we have derived, if minus K 1 plus z inverse E 0 z and what was E 1 z, 1 minus K 1 z inverse into E 0 z minus K 2 into minus K 1 plus z inverse E 0 z, can I write this in some form of this function.

Because, my E 0 z is basically x z, so my aim is to calculate A 2 z, which is E 2 z by E 0 z, so can I write this if I take z common, this is z inverse into 1 minus K 1 z. So, what was 1 minus K 1 z inverse is appearing as 1 minus K 1 z and K 2 z to the power minus 1, so can I write this as this is what, this one was A 1 z. So, I can write this as A 1 z minus K 2 z inverse A 1 z inverse, z replaced by z inverse can I write like this.

So, generalizing this we can write A i z as A i minus 1 z minus K i z to the power minus i A i minus 1 z inverse, if this is A 1 this is also A 1, so A i minus 1 A i minus 1, z is replaced by z inverse. In this bracketed quantity, if I replace z by z inverse I get this, and there will be accumulation of z inverse there, so K i z i inverse z to the minus i, it should be z to the minus 1, a friend suggest that it should be minus 1. What is it should it be z to the minus i or z to the minus 1 i minus of 1, try with A 3, A 4 and then generalize it leave it as an exercise, similarly A tilde let us see A 2 tilde we have computed A 2 z.

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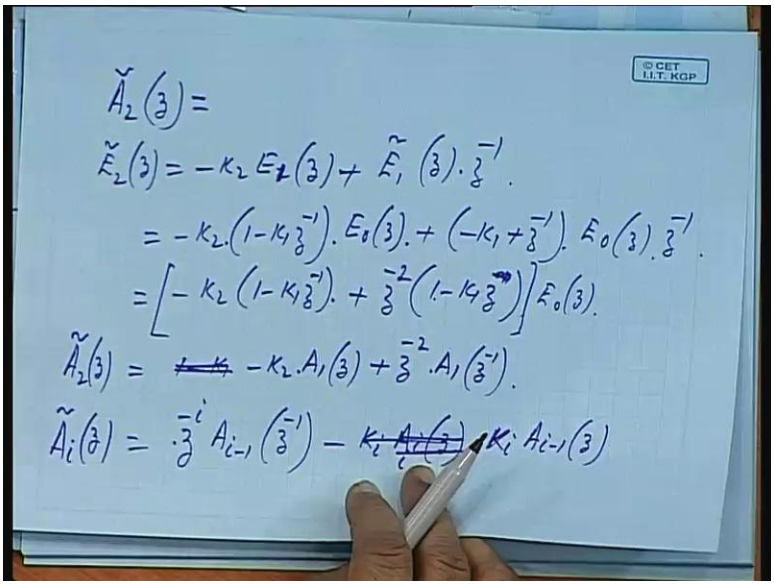

A 2 tilde z is how much, so E 2 tilde, if you write first E 2 tilde z, ((Refer Time: 51:19)) A 1 z inverse is z to the power minus 1 into 1 minus K 1 z is equal to z to the power minus 1 into A 1 z inverse. A 1 tilde is A 1 z inverse z to the power minus 1, what it be E 2 z, E 2 tilde z is if you remember E 2 will be minus K 1 minus K 2 E 1 z minus K 2 E 1 z plus E 1 tilde z z to the power minus 1.

And now let us substitute minus K 2 into E 1 z was 1 minus K 1 z inverse into E 0 z plus E 1 tilde z E 1 tilde z is minus K 1 plus z inverse E 0 z into z inverse. So, that gives me minus K 2 into 1 minus K 1 z inverse plus this I can write as z to the power minus 2 1 minus K 1 z inverse into E 0 z. Therefore what will be A 2 tilde z, so 1 minus K 1 z inverse is common, 1 minus there is some mistake here this one, so it is 1 minus K 1 z inverse is what, ((Refer Time: 54:22)) 1 minus K 1 z inverse is A 1 z.

So, it is minus K 2 into A 1 z plus z to the power minus 2 into A 1 z inverse are you getting this. So generalize this A i tilde z, it should be z to the power minus i A i minus 1 z inverse minus K i A i A i minus 1, A i minus 1 strike this and write minus K i A i minus 1 z. Now, based on these two relationships that is the iterative relations that you have got, the recursive relation that you have got for A i z and A i tilde z, we have got what is that A i z.

((Refer Time: 56:08)) A i z as A i minus 1 z minus K i z to the power minus i A i minus 1, so actually we are interested in only A i, not A i tilde, A i tilde has come as a byproduct for solving this you needed A i tilde. So, finally, this is the relationship for A i with respect to it is previous values, now from here will find out the polynomial coefficients. So, given a polynomial what will be lattice constants or given a lattice structure what will be the polynomial, so in the next class will take up from this ((Refer Time: 56:51)). How to determine? Finally A z at the N th stage, given the lattice constants K 1, K 2 up to K n or given a polynomial how do you determine the lattice constants.

Thank you very much.