Digital Signal Processing

Prof. T. K. Basu
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 20
Tutorial & Introduction to Computer Aided Design of Filters

We shall start with a tutorial exercise and then we will go to introduction to computer aided design, design of filters.

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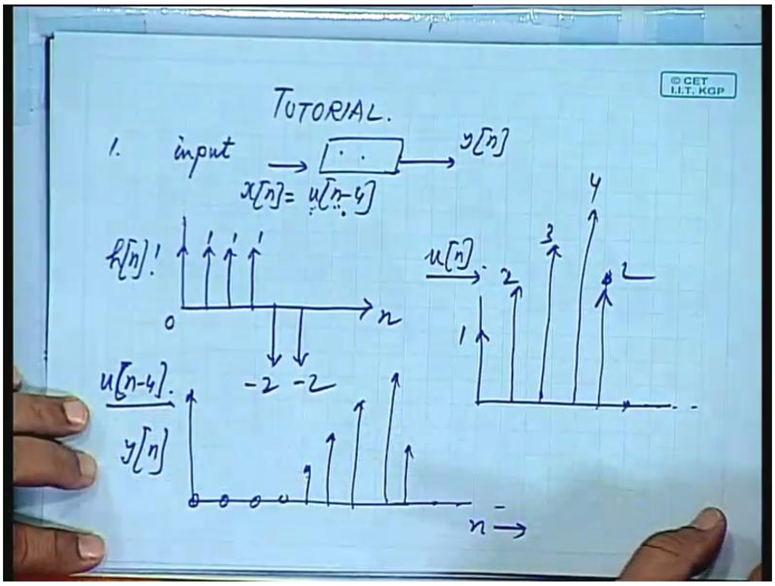

So, the first problem is, this will be the problems; that we have already gone through a similar problems. The input to a system is x n given by u n minus 4 and you have to determine the output. You are given the impulse response as minus 2 minus 2, 1, 1, 1, 1, okay. So this is the impulse response h n and you have to determine the output y n. Had it been u n? Had it been u n then it is a simple step.

So, given a given an impulse response, how do you compute the step response? Keep on adding, that is the integration. So, here it will be corresponding to u n, what will be the output y n if the input is u n? Output will be 1 then 2, this 1 plus 1 then 3 then 4 then 4 minus 2, 2 okay and then it will be 0, okay after that it will all be 0. So, it is magnitude then are to the scale one, two, three, four, two, zero.

Now, if I take u n minus 4 and if it is an LTI system; that is time invariance system then it will be shifted by 4 steps, so it will be finally zero, zero, zero, zero then one, two, three, four, two then zero. So, this will be the output, okay. This is the output corresponding to an input of u n minus four, all right.

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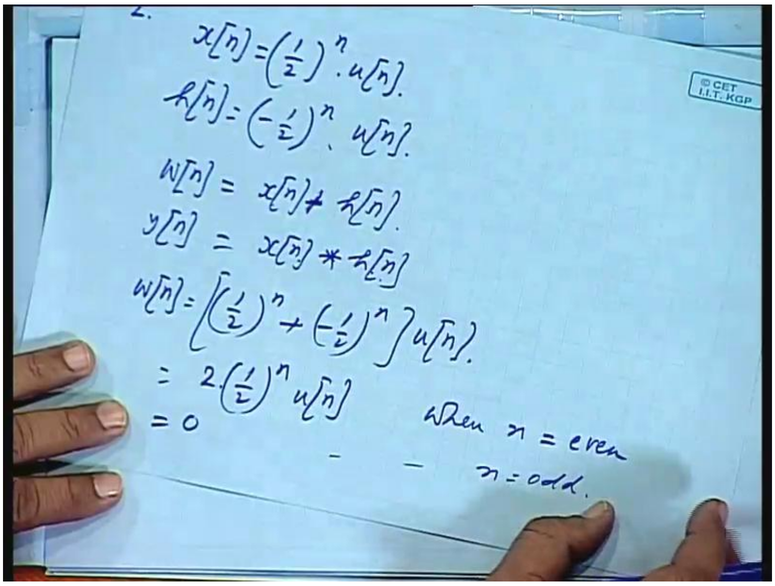

Next, the second example is something very interesting, half to the power n u n. This is x n and h n is equal to minus half to the power n u n, okay. So, you are required to find out the output corresponding to this w n; corresponding to x n plus h n and also the output y n when x n is convolve with h n, is that all right?

Now, you can take these two signals together, w n will be half to the power n plus minus half to the power n u n, okay. Obviously when n is odd, they will get cancelled. So, this will be when n is even they will be same; so 2 times half to the power n u n when n is even, equal to 0 when n is odd, okay. One may similarly find out y n by taking the convolution, okay. There is an alternative approach, take the z transform what you get?

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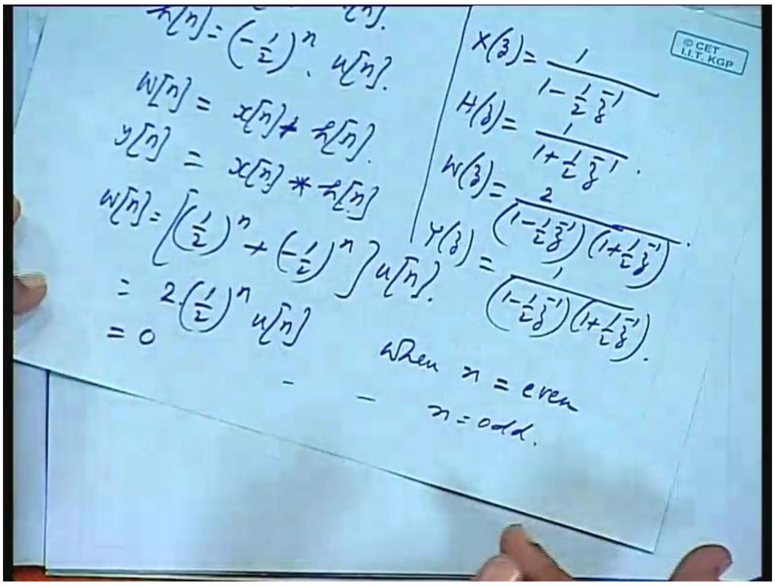

If I take the z transform, X z is 1 by 1 minus half z inverse. H z will be 1 plus half z inverse; if you add them together, W z is equal to half will get cancelled, 2 by 1 minus half z inverse into 1 plus half z inverse, is that all right and what about Y z? It will be product of the 2 which will be just half of W z. If you take the product it will be 1 by 1 minus half z inverse into 1 plus half z inverse, okay. So, it is half of this. So, y n will be half of W n.

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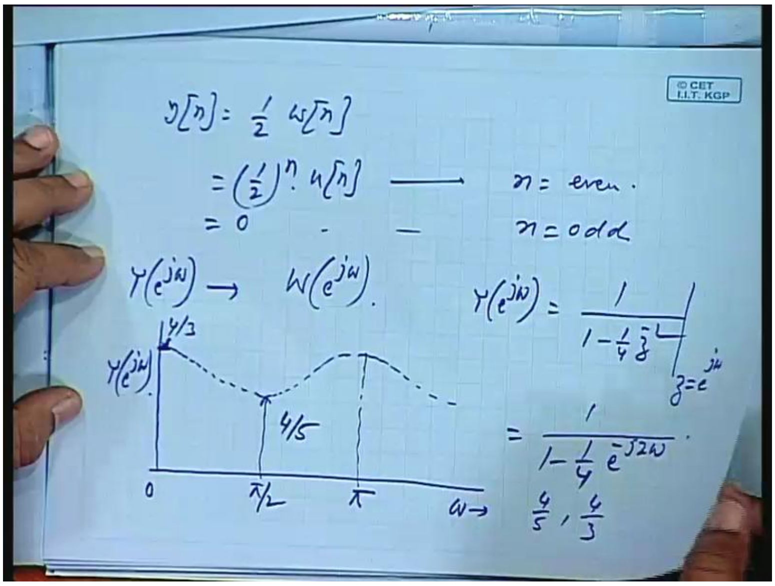

So, W n you have already computed; so without taking the convolution, you can straight away calculate from here y n is equal to half of W n, is equal to half to the power n u n when n is even and equal to 0 when n is odd, okay. Then the next question is what would be the frequency plot of this? That is we are required to determine this plot against omega and similarly; W, since W is just twice the value of Y. So, the plots are identical, they just scaled by a factor of 2, okay.

So, let me plot just one of them. Y e to the power j omega, what is it? 1 minus half, okay. Let me write on this side expression 1 by 1 minus x into 1 plus x; so 1 minus half to the power 2, 1 by 4 z to the power minus 2 and z evaluated at e to the power j omega, is that all right? So, 1 by 1 minus 1 by 4 e to the power minus j 2 omega, okay. What are the two extreme values of this? One is 1 by 4 plus 1 by 4, the other one is minus 1 by 4. So, if I take plus 1 by 4, it will be 4 by 5. If I take minus 1 by 4, it will be 4 by 3.

So, it will be starting with a value 4 by 3 when omega is 0, omega is 0 this is 1; so 1 by 1 minus 4, so 4 by 3. And it will be coming to minimum value, minimum value when this quantity becomes plus 1 by 4. So, it will be going like this and this minimum value is 5 by sorry, 4 by 5, 0.8. So, 4 by 3 is this magnitude 4 by 5 is this magnitude and it oscillates. What would be the frequency? Twice omega, so it comes back again in this position after an angle pi measure to pi, is that all right? So, this multiplied by 2 will be W.

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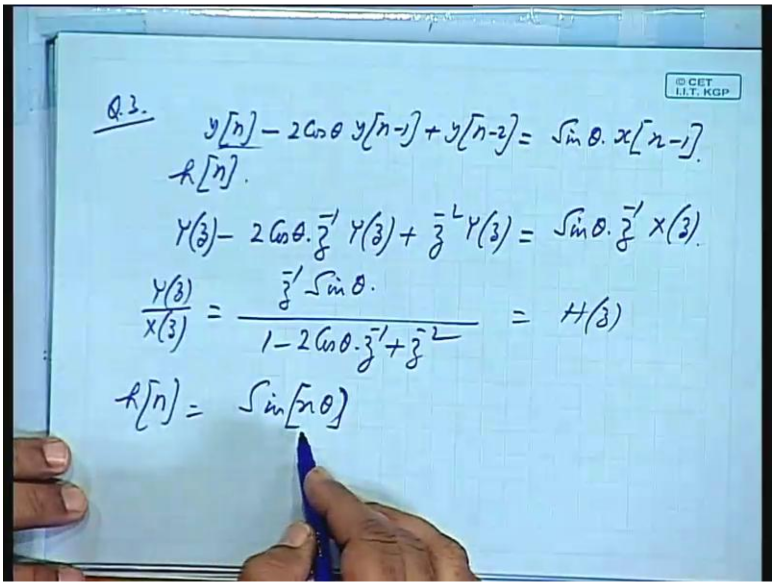

Next we have another question, y n minus twice cos theta y n minus 1 plus y n minus 2 equal to sine theta into x n minus 1, okay. What would be the impulse response h n corresponding to this difference equation? If you take z transform, this be Y z minus twice cos theta z inverse Y z plus z to the power minus 2 Y z, okay equal to sine theta into z inverse into X z.

So, Y by X will be z inverse sine theta divided by 1 minus twice cos theta z inverse plus z to the power minus 2 therefore; and this is H z, so what would be h n? Inverse of this is or familiar expression sine n theta, okay. So, this is the difference equation for sine n theta. Next we have you are ask to calculate the step response for a causal system, H z equal to 1 minus z cube by 1 minus z to the power 4, okay; 1 minus z cube by 1 minus z to the power 4.

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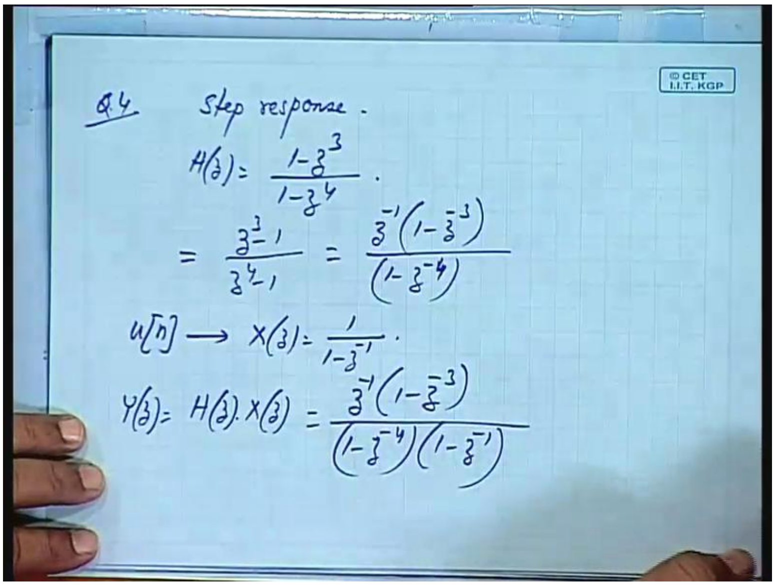

Now, if you look at this; one may write z cube minus 1 by z to the power 4 minus 1 and after that, if I take z to the power 4 common, z to the power 4 common then this would be z inverse 1 minus z to the power minus 3 by 1 minus z to the power minus 4, correct me if I am wrong, is that all right?

I have just taken z to the power 4 common here, z to the power 3 common here and so that becomes z to the minus 1 and this. Therefore what would be the input? If it is u n corresponding X z is 1 by 1 minus z inverse, is that all right. So, if I multiply output Y z will be H z into X z, okay. So, z to the power minus 1 into 1 minus z to the power minus 3 divided by 1 minus z to the power minus 4 into 1 minus z to the power minus 1.

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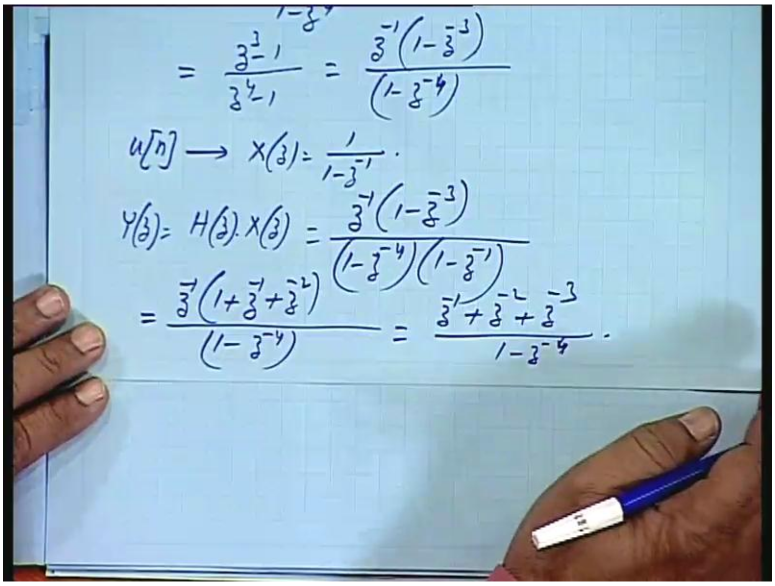

And that is equal to 1 minus z to the power; minus 1 I can cancel here, it will be z to the power minus 1. I will be left with 1 plus z to the power minus 1 plus z to the power minus 2 by 1 minus z to the power minus 4, okay. That gives me, z to the power minus 1 plus z to the power minus 2 plus z to the power minus 3 divided by 1 minus z to the power minus 4, is that all right?

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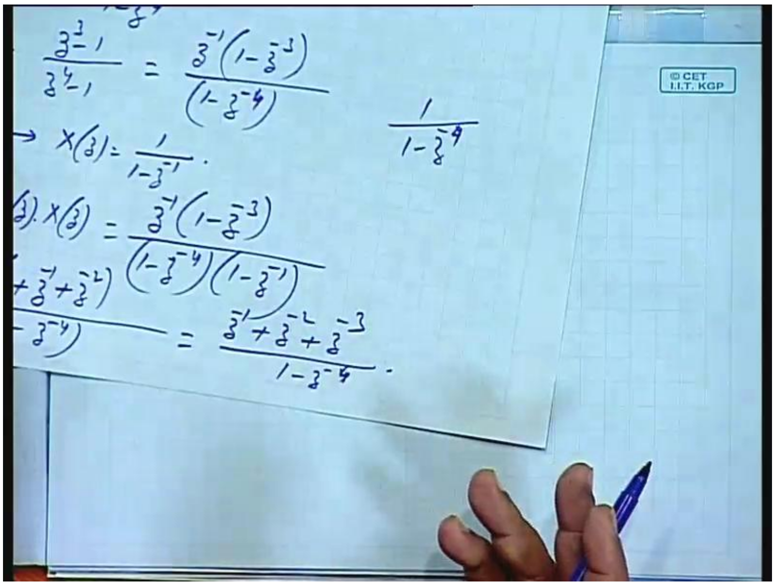

Now let us compute from here, what would be the response due to 1 minus z to the power minus 4? Whatever response we get, we will have 1 shift, 2 shifts and 3 shifts and add them together, okay. So, corresponding to 1 by 1 minus z to the power minus 4, what will be z inverse of this?

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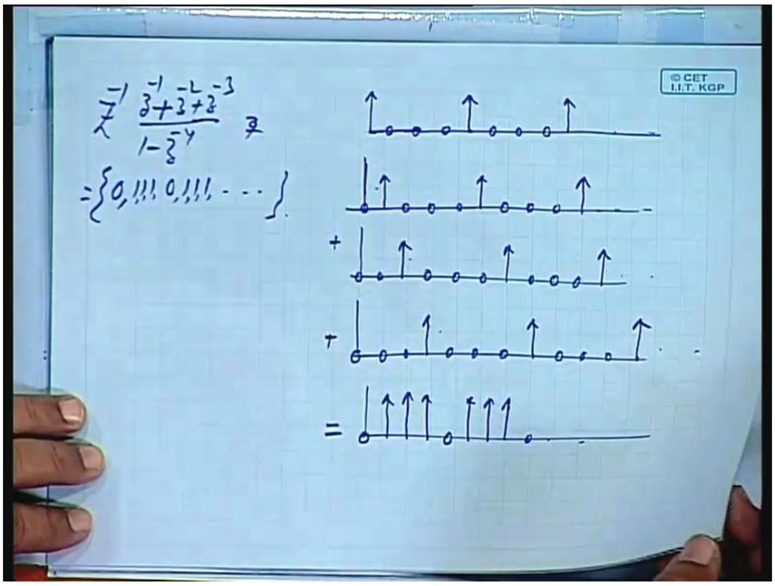

If you remember, earlier we had discussed in terms of the sequence; if you write straight in terms of the sequence, it will be 1 then 0, 0, 0 will it be minus 1 or plus 1? Again plus 1, because it is minus here all right then again 0, 0, 0 is that all right, again 1 and so on. Am I, all right?

Then if I have shifted, if I have this same sequence shifted by one step, two steps and three steps and add them together; so I will get, the first shift will be 0, 1, 0, 0, 0, 1, 0, 0, 0,1 and so on. Plus shift of two steps, 0, 0 then 1, 0, 0, 0, 1, 0, 0, 0, 1 and so on. Plus this term, the three shifts; 1, 2, 3 then 1, 0, 0, 0, 1, 0, 0, 0, 1 and so on. So that finally gives me, this plus this plus this gives me; 0 then 1, 1, 1 then again 0, 1, 1, 1 again 0 and so on. So, this is the sequence.

So, if I am permitted to write in terms of the sequence; 0, 1, 1, 1 then again 0, 1, 1, 1 and so on. That will be the inverse of the original one; that is z inverse plus z to the power minus 2 plus z to the power minus 3, okay. It will be this sequence. If one wants to write in a close form that is instead of giving the sequence that also you can write.

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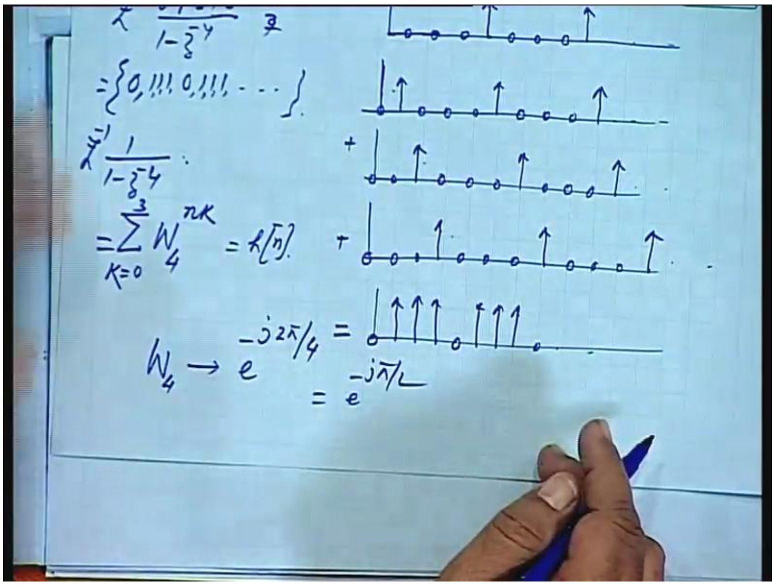

What would be the expression for this, z inverse of this? W 4 n k summation, k varying from 0 to 3 that will be equal to h n, is that all right; where W is e to the power minus j 2 pi by, if I write n then it will be 2 pi n. So, here it is 2 pi by 4 which is e to the power minus j pi by 2, check whether you get the same result.

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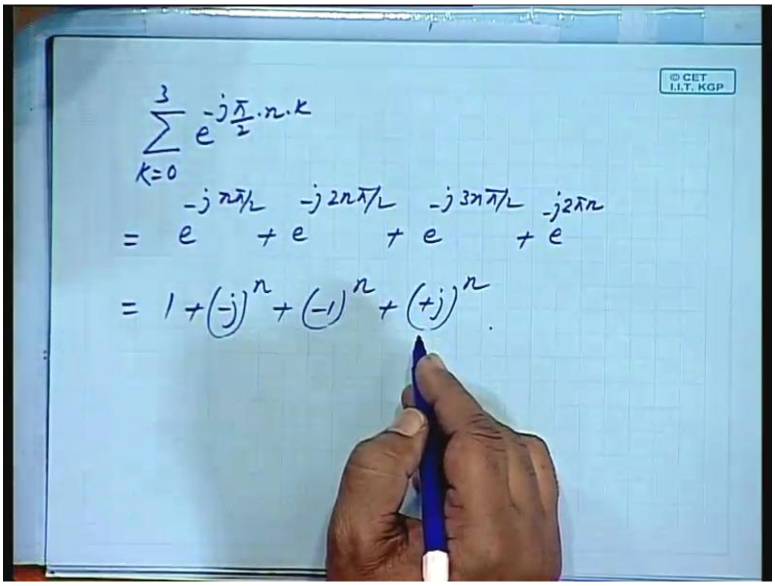

Summation e to the power minus j pi by 2 into n into k, where k varies from 0 to 3 gives me e to the power minus j n pi by 2 plus e to the power minus j 2 n pi by 2 plus e to the power minus j 3 n pi by 2 plus e to the power minus j 4 n pi by 2, means 1, okay. So, basically it is a multiple of 2 pi, all right. So, it will be 2 pi n which is 1. So, it is 1 plus e to the power minus j n pi by 2 is minus pi by 2; so minus j to the power n plus minus 1 to the power n plus plus j to the power n, correct me if I am wrong, is that all right?

Now, let us test it with some values whether we get this sequence or not. n is equal to 0, n is equal to 0 will give me 1 plus 1 plus 1 plus 1. So there has to be scaling factor, 1 by 4 okay, 1 by 4, so that gives me for n is equal to 0; therefore h n becomes 1 when I put n is equal to 1, this is 1 minus j minus 1 plus j it becomes 0. Sorry n is equal to 0; it is 1, 1, 1, 1. n is equal to 2, this becomes 1 then minus 1, 1 again minus 1 so again 0. h 3, similarly h 3 will also be 0, are you all getting that? And then h 4 will become 1, so we are getting this sequence, okay.

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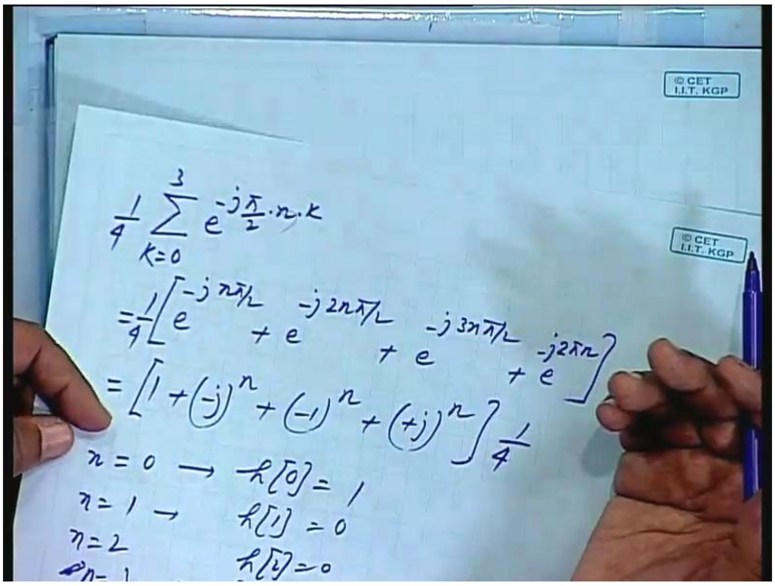

So in a close form, if you want to write just the analytical expression not a sequence like this; what would be the mathematical expression for this sequence? For the main sequence, it is this. So, this has to be shifted by one step, two steps and three steps and add it together, is that all right?

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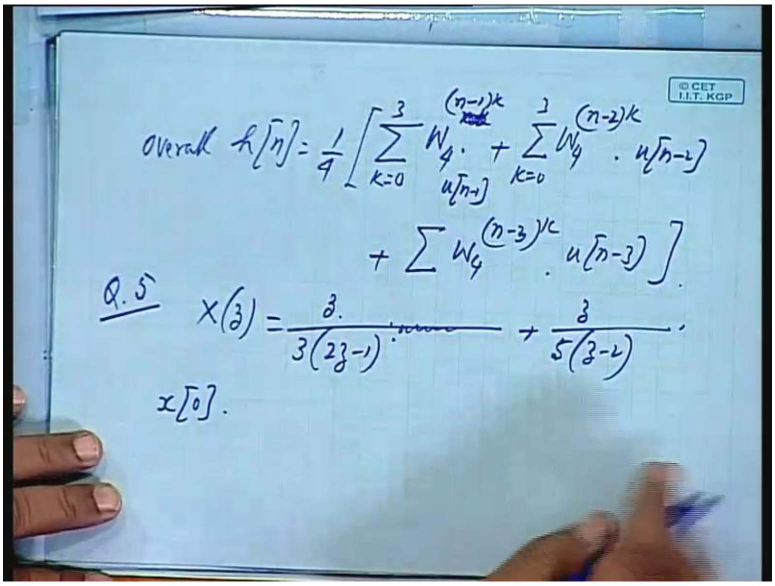

So it will be, 1 by 4 therefore actual h n overall h n; I should write as 1 by 4 summation W 4 n k, k varying from 0 to 3 plus summation W 4, k equal to 0 to 3 n minus 1 k. Actually this itself should be shifted, so n minus 1, n minus 2, any question? I should multiply by u n minus 1, u n minus 2, is it not? So, let me write this has to be multiplied by u n minus 1, this has to be multiplied by u n minus 2; similarly the third one, W 4 n minus 3 k u n minus 3 this is the overall sum, is that all right?

Next question, question number 5 is; determine the value of X 0, given X z equal to z by 3 into 2 z minus 1 okay, z by 3 into 2 z minus 1 plus z by 5 into z minus 2, all right? Determine the value of x 0; determine the value of x 0.

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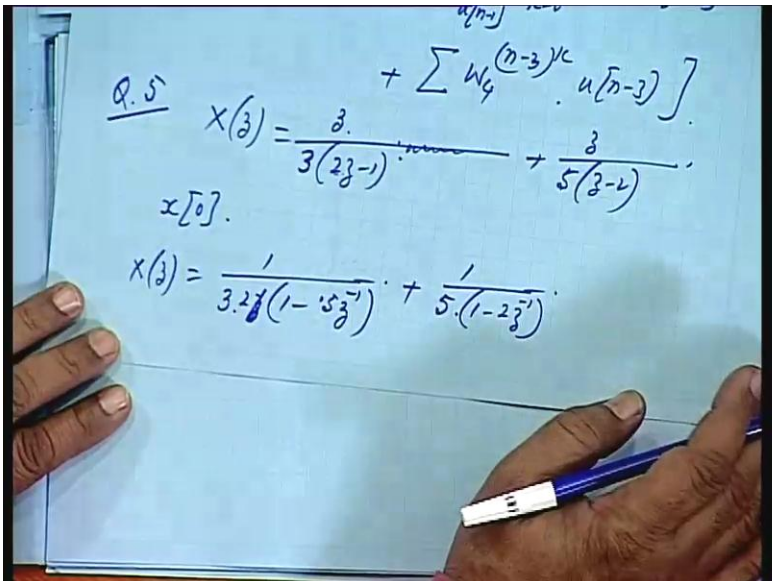

So, I may write X z as 1 by this is, if I take 3, if I take z common; okay, 2 z common what we get? 1 minus 0.5 z inverse, is that all right? z divided by so z will go, plus similarly here 1 by if I now take 5, 1 minus 2 z inverse, all right. This is a causal system. If you break it up, if you break it up the how do you get the initial value? z to the power minus 1, z to the power minus 2, they are all made zero all right; that means z is made infinity, is that all right? So if you put z is equal to infinity, you will to get this. If I made z is equal to infinity right here, this will become z by 6 z, so 1 by 6.

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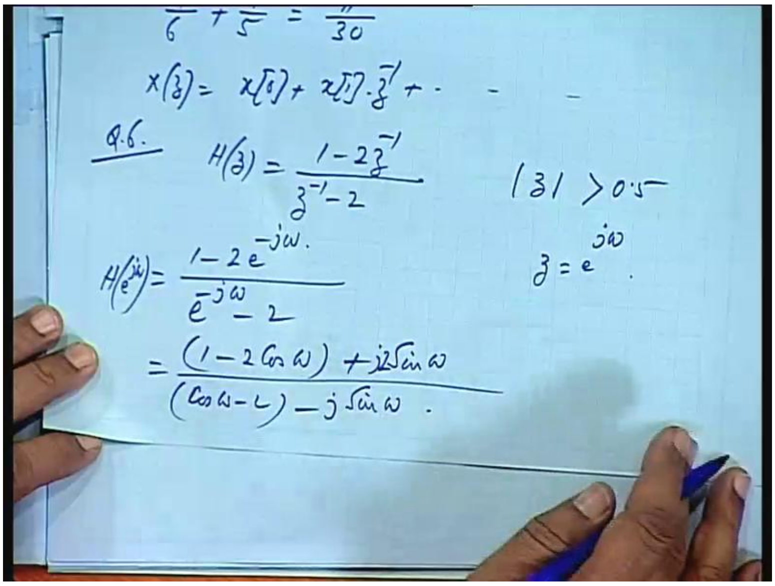

1 by 6 plus similarly this becomes 1 by 5, okay. So, that is 11 by 30, all right. See, X z you have written as x naught plus x 1 into z inverse and so on. So, all these z inverse z to the power minus 2 etcetera are made 0; that means z inverse is made zero, z is made infinity. So, this will give you directly X value.

Next question is sketch the magnitude plot of H z equal to 1 minus 2 z inverse divided by z inverse minus 2 and z magnitude is greater than 0.5, okay. So, this 1 minus 2 z inverse by z inverse 2; if I put z is equal to e to the power j omega, what we get in the numerator? 1 minus 2 e to the power minus j omega divided by e to the power minus j omega minus 2; take the real and imaginary parts, so it will be 1 minus 2 cosine omega plus j sine omega 2 sine omega, divided by cosine omega minus 2 minus j sine omega, all right.

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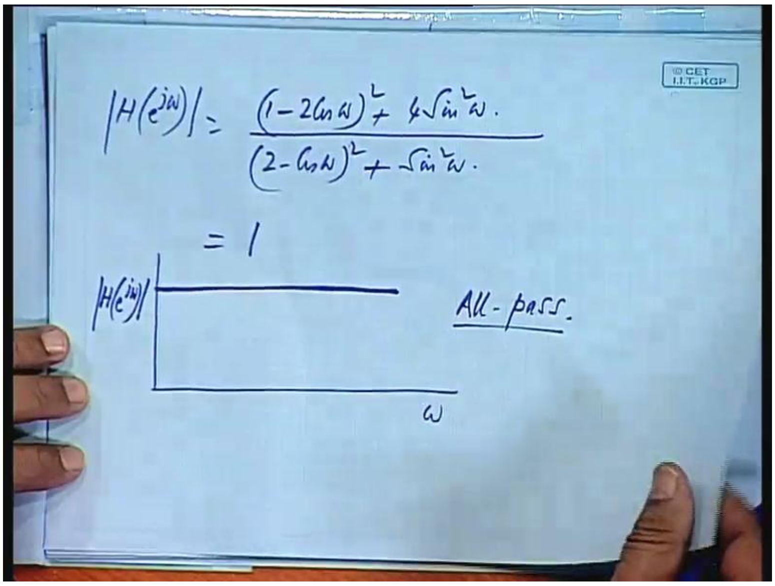

If I take the magnitude, if I take the magnitude it will be 1 minus 2 cosine omega square, 1 minus 2 cosine omega whole square plus 4 sine square omega divided by 2 minus cosine omega whole square plus sine square omega which gives me very interesting result, 1. So, the magnitude in this case will be 1. Phase you can make out from here, tan inverse twice sine omega by 1 minus 2 cosine omega and tan inverse sine omega by 2 minus cosine omega, difference of the two.

This phase will be you have to compare for one or two sample values, it will be again a periodic function. A filter which has such a characteristic, a flat gain is known as and all - pass filter. So, basically it was an All-pass filter. What is the peculiarity about the All-pass filter?

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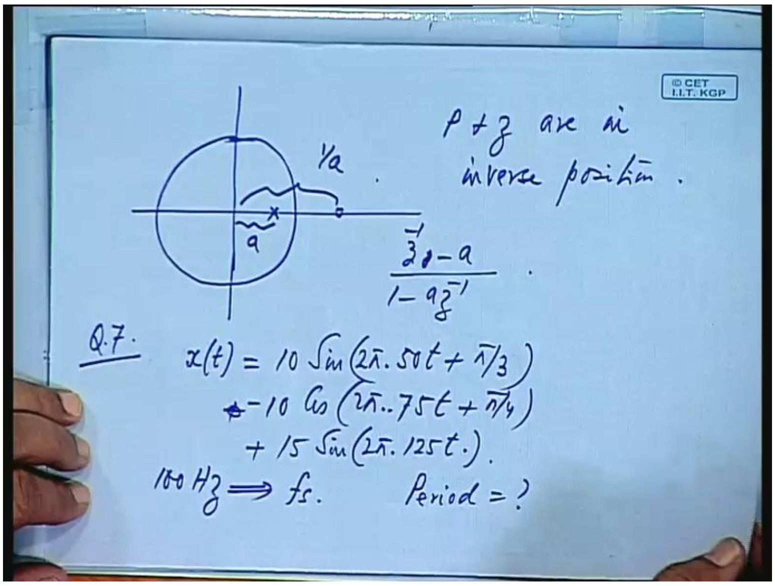

An All-pass filter has poles and zeros in inverse position; that is if there is a pole here at some distance a then there will be a zero at 1 by a, all right. So, poles and zeros are in inverse position. So, it will be appearing like this, say 1 by 1 minus a z inverse then this one will be z inverse minus a, function will be of this type, okay.

Next question is; the signal is given x t equal to 10 sine 2 pi into 50 t plus pi by 3 plus okay minus 10 cosine 2 pi into 75 t plus pi by 4 plus 15 sine 2 pi into 125 t okay. And it is sampled that a frequency of 100 Hz frequency of sample, you are required to determine the period of the signal. How much is the period, all right.

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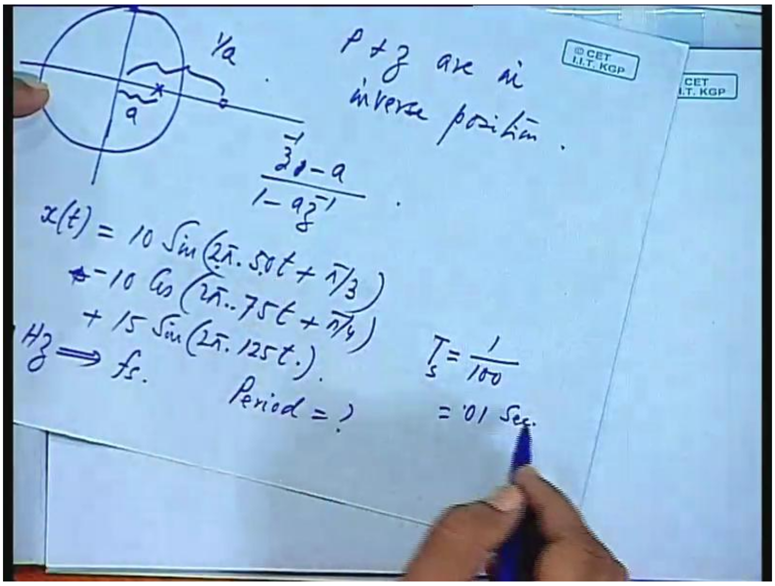

So, if you are given the sampling frequency f s that means; the time sampling time is 1 by 100, that is 0.01 second, is it not? So, we can write x t as 2 pi into 50 into n times T, so 0.01 T.

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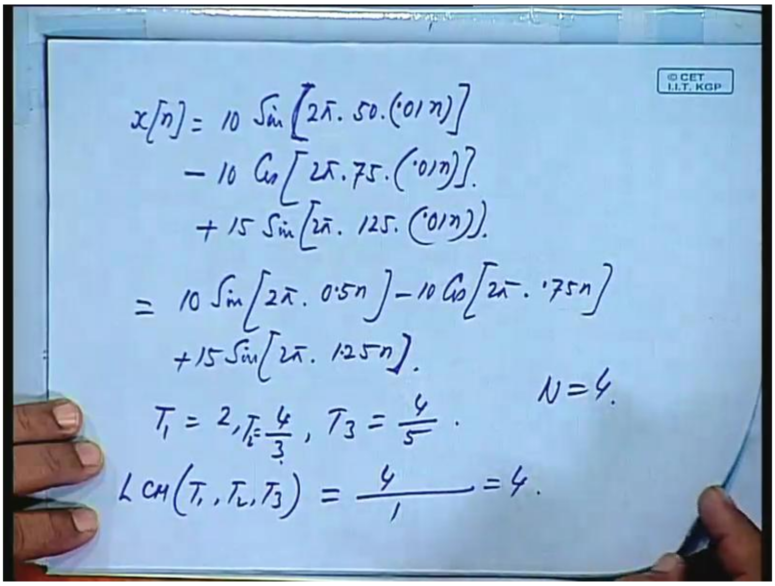

So, x n will become 10 sine 10 sine 2 pi 50 into 0.01 n plus okay minus 10 of cosine 2 pi 75 into 0.01 n plus 15 sine 2 pi into 125 into 0.01 n. So, that gives me 10 sine 2 pi into, how much is this? 0.5? 5? pi n, if I write 2 pi into something then it will be 0.5 n minus 10 cosine 2 pi into 0.75 n and then plus 15 sine of 2 pi into 1.25 n, is that all right? Now, what is the period of these three discrete samples, these three discrete components? Period is; now what is the period of this, period of this? First one I will call it T 1 is 2. Next one, it can be 4, 4 by 3 that is T 2, and T 3, 4 by 5. So, you have to take the LCM of these three periods T 1, T 2, T 3 which means LCM of 2, 4 and 4 and HCF of 3, 5 and 1 so 1, okay. So, that is equal to 4.

So period N is 4, are you able to retain, recover the frequencies from such sampled signals? What is the sampling frequency? 100 Hz, what are the frequencies present in the signals? 5o, 75 and 125, all right. 125 is the frequency and your frequency of sampling is 100 Hz, your sampling at a very slow rate. This is just meeting the requirement of that Hurwitz criterion.

Others, for other two signals this is not meeting the requirement. So, you will have period T which does not reflect the signal that will be contained in, I mean the components that will be contained in the signal. What should be, to recover the original signal what should be the frequency of sampling? At least 250, is that all right? So, can you try at home, if the sampling is at 250, what is the period?

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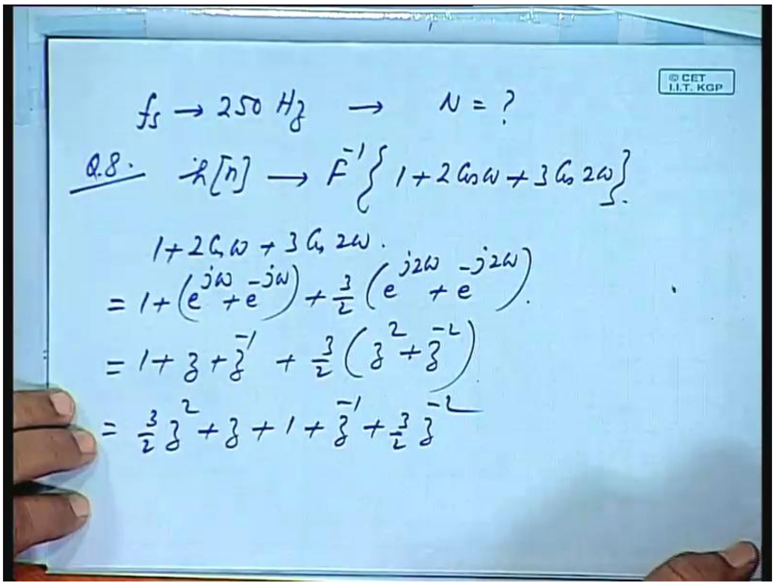

If the sampling rate is now 250 hertz, if the sampling rate f s is made 250 hertz, what would be the period N, for the same signal, okay? Try at home and let me know. Next we have another question, determine the sequence h n which will be the inverse DTFT; that is Fourier inverse of 1 plus 2 cosine omega plus 3 cosine 2 omega, Fourier inverse of this; DTFT you are required to find out.

So, let us see 1 plus 2 cosine omega plus 3 cosine 2omega; I can write this as 1 plus twice cosine omega as, e to the power j omega plus e to the power minus j omega plus 3 by 2 e to the power j 2 omega plus e to the power minus j 2 omega. For each e to the power j omega, we put z, all right.

Then this will be 1 plus z plus z inverse plus 3 by 2 into z square plus z to the power minus 2. So, you get a sequence 3 by 2 z to the power 2 plus z plus 1 plus z to the power minus 1 plus 3 by 2 z to the power minus 2, in the descending order of z I have written this.

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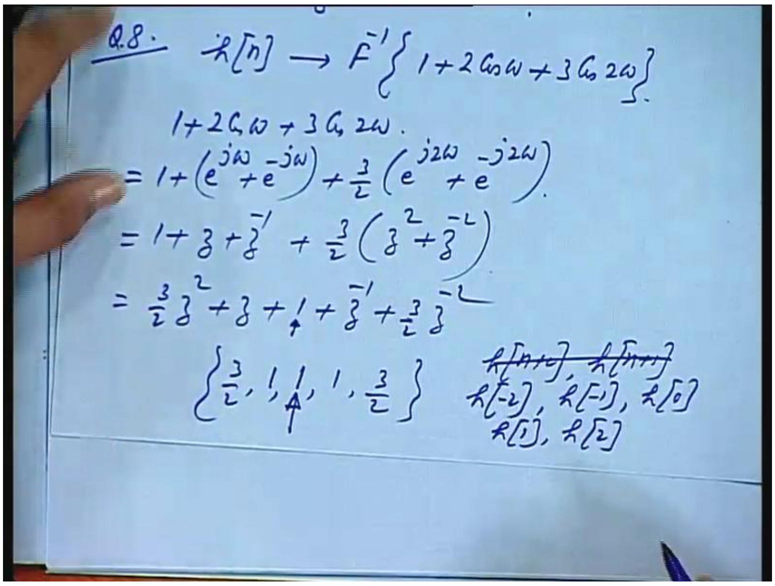

So, that gives me straight away the sequence as; this is the one with z to the power 0, so 1 before that 1, before that 3 by 2 then 1 and 3 by 2. So the arrow, this is the h 0; so it will start with h n plus 2 which is 3 b 2, h n plus 1 which is 1 then h 0 has 1, h 1 has 1 sorry not h n plus 2 sorry, h how do I write? h minus 2 then h minus 1 then h 0, h 1, h 2 okay; so these are the values.

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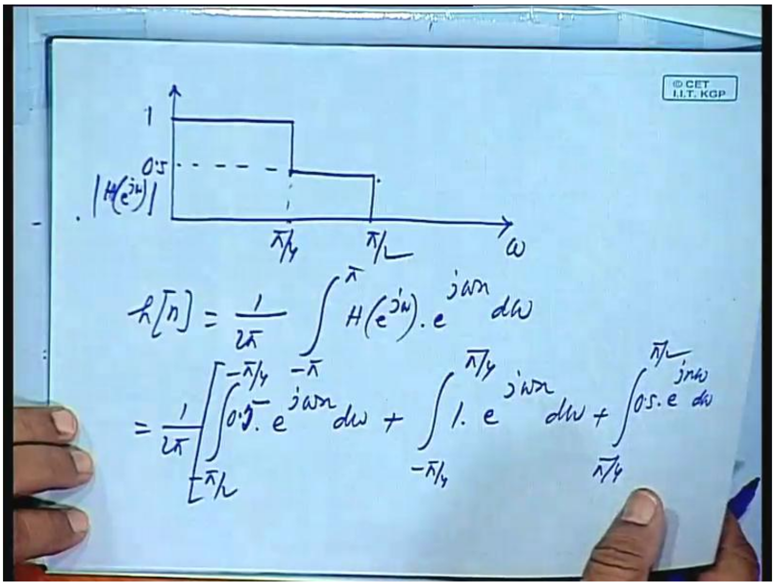

Next, you are asked to design a very simple FIR filter, 1, 0.5 is a magnitude. As you know for the FIR filter design, we calculate the expression for h n; we try to obtain by integrating it over a range of one period 2 pi, from minus pi to plus pi, a given function as it is, because we will be providing a linear phase afterwards, by just having a shift and e to the power j omega n.

In this case, if I produce it backward on the negative side; it will be symmetric, so I will take it, I need to take only from minus pi by 2 to minus pi by 4 and then minus pi by 4 to plus pi by 4 then plus pi by 4 to plus pi by 2, is that all right? So, I will take it this way; minus pi by 2 to minus pi by 4, it is 1 e to the power j omega n d omega that will be half. Thank you very much, so it will be 0.5.

Then minus pi by 4 to plus pi by 4, it is 1; into e to the power j omega n d omega, I will take and then again 0.5 into e to the power j n omega d omega, it will be from pi by 4 to pi by 2, is that all right? Now you are ask to, calculate a nine point sequence that is a length of nine. So, how many points should it be shifted by?

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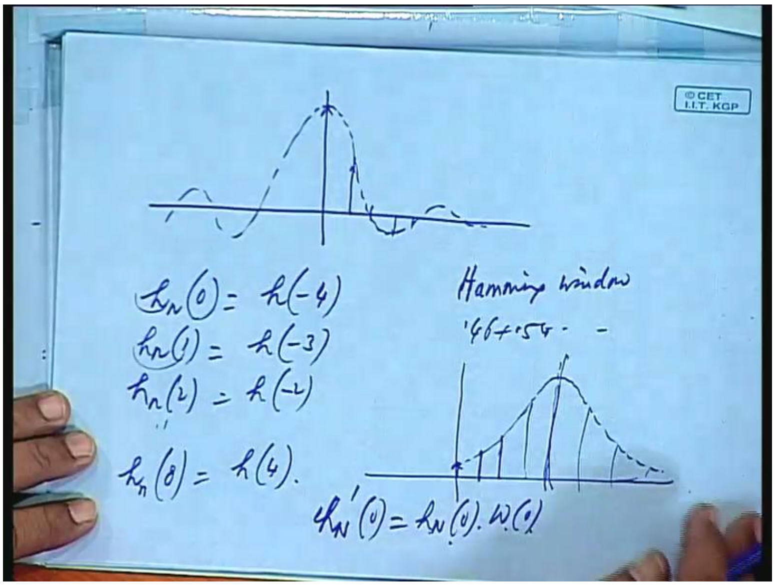

So, I will compute whatever I presume say; the function overall function after integration looks like this, okay. So, I will compute h 0, h 1, h 2, h 3, h 4, four on this side, four on this side minus h minus eight, and h zero include it, included that will be total nine points sequence. Not nine plus nine and one, all right. I do not want nineteen point sequences.

So if the sequence length is given; I say 9, normally we give odd number. So that, minus one the central one will be just once; others are just mirror images, so divide it by 2 after subtracting 1 that gives you, the number of points to be calculated. Put them on this side symmetrical number then give a shift.

So, the new h 0, I will call it h new h 1, h new 1 etcetera. New h 0 will be old h minus 4; similarly old h minus 3, all right? h n 2 will be, old h minus 2 and so on. And similarly new h n 8 will be old h 4, is that all right? After you have obtained these, you have been asked to use hamming window. So, window function, hamming window function is given to you okay, all right; 0.46 plus 0.52 something like that, 5, 4 cosine okay, it looks like this.

So, there are values, these nine values you have to take from here; multiply by these factors, these coefficients that will be your actual realized window function, a realized filter function. So, I will get h N dashed 0 which is nothing but h N 0 into W 0, all right. Similarly, h N dashed 1 will be h N 1 into W 1, is that all right? Now, we have just taken some varieties of problems for discussion, we will come to the computer aided design. So, before that let me just describe in brief, what are the structures that we have?

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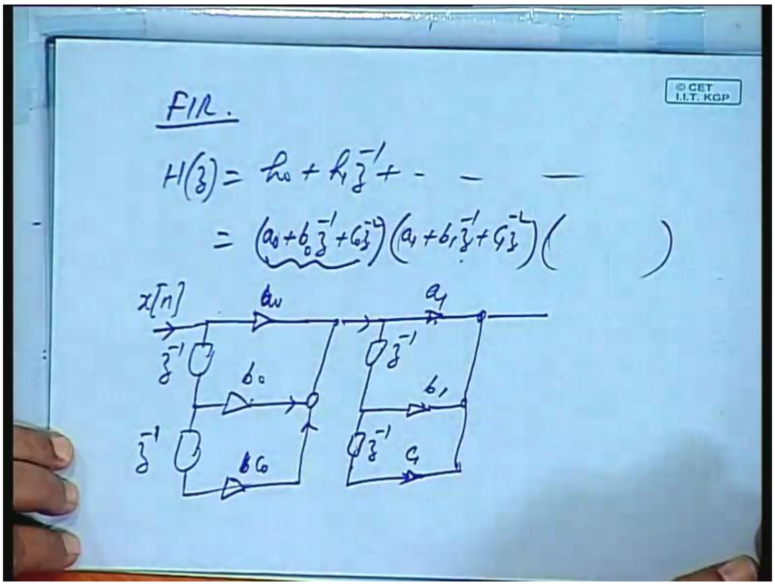

What are the possible structures for realization of an FIR filters? For an FIR, you are given a function H z, so h 0 plus h 1 z inverse and so on. I can always factorize this in the form of quadratics, all right. Say, something like a 0 plus b 0 z inverse plus c 0 z to the power minus 2, a 1 plus b 1 z inverse plus c 1 z to the power minus 2, again a quadratic like this. One may go for quadratics.

If it is an odd part, last one will be a linear function, okay. What is the advantage? Let us see. If I take in a product form, a 0 plus b 0 z inverse plus c 0 z inverse z to the power minus 2; what will it be like? I have a signal x n. I take a derivative z 1, again derivative z to the sorry, a delay z to the power minus 2 and then multiplied by b 0, multiply this by c 0 add them. Just a small circle will be sufficient to denote an addition and then a 0.

So, this gives me the output, coming out of the first block. Again, you have z inverse, b 1 z inverse, c 1 this is a 1 and so on. So, for each quadratic we have such blocks all right. In case now these parameters, you are trying to finally represent by a finite register length; memory is finite, so there will be some truncation error, all right. But, suppose any error in these coefficients, any of the coefficients will alter the roots of only these that is roots associated only with this particular quadratic. It will not affecting any other root, all right.

So, we are trying to make the roots sensitivity restricted. Roots coming out of this quadratic will be restricted; the sensitivity will be restricted to only these three parameters all right. Similarly for the second one, the roots will be restricted to only these three parameters, if they change then only the roots will change.

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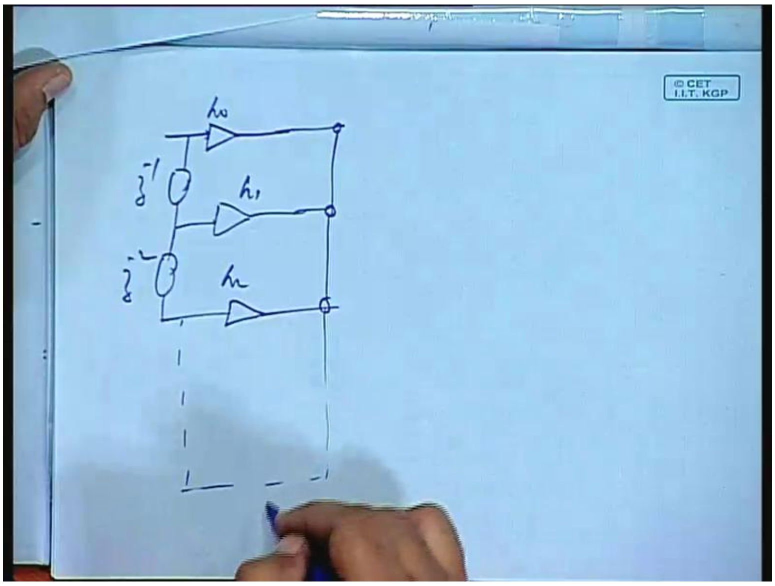

Had I had a realization like this h 0, okay, h 1, h 2 and so on? In a single shot, if I had all the elements together mind you, number of delay elements will be identical, all right. Only thing, probably this will be, I mean this will be easier to implement, it might appear, okay. But then change in any of these will be affecting all the roots, okay.

If any coefficient is affected then all the roots will be affected, simultaneously. So, here the sensitivity can be restricted to only a limited number of parameters, limited number of coefficients by this realization, okay. Similarly in case of an IIR filter; I have H z equal to say all right, b 0 plus b 1 z inverse plus b 2 z to the power minus 2 and so on, divided by a 0 plus a 1 z inverse plus a 2 z to the power minus 2 and so on, polynomials both in numerator and denominator.

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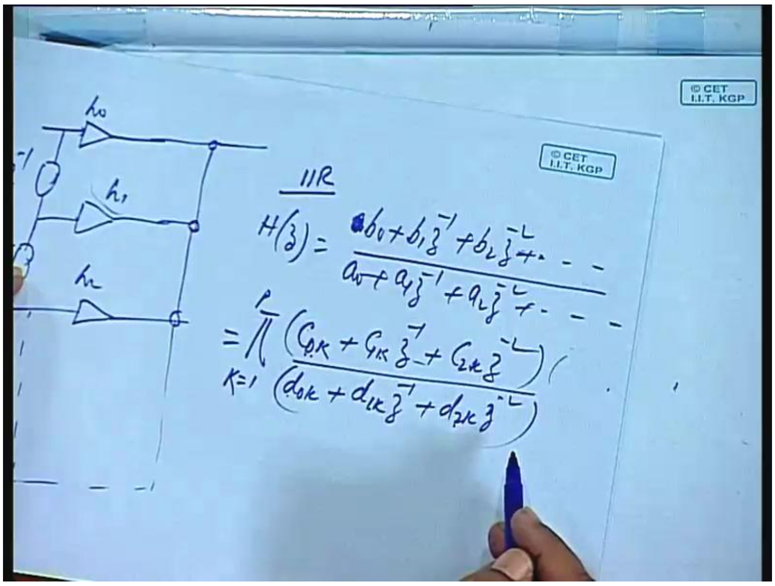

I may express these in terms of bi-quads okay; say I will call it c, because I have been used a and b terms so I will write like this, c 0 k plus c 1 k z to the power minus 1 plus c 2 k z to the power minus 2 divided by d 0 k plus d 1 k z to the power minus 1 plus d 2 k z to the power minus 2 okay where k varies from 1 to say any number p. That means, the number of such quadratic terms that we can get, it may so happen.

This is of lower degree, so you may not have all the coefficients present in a bi-quad; it can be just a constant divided by a quadratic. Now in a bi-quad again the poles and zeros corresponding to 1 bi-quad will be restricted, the the sensitivity will be restricted to only these coefficients, this is not the entire set of numerator and denominator coefficients, okay.

So, quite often it is this bi-quad forms which had used for computer aided design of of of an IIR filter. How is it done? The technique is very simple, one of the techniques is.

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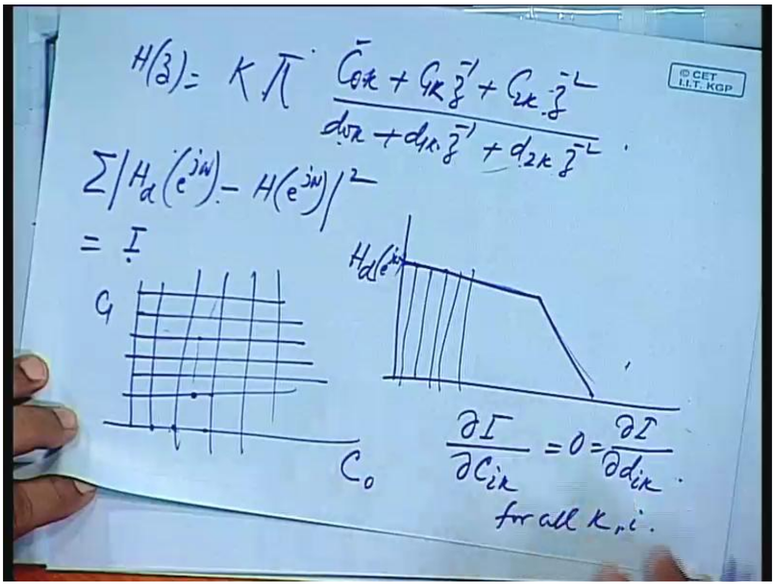

Say, you have H z equal to some constant into these quadratic coefficients, c 0 k plus c 1 k z inverse plus c 2 k z to the power minus 2 by d 0 k plus d 1 k z to the power minus 1 plus d 2 k z to the power minus 2.You assume some values of this coefficients okay to start with; then find out from the given specification H desired, minus H that you are using for a for improving in successive iterations.

So, I will just write H e to the power j omega square submitted over the entire frequency range. So, you take omega 0, omega 1, omega 2, omega 3 some discrete frequencies; obviously this is continuous function, integration of this is quite involved, so you do it by discrete mode, all right.

Suppose, this is the filter to be realized, filter to be realized need not be necessarily a low pass filter or a high pass filter or a conventional filter, it can have any desired characteristics. I want to realize this filter, okay. So, this is the characteristics of H d given, what should be my quadratic form? Suppose I start off with one bi- quad, I may have two such bi-quads, all right.

We associate some initial values to start with, with all these coefficients. If there are two then 6 plus 6, 12 coefficients are to be given some initial values and then you change one at a time. Change c naught k to some, change it by some positive value then again compute it, compute the error, all right. We take discrete frequencies like this, all right, calculate the magnitude that is evaluated from here and the given magnitude.

So, the differences square it and keep on adding, all right. Say this integral is I, okay. This basically integration, we are doing it by discrete summation. Then change this and see what is the change in I; if it is reducing go in that direction, again change in the same direction, if you find it is increasing then go backward. So it is like a grid search, in two dimensions it looks like this.

Suppose there are two parameters c 0 and c 1, okay. So, what I am doing is, first you keep on taking some value then increase it, is a value of I changing? Is it reducing? Then go further. Is it increasing? Then come back, take middle way, all right. So, wherever you get the minimum point you stop there, in one or two iterations.

Then you change c 1, wherever you get the minimum you come there again change c 1, c 0 then again change c 1. So, you keep on changing one particular parameter at a time, okay. So, here you change for c 0 k then c 1 k then c 2 k then d 0 k then d 1 k then d 2 k again come back to this. So, in an iterative fashion you keep on doing it. So, you will get the minimum and you get the set of values.

So this is a very simple technique and this is quite often adopted in many other optimization methods, that is you take one parameter at a time. So, basically what we are trying to achieve is the performance index, its sensitivity with respect to del Cik equal to 0 equal to dik, all k’s for all k's and all i's. So, this is one method of computer aided design, okay. Next time will take up the other methods. Thank you very much