Digital Signal Processing
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Lecture - 2
Discrete Time Signal and System (Contd.)

Good morning friends.

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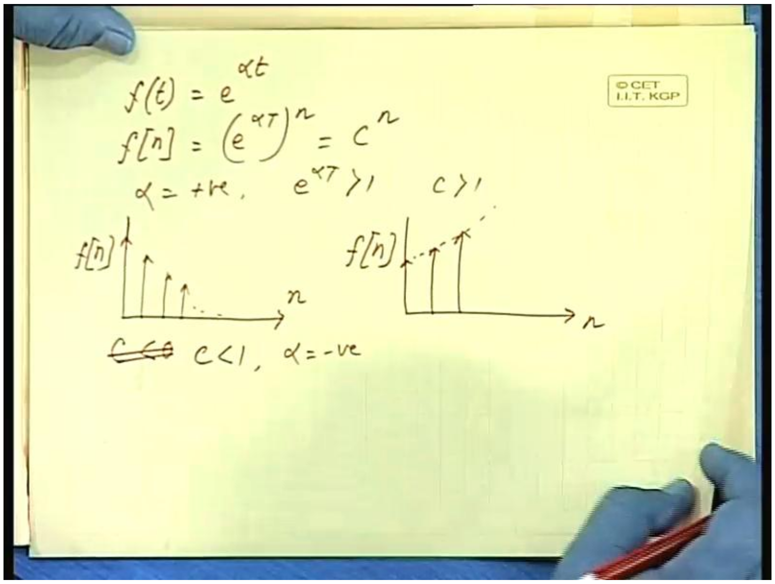

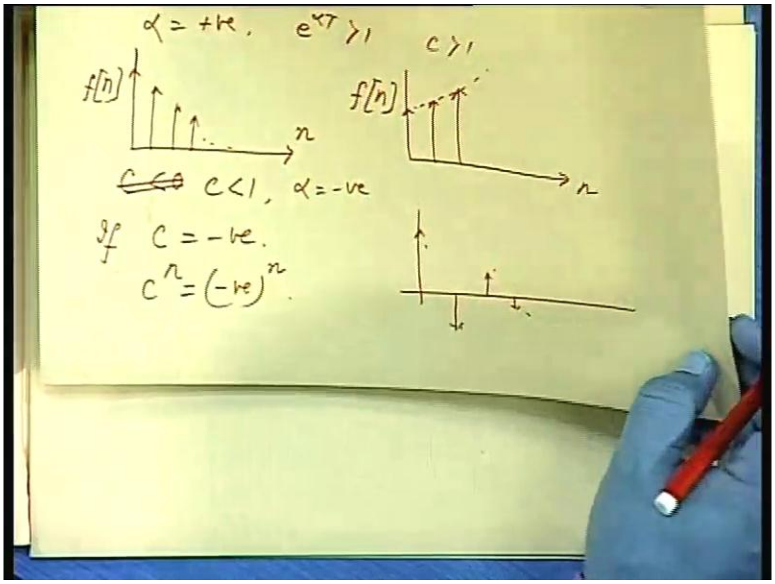

In the last class, we are observing for exponential functions of this current. The sampled version was f n e to the power alpha T to power n, so this we took as constant C. And if alpha is positive, we observed e to the power alpha T is greater than 1; that is C is greater than 1. So, the response was, well the function f n was something like this, okay. Thank you this is for C negative, C less than 1; C less than 1, that is alpha negative okay, it was like this. And for C greater than 1, it was increasing like this, okay.

If C is less than 0, C is negative; then C to the power n is some negative quantity, all right, to the power n, depending on the magnitude of that quantity. It may be decreasing in magnitude or increasing in magnitude but it will be appearing alternately with alternate signs. One of our friends had asked when do you get C negative; for alpha positive values or negative values, you do not get C negative, is it not? C was less than one or greater than one, but when do you get C negative, okay. Thank you very much.

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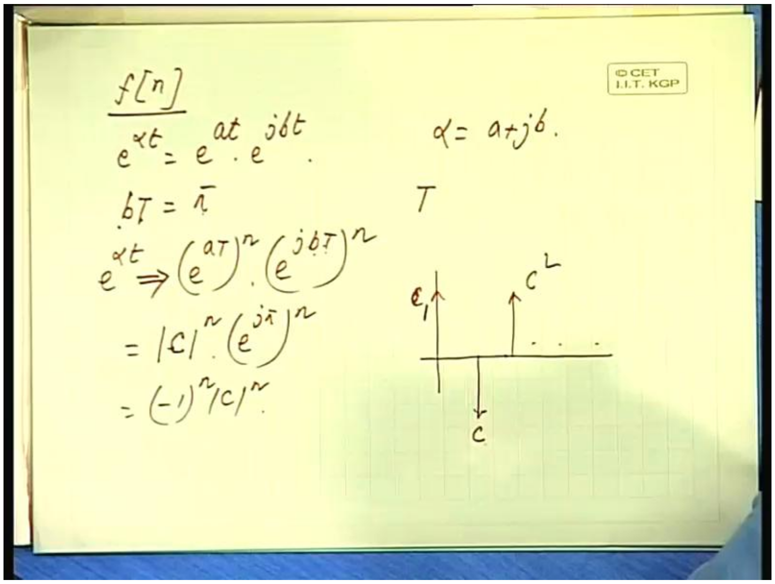

It is for a complex function. Suppose we have, that is alpha equal to a plus j b, all right; it can be negative or positive depending on that, it will be exponentially increasing or decreasing. Let us select the sampling time such that, b into T is say pi. We select the sampling time T, such that the product of T, at the frequency term b is equal to pi, then what happens? e to the power alpha t, if I take the sampled values; would be e to the power a T to the power n, e to the power j b T to the power n, okay. So, this is some magnitude C to the power n, e to the power j b T and b T is pi; therefore, this is e to the power j pi to the power n. And what is e to the power j pi, minus 1.

So minus 1 to the power n, magnitude C to the power n, is it all right? So, it will be C depending upon the magnitude of C, if it is greater than one then it will be more; it will start from one, then C, then C squared and so on, okay.

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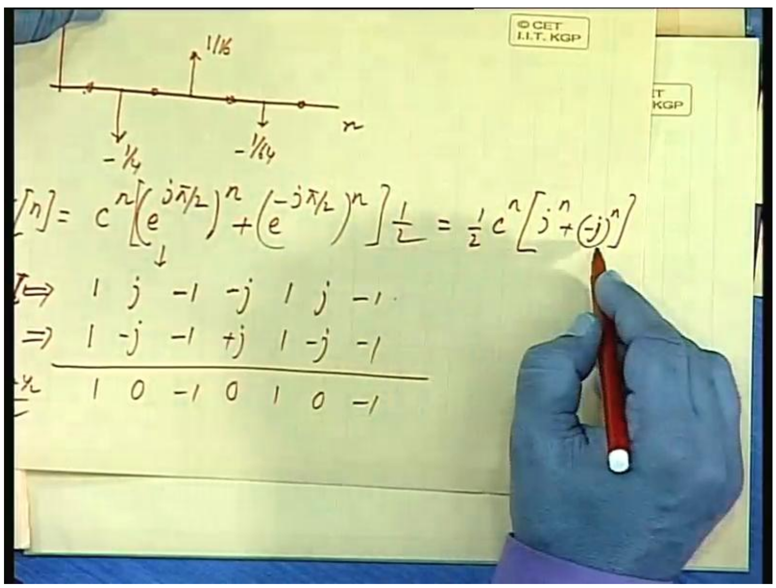

Now, suppose we have a function f n. Say, this is 1, 0, minus 1 by 4, 0 then 1 by 16, these are not shown to the scale; so I am just writing the values of this, it is gradually diminishing in magnitude, 1 by 64 and so on. And there coming alternately plus minus with zeros appearing in-between, okay. So, it is 1, 0, minus 1 by 4, 0, 1 by 16 and so on.

What will be representation of such a function? Could someone suggest?
p i by two, okay.

So, C to the power n, e to the power j pi by 2, let us try e to the power j pi by 2 n, you need this. Okay, let us see, C to the the power n, will take care of the magnitude part. Let us see, e to the power j pi by 2 n, what it gives. e to the power j pi by 2 n, what are the values, for different values of n? n is equal to zero, it is 1. n is equal to one, e to the power j pi by 2, that will be j. And then minus 1, then again minus j, then plus 1, then j and so on, okay.

So, that does not satisfy this. I have to have another term; I will to have another term which I can write, I will compliment this j's. If I add them together and then take the average, I will get so I call this as, some y 1, y 1 series and y 2 series. So, y 1 plus y 2 by 2 will give me, the desired series, okay. So, it will not be e to the power j pi by 2 n alone; it will be and average of this, is it all right?

Could it be 1 to the power n, n minus 1 to the power n? Instead of pi, if I take 1 to the power n, n and minus 1 to the power n. So, basically this is half of C to the power n. This is j, so j to the power n plus minus j to the power n, is it all right? This is j to the power n plus minus j to the power n, half of that.

Next question is, could it have been 1 to the power n and minus n to the power n? Let us see.

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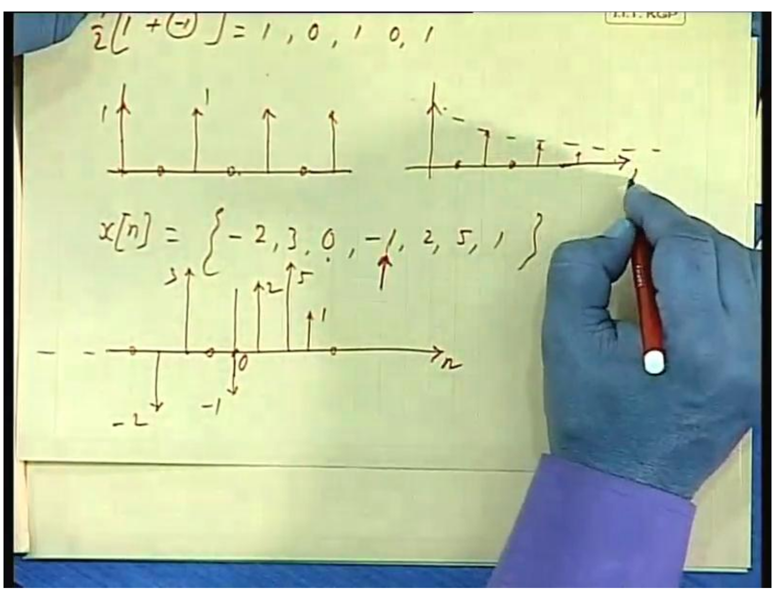

Half of this, what is it give me? 1 then if I put n is equal to 0, n is equal to 0, 1. n is equal to 1, n is equal to 2; again 0, 1, so this will give me a series; 1, 0, 1, 0, 1, 0 and so on.

So, far as this term is concerned equivalent of this term; that gets multiplied by C to the power n, so it will be modulated. So, it will either exponentially decreasing or increasing, but the terms that will be appearing will be alternately coming it positive signs and in-between there will be zeros. So, if you have signals of this kind, where C is less than 1; it will be decreasing and if there are zeros in-between, it will be like this, okay.

We will get back to this kind of exercise again later on. X n, the series is normally written; if it is a finite series then say, minus 2, 3, 0, minus 1, 2, 5, 1 and this is the finite series, I show by an arrow. Suppose there is an arrow here, that means this is the origin. So this is in the negative region of time, this is in the positive region of time, that is the sequence is defined both in negative and positive directions.

So this is, at this point it is minus 1, 2, 5 then up to this scale; 1, 0 if this terminated here, that means here after this is all zeros, 0 then 3 minus 2 and so on, rest all zeros. So by showing an arrow here, you show the location of the origin okay, this is origin.

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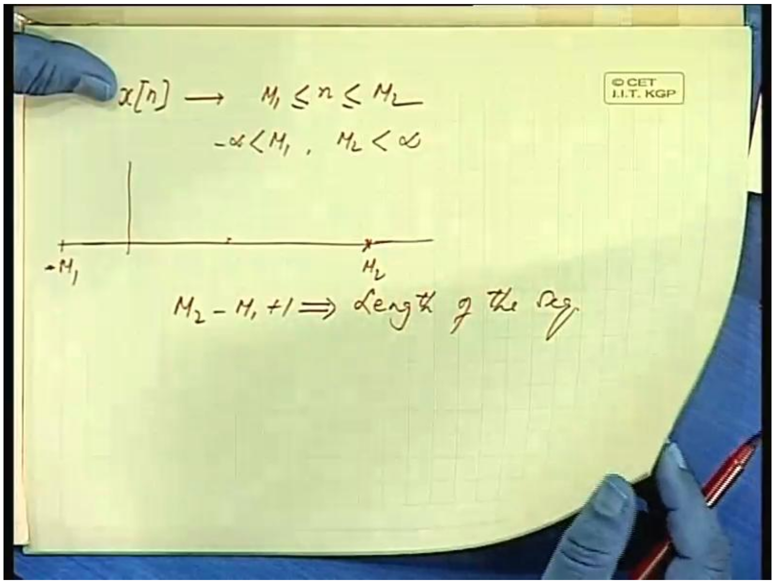

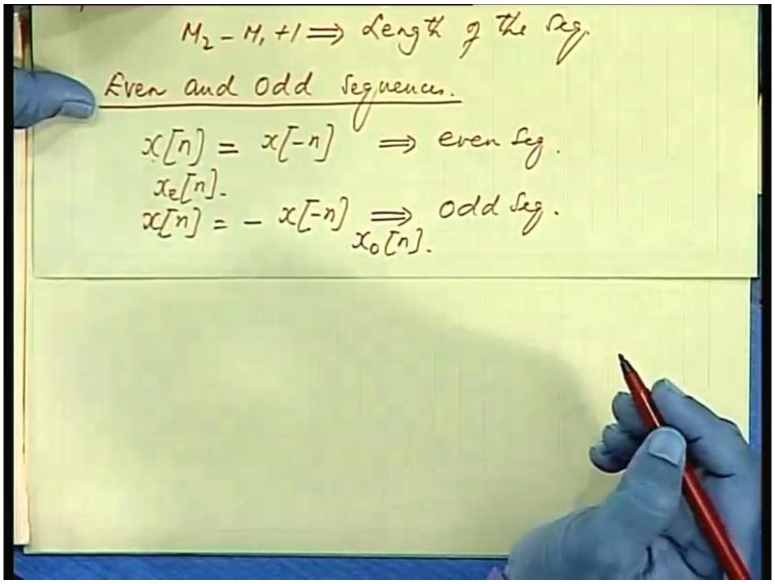

The sequence X n, may be defined over a range, is a finite range; where M 1 is greater than minus infinite and M 2 less than plus infinity, that means between some M 1 it may be in the negative region of time, and some M 2, it is defined all right, like the previous one. Here, this is 0, minus 1, minus 2, minus 3, so M 1 was up to minus 3. M 2 was up to plus 3.

So, what is the sequence length? What will be the sequence length? One two three four five six seven, all right, so three plus three minus of minus three plus one, okay. So, M 2 minus M 1 plus 1, this will be the length of the sequence, okay. M 1 could have been here also then also M 2 minus M 1 would have been this much.

Like in the continuous domain, we also defined here in the discrete systems, even and odd functions or even and odd sequences. So any sequence X n, if it is an even sequence then it will be same as X minus n. So, this is the condition for even sequence; such sequences we shall write as, X e n, x even n. Similarly an odd sequence, we will follow this relationship and we will denote these as X o n, okay.

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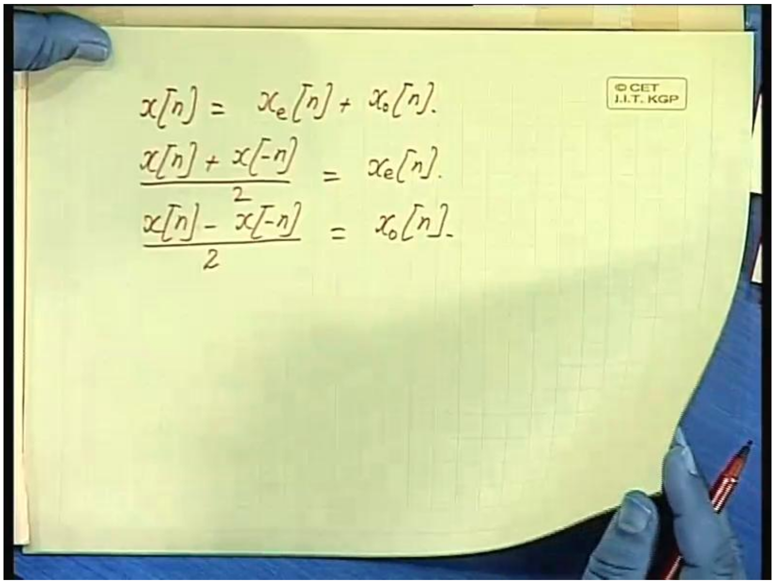

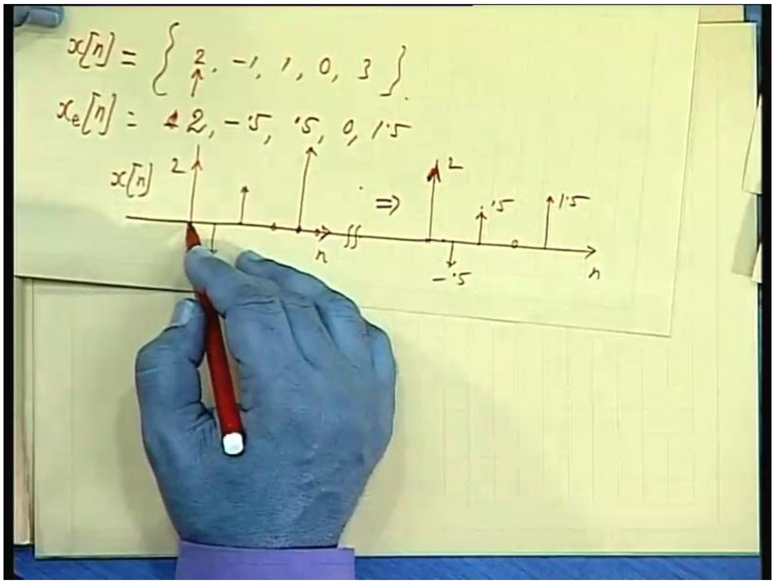

Any sequence X n, can be written as a sum of an even component and then odd component, okay. Now, how do you find out the even component? You take X n plus X of minus n, divided by 2; that gives you X e n, okay. Similarly, X n minus of X of minus n, divided by 2 will give you X o n.

Let us take an example. We have given X n, as say 2, minus 1, 1, 0, 3 what will be the even sequence? Would someone tell me? What are values in the negative region of time, all zeros? If I have origin here, okay on this side of, this arrow left side it is all zero, so it will be half of this sequence plus its negative region values, divided by two. So, it will be half of all the values. Should it be the half? Any disagreement? First one? 2, then minus 0.5, then 0.5 then 0 then 1.5, is it okay?

So, I have a sequence, graphically if we show, if this is 2, minus 1, 1, 0 and 3, it’s corresponding even part, this is X n, this is n, okay all right, it is a break here. Then even part will be 2 then it is minus 0.5 then plus 0.5 then 0 then 1.5. So, except the first value; others all half, the first value is remaining as it is, all right because when you add with this X, minus zero is also same as X plus zero so, that divided by 2, means the original value. So all other values will be reduced to half, what with the odd sequence?

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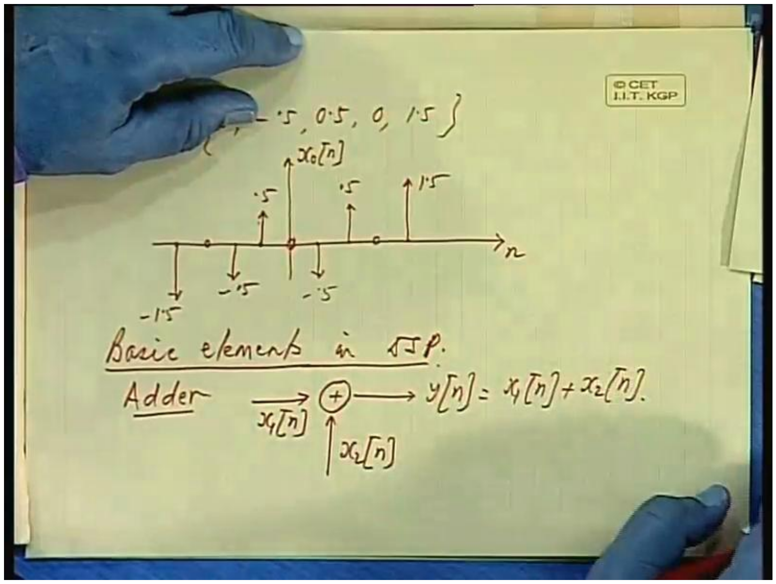

X odd, it will be 2 minus 2, 0 then this one will be the same, so minus 0.5 plus 0.5, 0, 1.5. So, the odd sequence be 0, minus 0.5 plus 0.5, 0 and 1.5, okay. Say, except for the first value others are same. So, you mean to say except the first value and even function and odd function will appear identical. So where is the difference?

So, even function is though; we are showing it only in the positive region of time, it is also having its counterpart in the negative region which has not been shown. So, the total function is a mirror image of this plus this itself, okay. So X n, if you take a mirror image on this side and then club with this that will be the even part X e n. Similarly, if I take a mirror image of this twice, once about y axis then again about x axis; so I get its replica on this side, so first quadrant and third quadrant symmetry.

So it will be, for example odd function; if you take X o n, the total function will be 0.5, minus 0.5, 0 and minus 1.5, okay. Sorry. So, this is the total function. Normally, we do not show this part, it is understood; it will be having these values, in the negative region it will have just negative of these values. Similarly, the even part we do not show, the left hand side but it is also having these values.

So, even function and odd function will be met much longer in sequence compared to original sequence; because the original sequence was only up to this, okay. The other values on this side were zero. Now, you take some basic elements used for signal processing operations. One is an adder, you have a sequence X 1 n, X 2 n, the output is X 1 n plus X 2 n okay. Then you will have a multiplier; X 1 n symbol for a multiplier is like this.

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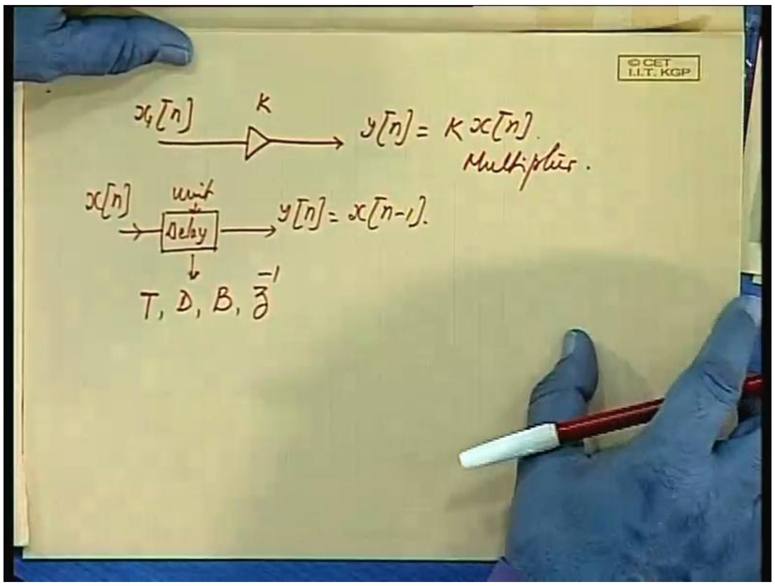

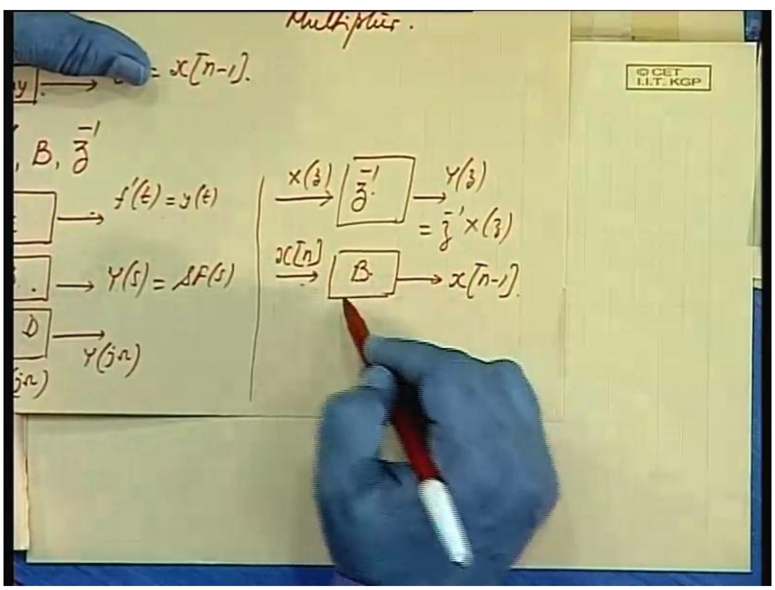

So, multiplier K, so output y n gets multiplied by a scalar K, k times, x n okay. So, this is a multiplier. Then we have the most important element that is a delay element. If this is x n, if this is a unit delay, one sampling time delay then y n, the output will be x n minus 1, okay, if it is a unit delay. The symbol for a unit delay used in different books are; sometimes it is written as, capital T that is one sampling time, sometimes capital D that is a delay standard one delay.

Mathematicians, they use backward backward shift operator B. And in the transform domain, we also use in text books Z to the power minus 1 okay, in terms of Z transform operator. It is similar to using or in in a derivative circuit. In many books they write d d t; if you have f t here, you get f dash t here in an analogue domain, okay. In the control books, they replace it by s. And mathematicians will find they write as p, d d t operator. In some books, they write heavy side operator, capital D. They are one and the same thing, only thing is in different domains, we use different symbols.

If, you are working in the time domain only then you write d by d t but the moment you use this as a Laplace operator s, then you should not write f t; this is not correct, you should write its transform F s and corresponding output, whatever it is. Suppose, this was y t then I call it Y s and not y t. Y s it happens to be s times F s in this case, okay. So, the symbol use of the symbol is depending on what domain you are using. In the frequency domain, similarly this will be; if I write F j omega and this Y j omega then this will be straight away j omega, okay. s replaced by j omega in the continuous domain.

However, in the discrete domain in the discrete domain; this is very commonly used symbol, though technically it should be used only when you are using the variables in the Z domain. If I am using X z then this has a meaning, Y z will be, z inverse of X z. I should not write X n is in the time domain then, I should use either B or capital T, okay. So, B means backward shift operation that will give me X n minus 1, which is y n okay. But, anyway most of the text books you find, they use z inverse even in the time domain okay. So, this is a common slip but conventionally it is acceptable; so we will also use z inverse as an element, if required. All these symbols will be used sometime or rather, they are one and the same thing.

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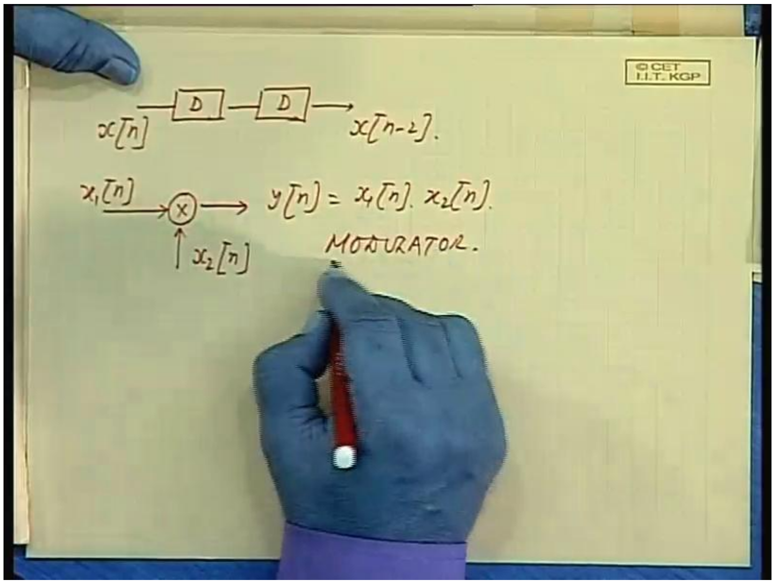

So, if you have two delay elements, if I show two delay elements; if this is x n then what will be the output? x n minus 2, okay. Next, you have another kind of multiplication that is; x 1 n gets multiple multiplied by x 2 n, so y n output is x 1 n multiplied by x 2 n is a modulator, okay. You must have studied the amplitude modulation, frequency modulation, all right.

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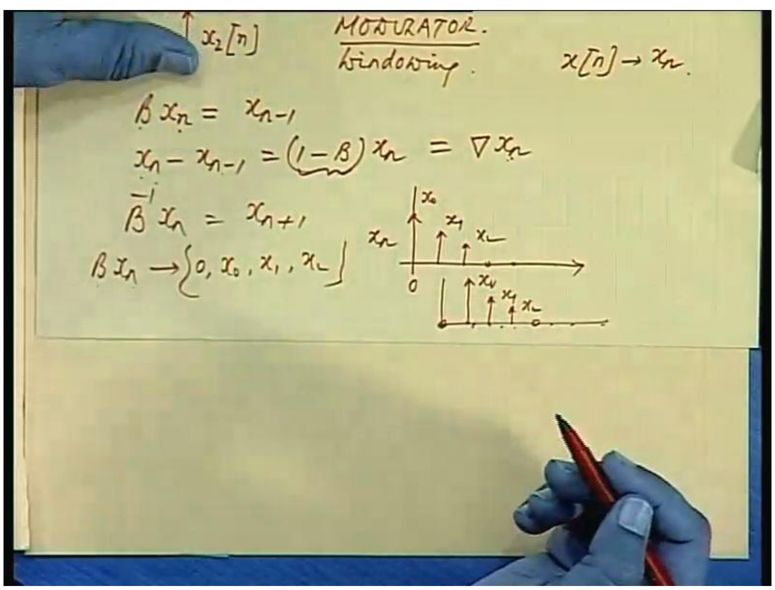

This is also used in windowing windowing operation; that means a function x 1 n, we passed through a window window function x 2 n where the function values will be multiplied by, it will be scaled by different amount, depending on the values of x 2 n, okay. So, basically what you are trying to, you are trying to prove; x 1 n by different amounts, at different instant, okay. That proving function is x 2 n that is windowing.

So, what is this backward shift operation? Therefore, on x n, x n is a sequence; I can write x n, sometimes we also use x n all right. So, B on x n will give me x n minus 1, okay. So, what is x n minus x n minus 1? 1 minus B times x n, so this we call; as differencing operation on x n. So, on x n if I apply a differencing operator, I get the difference between x n and x n minus 1; it is very similar to your derivative operation.

If I have B x n as x n minus 1; what will be a forward shift operation? Forward shift will be, B inverse x n, which will be x n plus 1, okay. What is this shift really? Meaning, suppose you have a sequence x n; this is say at 0, x 1, x 2, rest are all 0, this is a sequence x 0 may be five, four, three. If, I give it a shift, B of this represents the general x n, B of x n will give me; this whole thing is shifted to the right, shifted to the right means first instant, it will be 0 then only x 0 will appear, the x 1, x 2 and then 0. So, x 2 starts with a 0, if I give two shifts then 0 0 then x 0, x 1, x 2.

So, B of x n will be shifting the sequence; such that this will be the values. Now, 0, x 1, x 2, x 0, x 1, x 2.

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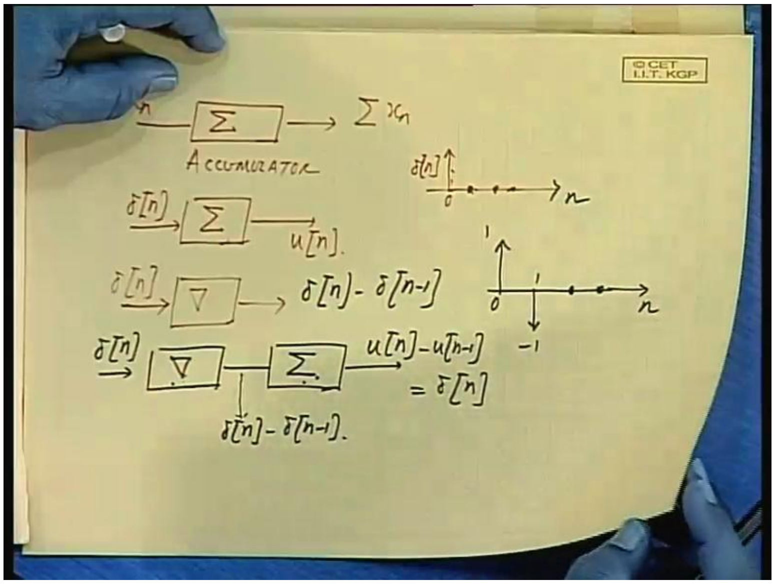

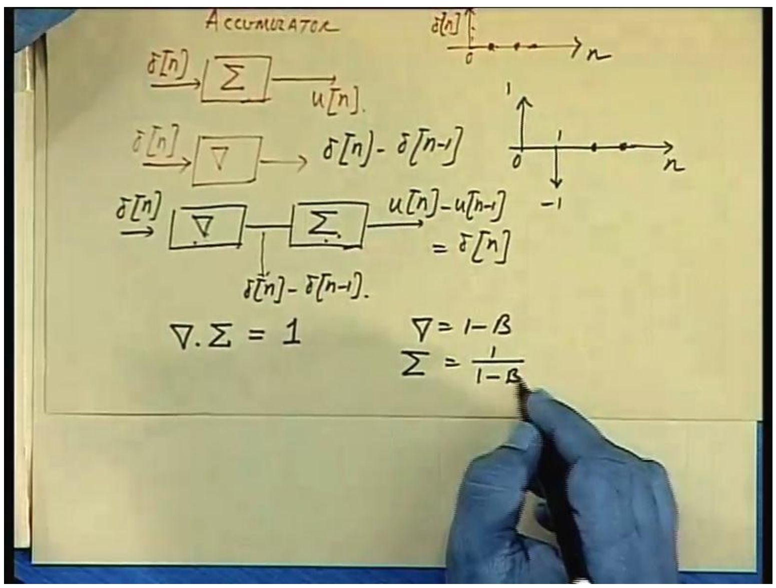

There is another operator, a summator or an accumulator, accumulator is a proper name. So, what you get is summation of x n, all right; that is x 0, it is similar to your integration, summation of the sequence up to that point n. What kind of output you get, if I excide the system with a delta function? It is delta n is this one; okay this is delta n, rest are all 0. Now, in an accumulator what will be the output? At this instant, it is one, next instant also it is one, one plus zero, one plus, zero plus zero and so on. So, the output will be u n, okay; it is similar to integration.

If I now, use it because a differencing operator, differencing operator what do you get? Delta n minus delta n minus 1, is it not? What will it look like? Okay, this is delta n on and minus 1; if I use a delta n and a delta function and then an accumulator, what will be the output?

So, this one is delta n minus delta n minus 1, and a summator delta n will give me u n. And this will give me u n minus 1. So, this will be u n minus, u n minus 1 which is nothing but again delta function, all right. This is delta n minus delta n minus 1, so delta n is coming back as output.
So, one is the inverse of the other, all right. So, accumulator is the inverse of a differentiator, difference in differencing operator okay.

So, delta and sigma; if we use these notations is equal to unit, all right and delta was 1 minus B and therefore sigma will be, can you prove it? Can I prove it? Let us see, what is sigma?

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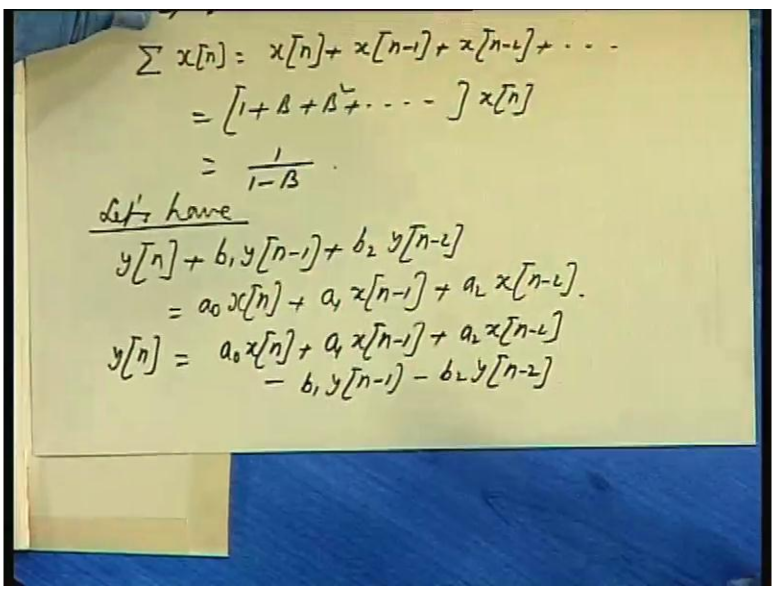

It is one plus; see sigma, this operator operating on x n, sigma operating on x n, nearly x n plus, is it not, all the values, p s values? So, this is 1 plus, this is B plus, B square and infinite sequence and this is a g p series; we can write this as 1 by 1 minus B, okay.

Now, let us use these elements, let us see; how these elements can be used, to represent a different equation or a general equation. Suppose, you have a difference equation in this form, will discuss later on how such equations can be obtained, okay.

Suppose, we have an equation y n plus b 1 y n minus 1 plus b 2 y n minus 2 is equal to a 0 x n a 1 x n minus 1 plus a 2 x n minus 2. One may write, where y is the output and x is the input, okay. So, you can write y n is equal to a 0 x n plus a 1 x n minus 1 plus a 2 x n minus 2 minus b 1 y n minus 1 minus b 2 y n minus 2, do you agree? So, from there, you can see a 0 x n.

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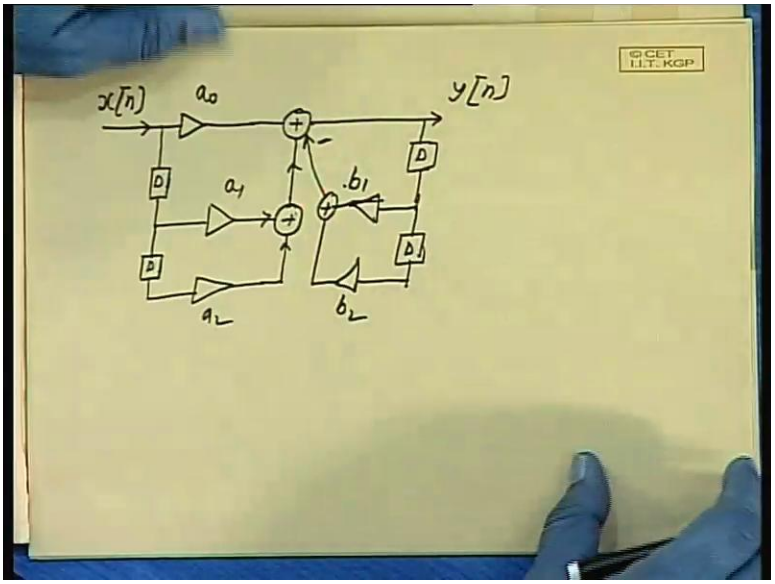

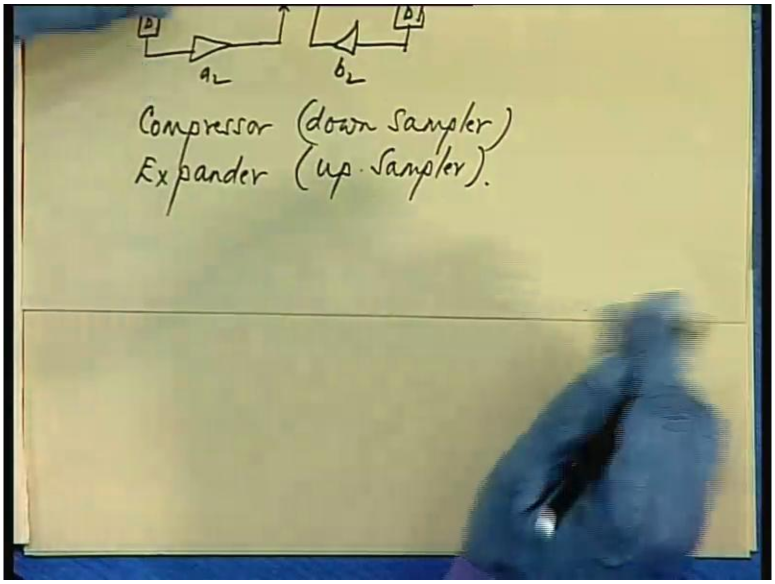

Suppose, x n is the input then that gets multiplied by a constant a 0, added with a 1 times one delayed function. So, I take x n here, I tap it here, put a delay element, you can write d or capital D then multiplied by a 1. Then again take another delay, multiplied by a 2, okay. So, this is a 2 times x n minus 2, plus a 1 times x n minus 1. So, they are added together and then finally added with a 0 times x n.

This is suppose, this is output y n then I am yet to add these two terms. So, that y n from the output side, again I can have a delay; b 1 then b 2, I can add them together and then they are to be negated, okay. So, these two from these two, if I stretch channels and added signals; will be giving you y n, okay. So there is a multiplier, adder, delay.

There are some more special functions, operators, we use for changing the sampling rate, all right. One is a compressor or down sampler and the other one is up sampler or expander, okay. When we compress it?

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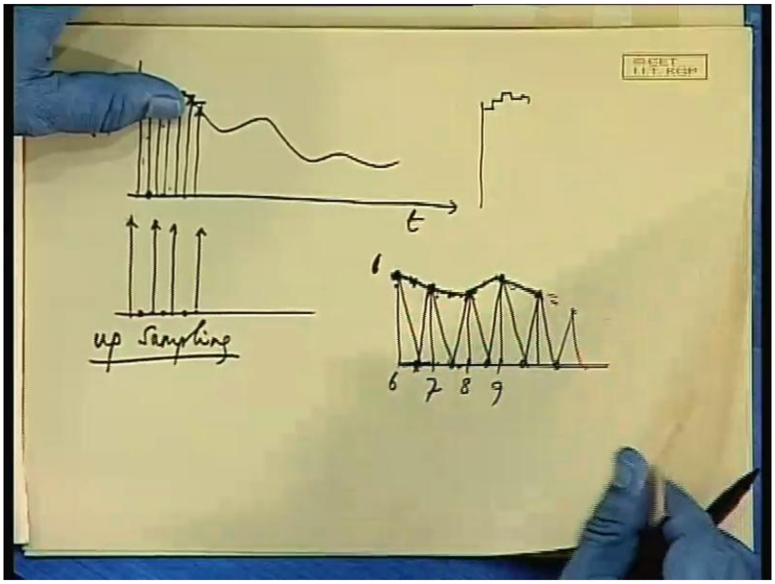

Suppose, okay let me take the signal here, there is a signal here; this is a continuous domain signal x t, your sampling at very close intervals, if you have very close intervals of sampling then you are almost recovering the continuous domain function, okay depending on the degree of that sampling rate.

Now, if you are very very close and if you are using sample and hold elements that means; your approximate approximating this function by this type of function, okay, these size constant functions. And if they are very very close then that error will be minimise. Now, suppose the function is changing very slowly, all right then you can relax the sampling rate, you can reduce the sampling rate.

So, you drop, say we take only alternate values whatever it be this, in-between these values; I made zeros, will it be same? If the function was changing very slowly and if I sample it instead of at this high rate, I have got these sample values. Now, I drop in-between, these values we drop that means, you make them zeros, okay. Then we get samples like this; that may represent the original signal to the satisfactory level.

We could have dropped every third or sorry, we could have dropped two consecutive values, that is; we could have taken only the, only every third value x zero then x three then x six and so on. And make x one, x two as zeros, we could have done that. And then you compress it, then you compress it back to its original sampling time; then you get the same wave, seen in a very compressed form, okay.

You may not lose much of information, if the sampling rate original sampling rate was very high or compared to the sampling rate, their signal is changing very slowly, okay. So, you can keep on trying to decimate to down sample, keep on doing it till you find that important information are getting lost, till that time you can do these. Will get back to this problem latter on, what are the conditions for fixing the sampling rate, for a given signal that we shall discuss in subsequent discussions.

So the basically, in down sampling we drop the values. These are dropped and then again these are compressed. In up sampling, we insert some artificial values, zeros; so, you are given a sequence say for example, you may be given the power demand for campus at regular intervals for one hour, we have the records, at say 6 o' clock, 7 o' clock, 8 o' clock, in the morning 9 and so on, I have these values.

The demand in megavolt may be like this. One may insert some zero values, because they were not defined, because they were not defined; you can insert zeros also, will it be same? Basically, this denotes the profile of the actual variable to a certain extent all right, because the demand is not very varying very fast; so, we we can sample it at a at an interval of one hour, now if somebody puts zeros here, though initially, we said that here it is not defined but it cannot be zero, is it not?

Now, if I try to make a zero here and then the profile will be like this, is it not? It is not same as this. You are bringing it down to zero, so there is a sharp change taking place which is not representing the original sequence. A sharp change means high frequency components, so you have to eliminate this.

So, if this sequence is pass through a low pass filter then you can get back more or less the original sequence. One may insert some values by linearly interpolating these. That is take the values in-between, between 6 o'clock and 7 o' clock whatever values you want, at six thirty, it will be the average of the data that you get at these hours. I add these plus these by two, these plus these by two; this is a simple linear interpolation of second order, okay. Then probably, it will represent to a certain extent, the original profile, okay.

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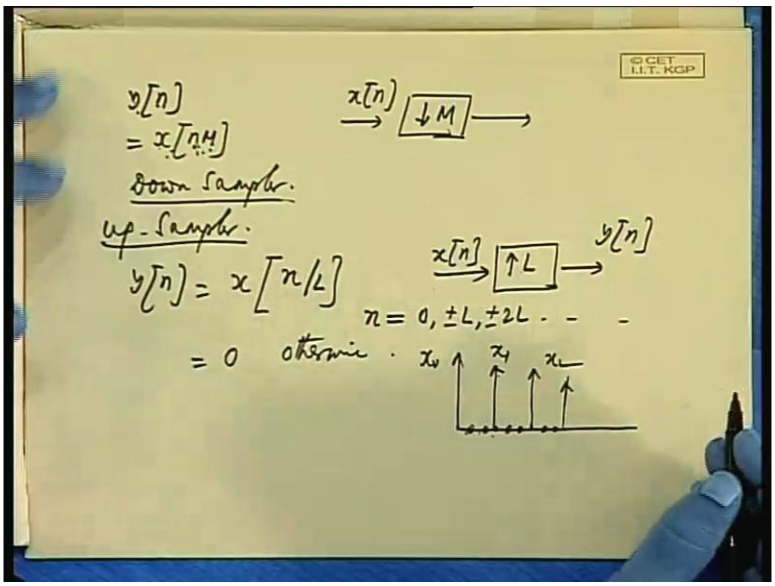

So, for a compressor y n, if this is the output of a compressor; this is x n and we are down sampling by M, it is x n M, okay. And rest of the values are all zero, only the Mth value is counted, okay. Suppose, M is equal to five then n you put as one, it will be one into five, so x five will be treated as y one, x zero as y zero, x five as y one, x ten as y two and so on, so this is for a down sampler.

In case of an up sampler, y n will be x n by L where symbol is, x n is the input, this is the symbol for the up sampler; it is your increasing the sampling rate. And y n, x is equal to n by L where n is equal to 0 plus minus L plus minus 2L and so on and is equal to 0 otherwise; that means if this is your x 0, x 1, x 0 2 and so on, okay. Then you are inserting zeros here and you are generating a new sequence. So, x 1 whatever was x ten, sorry x 1 will be now, say x three or x four, depending on the number of zeros that you are inserting; that is the factor L, okay. We will stop here, today, next time we take up from this point, okay. Thank you very much.