Digital Signal Processing
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Lecture - 17
IIR Filters (Contd.)

We are discussing last time about IIR filters.

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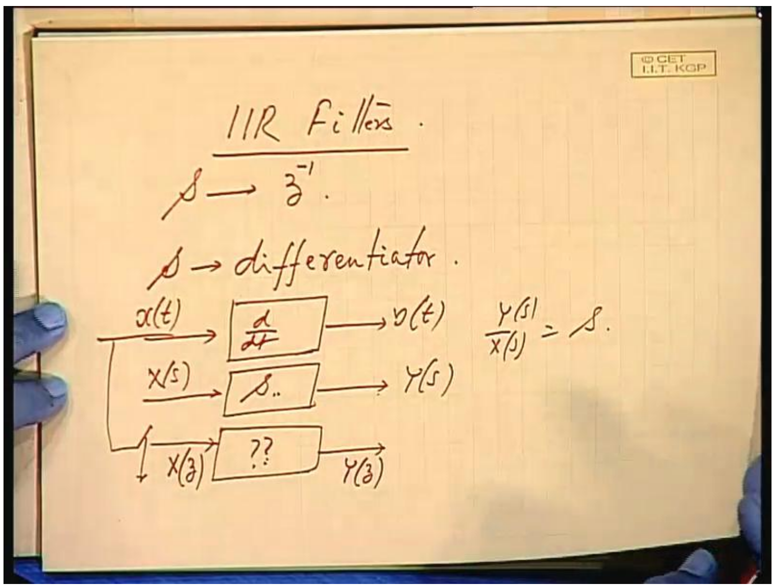

Now, the design of IIR filters is based on the design of analogue filters, okay. It is a very simple technique. Analogue filter design is highly standardised. So from the specifications, you design any of the analogue filters and then make a substitution for s a function of z or in terms of z inverse. If you can express s in terms of z inverse, then you just make a substitution for s the function of z and then that becomes a filter.

Now, shall we taking up some of the simple substitution that can be made and then see the limitations of different substitutions. And then again we will come back to, a little details of analogue filter design. Now, as I was mentioning last time the approximations that we can make are like this; s can be treated as a differentiator or differentiating element.

That means, if I have in the time domain a function x t and corresponding output is y t then in the transform domain; if I take X s, this is Y s then this element will be s. This was d by d t operator operating on the input x t, giving me an output y t. And this can be used as a multiplier algebraic multiplier s, in the s domain okay. So, Y by X in the s domain becomes s.

Now in discrete domain, the same signal if it is discretized with a regular switching; I call this X n and in the Z domain, this is X z and this is correspondingly Y z. What is this function? So, whatever appears here; that will be used to substitute for s, this is our primary goal. Now, let us see when we talk about differentiation; one of the approximations is as I was mentioning earlier.

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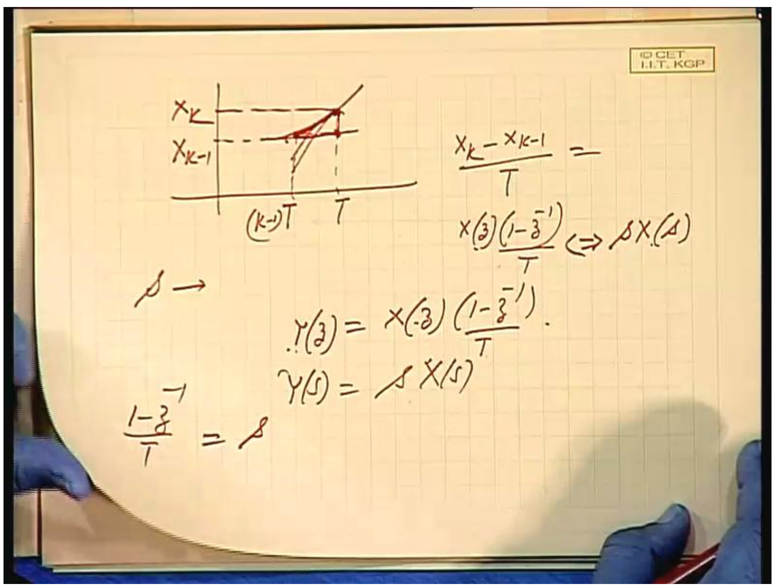

One of the approximations that I, we can make first order approximation is this difference; this divided by this can be taken as tangent at this point, that means the previous value and the increment from the previous value in the function, okay. So, if sorry if this is K minus 1 T and if this is T, if this is X K minus 1, if this is X K then X K minus X K minus 1 by T is a slope of this, all right.

So, if I have a curve like this, if I take okay so I will come to that later. So, basically the operator s has been replaced by; if I take the z transform of this, what do i get? X z into 1 minus z inverse by T, okay. So, X z into 1 minus z inverse by T is equivalent to in the s domain, continuous domain is equivalent to s times X s, all right; derivative of X is in the discrete domain derivative of X on this, so we are trying to equate these two quantities.

So, reverse side X z. Rather if I say, Y z is a corresponding output; Y z is in one case, X z into 1 minus z inverse by T. If this product can be taken as the output Y and Y s is s times X s these domain. So, what we are equating is Y by X, the transfer function in the first case is 1 minus z inverse by T. In the second case, it is s. So, s is 1 minus z inverse by T, this is 1 substitution, okay.

If I go for the other substitution that is if I take the forward value then it will be X K plus 1 minus X K by T; this will be the corresponding Y K or 1 say this is z times X z, okay.

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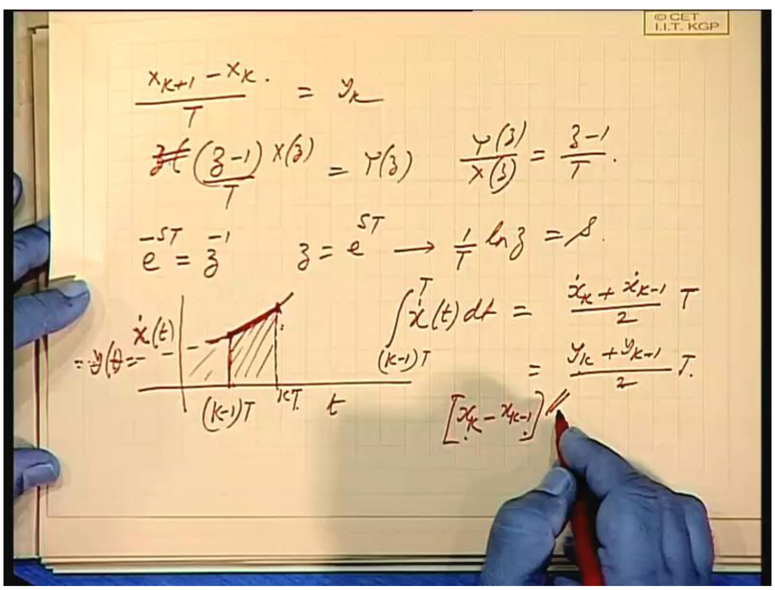

z minus 1 into X z by T is equal to Y z or Y by X in the z domain is z minus 1 by T, this is another substitution, thus this is another approximation. Now, we shall see today some higher order substitutions, okay. And will see the limitations of these substitutions. Actually, your relationship is e to the power minus S T was; z to the power minus 1 or z is equal to e to the power S T, this is actual substitution. If I take log, it will be 1 by T l n z equal to s.

So, s is actually 1 by T l n z and we are making approximations of this kind. Let us see the integration operation. Suppose, we take this is derivative of X, X dot t and this is t, okay. So, what will this area under the curve give me, is x t. Now, if I want to measure between K minus 1th instant to K th instant, this area, okay.

So that is X dot t integral d t, integrated between K minus 1 into T to T. This will be this area. And if we approximate it, if we approximate it by a straight line here; this is a first order approximation, okay. If I make a straight line approximation here, what is the area under this curve? It is this plus this divided by 2 into T. So, it will be x K plus x K minus 1 by 2 into T, is that all right? This is okay? It is the area for trapezium. Okay.

So, x dot K plus x dot K minus 1, okay. x dot K if I call this as y t; so I can write this as y K plus y K minus 1 by 2 into T, okay. And that is how much? From the x side, it is slope; yes x K minus x K minus 1 by, no. x K minus x, K minus 1 because; what is x K? It is the integration from 0 or minus infinity to this point, area under the entire curve and what is X K minus 1; area under the curve, up to this point.

See, if I subtract one from the other, I get this hast area. So, it is this difference all right which is equal to Y K plus Y K minus 1 by 2 into T. If I now take, z transform on both sides, z transform on both sides I get X K will be X z; I get 1 minus z inverse and this side I get, 1 plus z inverse by 2 into T into Y z, okay or Y by X in the z domain will be 2 by T into 1 minus z inverse by 1 plus z inverse, is it all right?

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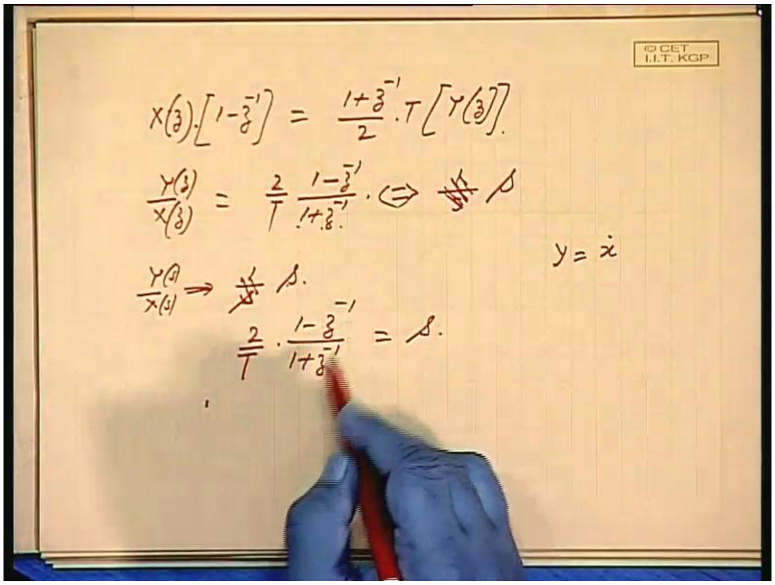

So, this is known as bilinear transformation and this is what? Y by X is what, what is Y by X? What is Y by X? It is 1 by s. We are performing an integration operation, all right. So, Y by X is 1 by s in the s domain, so this is equivalent to 1 by s, is that all right, no? Is this okay? What have we done? 1 minus z inverse by 1 plus z inverse into 2 by T is Y by X.

Y is equal to x dot all right; so it should be equal to s, it should be equal to s. So 2 by T, 1 minus z inverse by 1 plus z inverse is equal to s or 1 by s okay; s is this much. Wherever we find and s wherever we find s, will make a substitution for 2 by T 1 minus z inverse by 1 plus z inverse. Earlier, we are making a substitution of 1 minus z inverse by T or z minus 1 by T. So, this is a third approximation, okay.

So like this, we can go for higher order substitutions. So, basically you are expressing X K in terms of x K minus 1, all right and then trying to find out a relation.

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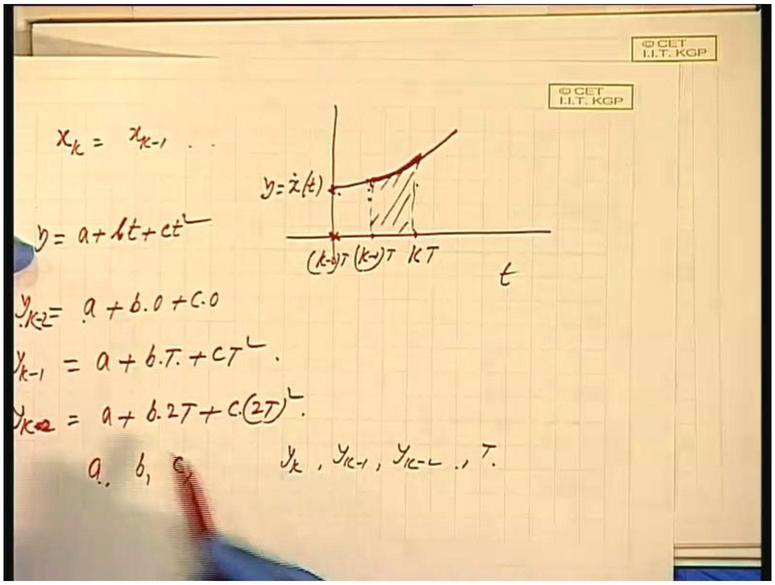

Suppose we take, we made a straight line approximation here for this curve; all right and area under the curve was integration of that, okay. Suppose, we take x dot t, that is y which is equal x dot t against t, we make a second order approximation. Say, we choose this as the origin, so y is equal to say a plus b t plus c t square; if we make such a substitution, that means instead of going back by one step expressing the value of the variable at this instant in terms of the previous value, we take previous two values all right. So this will now, be will be a part of a parabola, not a straight line.

So, if we take these and if we fix our origin here that is; at t is equal to 0, y is equal to say y K minus 2. This is K T, this is K minus 1 T and this is K minus 2 T. So, Y K is equal to a plus b into 0 plus c into 0. So, a is equal to Y K sorry; Y K minus 2, Y K minus 2 is equal to a. Y K minus 1 is equal to a plus b into T plus c into T square. And Y K minus 2 sorry, K is equal to a plus b into 2 T plus c into 2 T square, okay.

So, you can finally get a b and c in terms of these three values, okay. These three values; Y K, Y K minus 1, Y K minus 2 in terms of this and the T s, all right. So once you get these, make a substitution then you find out this area. Once again there is integration of x dot t, all right. x dot t

I have already got in terms of a b c and a b c's are already known; so from there for a second order we can verify for yourself, X K finally can be written in terms of, I will write only the final answer here, we are not going to get the detailed derivation of all these.

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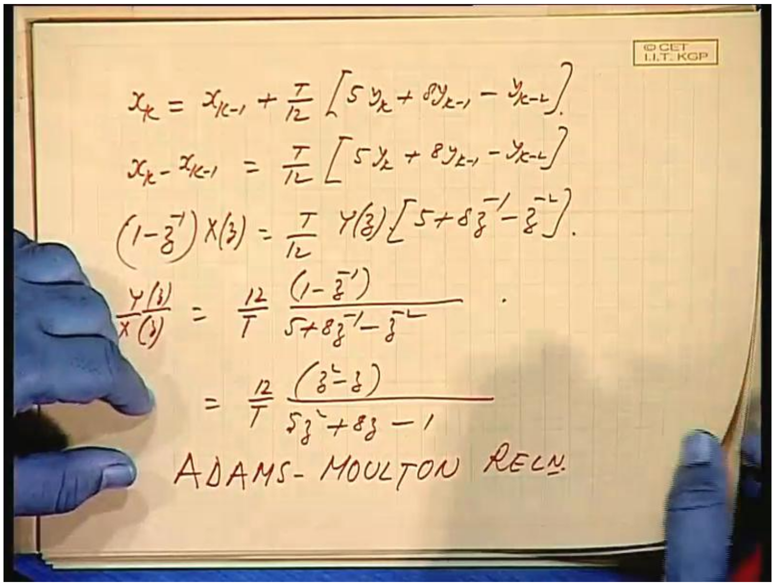

So, I give you T by 12 5 times Y K plus 8 times Y K minus 1 minus Y K minus 2, it can be shown to be of this type, okay. So, X K minus X K minus 1 will be equal to T by 12 5 Y K plus 8 Y K minus 1 minus Y K minus 2. What you are doing here? Here is now, we will be integrating this from K minus 1 to K and get the area all right; and that should be equated to Y K minus, X K minus X K minus 1.

So, this is what we have done. So, if it is z transform; 1 minus z inverse into X z is equal to T by 12 Y z. This side you will get 5 plus 8 z inverse minus z to the power minus 2, okay. So, Y z by X z, Y z by X z will be 12 by T, is that all right? 1 minus z inverse by 5 plus 8 z inverse minus z to the power minus 2, this is a second order approximation; you can multiply by z square, so it will be 12 by T z square minus z divided by 5 z square plus 8 z minus 1. So, this is a second order approximation.

So, like that you can go for a cubic parabola; in this a plus b t plus c t square plus d t cube, you take previous three points and once again you get hold of those value of a b c d and then get the expression for the area, all right. And you will get a higher order term. So, this is known as Adams Moulton relation, Adams Moulton relation or transformation.

So, bilinear transformation is one reversion of this transformation. Now, what is so good about these transforms, compare to the simpler approximations, 1 minus z inverse by t?

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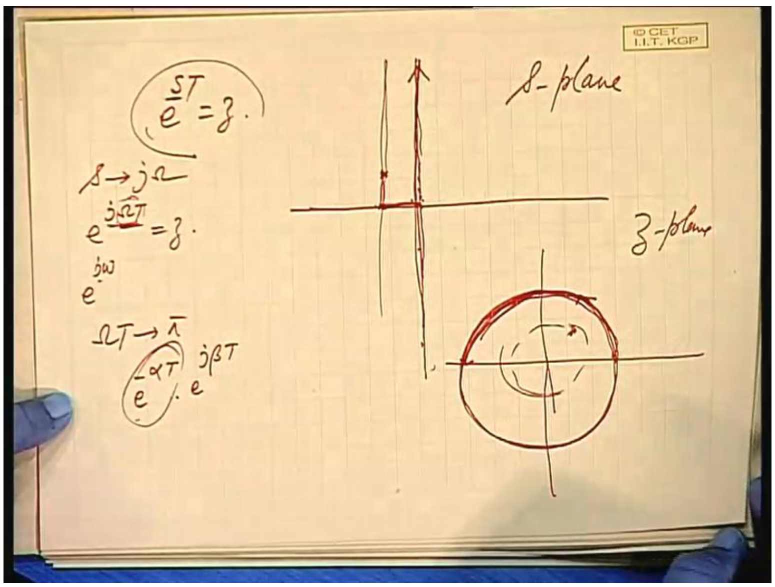

So let us see, e to the power S T equal to z; if this is a real transform from z to s, where do I get the imaginary axis of the s plane mat in the z plane? When I vary s equal to j omega, s is made j omega. So, e to the power j omega T is z, is it not? So, z becomes and this omega into T is omega, all right.

So, this particular line imaginary axis is mapped in the z plane as a unit circle. e to the power j omega is unit circle; its magnitude is always unity, is it not? So, we are travelling along this with different values of omega here, okay we are travelling along this. This is zero to minus infinity okay, and this 0 to plus infinity.

So the entire imaginary axis, entire imaginary axis is mapped here on this circle, is it all right? How many times? Infinite number of times. Whenever I make omega T is equal to p i, I come up to this; that means it is dependent on sampling frequency, sampling time T will decide how many circles how many, how often I will make the circles here, all right.

So, it is basically mapped as an infinite number of circles, ideally okay. So, a point on the left hand side of this will be a point inside this; okay that you can see any point here is minus alpha t plus j beta t. If I made a substitute, substitution here instead of j omega T s; I write e to the power minus alpha T and e to the power j beta T. e to the power minus alpha T will be less than 1, all right. So, it will be inside, it will be a circle like this; if I go along this, it will be a circle like this.

So, all the points in the left hand side will be within this unit circle, in this substitution. This is actual substitution. Now, let us see our next approximation.

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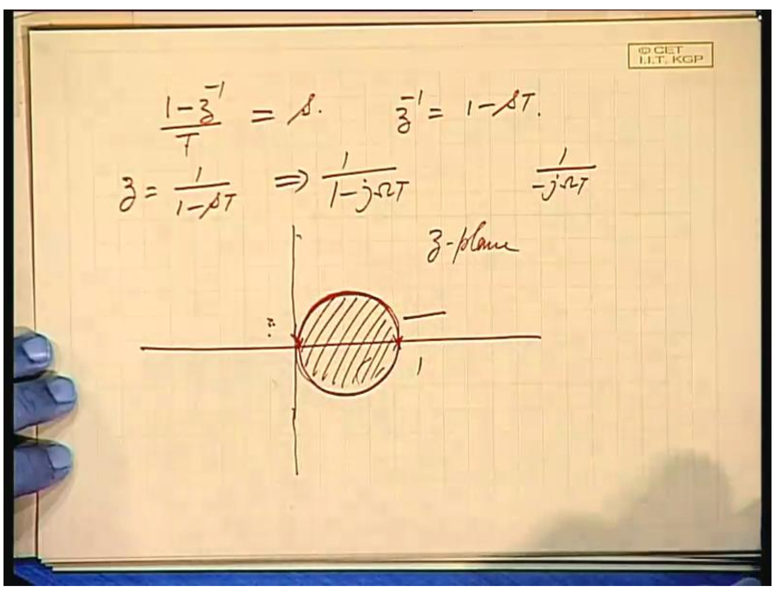

1 that is we made, 1 minus z inverse by T as equal to s. So, z inverse is 1 minus s T, is it so? So, z is equal to 1 by 1 minus s T, okay. So, if I make s equal to j omega; so that gives me, 1 by 1

minus j omega T. Now, if I now traverse again along this path in the s plane, that means I make omega vary from 0 to infinity or minus infinity to plus infinity; what will be the mapping in the z plane?

If we take omega very very small, it will be 1, 1 okay. When omega tends to infinity, a very large value; this will be tending to minus j omega T, omega tending to infinity. So, magnitude wise it will be very very small and minus 1 upon j is plus j; so it will be approaching 0 value from plus j side. So, it will be tangentially approaching zero like this, is that so, when I go to plus infinity.

When I go to minus infinity; similarly it will be like this, okay. So, basically minus infinity to 0 to plus infinity, this will be the circle. How many times it is making the circle? Only once. Earlier, the circle was having a centre at the origin. And how much was this? 1. This was? 1, all right. Now, I am getting a circle like this; well within the original circle. So, this was actually the stable region but with this approximation, I am constrained to believe that this is the stable region because mapping of imaginary axis of the s plane is giving me this. So, it is a smaller circle, its highly pessimistic, very very conservative.

This point is conceived as unstable point, outside the stable zone; whereas it is not so, in reality. It is very much stable but we are not able to see, whether this point is stable or not. We are feeling that, it is in the unstable zone, all right. So, the stability is highly conservative. The estimate of the stability of the system is highly conservative. Let us see, the other approximation. Any question on this?

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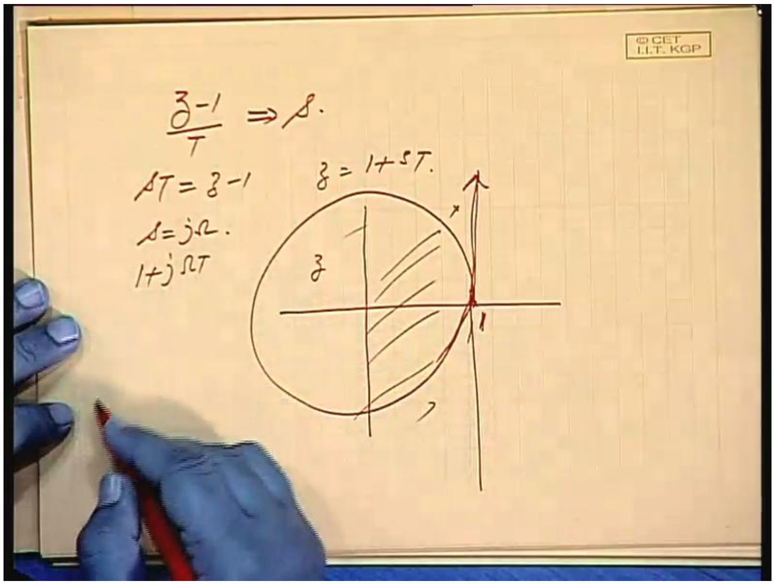

The next approximation was, z minus 1 by T equivalent to s. s T is equal to z minus 1 or z is equal to 1 plus s T, is that so? So, now if I make s is equal to j omega, s equal to j omega; so it is 1 plus j omega T, what is it? z when I take this, this is 1 and it is a straight line, imaginary axis. So, this is the centre region is stable but actually this is 1, actually the stable region is only this unit circle.

So, you are feeling you are safe, you are in the stable zone but actually you are not. Actually stable zone is this much, so it is highly optimistic, okay. So, this is also not a very correct estimate. Earlier one was pessimistic, but it was guaranteeing stability of the system, this will not guarantee stability of the system. So, let us see what bilinear transform does.

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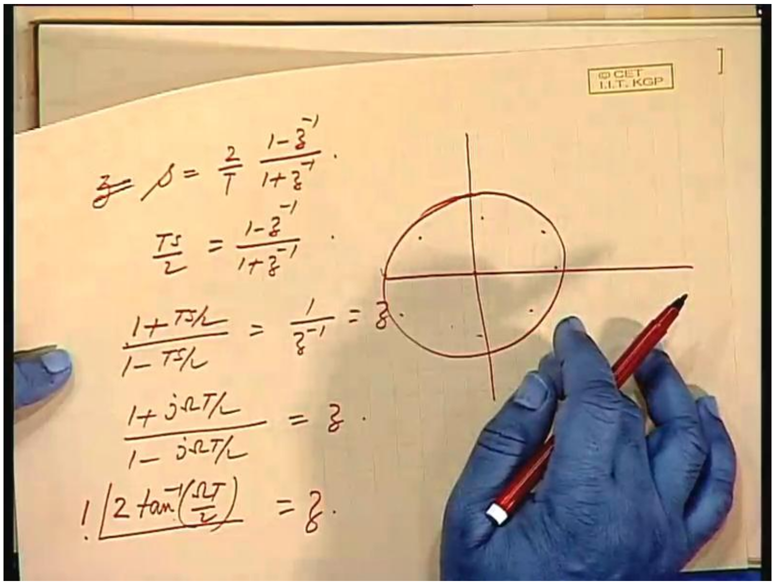

z is now approximated as sorry, s is approximated as 2 by T 1 minus z inverse by 1 plus z inverse. Let us see T s by 2, T s by 2 is equal to 1 minus z inverse by 1 plus z inverse. So, if you make componendo and dividendo into 1 plus T s by 2, correct me if I am wrong. If I add, okay 1 minus T s by 2 is 1 by z inverse sorry, is that so?

So that gives me, z. So, it is 1 plus j omega T by 2 by 1 minus j omega T by 2 which gives me z. When I make a substitution s is equal to j omega, I will get corresponding mapping in the z plane, okay. So, what is the magnitude? What is the magnitude of this? And what is the angle? See, here the angle is 10 inverse omega T by 2, here minus of that so that will become plus. So, this twice 10 inverse omega T by 2, this is the angle.

And the magnitude is always 1; it is 1 plus j x 1 minus j x, so magnitude is always 1. So, this is the well of z that means; it will represent again a unit circle, magnitude is 1 and the angle is changing. Now, here when I take omega tending into infinity, omega tending into infinity; this will give me p i by 2, 2 into p i by 2 that will be p i. Similarly from the negative side, if I go to minus omega; so it will come from 0 to minus p i. So, this entire circle will be described only once under bilinear transformation, all right. Whereas in the actual transformation; z is equal to e to the power s T, it is making a large number of circling around the origin.

Now, it is only a single circle but so far as the stability of the points are concerned; we are describing the same limits of stability, all right. This is advantage of bilinear transformation; you do not distort the stable zone. So, we have gone for a substitution like this, s is equal to 1 minus z inverse into 2 by T, 1 plus z inverse.

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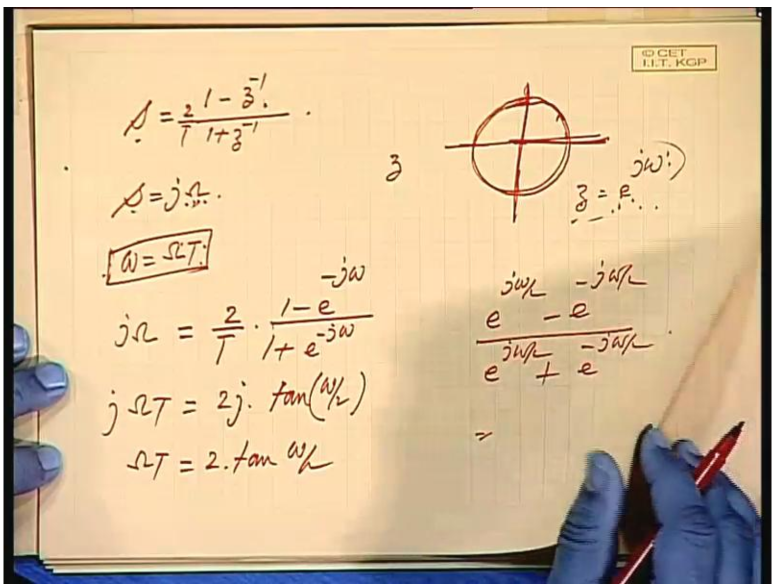

If I make, s is equal to j omega on this side and if I go along unity circle, if I substitute z is equal to e to the power j omega all right; in this substitution, bilinear substitution bilinear transformation, I want to see the relationship between the two frequencies, okay. Ideally, ideally omega should be equal to omega into t okay, is that relationship maintained in bilinear transformation or is there is any distortion?

So, you would like to see what distortion takes place in these? So, according to our substitution in the frequency domain, discrete frequency domain z is equal to e to the power j omega; so we will make a substitution on this side and we will make corresponding substitution s equal to j times omega for the continuous domain, we would like to establish a relationship between capital omega and small omega.

So, this is the ideal relationship since this is an approximation between s and z; so we would also like to see the transformation; what the distortion it makes in the frequency domain. So, on this side I get j omega. We said, we have made yes; I can make a substitution here itself, okay. Yes, we have already calculated that.

So, z is e to the power j omega okay; we have already done that, 2 by T 1 minus e to the power minus j omega by 1 plus e to the power minus j omega. So, we saw here e to the power j omega, all right. Omega into T j times is equal to 2 into 1 minus e to the power j omega; if I multiply by e to the power minus j omega by 2, it will become e to the power j omega by 2 minus e to the power minus j omega by 2 divided by e to the power j omega by 2 minus e to the power minus j omega by 2, okay plus.

So, this will be sin and this will be cos; there will be two j term coming out of this, that j will get cancelled, okay. 2 and 2 will get cancelled, so this will be 2 j sin by cos tan omega by 2 okay. Or omega T is equal to 2 into tan of omega by 2; this is the relationship between small omega and capital omega.

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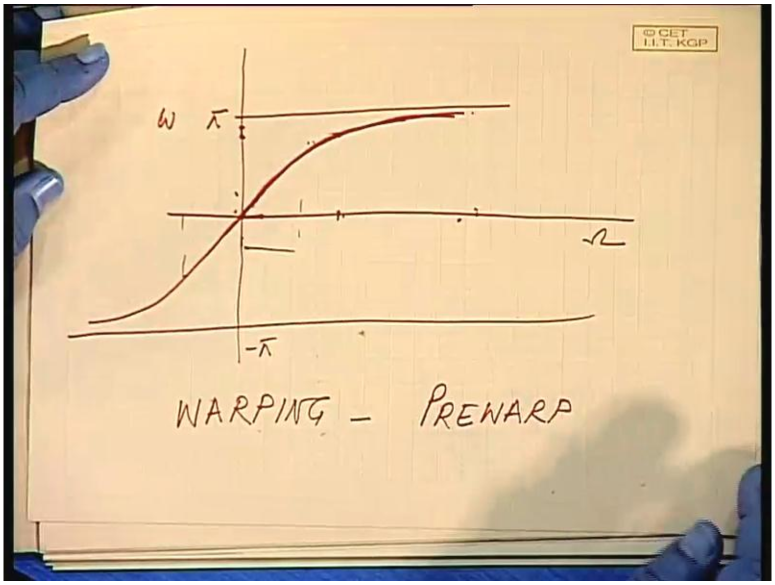

So, if I make a plot of omega and small omega. When omega is small, when omega is small so it is omega by T by 2 equal to tangent omega by 2 or omega is equal to 2 times tan inverse omega T by 2, okay. This is what we got earlier also, the same answer; this was nothing but e to the power j omega, so this small omega, j times omega was angle here, so small omega is basically this.

This is what we have re-derived, anyway. So, this is when omega is very very small; this is a small quantity tan inverse x is 10 into x, so it will be straight line. 2 times x by 2, so basically omega is equal to just capital omega T, this is the ideal relationship. But when omega tends to infinity, this becomes tan inverse infinity which is p i by 2.

So, after some time it will get saturated, similarly on this side. So, when omega tends to infinity this tends to p i by 2 into 2, so this will be p i. Similarly, this side it will be minus p i. So, this is known as warping that means; if I give a signal, if I give a signal say of this much frequency, correspondingly this frequency is compressed. I increase the frequency five times, correspondingly there is a slight change in the frequency at the high level; that means at the higher level there is a compression of frequencies, all right.

At a very low level, it is proportional but at a high level it is distorted. It is distorted like this, say beyond thirty degrees or so you will find the distortion is quite prominent. So when we give a specification for any filter, we must ensure; we must ensure that such a distortions do not take place, this is known as warping phenomena.

You must have seen in ply woods, they say that the top of the table; when it rains or if it gets wet after sometime there will be the wrapping, that means you are trying to compress, you are trying to compress the surface. Similarly, here you are trying to compress the frequency original frequency so much but in the discrete domain, you are trying to compress the frequency. So, this is known as warping.

So, you have to pre-work the specification before starting the design in the analogue domain; you stretch it, you stretch these and then give your specifications, design the filter and then you make a substitution, such that after this warping it comes to the normal specification, all right. It needs a normal requirement. So, that is the idea of pre-warping. So, to counter warping phenomena in the design stage, we pre-warp the specifications.

So, today we take up one or two simple analogue design and then see after substitution, what kind of functions we get. Suppose, you are given specification in the analogue domain; we will design the filter in the analogue domain and then we will make the substitution like, simple bilinear substitution.

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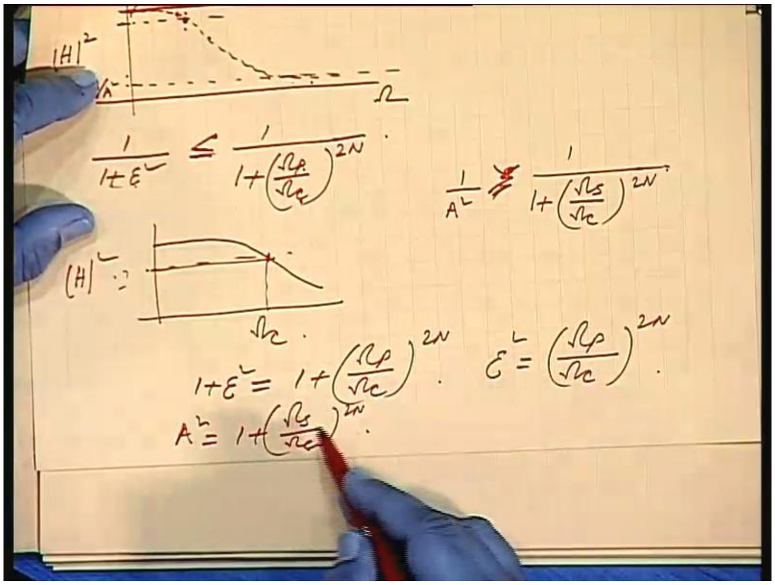

As I was mentioning the other day, low pass filter for example will be specified in terms of some tolerance in the pass band and something in the stop band. So, this we are referring as say; if it is 8 H magnitude square then this was 1 by A square and this was 1 by 1 plus epsilon square. If this magnitude is A A square, then this is how we are specifying the filter.

So, 1 by 1 plus epsilon square will be 1 by 1 plus omega p by omega c to the power 2 N, all right. See, Butterworth filter will be specified, a Butterworth filter is specified in terms of frequency cut off frequency omega c, all right. At omega c, the gain is 0.7 or seven or H square magnitude will become half.

But in the actual specification, suppose we do not specify that particular frequency; that frequency is unknown but I specify some other frequency, call it pass band frequency at which the gain is so much, all right. That means the gain is 1 by 1 plus epsilon square. Then you have to search for omega p, omega c. Omega c is not given, omega p is given. So, this will be the relationship.

Similarly, in the stop band I specify 1 by A square, so 1 by A square is equal 1 by 1 plus omega s by omega c to the power 2 N. Normally, we suggest that 1 by 1 plus epsilon square should be; that means this must be at least this much, this gain should be greater than or equal to this much. It can be here, here, above this but it should not go below this, okay. Similarly, this should not be above 1 by A square, in the stop band; it should be less than? Greater than.

Should it be 1 by greater than? This should be less than 1 by A square or 1 by A square should be greater than this, okay. So, it should be well within this and this should be well within this tolerance limit. So, if you try to equate these two; so from here I get 1 plus epsilon square equal to 1 plus omega p by omega c to the power 2 N or epsilon square is equal to omega p by omega c to the power 2 N, okay.

And from here A square is equal to 1 plus omega s by omega c to the power 2 N. So, you can eliminate omega c from here. So, what I get is A square minus 1 by epsilon square is equal to omega s by omega p to the power 2 N.

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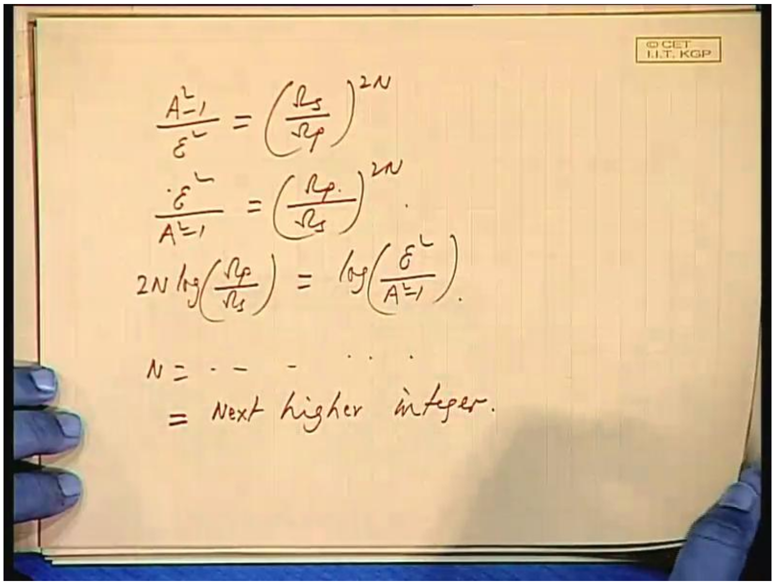

Omega c gets cancelled. Omega c to the power 2 N is common, so that it get cancelled. So, I will get this, okay. So from the specifications given, you can see if you take log, okay we can invert it; epsilon square by A square minus 1 is equal to omega p by omega s to the power 2 N. 2 N log omega p by omega s or I could have written this, anyway; omega p by omega s is equal to log of epsilon square by A square minus 1.

So, from there you can calculate N. N whatever values you get, approximate by the next integer. So take next higher integer, this is a design procedure. Once you have got this higher integer value then obviously these specifications were in equality specifications. If you equate them then both cannot be satisfied with the same value of N, both have been satisfied with a fractional value of N, is it not.

So, substitution of another value of N, approximate value of N will not satisfy both the equation. Now which one to be satisfied? If we take the second equation, you are solving this at this point, at this specified stop band, this value comes within this but then omega c is; there are two solutions of omega c. See, N has, N has been solved, other things are specified. So, what you are searching for is a solution for omega c.

So, there are two possible values depending on in which you are substituting N. So, this will give you some value, this will give you some other value. If we substitute it here, if we substitute it here will get an omega c correspondingly, such that it is well within that is the pass band frequency is pushed. Omega c will be a little higher, pass band pass band frequency will be pushed inside. So, you are very much safe on the pass band side.

If you make a substitution in the first equation then it is stop band which will be going further, okay. So there is more advantage; if you want to, after all we are more interested in the pass band restriction. So, we make the second substitution normally and get hold of omega c. Our interest is to find out N and correspondingly omega c; once you know these two then your design is complete, because in the Butterworth filter finally we have got H equal to 1 plus omega by omega c to the power 2 N.

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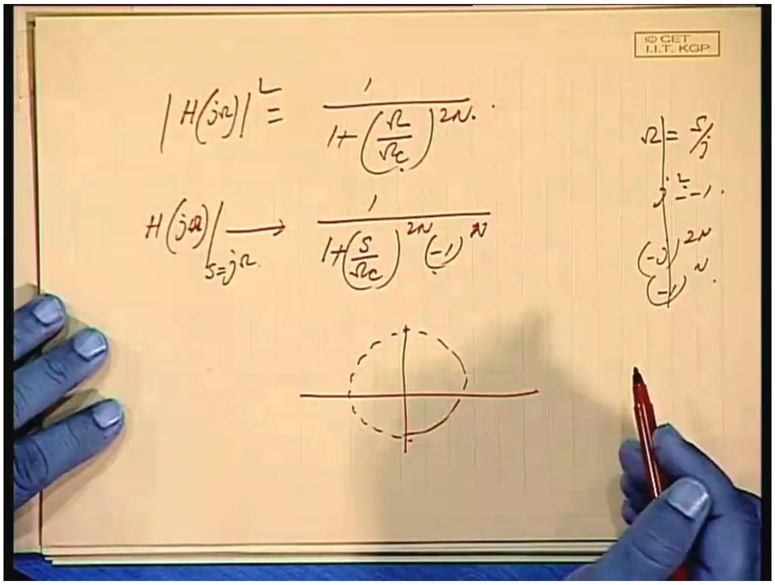

So, N has been solved, omega c has been solved. Once you know these two; H j omega sorry, j omega at s is equal to j omega. That means, omega you make a substitution of s by j, all right. So, this one is obtained from here. I had mentioned it earlier; you make a substitution of omega equal to s by j, so you get a function of s minus 1 to the power 2 N. j square equal to minus, so it is j to the power 2 N which is minus 1 to the power N, okay.

So, if you make a substitution like this then get hold of left of plane roots; this will have 2 N numbers of roots, N on the left hand side, N on the right hand side. So from there you calculate the left hand side roots and correspondingly generate the polynomial, so you get the filter function. Once you know the filter function; you may a substitution of that s is equal to 2 by T 1 minus z inverse by 1 plus z inverse, you get the discrete domain filter, it is very straight forward.

So, let us take one simple example; of say third order Butterworth filter all right that is N is equal to, suppose from here we get N approximately 2.8. So, we will go to the next higher value, N is equal to 3. So, N is equal to 3.

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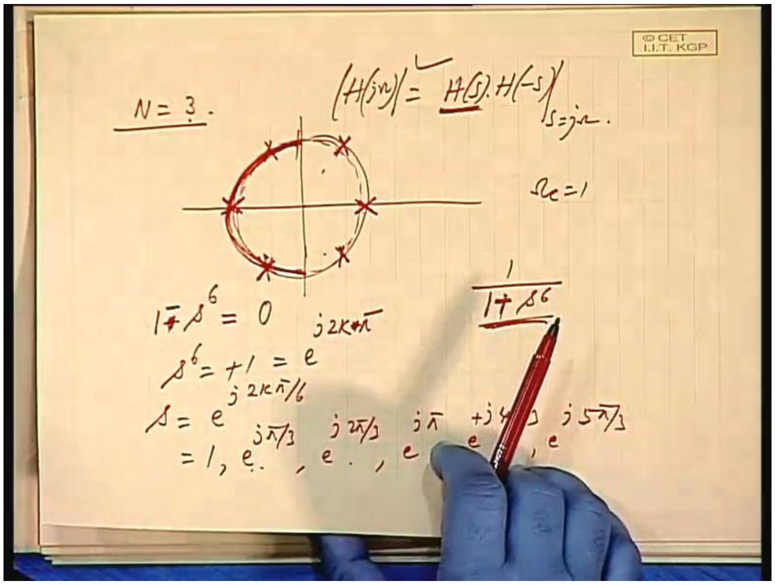

Suppose, N is equal to 3 we get then N is equal to 3 will give me; roots on the if I take the normalized frequency first, omega c let us for the time being take, let us take as 1 then 1 plus s to the power 6, 3, two into 3, 6 is equal to 0. This is the function you get, s to the power 6, 1 plus s to the power 6, okay. And you find out the roots of this; so equate this to 0, so s to the power 6 is minus 1 which means, is that all right? Should it be s to the power plus 1 plus s to the power 6 or minus?

It should be minus, is it not? N is equal to 3, so this should be minus. So for all odd orders, it should be 1 minus s to the power 6, if it is fifth order 1 minus s to the power 10 and so on. So, s to the power 6 is plus 1. So, e to the power j, 2 K plus 2 K into p i, okay. So, s will be e to the power j 2 K p i by 6 for different values of K. So, K equal to 0 that gives me 1. K equal to 1, 2 p i by 6 that is p i by 3; so e to the power j, p i by 3.

Next, e to the power j 2 p i by 3, e to the power j p i, e to the power j 4 p i by 3 sorry, e to the power j; next 5 p i by 3, okay? One, two, three, four, five, six so; the values are 1 then e to the power j p i by 3, here sixty degrees. e to the power j 2 p i by 3, here. e to the power j p i, here. e to the power j 4 p i by 3 and 5 p i by 3, is that all right. Out of these, we will find their symmetrically distributed, left of left of plane roots are just mirror images of this and you drop the right of ten roots, okay.

So, because it is H s into H minus s when I put s equal to j omega which has given me; H j omega magnitude square, is it not? So, I am trying to find out H s. So the correspond, since it is a stable stable system, it is a stable filter; so the roots are necessarily on the left of left of plane, so we take only these roots. So, this will give me correspondingly H s.

So, we have taken omega c to be equal to 1. So, you make a substitution now; after you have got hold of this, how much is the? I mean what is the function corresponding to this set of roots? One is s minus 1, minus minus 1. So, the polynomials will be H s I will write analogue, s minus p 1 which is at minus 1.

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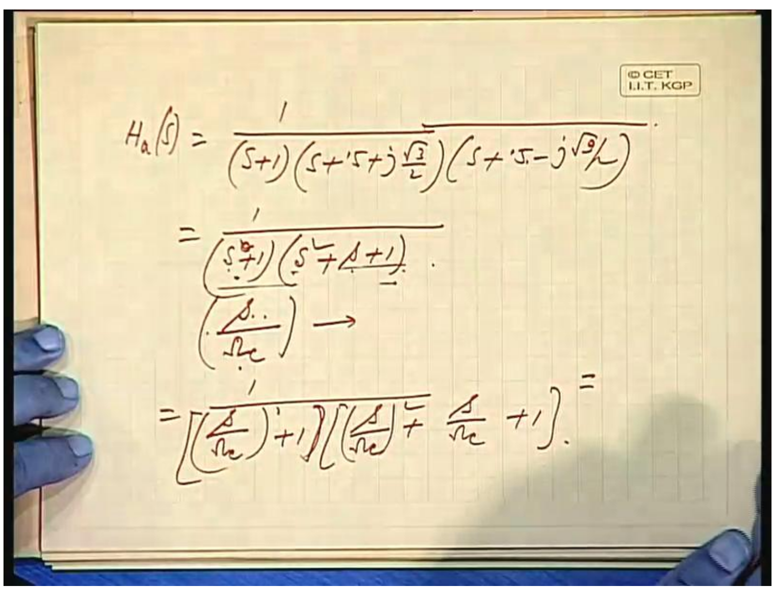

So, the corresponding factor will be s plus 1. This is minus 0.5 plus j 0.86 is root 3 by 2. Similarly minus 0.5, j minus j 0.866. So, s plus 0.5 plus j root 3 by 2 into s plus 0.5 minus j root 3 by 2; which means 1 by s plus sorry, s plus 1 into s square plus 0.5 into s plus 1. There is no scope for minus s, all right. Aurrawthervitch Criteria says polynomial should not have any negative coefficients, if it has to be stable. So, in a stable filter you always get plus terms.

So, this is my analogue filter. If I make a substitution of s by omega, omega c as new value of s; than this s I will replace by s by omega c. This was designed with omega c is equal to 1, now I de-normalise it, okay. So, put in place of s, s by omega c. So, it will be s by omega c plus 1 into s by omega c plus Whole Square plus s by omega c plus 1.

So, this will be the actual filter, analogue filter. And now you make a substitution, 2 by T 1 minus z inverse plus 1 plus z inverse, we will get the desire filter. Okay, we stop here for today and in the next class; we take up few analogue filter design problems. The next type of filter, that we shall be considering is shavishade filter. Thank you very much.