Digital Signal Processing
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Lecture - 12
Discrete Fourier Transform (DFT) (Contd.)

We shall continue our discussions, on discrete Fourier transforms; how to make use of it in the computation of some very complex sequences, say last time we have seen in one go you can find out the discrete Fourier transform of two sequences of N point length. Now, today we will take up a sequence of 2 N points, with the help of N point DFTs okay, this will reduce our burden.

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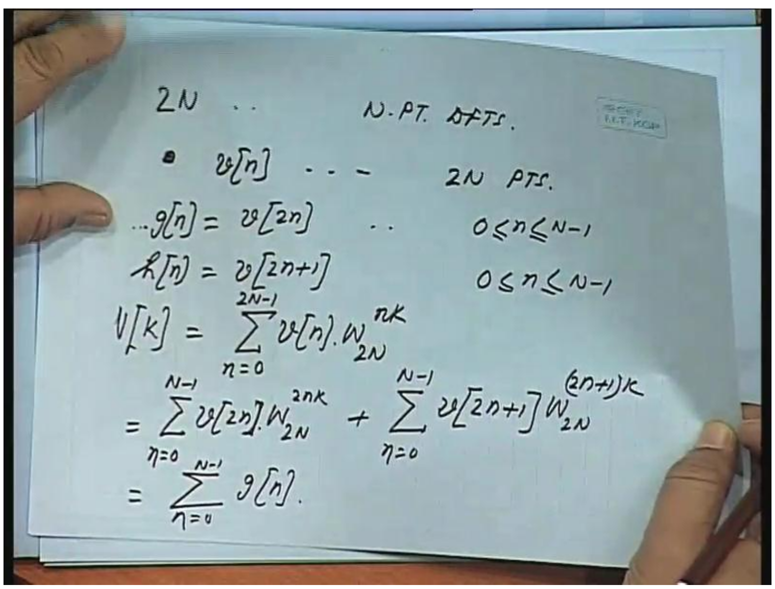

We will see later on, how the burden can be reduced by reducing the size of the sequence length. Now, if you have a sequence say v n of 2 N points, I break it up into two sequences all right; I break it up into two sequences, one is pick up the even numbers, the other one is pick up the odd numbers all right.

So, it will be sequence two sequences of length N. So, let us define a sequence g n which will be v 2 n okay, so 0 n N minus 1, okay and h n another sequence h n, v 2 n plus 1 okay so 0 n N minus 1. So, what will be V K equal to; V K is v n W 2 N n k, is it not? 2 N point DFT. So, I will multiply by W 2 N n k, small n varying from 0 to 2 N minus 1.

This can be broken up into, if I segregate the even and odd parts; W 2 N 2 n K, n is equal to 0 to N minus 1 plus, correct me if I am wrong, v 2 n plus 1, W 2 N 2 n plus 1 into Km is that all right? And this itself, we are defining as g n and h n.

So, I might as well write this as, summation over n g n. Now, 2 N and 2 n K, 2 will get cancelled. It is like this, suppose I have this entire circle of, say two p i angle divided into six parts; so, W 6 will be the elemental angular shift of sixty degrees, three sixty divided by six, six divisions all right.

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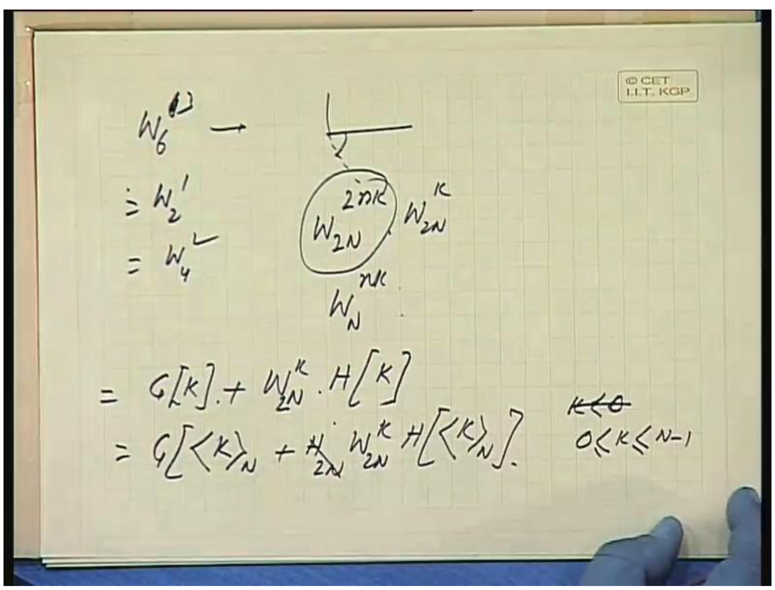

Now, if I take three out of six, it is as good as one out of two, it is like this. So, W 6 3 is sorry, 6 3 is same as W 2 1, is same as W 4 2 means two out of four is same as that. So, this can be written as W N n K plus, similarly this one, what will be this one? h n W 2 N, W 2 N this is 2 n plus 1 into K, 2 n plus 1 into K I can always write as; 2 n K into W 2 N K, I can separate it and this is once again W N n K, all right.

So, I will write W N n K and a separate multiplier, W 2 N K n is equal to 0 to N minus 1, okay. And what is this? We define this as, G K okay plus this is a multiplier W 2 N K and then H K okay. But now, you see this K should vary, actually I am going to determine K for all the 2 n points, all right is not. K should vary from 0 to 2 n, whereas I am having an N point DFT, all right.

So, after say suppose N is eight, so we are taking sixteen point DFTs. So after the first state, ninth, tenth etcetera should again be computed with some renumbering. So, I will write this as okay; H K to be more precise G K N all right, plus H 2 N sorry, W 2 N K H K N okay, and K varying from sorry 0 K N minus 1. So, there are N number of K s, all right, capital G 1, G 2, I cannot find for an from an eight point sequence, sixteen points DFT.

So, from eight point, eight point sequence I can get an eight point DFT, with the help of 2 eight point DFTs I am going to generate a 16 point DFT. So, as you cross the eighth point, then ninth point onward again this will be numbered but that will be taken care the sign will be taken care of by this okay; that we will see in details, how it works when we go to fast Fourier transform. That is the faster algorithm for DFTs, for the time being we are not going to discuss that. We are going to dwell with, dwell on something else; will go to FFT later on.

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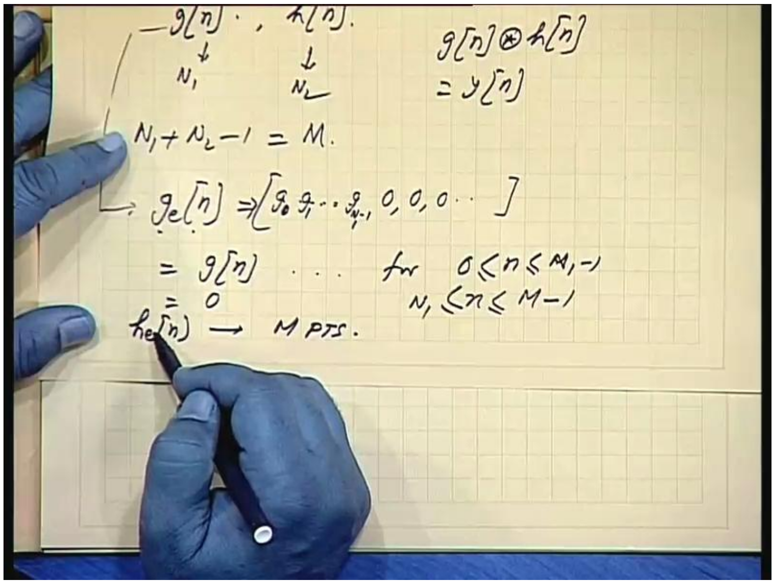

Now, we will take up linear convolution of two finite sequences with the help of DFT. Now, computation of DFT is much simpler all right. If you are having an N point sequence then you can always get an N point DFT all right; now unlike the DTFT, that is in the continuous domain of frequency where you have infinite number of frequencies all right.

So, here the computation will be restricted to only those many points. So, how do you get convolution of two sequences in the time domain, say g n and h n. Two sequences of lengths; say this one is of length N 1, this one is of length N 2, so what will be the product length? If I take g n a convolution product is say y n, so y n will be of length N 1 plus N 2 minus 1 okay; let us call it some M all right, so there is a very simple technique, g n you extend, you stretch it to some extended sequence g n which will be first N 1 points and then padded with zeros.

So, that the length, sorry say it is like g 1, g 0, g 1, g 2 etcetera g N minus N 1 minus 1 and then 0, 0, 0 up to N points. Total number of points is n, all right. So, I will define this g e n, as equal to g n for 0 n N M minus N 1 minus 1 okay. And equal to 0 for n M minus 1 to N 1, okay. Similarly, you stretch this h n to an equivalent, a very similar kind of a sequence h e n of M points, all right by padding with zeros.

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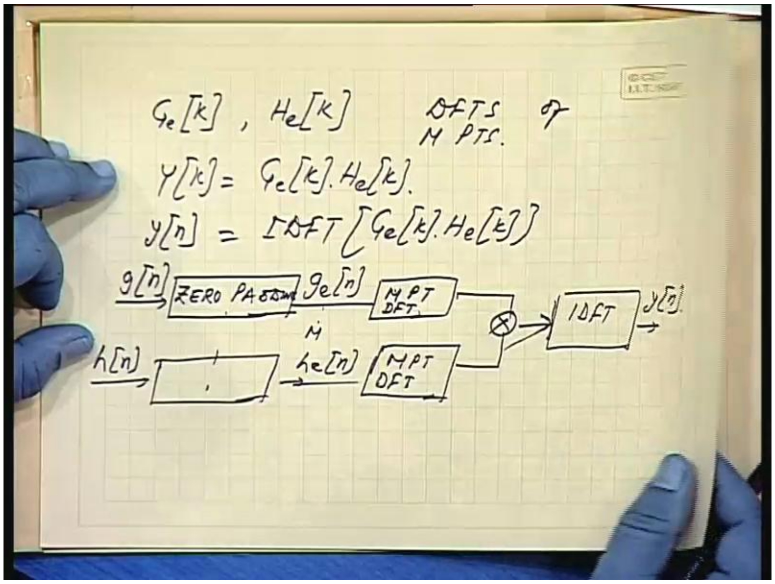

Now, for g e n, you can get DTFT, a DFT. So, let G e K with a DFT; corresponding to the similarly H e K with the DFTs of this M points, these are the two M point DFTs, okay. Now, we know the property if there are two periodic sequences or two sequences of lengths identical lengths, if you take their products if you take their products and then that is if you take the DFTs and then take the product. If you take the inverse we can get back the linear convolution; provided their padded with sufficient number of zeros, otherwise you have to go for circular convolution, is it not.

So, here Y K is G e K into H e K all right. So, y n will be IDFT, take now the inverse of this, G e K H e K is as simple as that. So, the procedure is like this. Now, all of you please try to develop a program, it is there also in the text books, some Matlab programs to help you. So, follow these steps and then. So, g n go for 0 padding, you get g e n then write a DFT program g e n padding up to M points, okay, so take M point DFT.

Similarly, a h n pad with 0, you get h e n and again you go for M point DFT. Then take the product, what you get is Y K all right. And then take the IDFT, you will get the sequence y n, it is very simple. Now there is a very practical situation, when you have a continuously flowing sequence of data that is an infinite sequence x n is coming, all right.

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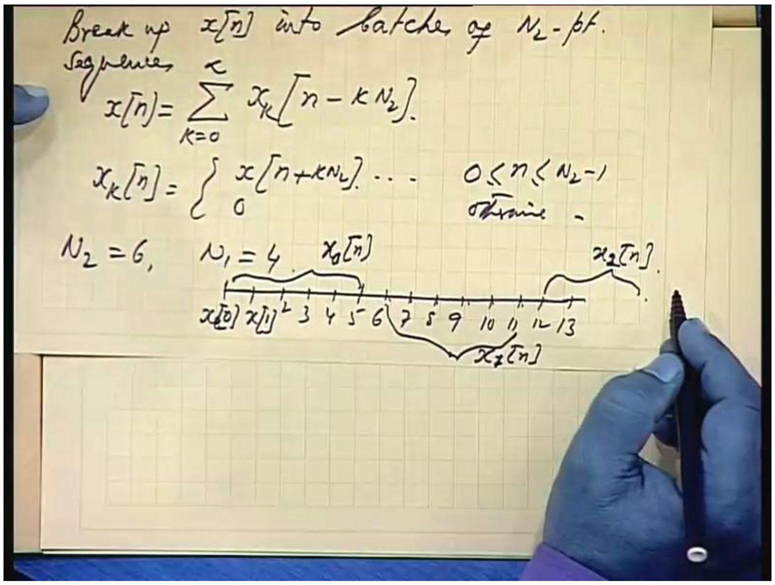

And it is convolved with a filter of finite length h n, okay. This filter is having a finite length, say N 1 all right, how do you compute the DFT? How do you find out, say the convolution all right, I want a convolution that means, I am basically filtering the data. Data sequence is x x n, I am filtering it with a filter h n, all right. What will be the filtered sequence? And that will also be a continuous sequence; unlike the previous one where we had two fixed lengths all right, here it is a one is an infinite length, other one is a fixed length, okay. How do you do that?

Now, here you break up the input data into batches all right, of some finite length. Let that we some N two break up x n into batches of say, N 2 point sequences, okay. So, you define x n as summation, there will be infinite number of batches, x I put K here, x K n minus K N 2, okay. So, every N 2 set will be forming a set number, a batch number x 1, x 2, x 3 and so on okay or in other words; I can write x K n is nothing but x n plus K N 2 for 0 n N 2 minus 1, okay and 0 otherwise, so it is like this.

Suppose, N 2 is equal to 6; we take an example and N 1 is equal to 4, N 2 is equal to 6 and N 1 is equal to 4, how does it look like? So, I will say take a sequence like this; so this is x 0, I will put x 0 okay, x I will write x 0, x 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 and so on. These are the points x 2, x 3 I am not writing x everywhere, these are all the sequence points of x.

So 0 to 5, this will form the first batch. This is X 1 n then from 6 to 1, 2, 3, 4, 5, 6, so 11, this will be X 2 n; index is always n, n varies from 0 to 6, 0 to 5. Then from 12 onward up to 17, it will be X 3 n and so on, okay. A first one sorry, thank you very much, this will be X 0, this one will be X 1, this one will be X 2 and so on, okay.

So if I convolve, if I convolve with h n, there will be a spill over because h n is having a sequence length of 4; so 4 plus 6, 10 minus 1 sequence length will be 9, that means including 0, it will go up to 8, okay. So, let us draw this a little more clearly, a little more clearly here, let me write here.

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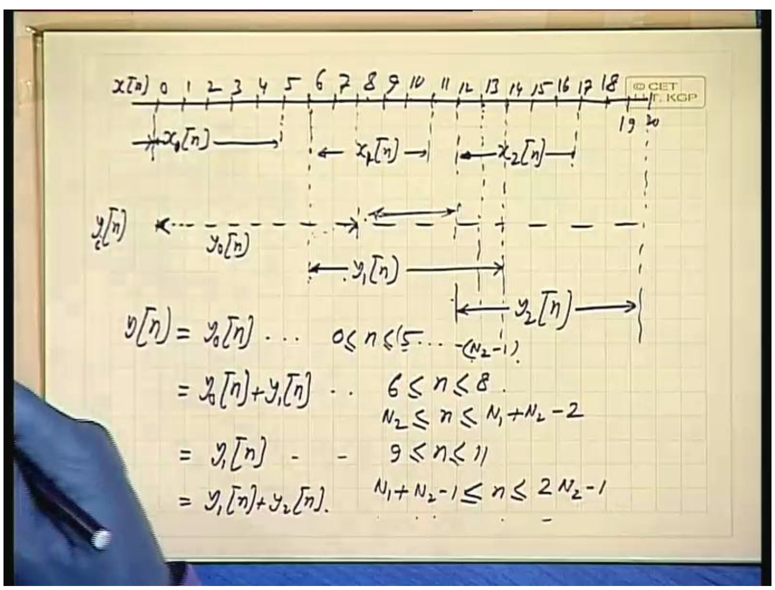

This is x n 0, let me mark it on this, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 and so on 16, 17, 18 and so on. This is x n. So, I have broken it up, up to this is x 1 n, x 0 n. Then from here to here, x 2 n then x 1 n then x 2 n up to 17, okay. So, if I write y n sequence, sorry y 0; I write y, I so this is y 0 which will go up to, sorry excuse me up to this y 0 n, okay where will y 1 start from? 6 y 1 where did it will go up to; included 13? Length is 9, length is 9 so 6 plus 9, it should be 14, is that so?

6, 7, one two three four five six seven eight nine, so it should include 14, okay. Again the third one starts from 12, okay. This is y 2 n goes up to 19, 20 is that all right. Not nineteen? Check that all of them are having a length of 9, so this is up to 8, 9 and then 8 to 15, 6 to 14 and then 12 to 20, okay. So, I can write the output y n. Now, you can see, y n will be equal to y 0 n; for 0 n 5 up to the first six points, it will be as it is, y n is this much.

Next up to this point, it is y 0 n plus y 1 n equal to y 0 n plus y 1 n for 6 n 8. So, what is it in general? It is up to N 2 minus 1; this 5 corresponds to N 2 minus 1, the sequence length okay of 6, is it not. Next one, what is this point? N 2. So, I will write the general version simultaneously and what is this; N 1 plus N 2 minus 2, is that so? Equal to, next? It will be only y 1 n for some time, only y 1 n from this point to this point all right, from nine to twelve.

So, 9 to 11 sure? So, what should be this; N 1 plus N 2 minus 1, this was ending at minus 2, so it it should be again from minus 1 to, how much is this? This is 2 into N 2 minus 1. So, you can write this sequence, again next; it will be y 1 n plus y 2 n, okay. Now, you can fill it up, you can write, keep on writing like this.

So here, this is add and safe; that is you take the six points, there are six points, so you have extended it up to so many points n points and you have also taken for h n, the four points so 6 plus 4 minus 1, so nine point length you have got. The DFTs, you have computed for this nine point sequences then take the product, then take the IDFT, you get a nine point sequence.

Out of that, nine point sequence you first consider only the first six that is N 2 minus one points, rest of it you save all right. 6, 7, 8 these three points you save then you compute for the second batch, again go by the same process that is padded with zeros and then take the DFTs. Two finite sequence length, just now we have discussed you pad with zeros; make them of equal length nine and then take the DFTs, take the product, take the IDFT, again you get a nine point sequence out of which the first three will have to be added with a previously saved value, all right. So, you add and save.

So, this these three points you add, rest of it you save. Again, you go for the third batch and so on, keep on doing it all right, so you get the output sequence all right. There is another method, so I will take it up later on; if we get time because there are many more important topics you can read of from the book, otherwise. Let us take some very typical examples for considering the DFTs.

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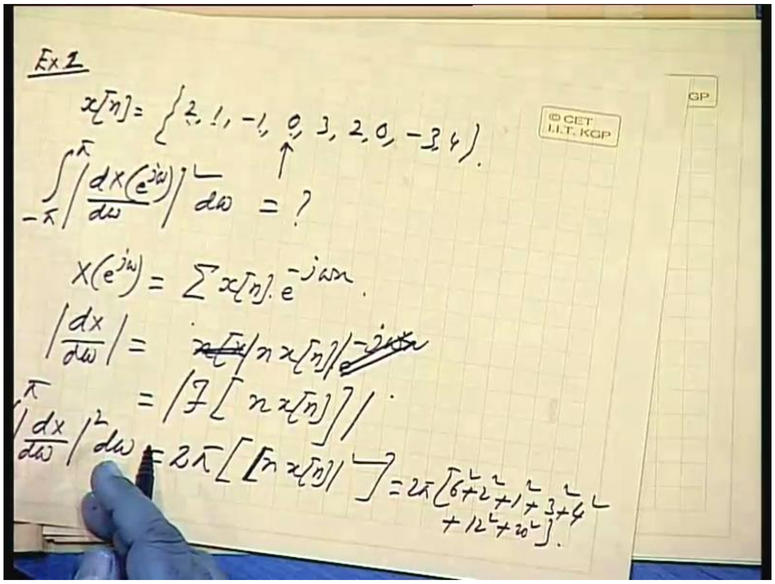

Just now you have seen, okay example one. We are given a sequence x n say, 2, 1, minus 1, 0, 3, 2, 0, minus 3, 4, this is a origin. You are asked to, calculate the value of say, this integral d x e to the power j omega by d omega square d omega. So, how much is this? We know X is equal to sigma x n e to the power minus j omega n, so if you take it derivative what do you get? n x n, n x n e to the power minus j omega n, anything else?

Minus j will be there, if I take its magnitude, okay so basically is it not the Fourier transforms of n x n magnitude? Yes, when I take a magnitude then it gets only this part; so this will vanish and j also will go okay, so I strike it off. There was j term, so j will also go, this is the magnitude basically; it is a Fourier transform of this, with a j term.

So, so how much is d x by d omega, square d omega minus p i to plus p i, you remember Parseval's theorem; discuss last time, energy content. This represents basically, energy content of the signal n x n, in the frequency domain it is this and in the time domain it will be this n x n square, okay. So, if you compute this, this is 0, minus 1, minus 2, minus 3, so minus 3 into 2 square so minus 6 square; so that becomes, is that all right?

This 2 p i into n x n square, so this is 1, 2, 3; so 6 square, it is n into x n whole thing square, mind you, plus 1 into 2, so 2 square plus 1 square plus 0 plus 3 square plus 2 into 2 into 2, 4 square plus 0, plus how much is this? 2, 3, 4, 4 into 3, 12 square plus 5 into 4, 20 squared whatever be that value, that will be the value of this, okay.

Let us take up another example, is this clear? We are just, simply applying Parseval's theorem.

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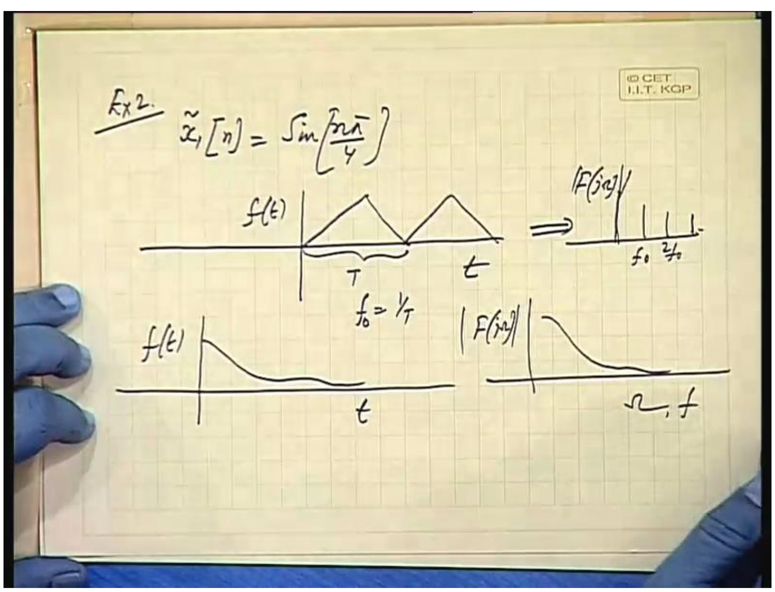

Suppose, this is a periodic sequence x 1 n which is given as sin n p i by 4 okay, all right. For a periodic sequence, what you get is a periodic transform, we call it discrete Fourier series, okay. The relationship is exactly identical; that is if you have a the I think I discussed it earlier, if you have a periodic function, say in the time domain, correspondingly in the frequency domain, you get lines all right. Corresponding to this frequency 1 by T, corresponding frequency is 1 by T, so say f 0, so it will be at f 0, twice f 0 and so on, as you have got in Fourier series.

This F j omega magnitude, if you are having a discrete function. Okay, if you are having a a periodic signal, if you are having an a periodic signal, here you get F j omega a continuous function. You can write either in terms of frequency or radian frequency, okay. You will get instead of lines, we will get a continuous function whatever be that shape.

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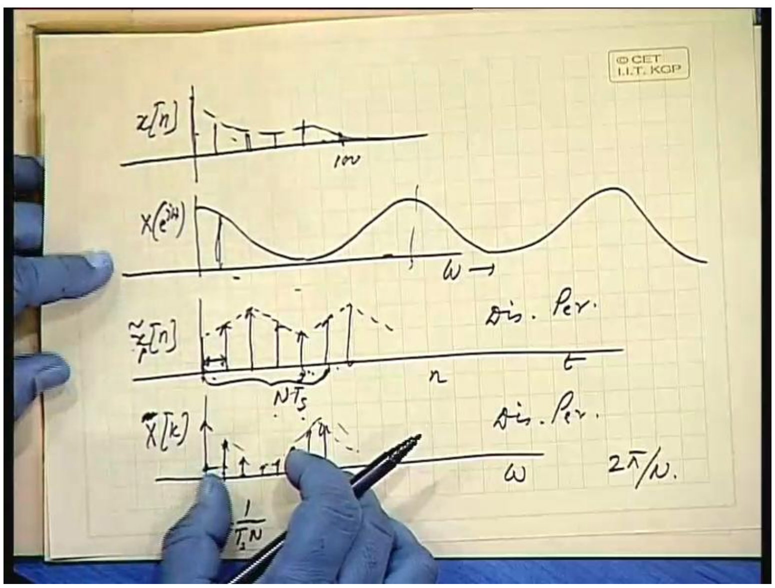

Now, if you discretize this, if you have a discrete function in the time domain, like this. If it is of say, finite length or is in if it is infinite length, if it is converging then in the frequency domain you get periodic functions. So, if one is discrete in one domain, then the in the other domain it is periodic; this is what we have seen, if it is discrete in the frequency domain, in the time domain, it is periodic in case of DTFT, this is x n, it is like this.

Now, next comes if you are having a periodic function in the time domain, periodic as well as discrete okay. Let, it be like this like this; suppose this is x n, I show it by x tilde n, that is a periodic function then in the frequency domain, again this will be periodic all right because it is discrete, it has to be periodic, because it is periodic then it has to be discrete. So, this will also be discrete and periodic, okay.

So, here it may be say like this; I do not know the exact nature, how it will vary but it also may be like this and so on, okay. So, if this is discrete and periodic, then this will also be discrete and periodic okay. In one period, in one period if there are N points in the time domain then this time length is N into T, is it not? T is a sampling time.

So, N numbers of points are covered. Say, in one period whatever will be that, period, okay. So, N T is a total time. So, what is a corresponding frequency here? Base frequency will be, this f 0 will correspond to 1 by N T okay and what is this frequency? This is a sampling time T s if I call it. So, what is the periodicity here corresponding to 1 by T s, all right. So, this was N T s and this is 1 by T s, so what is the ratio, N all right.

That means, there will be in the frequency domain, this is 0 to 2 p i between 0 and 2 p i there will be exactly n number of points accommodated and there will be again, at regular intervals because if you remember; the harmonics will come at regular intervals of f naught, is it not, so here also the distance is the same frequency f naught, that is a corresponding to the sampling frequency.

So, sampling frequency will be the base and it will be N multiples of sampling frequency which will form 2 p i, so the sampling frequency is again 2 p i by N. So, in N point sequence of a periodic function will be represented by N point sequence, again a periodic function in the frequency domain, okay. And these N points we find are exactly those points, as you have seen in the case of DFT; that means for describing any discrete system of finite length, say if there are 100 points I need only, I need not have the entire spectrum from minus infinity to plus infinity, I can select only hundred points between a span of 0 and 2, p i, okay.

So these points are 2 p i at an interval of 2 p i by N and here also, I find it is same as 2 p i by N and the relationship is exactly identical. So from the periodic Fourier transform, which we call it DFS discrete Fourier series; basically you just remove the hat it becomes DFT of N point finite sequence, which covers one period of the periodic function, is that all right.

So, for a periodic function x tilde n, some people write x p n, to be to be most stress on the periodicity. So, for a periodic function if I take just one period and take that a periodic sequence of finite length sequence, the corresponding DFT will be exactly same as X tilde K for one period, okay.

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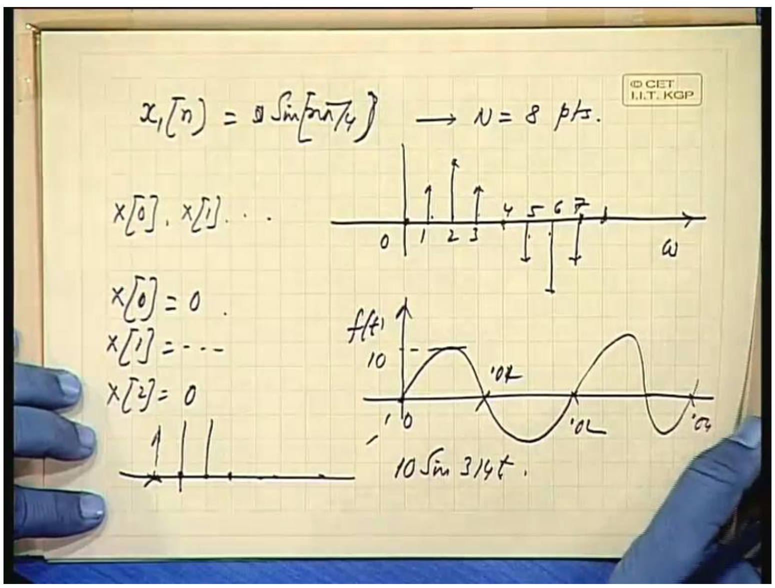

Now in our case, in this example x 1 n is a is one such period of a periodic function, sin n p i by 4, so how many points are there? How many point sequences? n p i by 4, no? Now, we are taking just one period of the periodic sequence, so what is that one period covering? How many times, how many sequence points are there? 8 p i by 4, is a basic unit.

So, there are basically 8 points. You can write those eight points, what will be the DFTs? So, N is equal to 0 is sin 0, n is equal to 1 is sin p i by 4, 1 by root 2, Okay. Then 1, then again 1 by root

2, then 0, then minus 1 by root 2, minus 1, minus 1 by root 2 and 0, okay. These are the eight points, this is 2 p i. So, 1, 2, 3, 4, 5, 6, 7, 8 all right, 1, 2, 3, 4, 5, 6, no that is 7 including 0, there are eight points.

So, what will be X K? Could someone tell me, what will be X 0, X 1? What would be the values? One way of doing it is, compute it. You need to compute this. Okay, X 0 is how much? You can see, its average value o, is not? x 0 corresponds to basically some of these, so that is equal to 0, okay.

Now, could someone tell me what would be X 1? What will be X 2 without computing; can somebody guess, without computing? Okay, before we go to this problem, this a periodic function all right, can you write the Fourier series for this Fourier series? Can you show the Fourier spectrum, all of you please try what will be the Fourier, Fourier spectrum of this?

Suppose, this is corresponding to fifty hertz in the time domain, sorry 0 2, 0 1, 0 4 and so on. This is 0; this is f t, so f t is equal to? Say this magnitude is 10, then what is it? What is this? 10 sin 314 t, corresponding to 50 hertz, 2 p i f t, okay. So, what are the, what will be the frequency spectrum? Frequency spectrum, no D C value, only at 50 hertz there is spike that is all, okay.

If you take the complex frequency spectrum, it will be plus 50 and minus 50 j, is it not, if you take complex Fourier series. So, for a pure sinusoid there is only one frequency, is not? So, if the periodic sequence happens to be a part of only a single frequency then what are the elements that will be present? So, this will be 0, this will be 0, except X 1, only the first harmonic; you call it first harmonic or fundamental, only that will be present, is it not.

So, this will be finite others are all 0, that will be all right, what about this corresponding to this? So, what are the properties of DFT? There is symmetry from the other end, so there will be another frequency corresponding to X 7. There are eight points, I write 0, 1, 2, 3, 4, 5, 6, 7 all right you wrap it, 0 is the centre all right.

So, this is 1, this 1 will be 7. So, 1 and 7 will be identical a complex conjugate, all right. Then 2 and 6, 3 and 5 and 4 will be exactly opposite to that; X 0 is a real quantity, 4 will also be a real quantity. In an eight point sequence, X 4 will also be, always 0, that you can see for a real sequence for a real sequence; why because W N 4 is either minus 1, and if you take its power it will be minus 1, plus 1, minus 1, plus 1.

So, basically the sequence points will be added either with plus 1 signs or minus 1 signs; alternately there will be minus 1, plus 1, minus 1 and add them together, that will be always real. So, X 0 and X 4 both will be real okay. And in this case, only X 1 and X 7 will be present and you can compute those, okay.

So, if you are given sequences like this, by inspection if you can make out; there is only one frequency existing here then you can write down which frequency it is.

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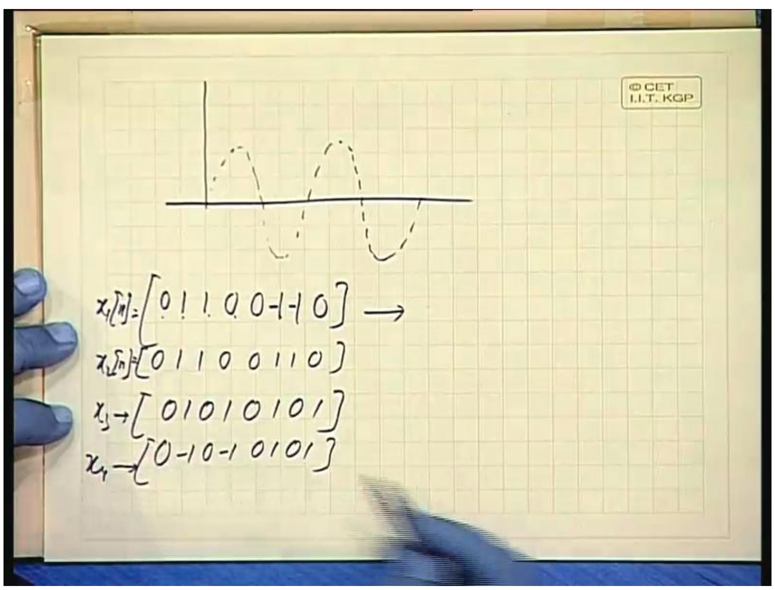

For example, if I give a sequence, if I give a sequence of say 0, 1, 1, 0 okay, 0, 1, one two three four five six seven, 0 apparently it is difficult to make out, what it is okay, okay. You try this at home, take it as an exercise what are the components, X K. Suppose, x n is this eight point sequence, what will be x n, sorry I forgot to put minus signs, sorry minus signs.

And 0, 1, 1, 0, 0, 1, 1, 0 again another sequence, this is x 1 n; this is x 2 n, okay. Then 0, 1, 0, 1, 0, 1, 0, 1, and 0 minus 1, 0 minus 1, 0 plus 1, 0 plus 1, take this is x 3, this is x 4, these are the four sequences, you try to find out the DFTs of these four sequences. This is symmetry; some type of symmetry from where, you can find out what are the harmonics and again, I mean from there you can compute the corresponding harmonics, okay.

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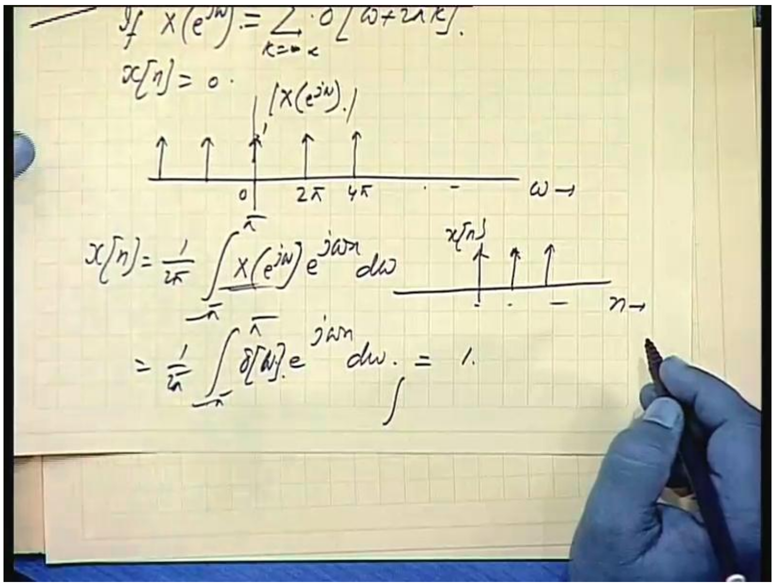

Next, suppose we will take up another example. If, sigma delta omega plus 2 p i K, what will be x n? It is a very typical type of function, what is this, for all the values of K 0 to minus infinity to plus infinity, okay? So, basically it is a periodic function at intervals of 2 p i, this is in omega. You are given this is magnitude one, all right, what will be x n?

Now, how do you compute x n? x n is 1 by 2 p i, 1 by 2 p i, you cover X e to the power j omega, e to the power j omega n d omega from minus p i to plus p i, okay. So, in one period this is coming only once. So, how much is it? It will be 1, e to the power j omega n, this becomes 1? Yes. See, what does it mean? X n is also sequences like this, check-up whether the factor 2 p i comes or it gets cancelled.

Now, what is x, sorry not 1, it is 1 into this I should put just delta omega over one period; which is existing only at this point all right, which is existing only at this point, it is 0 here. So, how much is it? Integrate and then find out. How do you get the integration of f t delta t; say minus any A to B, how much is this? Could someone tell me?

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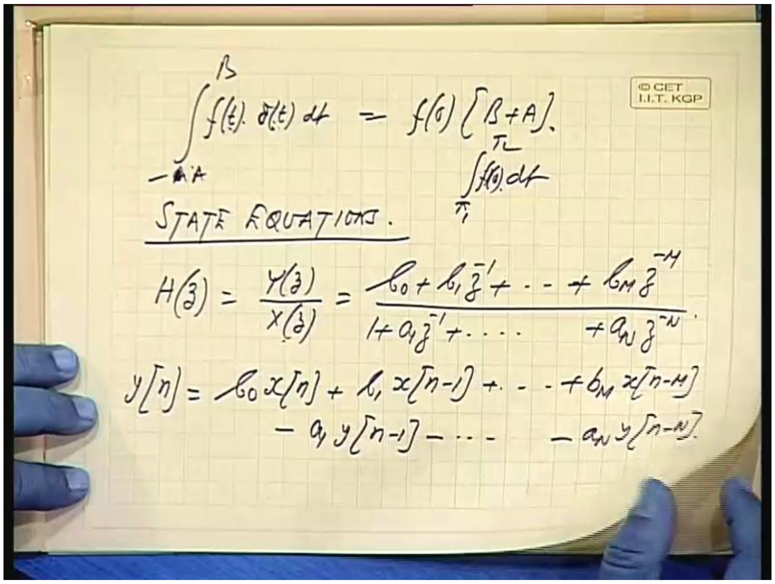

If I multiply a function by delta function, what is it? It is the magnitude of that function at T equal to 0, because delta t exists only at T equal to 0, it is a window through which you are trying to see a function and you can see only at delta t; means at T equal to 0, it is multiplied by an impulse. So, it is magnitude of the impulse of strength f naught, because it is visible only at t equal to 0, so it will be f naught, is it not?

Precisely, that is what we have done here. So, it will be value of this function by which you are multiplying this; at omega equal to 0, so that is equal to 1, 1 and 1 integrated from minus p i to plus p i, that will be 2 p I, so 2 p i will get cancelled. So, it is just 1. So it is, x n is a sequence of ones, is that all right? Now, you come to state equations… which one? This one? This integration? After this f t into; no of obviously, in this case it was minus p i to plus p i and that was getting cancelled okay, f naught into B plus A okay fine, is that all right.

What I wanted to stress on is, except for the limit basically, what you are integrating is 1 into d t and then put the limit set you want to t 2, so it will be t 2 minus t 1, all right. It amounts to that of course f naught I call it f naught, f naught is a constant you can take it out. So now, we will take up state equation.

Suppose, we are given a system whose transfer function is; we are once again going back to Z domain representation, Y z by X z, okay is equal to b naught plus b 1, z inverse etcetera plus b n z to the power minus M divided by 1 plus a 1 z inverse and so on plus a N z to the power minus N, sorry excuse me.

So, one may write this in the discrete domain as b 0, I multiplied by x cross multiplication and then transfer the delayed quantities of the output y on this side; so I get b 0, x n plus b 1 x n minus 1 and so on, plus b M x n minus M, minus because these are plus so transferring on this side, it will become minus a 1 y n minus 1 and so on, minus a N y n minus capital N, okay. Which means, now this is the equation it it is a general arma model; we discussed about arma model, is it not? General auto regressive and move moving average.

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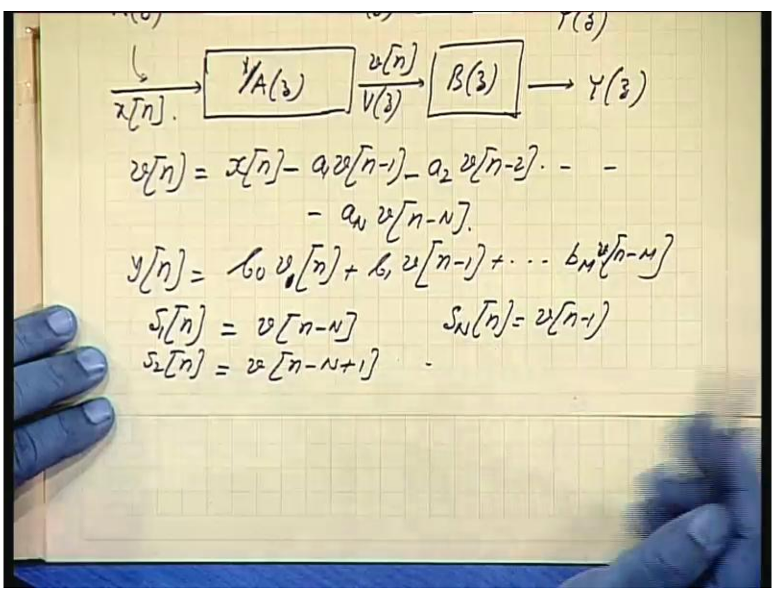

So, it is like this, one may take x n or in the Z domain; I can write X z and in the time domain it is b n or in the Z domain it is B z. Here you get, w n I, I can write w n which is convolution of b n with x n or W z; one may write which means, X z into B z in the Z domain it will be multiplied, in the time domain it will be convolved, okay.

I can segregate these two operations of A z by B z or B z by A z, as B z multiplied by 1 by A z, okay. So, this can be written as, A z and correspondingly it is a n in the time domain and it should be 1 by A z. Suppose, correspondingly it is some coefficients of n; so on this side we will get y n or Y z, one might as well reverse the order of these two hypothetical filters. I can consider them as two different filters, put together in cascade which is giving me B z by A z, all right.

So, I might as well interchange their positions and one may write, obviously the intermediate output will be different. Once again the same inputs, so I will write A z 1 by A z here and I get say v n as the output and B z here and then Y z is the corresponding output or Y n, okay. So, how do we write Y n, okay? One may write v n, let us see the second one all right. V n is equal to x n, if you write this 1 by A z if you expand, all right.

So, x n minus a 1 times, v n minus 1, minus a 2 times v n minus 2 and so on. There is nothing like a 0, because in the denominator we have already normalized with the coefficient 1 that has been incorporated in the numerator, okay. So, minus a n v n minus N okay. Then Y z or Y n, you may write in terms of v n, okay. It will be b 0 into v 1 n v 0 n? v 0 n plus b 1 into v 1, okay; basically it is v v n minus 1 and so on, okay. B M v n minus M, okay. We will write S 1 n, we choose some variables S 1 n as v n minus N. S 2 n as v, one step earlier n minus N plus 1 and so on.

S N n as v n minus 1, we start from the other end; this is S 1 n, this is S 2 n and so on okay, S 3 n, S 4 n like that and end appear as S N n, okay. Let us define after up to this, if I add one step with this that means; if I go to the future, if I go the future so S 1 n plus 1 will be V n minus N plus 1 which is same as S 2 n.

So, in state space, in control systems and networks you have seen; I can write x 1 dot is equal to x 2, x 2 dot is equal to x 3 and so on. I can define the derivatives of each state in terms of the other state, all right. So, exactly similar operations we will take up. So, we will take it up in the next class, we will complete this in the next class. Thank you very much.