Electrical Engineering Optimal Control

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Lecture - 19

Solution of LP Problems with Two Phase Method

So, last class we have considered the solve to solve a LP problem using in tabular form. We have consider an example, and that example we could not complete at that, at that day.

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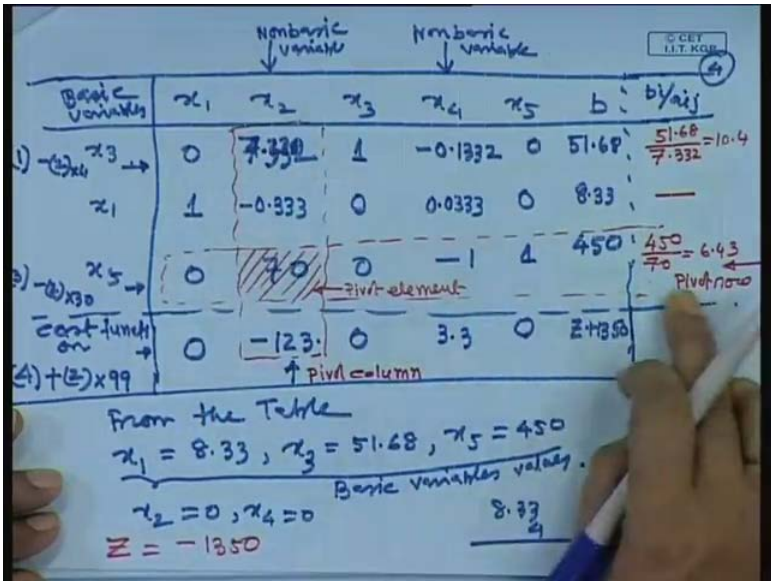

So, let us see from the tabular, table 2. And table 2, 70 we have considered in pivot element. Again, how pivot element is selected? First will see the cost function coefficient, in the cost function coefficient, which is the most negative coefficient, that will be treated as pivot column. So pivot column is selected, in other words x 2 is the our non-basic variable, which will entered as a basic variable. That is why x 2 is called entering basic variables.

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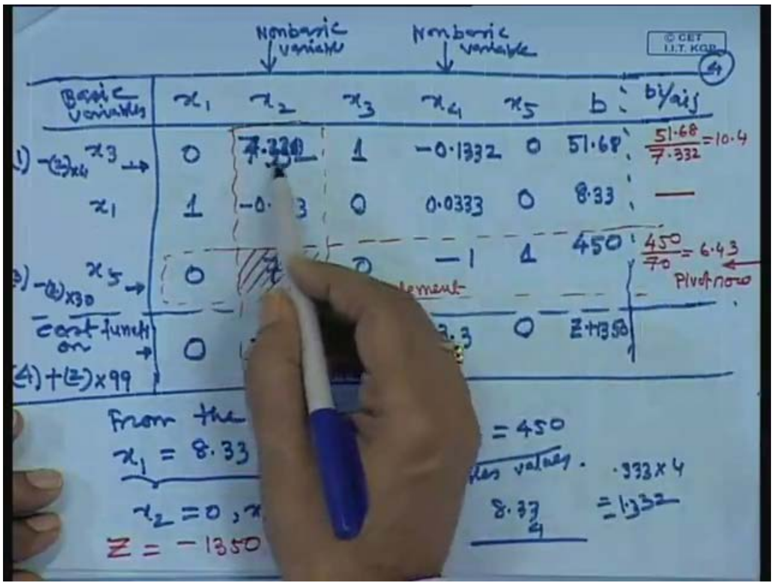

And how the pivot row is selected? Again, so once pivot column is selected, find out the ratio of b divided by x 2, with positive sign of x 2 coefficient. So in this situation, this 51 by 51.68 divided by 7.32 ratio, which is 10.4 and another is your minus. This minus sign will not help us to select the pivot element, if you select the pivot element minus of the 8.33 divided by minus 3, this will not help us to, what is called reduce the function value. If you consider x 1 is the living basic variable, it will not have.

So, minus sign will be, not will not be considered in order to select the pivot element. Sorry, pivot row. So, this is the only that, this is the pivot row we got it. Again, once you were got the pivot, out of this 2 ratio, minimum ratio is that one, 6.33 and that will be treated as a your pivot row. So, pivot row and pivot column will decide the pivot element. So 70 is the pivot element, so that equation 3 is to be divided by 70, in order to make the coefficient of x 2 equal to 1. So, other copies, other equations, equation number 1 and equation number 2, this coefficient must be 0. Coefficient x associated x 2 and x 2 in equation number 1 and 2 must be 0. And similarly, in cost function the coefficient associated with x 2 must be 0. These things, we can do it by the elementary row operations.

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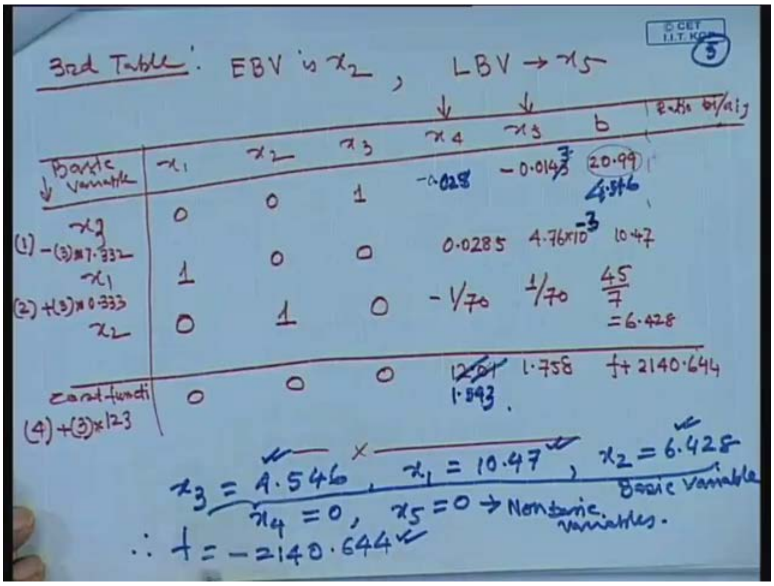

Once you normalise this one by 70, will get it this equation, eliminate x 2 coefficient, in order to eliminate that x 2 coefficient from equation number 1, 2 and 4, the type I have multiplied by equation number 3, with coefficient 7.332. And at separate it from the equation number 1. So, I have written 1 minus 3 multiplied by 7.332. If you subtract, that will come 0, 0, 1 minus 0.28 and minus 0147 and this last class we have written, 20.9 point. This is mistake, we have to write 4.546.

Similarly, this second equation, x 2 we can remove it by multiplying the equation 3. Equation 3 multiplying by this equation, you to after normalising this equation, you have to multiply it by 0.337 and added equation number 2, so 2 plus 3 again, then 0.337 this. So, add it, if you add it you will get it this equation. Last class, we written 10 to the bar minus 6, actually it will 10 to the bar minus 3.

So similarly, x 4 equation cost function equation, the x 2, we can eliminate by multiplying the equation, this equation by 123 and add with the equation number 4, this equation number 4. When you are taking equation number 3, correspond to this equation, okay. So this, you are doing. So after doing this one, look at the last row of the cost function, look last row of the cost function, this cost function, all coefficients are positive.

That means further we cannot reduce the cost function value, by changing one of the basic variable is a non-basic variable and one of the basic, non-basic variable is a basic variable. So, will stop our iteration of here and directly you can see, what is the cost? What is the function value, basic variable function value? So x 1 into 0 plus z 2 into 0 plus x 3 into 1 plus x 4 into this quantity, but x 4 is a non-basic variable. This value is 0, then x 5 into this.

Since x 5 is a non-basic variable, this is 0. x 5 value is 0. So, x 1 directly will get it 4.56. Similarly, x 1 we are getting directly is 10.47. Then x 2, non-basic variable value is x 2 is directly 4.62. And what is the cost function value? x 1 multiplied by 0 0 plus x 2 multiple with 0 0 plus x 3 multiplied by 0 0. x 4 is a non-basic variable value is 0. 0 multiplied by this non 0 quantity 0, this multiplied by 0 this equal to this quantity. So, this quantities will be 0, means f is equal to this one. So, our objective function value, minimum value of the function, objective function is that one.

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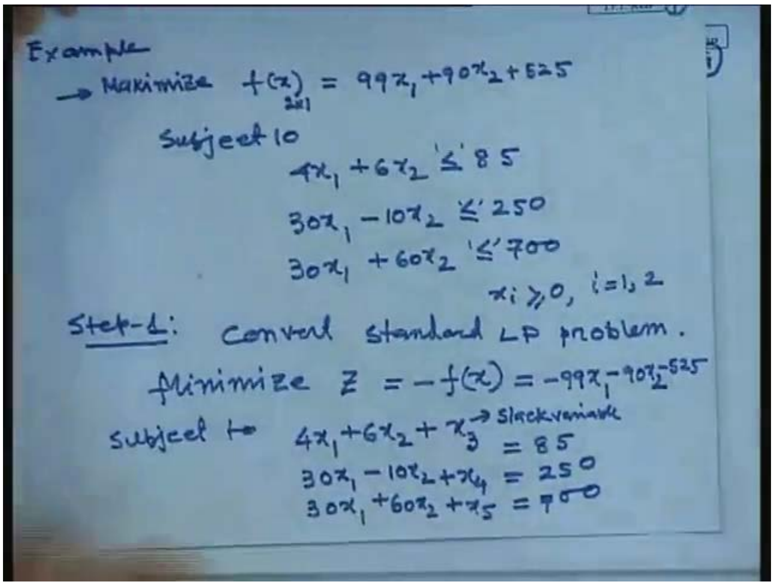

If you see this one and what we are supposed to find out? You say, our problem was maximize. Our problem, if you see the maximize f of x. Again, that minimize value of f of x, we got it. So, maximum value, maximize values of that one will be, f of x will be, that is not here. Last page, this one, the maximum value of f will be, what is this? f max value will be minus of minus 2 1 0.

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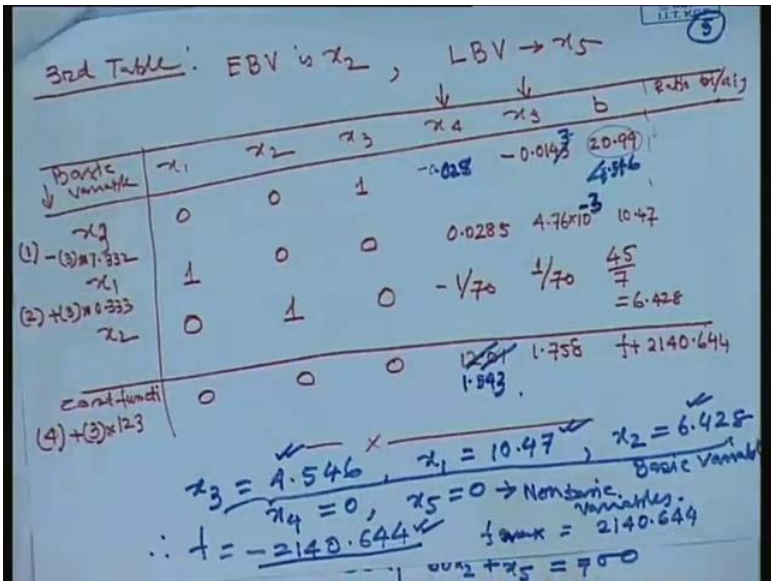

So, it will be 2140.644. That is our maximum value of that one will get it and what is this x 1 x 2 value? x 1 is 10.4, x 2 is 6.48. These are 2 values they are design variables, x 1 and x 2. So, these values are the, design variables are that one. These the design variables, this and this. And okay, you see this is actually, this be z not f. So, z is equal to this one and that f of x maximum value is that quantity. So, this is the way we solve the LP problem, using a tabular form. Now let us say, what is 2 phase method? The solving LPA, linear programming problem using 2 phase method ok. 2 phase, 2 phase simplex method for solution of LP problem.

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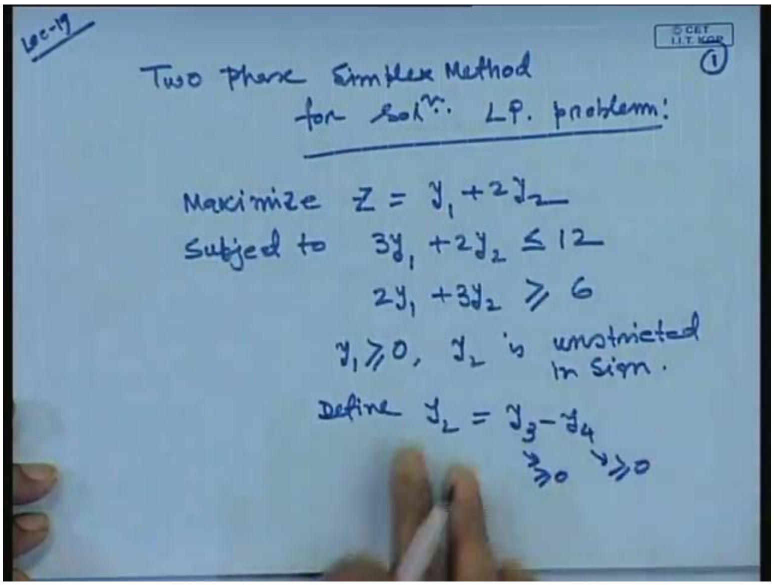

So, let us see what is this? Our problem is maximise z is equal to y 1 plus twice y 2, subject to 3 y 1 plus 2 y 2 is less 3 equal 12 and then 2 y 1 plus 3 y 2 is greater than equal to 6. Now you say, one inequality is less than equal to thing, another equality is greater than equal to 6. So, this type of inequality when it is there, greater than equal to some constant quantity, then we have to solve this problem by 2 phase simplex method. What is this? And it is also mentioned that y 1 is greater than equal to 0 and y 2 is unrestricted in sign. That means, this value can be negative, positive, anything 0 also.

So now, this because when we will solve this problem, using the standard LP method ok, by simplex method, you have to convert into standard LP problem. Then how you solve? Because y must be bigger than equal to 0. So, y we can really find, define y 2 as y 3 minus y 4 and y 3 is greater than equal to 0, y 4 is also greater than equal to 0. Now, if you replace y 3 by 2 new variable, y 3, y 2 by 2 new variable y 3 minus y 4 and both are greater than equal to 0. So, this quantity depending upon the value of y 4 or y 3, this will y 2 to may be positive, negative and 0. So, this problem we can really defined into a, what is called new variable form. If you define, x 1, y 1 is equal to x 1, x 1, I have given in by y 1.

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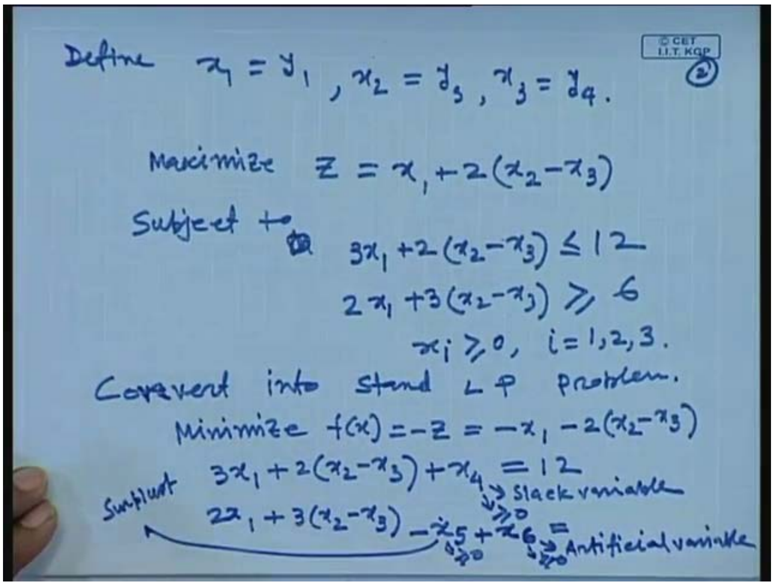

Then x 2, identify y 3 and x 4 sorry, x 3, x 2 as defined by y 4. So, our original problem, the original problem given this one, we can re write into this form. Maximise z, y 1plus y 2. y 1 plus 2 y 2. y 1 value is what? x 1 plus twice. y 2 value is what? If you say, y 2 value y 3 minus y 4. y 3 value we have considered x 2 and y 4 considered x 3, that is we defined here. So, this is our objective, in terms of new variable x 1, x 2, x 3 subject to f of x. That is maximise subject to, what is this? 3 y 1, 3 y 1 y 1. y 1 value I can write x 1. 2 y 2 means x 2 minus x 3 is less than 0. So, I will write 3 x 1 plus 2 x 2 minus x 3 is less than equal to 12. Then twice the second equation, inequality, twice x 1, twice x 1 plus 3, x 2 minus x 3 is greater than equal to 0.

2 x 1 plus 3 x 2 minus x 3 is greater than equal to 6. Now, we are writing x 1 is greater than equal 0, for i is equal to 1, 2, 3. So what is now? Problem is that, you convert into, then our solution starts from here. Convert into standard LP problem and standard LP problem you know, this is minimise f of x. What is f of x? Minus z. What is this one? Minus x 1, minus 2 x 2 minus x 3. And we have to convert all inequality conditions in a equality conditions with right hand side is a positive constant quantity.

So let us say, this one, this one we will write 3 x 1 plus 2 x 2 minus x 3 and this is less than 12. That means, something we have to add with this one. Let us called that new value x 4 is equal to 12 and right hand side you see positive quantity, is that we have to make it right inside, that equality condition, equality equation only to quantity, it is already positive. So, x 4 is thus our slack variables. The way we do it, we do the what is called standard LP problem, by adding slack variables and that slack variables value, greater than equal to 0. And second equation you see, second equation 2 x 1 plus 3 x 2 minus x 3 and that is greater than equal to, something we have to subtract, some variable x 5.

Suppose, if you make it x 5 is equal to that 6, then you will see of this, when you solve this equation x 1 x 3, this when it is 0, x 1 x 2 is 0, that values are coming minus. So, that creates a problem while solving the LP problem. So in addition to this one, I am adding with another new variable. That is called artificial variables, this is called artificial variable. This is called artificial variable and that artificial variable, well when we is greater than equal to 0. And this is also, it is call surplus variable. It is this variable x 5 with minus sign, it is call surplus variable. So, this is call surplus.

So, when this type of inequalities is there, we have a 2 variables we have to introduce. One is surplus variable another is artificial variable. If you just introduce surplus variable, you will see that, this x 5 variable value will after solving you may get it negative quantity and this is not , our constant is all are greater equal to 0. So, this will violate our solution, so in order to avoid that, one we used another artificial variable, that is called, the what is called x 6. Now, it is converted into our standard LP problems. Once it is convert into standards LP problem, we can solve this either in matrix form or we can solve by using tabular form. Let us say that, how it is solved by, this is solved by using the, what is called tabular form.

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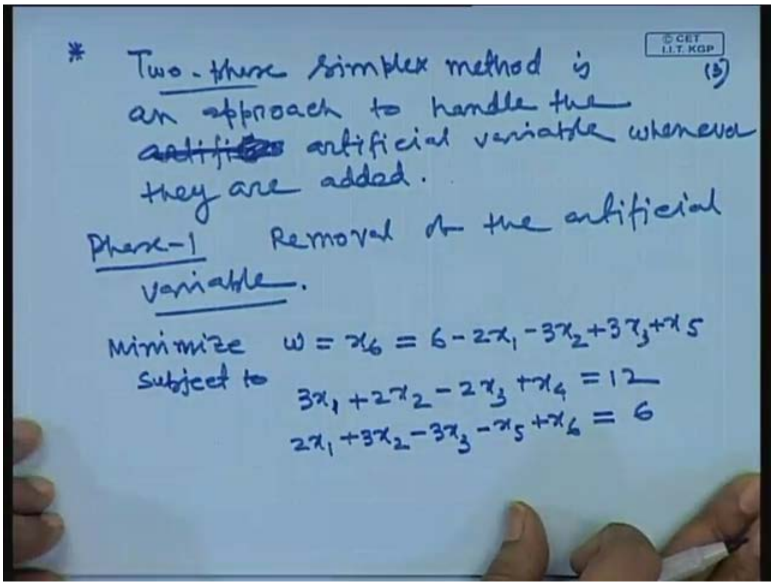

So, 2 phase method, 2 phase simplex method you can write, is an approach. 2 phase simplex method is an approach to handle the artificial variable, to handle the artificial variable, when they are added to the system equations. So, 2 phase simplex method is an approach, to handle the artificial variable, are t variable, whenever they are added that is.

Now, first phase that is called 2 phase, first phase will eliminate. First phase will eliminate artificial variable from the problem, that means how the artificial variable is going to be formed. So, the all artificial variable, all artificial variable is considered as a artificial function, artificial cost functions. Suppose were more than 1 artificial variables are there, we have to add all our artificial variable together and considered as a artificial objective function or cost function and that cost function you have to minimise.

In other words, that cost function value, artificial cost function value must be 0. Some of the all artificial variables must be 0. So, one phase you have to eliminate w, from the mathematical expressions. So let us say, how one can do this one. Phase 1, removal of artificial variable, so this, okay. So, artificial variable .So then how to remove this one? Suppose if you have a equation, this type of equation more than 1, let us call 2.

So, how many artificial variable are there? 2 artificial variable, in each equation 1 artificial is there and 2 equation, 2 artificial variable are there. This 2 artificial variable, you add together and treat as a objective function, in addition to the original objective function. So, we have to do simultaneously minimise, we have to minimise 2 objective function. First, you minimise the artificial objective function, once you minimise this one, then you start to minimise in phase 2, you minimise the what is call objective function of the original problems.

So, that is our steps to be followed. So let us see, in this example. What is our artificial function, where our artificial variable x 6. So, that x 6, the minimise w and w, I have consider x 6 and what is this x 6 expression? See from this one, x 6 is equal to, I can write 6 minus 2 x minus 3 into x minus 2 x plus x 5. So, our 6 minus 2 x, minus 3 x to plus 3 x 3. If you bracket you open the bracket, then plus x 5 is our artificial objective function. This is artificial objective function. Minimise this one, subject what? Subject to you see, this 2 constants, equality constants.

So, this equality constant, with 3 x 1, 3 x 1, 3 x 1 plus 2 x 2 minus 3 x 3 plus x 4. 3 x 1 plus 2 x 2 minus 2 x 3 plus x 4 is equal to 12. Then, second equation you see this equation, 2 x 1 plus 3 x 2 minus 3 x 3. 2 x 1 plus 3 x 2 minus 3 x 3 minus x 5 plus x 6 is equal to 6, okay. So, our problem is minimise this one, first phase. Minimises this one, subject to this one. Again, so if you write it this one in tabular form, this will look like this way. So, table 1, basic variables is a basic variables in directions.

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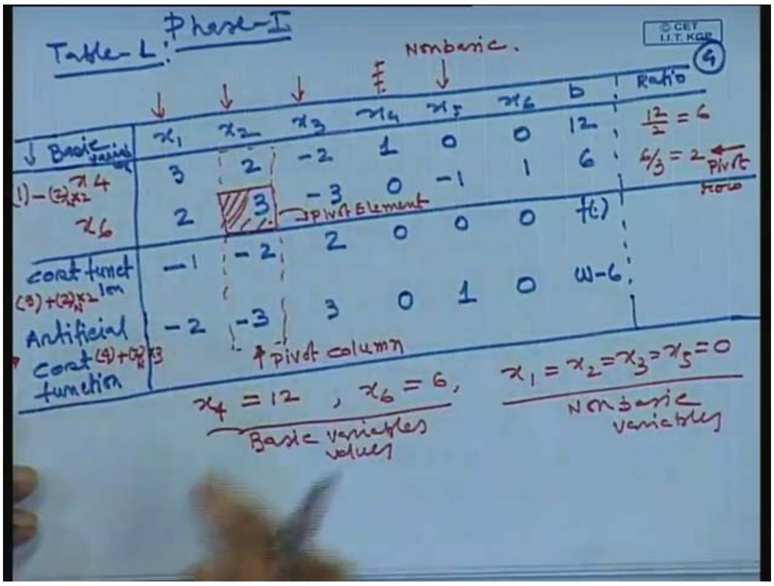

Then how many variables of their, if you see in this equation how many variables? x 1, x 2, dot, dot, x 6. So, 6 variables of their, so you write it x 1, x 2, x 3, dot, x 4, x 5, x 6, then b, then ratio. So, I will write first, this equality constants, 3 into x 1. So, it will come x 3. Next is your, see this one. Next is your, 2 into x 2 minus 2 into x 3 plus x 1. So, under 2, under x 2, 2 will be there with under x 3 column minus 2 is there under x 4 column 1 is there, x 5 x 6, more is the coefficients associated with x 5, x 6 is 0. So, I can write it that one, 2, minus 2, 1, 0,0. That right hand side is equal to 12. So, I will write this is equal to 12.

Similarly, the second equation, under x 1 2, I will write under x 2 3, under x 3 minus 3, under x 4 minus 1 x 5 minus 1 no x 4 is there, under x 4 is 0, under x 6 coefficient is 1 1. So, I can write it 2, 3 minus 3, 0 minus 1, 1, 6. So, we have 2 objective function, we have to give in phase 1. This is the phase 1. You can say this is the phase 1, phase 1 job is what, it will optimise. The optimize means, this will minimise the function value w, means of artificial objective function value, we have to minimise. In the other words, you have to make it 0 this one, by minimising this optimization problems, Lp problems, linear optimization problem. So, our first is cost function or non-normal original cost function.

Next is our artificial cost function. So, let us see our original cost function is what. If you see the our original cost function, that minimise f of x that minus x 1 minus x 2, bracket if you open plus x 3. So, under x 1 is minus 1, the original cost function. Under x 2, is minus 2, under x 3 is 0, 2 and then there is no x 3, x 4 is the there, if you see. x 4, x 5 x 6 is not there, so these all are 0, 0, 0, 0 and what is this your write inside of this one? These value is, these value is f of x. So, you will write, simply that f of x what is the function value of this one.

Next is our artificial variable. You see artificial variable objective function, what we have written it artificial objective function value expression ? Just now I mention here, is w is a, 2 x 1 minus 3 x 2 plus 3 x 3 plus x 5. Since, w is constant quantity, you can make it with the w minus, w minus 6 is equal to minus 2 x 1 minus 3 x 2 plus 3 x 3 plus x 5. So, if I write it this one, accordingly I will write under x 2 is minus 2 x 1 again, than minus 3. This is you see, this one you take this one is a minus 3 x 3, so minus 3 then plus 3 then x. There is no coefficient associate with here, there is no coefficient x 4, x 5 is one coefficient .So, x 5 one coefficient, x 6 there is no coefficient.

Then what is the function value? Because this 6 is constant you can take it left inside. w minus 6 with the objective function value. So minus 2, minus 3, 3,0,1,0. So, now start your table of the process of this. Now, is see first you have to identify, which one is the basic variables. Now, if you consider x 4, x 4 is exist in first equation of x 1 variable, belongs in equation number 1. It does not belong will stay in equation number 2. But x 5 you see, that x 5 does not contain in equation number 1, but it is with minus sign. So in order to make minus plus, then this will be minus. So, this we are taken as a basic variable. But x 6 contents only equation number 2 with positive, it does not contain any equation number 1. So, our basic variables are, basic variables are x 4 and x 6, is a basic variable, which is our remaining things are our non basic variables.

Because there we have 2 equations are there, 2 basic variable. And we have 6 variables are there, 6 minus 4, 6 minus 2 is 4 non-basic variables. So on non-basic variable generated denote with an arrow - x 1, x 2, x 3 and x 5 is non-basic variables. Now, you say what is the basic variables value immediately because non-basic variable value is 0. So, if you see this equation number 1, immediately you will get x 4 is equals to 12 and x 6 is equal to 6 and remaining x 1 is equal to x 2 is equal to 3 is equal to x 5 is equal to 0. Now, with this are the basic variables, basic variables values and is the most non-basic variables.

Now, which non-basic variable will act as a basic variable, that you are to find out by choosing the pivot and pivot row. Now, since I have to in phase 1, we have to optimise the artificial objective function or cost function, so you to concentrate only with the best on these coefficient, reduction reduced coefficient objective coefficient with artificial cost functions. Now, see this coefficient is minus, this coefficient is also minus. We have to consider the most negative, negative coefficient and most negative, negative coefficient is that one, okay, this one. So, this is our pivot column, because we have to minimise first w, then we have to minimise that our cost function. So, we have to concentrate only these last 2, which is artificial cost function.

So, this is the pivot column. That means, first x 2 entering as a basic variables, entering basic variable is which one, x 2. And which one is non-basic, basic variable enter as a non-basic variables? So again, you have to see, this ratio that we have to show 12 by 2 is equal to 12 by 2 is equal to your and 6 by x 3 is equal to 12 by 2 is equal to 6. Which one is your, what is call minimum ratio, this one again. This one is your minimum ratio. So, then what you do it? This is the pivot row, if you see the pivot row of this one, that means this is our pivot element. This is our pivot element. So, what do you do? We normalise this one, you normalise this thing by a 3.

That means, what I am doing and after that, you eliminate x 2 from equation number 1, equation number 3 cost function, similarly, equation number 4. So, what we can do it here? If you normalise this one, I am writing 1, equation number 1 minus equation number 2. T is equation, this equation number 2, you have to normalise first again. Then, multiplied by 2. So, you have to write equation number 2, normalised equation number 2 normalised, then multiplied by 2. N stands for normalised. Equation number 2, normalised by 3 or you can write equation number 2 divided by 3 into 2, then it subtract from equation number 1.

So if you do this one, this is for a equation number 2 and this after normalisation, what are equation you got it, that you multiplied by 2, add it with this one. So, that x 2 will be eliminated. Similarly, after normalisation this you multiplied by 3 add with this one. Then x 2 also it will eliminate, that means I can write it, if you see this one, that equation number 3 plus equation number 2 normalised by 3 this one, multiplied by what? This you have to multiplied by 2 added, then you will get it x 2 you will eliminate here, x 2 also eliminate here. And in this last equation, that equation number 4, you at you normalised this one by 3, then multiplied by 3. So equation number 2 normalised, multiplied by 3, then add with equation number 4, if you do a operation then ultimately you will get the table like this way.

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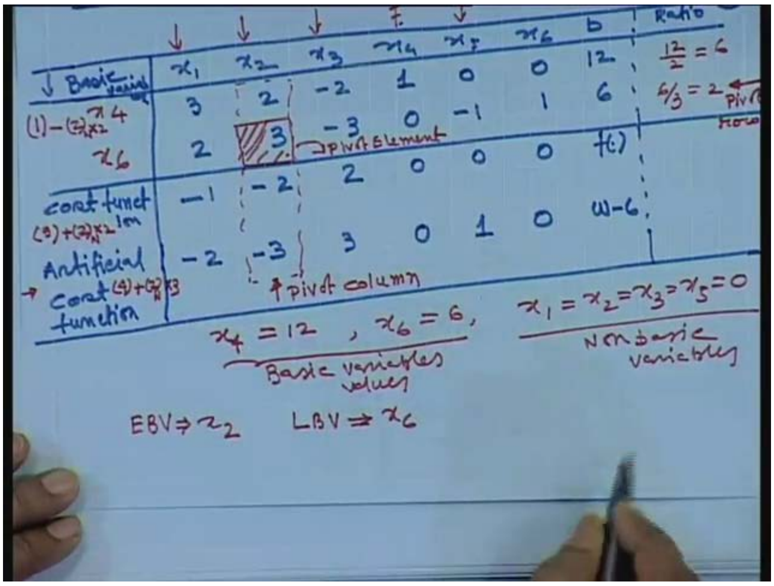

So, our you can say here, our with this one, pivot element selection, x entering basic variable is our x 2. Entering basic variable is our x 2. And entering leaving basic variable is your x 6. Now, you see the table 2. Basic variables, x 1, x 2, x 3 basic variable in this direction, x 3, x 4, x 5, x 6, b ratio.

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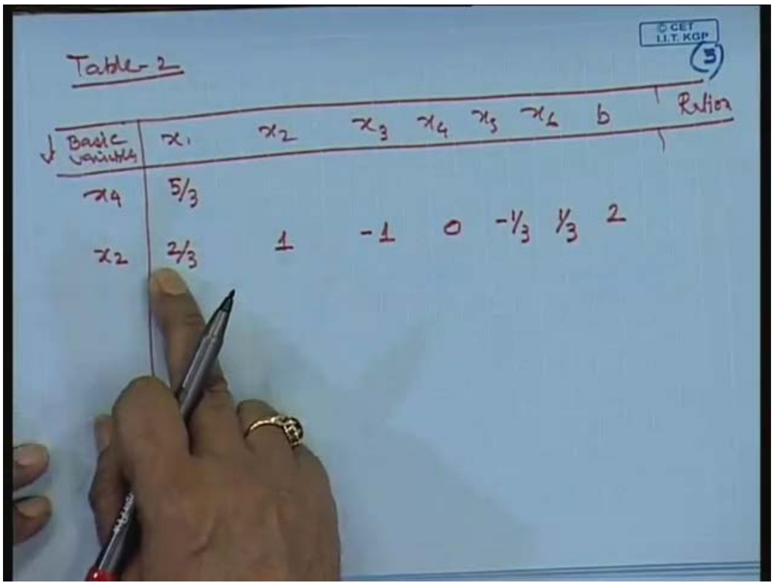

So, whatever the operation I asked you to do this one, that this equation you divide by 3, then it will be coming the second equation and our basic variables is here, what is this our basic variable? x 4, I am not changing x 6 is replaced by x 2. This is our basic variable, we are just explain it here. Then second equation, I have mentioned it divided and normalised by 3. So, this will be a 2 by 3, this will be 1, this will be minus 1, this will be 0, then x 5 coefficient will be minus 1 third. x 6 coefficient will be, x 6 coefficient will be 1 by 3. Then x coefficient will be 2, that b coefficient will be 2.

Now, whatever the operation I did it to do, if we do this one, then this will be coming 5 by 3. What I did it to do this one? You see, the equation number 2, after normalisation and a multiplied by 2. So, you multiplied by this 2, that means what you will get it multiplied by 2. That is 4 by 3, 4 by 3, 3 minus see, 3 minus, 3 minus this one. 3 minus 4 by 4 by 3. So, this one is coming, if you see this one, that I divided by these 2 by 3. So, multiplied by this thing is 2, 2 by 3, 4 by 3.

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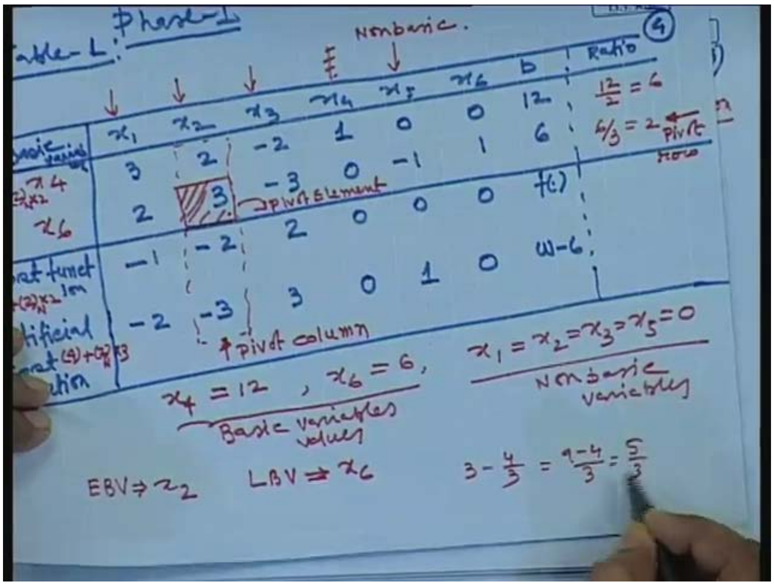

So, 4 by the this was is 3, 4 by 3. This is nothing but a 9 by 9 minus 4 by 3 is a 5 by 3. So, this is 5 by 3. Similarly, you do all this things. So 0, 0, 1 this will be a 3 by 2, 2 by 3, it will be a point or you can write 2 by 3 2 x 3.

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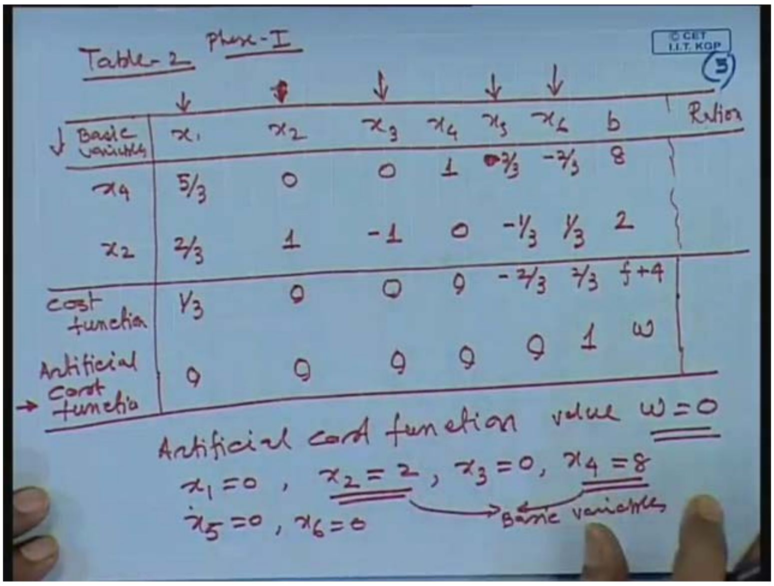

This is 2 by 3, then this is 8. Similarly, the cost function, what I did it that equation number 2, this after normalisation you multiplied by this thing by 2, multiplied by 2 add with equation number 3. So, if you do this one, you will get one third, 0 0 0 minus two third plus two third and this will be f plus 4 and this is the our artificial cost function artificial cost function. And what did they did, it for this one, you see this after normalisation, this is the this equation and this equation you multiplied by 3, that is what I did ,multiplied by 3 added with 4, add with this equation ,add with this equation. So, 4 with this equation, then you will get it that one 0 0 0 0 0 0, last is 1, then this is w, okay.

Now, you say the last because our job is to minimise this first in phase one, phase one, this phase one, first one to minimise that our w, means our cost, artificial cost function. You , last 2 of the artificial cost function, the reduced coefficient all are positive, this all are positive. That means, we cannot further reduce the value of the artificial cost function. Then, immediately I can write the artificial cost function, artificial cost function value w is equal to 0. How? x 1 into 0 0 plus x 2 into 0 0, x 3 into 0 0. Our basic variables are what, x 4 and x 2, others are non-basic variables. See, x 1 sorry not x 2 x, x 2 is non-basic variable, x 3, x 5, and x 6.

Now, x 6 is in non-basic variable. This into 1, means x is 0. So, w is equal to 0 we got it. So, at this stage, from the table 2, we can find from the table 2 this one, we can find that x 1 is non-basic variable, is 0. x 2 our basic variable, x 2 our basic variable, x 2 value is what will get it. See, x 2 value is 2. Directly you can see this is 2,this into this 0. Similarly, this into the x 2, this in non-basic variable 0, this basic variable into this. So, it is a 2 and then x 1 x 2, x 1 is 0. x 2 is 0, then x 3 is our non-basic variable 0, x 4 is basic variable straight away from this expression is 8.

So, these are the basic variables, these and this. Sorry 2, the basic variables. These are the basic variables, values and x 4 x 5 is equal to 0, x 6 is a non-basic variable 0. x 1 x 3 x 5 x 6 0. So, the w x 2 is non-basic variable this. So, after the end of the phase 1, that means we optimise the artificial objective function value, which is equal to 0. Even though we added x 6, x 6 value is 0, this artificial variability. So, at this at the end of this one, our basic solutions we got it, this one. Since w is 0, so this value x 6 value will not be considered in second phase, phase 2 from now on.

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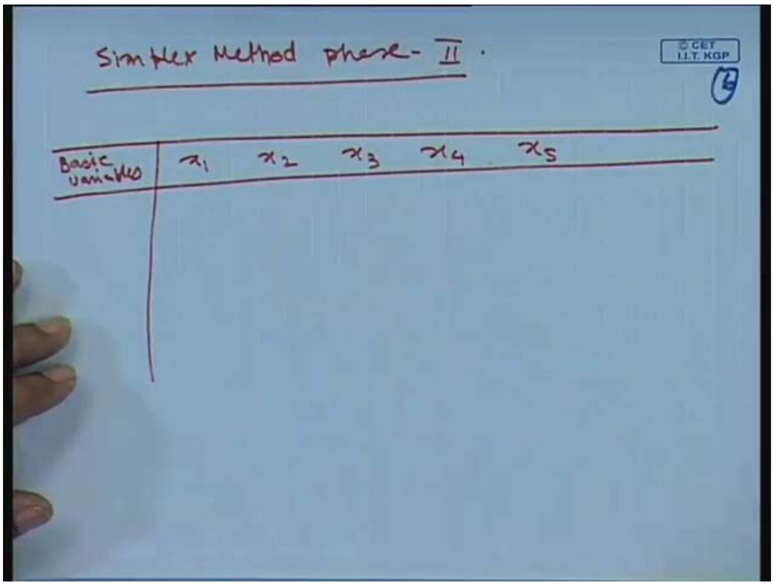

So, phase 2, simplex method phase 2. Simplex method phase 2. Since x w is, w is 0 means x 6 value is 0 you got. w is equal to x 6 we have consider this is. So, from this, the new table what will form it, x 6 you can form or you can keep it in the table, but ignored it any operation you do further from onwards. So I will take the table like this way, basic variables. See from this, the this equation. So, our these optimization value is 0.

Now our only left is x 6 0, okay x 6 0 I get it. But in turn during the phase 1, this our table of equation is now changing this one. Let us see this one, x 1 x 2 x 3 x 4 x 5, x 6 will not include in the table. Even if you including the table, you ignore it’s all operation from now onwards. So our last table if you see this one, I am reproducing here the our last table, omitting the last column as well as the artificial cost function again. Then the our problem is minimising the our cost function. So, I am writing only this portion, if you see from here up to this, up to this and including that one, this column also. This thing I am reproducing now. So, now our say 5 by, this is 5 by 3, 5 by 2. So, we are that the this is 5 by 3 is here, 5 by 3. Now, see this one, we are considering this table now. This were to now optimise that one this column.

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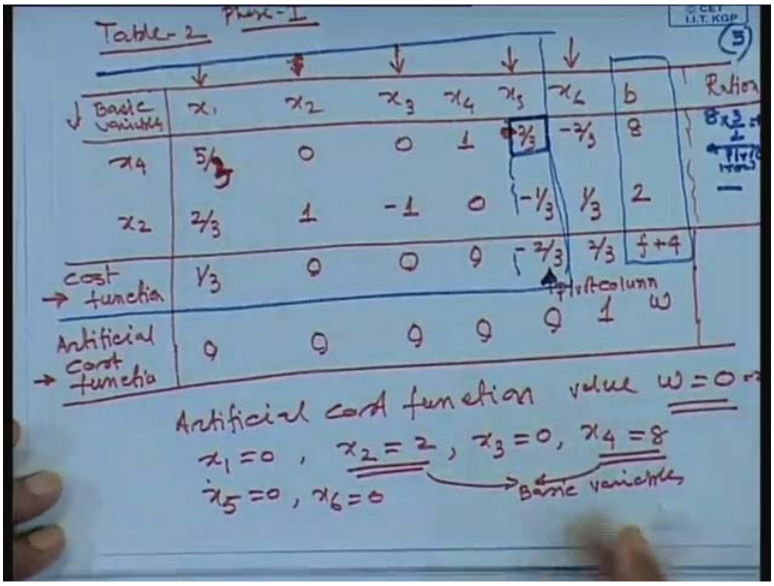

Now, this cost coefficient, you say whether we will wait further will be able to minimise the cost function value or not, that we have to see. How will see it that one? That is either with reduce cost coefficients, this is plus, this is plus, this is plus, this is plus only this minus. So, this quantity, at this that negative most negative are coefficient of the cost of objective function is along the x 5 variables. So, this is our pivot column this is our pivot column, if you see this column is our pivot column, this column.

Now, you find out the ratio, that means 8 by multiplied by 8 multiplied by 3 by 2, that it comes 12. Now, this is minus, ignore. That with this if you consider this is very our pivot row, then it this will not reduce our cost functions. Ok, this will go beyond our, what is called our region feasible region, if you consider this one. So these our, the other pivot this is the pivot row. So, our pivot element is that one. So, what will consider? That x 5 is entering basic variable and x 4 is living basic variables.

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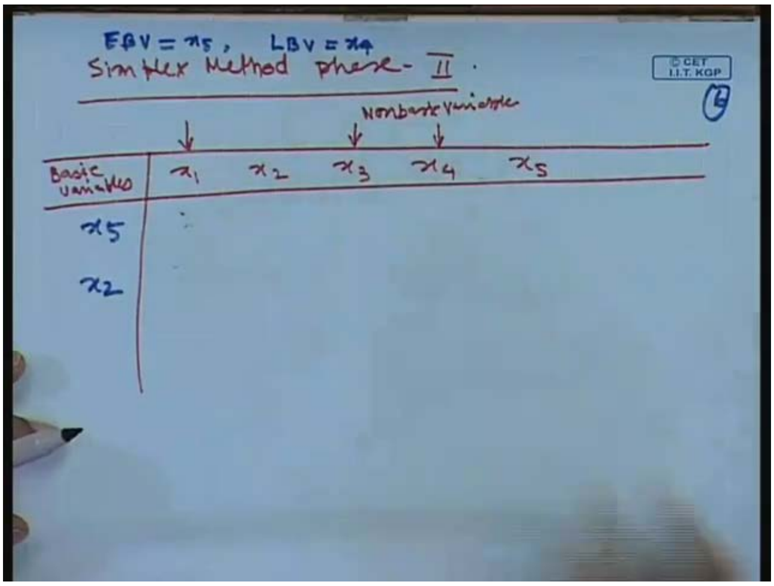

So, we can write it now here, that our before entering the phase 2, our entering basic entering basic variable is x 5, living basic variables, living basic variables is your that x 4. So, what we have to do here, in phase 2 you divide the equation gives, that menas normalised with this one, 2 by 3. So, if divide by 2 by 3, this will be 1 and this is will be 3 by 2 minus1 and this is 3 by 2, will be 5 by 2 that one. This is 3, this is 3. This 5 by 3. So, if you do this one, now basic variables is which one, say or basic variable x is living as a non-basic variable, entering x 5 is entering is a basic variable. So x 5 and x 2 is our basic variables and remaining are in the non-basic variable, x 1 x 3 and x 4, these are non-basic variables.

So, look this is, i divided by, what is this, i divided by 2 by 3, the normalised by 2 by 3 and you have to eliminate x 5 for me equation 2 and 3 that the cost function. Then what will do it, at to multiplied by after normalization of the multiplied by these one third, 1 by 3 and add this equation. So, x 5 is eliminated. Similarly, I multiplied by this 2 by 3 add this one.

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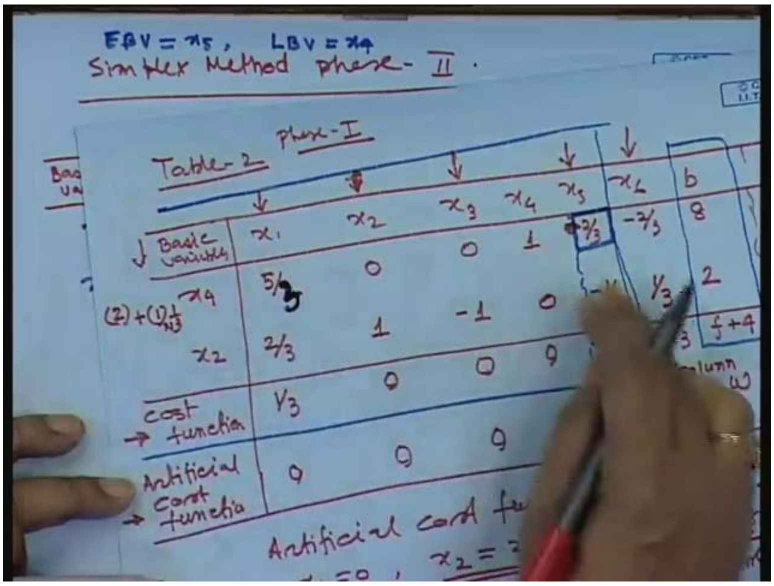

So, I am writing here that equation 2, equation 2, add equation 2 the multiplied by this that equation 1 to normalise by 2 by 3, then multiplied by one third. Whatever you will get it at, add with equation number 2. If you do this one, first you will normalize this one, that will give you give, if you normalise this one, that will give you 5 by 2, 5 by 2, 0, 0, 3 by 2, 1 and b and ratio.

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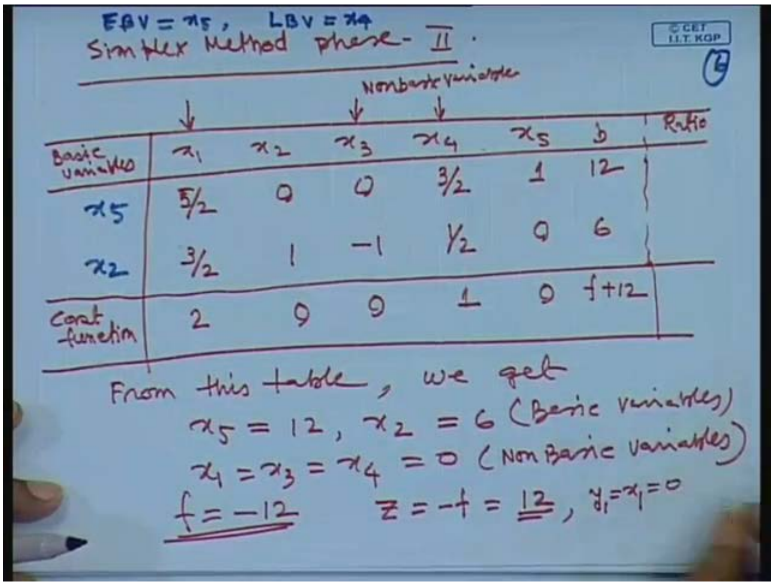

3 by 2, 1, minus 1, just a minute ok 1, minus 1 then it will get half 0 and this is 6, this is a 12. Cost function 2 0 0 1 0 f plus 1, you say what I did it this equation number 1, you

normalised 2 buy 3 and normalise by 2 by 3. So, x 5 coefficient will be 1, x 5 coefficient 1. I normalised 2 by 3 you see 3 by 2, this is 0, this is 0, this will be 5 by 2 this. Then, what I did, multiplied by that this is one third. This equation and multiplied by this equation and multiplied by one third and add with this equation, then you will get it this things.

Now, look whether is there any possibility to reduce the cost function further, by looking the reduce cost coefficient and reduce cost coefficient of the cost function object function all are positive. So, there is no chance of reducing the function value further. So we will stop our that the iteration in phase 2 here .Suppose were some of the coefficient is negative, then you find out the pivot column, pivot element that pivot row and then pivot element, proceeded in the similar manner.

So, from this table, so from this table, we get x 5 is equal to 12 x 2 is equal to 6, similar manner this the basic variables and what are the non-basic variable, x 1 is equal to x 3 is equal to x 4 is equal to 0 is a non-basic variables, non-basic variables. Then, what is the our function value, function value is minus 12 and our problem, if you see the earlier our problem, with this problem what we are to solve it here, is a that one maximize z. So, z value is what, z value is minus of minus of this one. So, it will be 12 plus and what is our corresponding, what is our corresponding y 1 y 2 all these things? y 1, I have considered x 1. So, y 1 value is same as x 1, this value is 0 and similarly, y 2 value is what?

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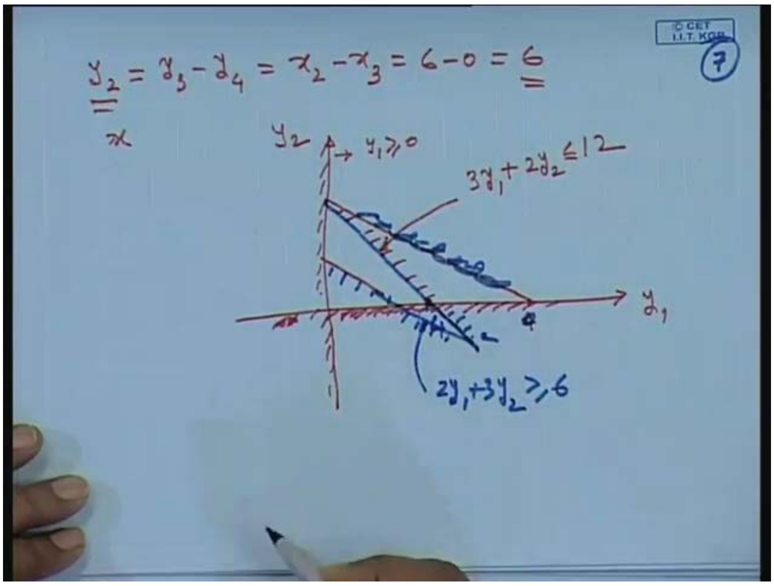

Y 2 value is nothing but, a y 3 minus y 4, is nothing but, a x 2 minus x 3 and x 2 value is what we got it, 6 and x 3 value is 0, x 2 value is 6. So, 6 minus 0 is equal to 6, that y 2 value is 6 and the other value x 2 values you got it x 3 and y 1 y 2 value you got it. So, y 2 value is 6 and y 1 value is from this one y value is this. So, graphically if you see this one, if you see the figuring graphically in original problem, you will see this one that our y 1 is greater than, if you see this one that y 1 is greater than 0. y 1, if you consider in this directions and y 2 is in this directions, y 2 can be positive and negative.

That means, this will be on the line and this region is, y 1 greater than equal to 0 and this region, that what is call and this region this region I do not need it, that top side or bottom side is y 2. If it is bottom top side, it is y 2 greater than 0 and bottom side is, y 1less than y 2 less than 0. So, our if you draw the equation number, this and this you will see you will get it something like this equation.

So, when y 2 is 0, then what we are getting here is 0 x, y 1 is 4 ,y 1 is 4. Then one y 1 is 2 y 2 is 6, though it is not drawn to the scale, let us call forget, this I am drawing the scale is this way, forget about this one. This is a 4, so this is our which side is this one so this is our, this expression is 3 y 1 plus 3 y 2, 2 y 2, 2 y 2 is less than equal to 12. And another equation, you see that one this will be a it can go extend like this way and this values is and that equation is twice y 1 plus 3 y 2 is greater than equal to 6. So, next class I will discuss the figure more clearly. We will stop it here.