Optimal Control
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Lecture - 17
Solution of Linear Programming Using Simplex Method - Algebraic Approach (contd…)

So, last class we have discussed how to solve standard LP problem linear programming problem by using algebraic approach. And then we have consider a numerical example which we could not complete, so we recap the example here.

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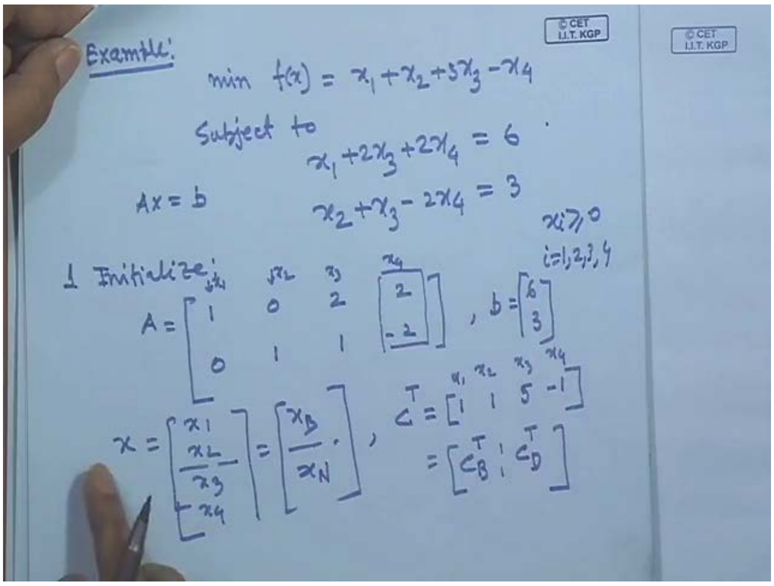

So, our problem is given the function with which you have to minimize subject to the constant which is in standard LP problems. So, what have to do first, so this two equation linear equation we have to write into matrix and vector form. That means we have to write in x is equal to the form that if you look this two expression, then A matrix is A matrix 1, 0, 2, 2 and next row in 0 1 2 minus 2 from this one and b is 6, 3.

Now, we have to do this thing into A that variables we have x 1, x 2, x 3, x 4 variables are there. So, we have to choose the which are the which are the what is called the basic variable and which are the non basic variable, this two equation state with that they are already in conical form. So, what is that x 1 x 2, I can consider as a basic variable and x 3, x 4 are non basic variable, so we have partition x 1 and x 2 this n x 3 x 4.

So, basic variables as we have used the standard notation x suffix be and non basic variable x suffix and so correspondingly the what is call the objective function that will write in terms of C transpose of x. Accordingly from this expression C transpose is this one coefficient of x 1 is 1, coefficient of x 2 is 1, coefficient of x 5 is x 3 is 5 and coefficient x 4 is minus 1. Then we have partitioned this C matrix this C transpose matrix into two parts that variables x 1 x 2 which is that means basic variables, which is with the C transpose matrix that we have partitioned denoted by C B transpose and this non basic variables associated with the non basic variables.

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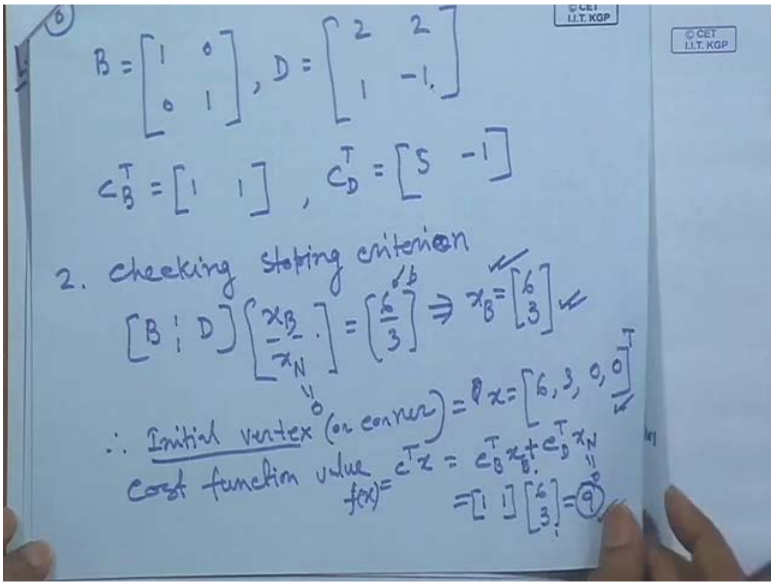

So, with this one we are immediately that identified that B and D matrix formed the matrix formed from the A matrix that we have formed that B matrix. We associate with the basic variables x 1, x 2 and D matrix associate with the non basic variables and C B transpose which is formed from the performed index or the objective function associate with the basic variables. C D is associate in the performed index associated with the C transposed matrix in the objective function C transpose is associate with the non basic variables.

So, first we have to find out that initial verdicts wise solution of this equation that we have derived earlier B into D because our matrix is partition into B into D form x B is x N is 0. So, x B into B plus D into x N is equal to b, b this is the b agree from there we got the initial solution or verdicts point is 6 by 2. So, this is the initial verdict, so our x 1 value is 6 x 2 value is 3 and that remaining two variables x 3, x 4 are the non basic variables. So, this is the solution for the initial partition, so correspondingly made it and find out the cost function value that is f of x value is that one, so that value is if I put the value of x B value C B transpose C D transpose x 1 is equal to 0.

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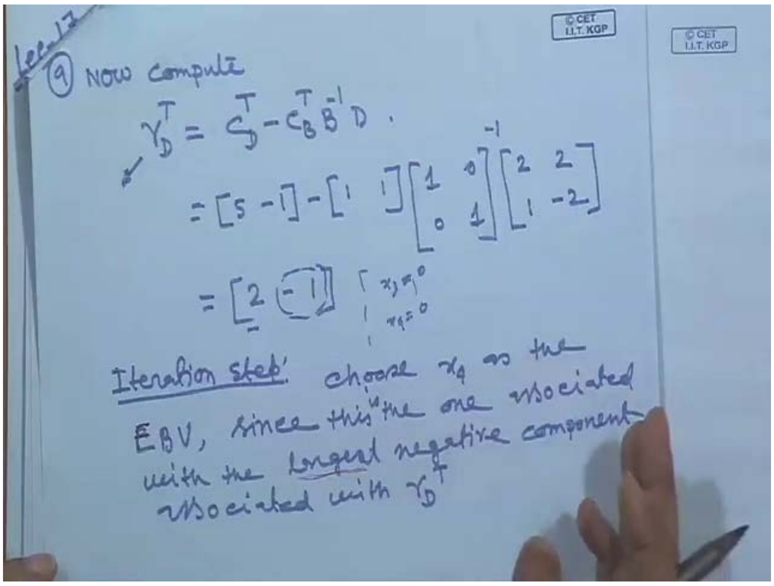

Then, I will get that objective function value is 9 then our question comes logically question whether those non basic variable if one of them is changed to a basic variables. The basic variable one of them is changed to the non basic variable whether the objective function value will decrease or not so that can be tested with the vector which we have derived or derived.

So, you know C D transposed is this one C B transpose this one than B in verse B is this one and that inverse is one of this, and then D is that one you see this one B is this one and D here is you see that is by mistake. It is a minus 2 this one is minus 2, so then D is that one, so if you simplify this one, we got the 2 minus 1 and that associate with the non basic variable x 3 and x 4.

Now, it is clear that in the in the objective function which can be expressed into a two parts, one is a r D transpose into x 3, x 4 you see from this expression this negative terms the negative value of that associate with x bar x 4. If you change this non basic variable to a basic variable agree then there is possibility to reduce what is called function value. So, our choice will be next our non basic variable x 4 will be will be converted into a what will be treated as a basic variable and form x 1 and x 2, which there are two basic variables are there out of which one will consider the non basic variable that we to decide.

So, from this one we can see this one choose x D is what entered basic variable from non basic variable. It is entering to a basic variable is one associate with the largest negative, this is the important largest negative component associate with R D transpose largest. Suppose, we have a more than two negative numbers were that out of these which one is largest that will consider as what is called entering basic variables because that vary that element if considered the largest negative component of r D.

If you consider as a basic variable that will reduce the function value vary compared to other negative values, it will reduce first or you can say reduced in the better way. So, this is last class we have discussed then we will see today that once we know that x 4 is the our non basic variable will be changed to a basic variable then we have to select between x 1 and x 2 which one will be the non basic variable.

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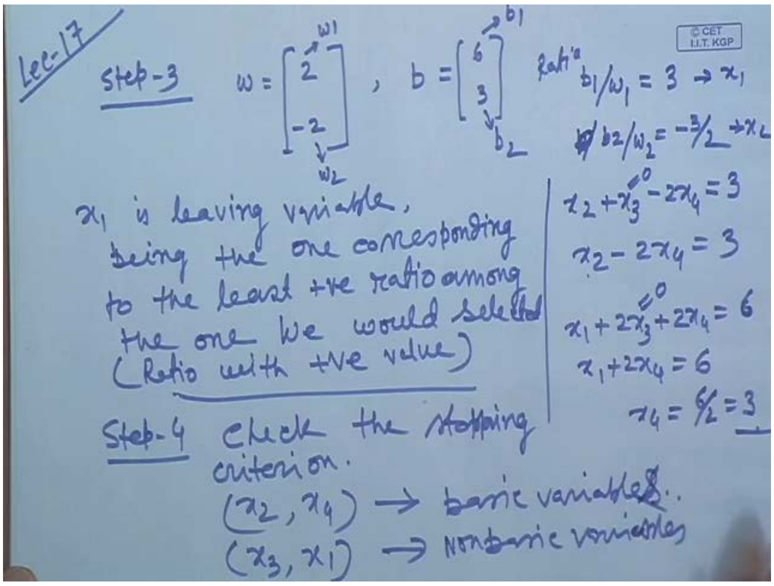

So, step three is so select w is equal to that is that you define w which is associated with the non basic variable x 4. The element associate with the non basic variable x 4 from our A matrix see this one our metrics is this one A matrix is that one and it is x 4 is now entering as a basic variable. So, this column will consider as w defined as a w, so that is very defined as a 2 to minus 2 is defined as a w and that is corresponding let us call this element is w 1 and this element is w 2. So, that corresponding A x is equal to B that equation basic set of linear equation that B we have a 6 3 which is good that we denoted by b 1 that we denoted by b 2.

Now, calculate the ratio b 1 by that b 1 by w 1 what is this values is equal to 3 and b 2 by b 2, b 2 by w 2 and this will be minus 3 by 2. So, we have to we have to find out the ratio and out of this we have to consider only positive one, so our next step is to find a because this is associate with the their w is associate to x 4, we our considering the basic variable. Now, we have to select from x 1, x 2 which one is the basic what is called which one will be non basic variables.

So, this ratio that corresponding to x 1 and that corresponding to x 2 which will entered as a non basic variable. So, see this one that we have to consider the positive quantity ratio which one is the largest or least of this one, so that we have to consider least ratio and positive that corresponding variable will be considered as a non basic variable.

Now, you can see also from here how you are taking this one day earlier also we have discussed, but if you see this expression, let us call this expression this expression, let us call first this expression of this one. So, this one if you write it let us see here and x 2 plus x 3 minus 2, x 4 is equal to 3, but our x 3 x 4 previously it was a non basic variable and from there we have selected x 4 will be a basic variable, so this value cannot be 0.

So, this remain as a non basic this is 0, so we have a now we have a equation x 2 is equal to minus 2 x 4 is equal to 3 another equation from the first equation you see that x 1 plus 2 x 2 3 minus plus 2 x 4 is equal to 6. This is now we have considered is a basic variable non basic variable change to basic variable and this is non basic variable, so we have a expression for x 1 plus 2 x 4 is equal to 6.

Now, look out of these two variables at x 1, if you consider that x 2 is the non basic variable than x 4 value is coming negative, so it cannot be that 4 is greater than 0 since x 4 is greater than 2. So, x 2 cannot enter as a non basic variable now in this case, sorry that is x 2 if it is a non basic variable x 2, if a non basic that x 2 is 0 the x 4 is equal to negative quantity.

So, this cannot be a non basic variable suppose this is a non basic variable x 1 is treated as a non basic variable than x 4 value is coming to 6 by 2 is equal to 3 that what we were checking the ratio see b 1 by w 1 b 1 by b is 6 by w 1 is 2, so this that way we are checking. Now, we have selected x 1 is x 1 is leaving variable being the one corresponding to the least positive ratio among the 1's we would selected ratio with positive values with positive value.

So, this next step is step four now we have selected every collect that x 1 and x 3 are non basic variable next equation is non basic variable and x 2 and x 4 is the basic variable. So, we have to check the stopping criteria check the stopping check the stopping criterion, so what is this one if you rebate x 2 and x 4 is the basic variable x 3 and x 1 are non basic variables. So, we have arrived right now once again you have from ax is equal to be now you identify from A x is equal to b.

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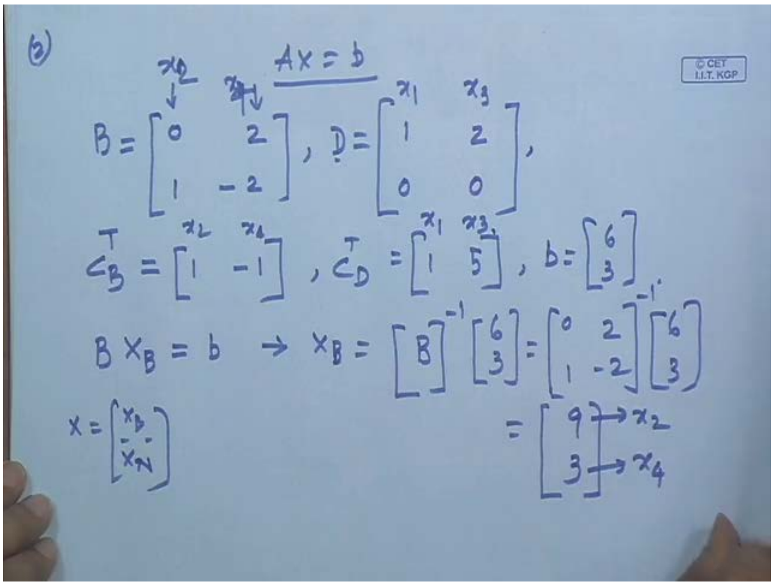

So, we have a matrix A x is equal to b identify which variables are basic variables and non basic variables we have seen x 1 and x 4 are the basic variables and correspondingly we can write that what is called b matrix from a matrix B case is 0 2 1 minus 2. So, this corresponding to x 1 column this corresponding to x 2 this is formed a matrix A, you have a 4 columns and columns associate with x 1. I have written here because that is a basic variables column x is associate with the x 4 this is x 4 is the basic variable that column. I have retained then d from the metrics column associate with the non basic variable that is x 2 basic, sorry this is a 2, this is a basic variables are x 2 and x 4 and this is the non basic variables is what x 1 and x 3.

So, column associate with x 1 in matrix A are 1 0 and this is the non basic variable column associated in A matrix associate with the non basic variable is 2 0. So, once again I repeat since x 2, x 4 are the basic variable from A matrix I am picking up the columns associate with the basic variables that is x 2 and x 4 from A matrix. Similarly, non basic variable x 3 and x 1, x 3 from the matrix A I have picked up the D matrix.

So, you know A D correspondingly the performance objective function I found out the C B transpose. That is corresponding to the variable non basic variables associated in the objective function 1 minus 1 that is x 2, x 4 and C D transpose that is associate with 1 5 that is x 1, x 3 see this one if you look at this one that this x 1 and x 3 are under non basic variable the coefficient associate at the 1 and 5. So, I have written 1, 5 and the basic variable x 2 and x 4 the coefficient associate with this one is 1 and minus 1 when you have considered this in matrix what is the vector form vector notation form this C transpose is split up into C B transpose and C D.

So, that is what A bas and once you written this one and B is equal to 6 3 immediately, I can find out that x into x B be is equal to your B. Therefore, x B is equal to B transpose B inverse not transpose into 6 3 B inverse what is that 1 0 1 2 minus 2 whole inverse 6 3. So, if you do the inverse of this one, you will get this value after multiplication after everything you will get nine three this corresponding value is this corresponding this is corresponding to x 2.

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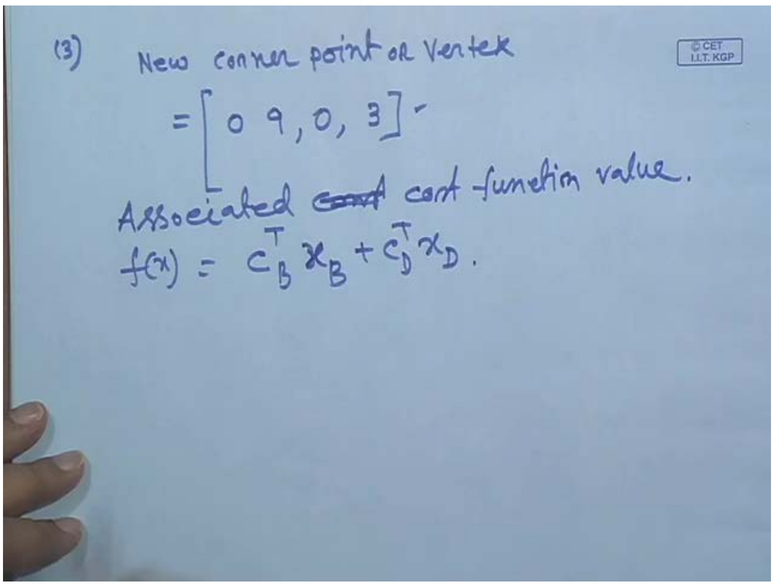

This is corresponding to x 4 the way we have partition b is x B x we have partition into a x B come x D or x N we have partition x B is equal to x, so our basic variables basic variables are x 2 and x 4. So, we got this one next once you got this one that our basic variable then you have to find out the objective function value, weather the function value is reduced or not, but in two sense is not necessary, so you can check.

So, our new corner point if you see our new cornered point our verdicts is what is our x 1 x 1 is entered as if you see the x 1, x 1 is a non basic variables and x 3 is a non basic variables. So, our new cornered vector is 0 then x to value I give basic variable x 2 value is 9 then x 3 value is your 0 by the non basic variable and x 3 value. We got it 3 agree that is x is value you got it 3, so here the x 3, x 4 value you got it 3, so this is the new cornered variable. Now, we have to see the check associate associated cost function cost function value, so you know f of x if you recollect C B transpose x B plus C D transpose x D.

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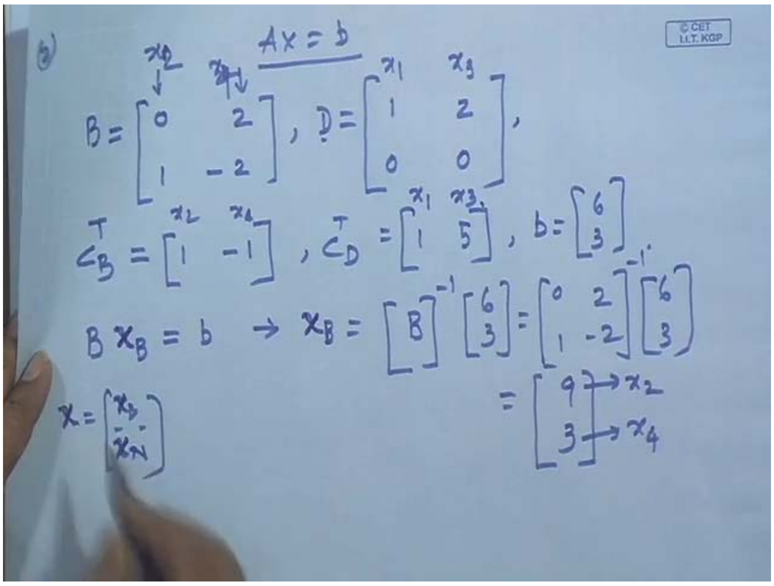

So, this notation I have been to a small x you can write it a small x B small x B and this small this is small that throughout the text keep it x.

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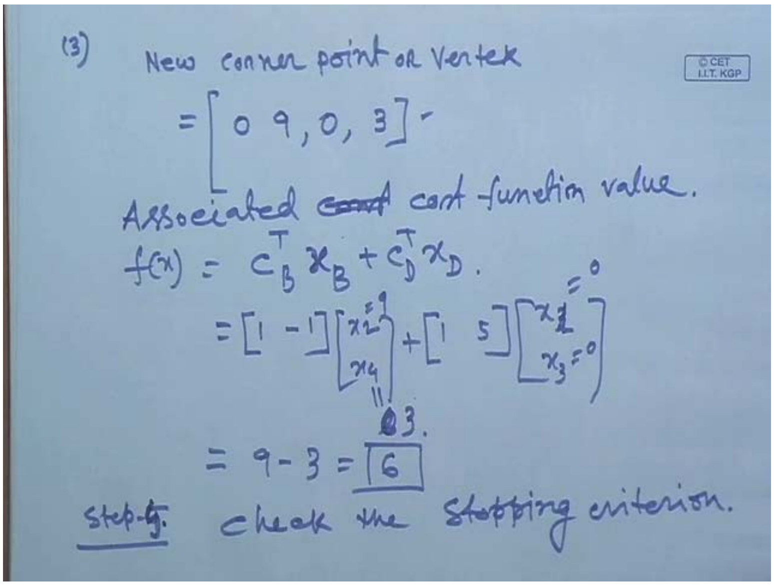

So, put the value of C B, C B means what the coefficient associated with in associated with the basic variables in the objective function so that value is 1 minus 1 C B transpose than it is A x to and x 4 than CD is your 1 5 than x 3. This x 1 and x 3, x 1 and x 3 and that value is 0, this value is 0 and this value you got it 9 and this value is 6, sorry 3. So, if you do this one that is 9 minus 3 is equal to 6 you see that C value is C B value C D value, I just put it in the objective function this.

So, again what you have to check the what is called your stopping criteria that you see the stopping criteria next check the step four, not step four this step five, step five check the stopping criteria what is the stopping criteria again, you find out our D transpose last example.

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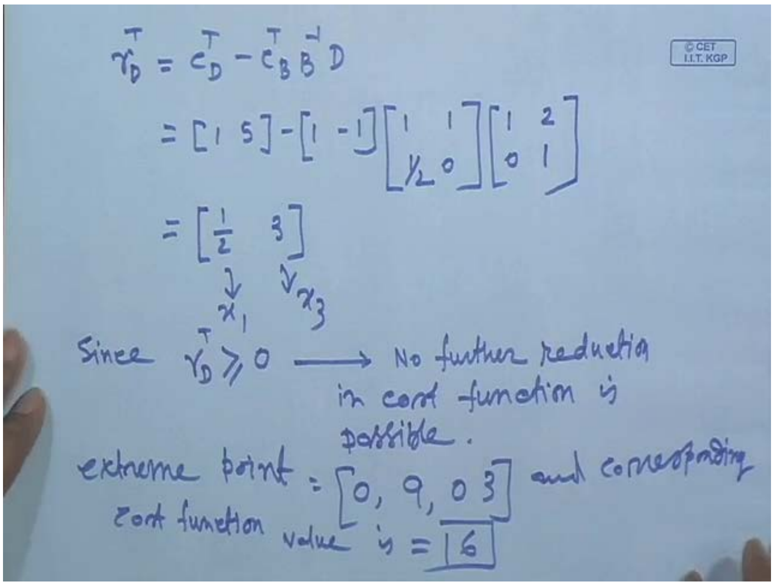

You see R D transpose, so what is the C D transpose minus C B transpose B inverse D, you put the value of C D transpose 1, 5 then C B 1 minus 1 then B inverse value I am taking the inverse of this 1, 0, 1, 1 half 0 this is the B inverse. Then your D is 0, 1, 0, 2 1 D value is same that d value you see it D value we got it. Now, D value what we got it this that is corresponding to x 1 and x 3 what is x 1 and x 3 this x 1, and x 3 x 1 and x 3, so this 1, 2, 1.

So, this value is 1, not this value is 1, so if you see this expression a matrixes 1, 0, 2, 1, so this is 2, 1, so and if you compete this one ultimately you will get half 3 and that corresponding to x 1 and that corresponding to x 3 which are non basic variables and see the non basic variables are positive. So, there is no negative term associate in the R D transpose, so it indicates that there is further not possible to reduce the value of the objective function.

So, our objective function remains same as earlier that means, since R D transpose is greater than equal to 0, this indicates no further reduction in cost function is possible. So, our optimum value of the function as we got in the last stage that is this one optimum value what we got that that will be the optimum value function at the verdicts. So, our optimal extreme point is 0, 9, 0, 3 and the corresponding cost function and corresponding cost function value is 6. So, this is the solution of this one, so basically first we have to find out the basic variables and non basic variables then we have to compete the new mum solution of verdicts of this one. Once you find out the verdicts then you compete the value of what is the objective value of the function what is called further equation is required or not by comparing R D transpose with this one.

Once you identify the B D, C B, C D then you can compete that one is the elements of our D is containing the negative terms than take the largest negative one that is what which complemented having the largest negative element. That one will be the corresponding what is called what is called the non basic variable to the basic variables, so this is the problem. Let us see an example, with this example we have shown how to sort out or how to solve the problems and graphically we will see.

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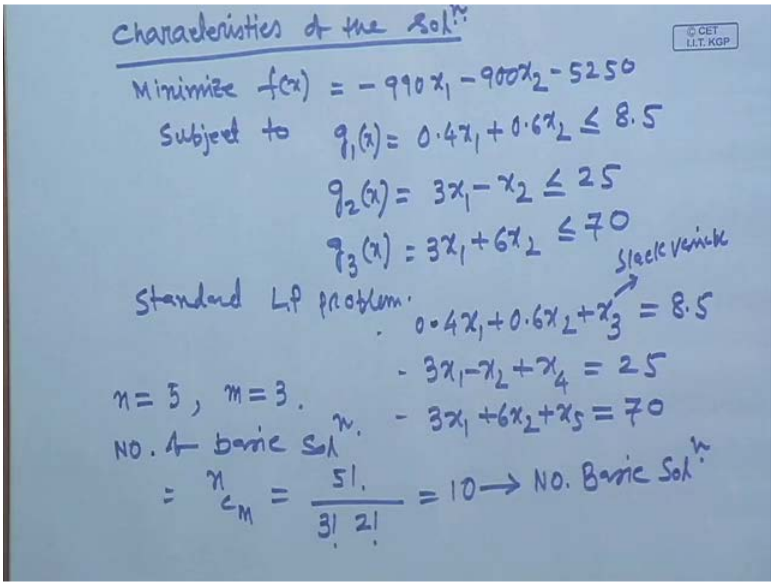

Now, characteristics of the solution, so minimize f of x is let us call that our objective function is given by minus 990, 0, 990 into x 1 900 x 2 then 5,250 this is our objective function and subject to g 1 of x is equal to 0.4 x 1 plus 0.6 x 2 is less then equal to 8.5. Then g 2 of x is equal to 3 x 1 minus x 2 then 25 g 3 of x is equal to 3 x 1 plus 6 x 2 less than equal to 7. So, one can easily form this is you can convert you can convert what is called standard LP problem, standard LP problem.

So, 0.4 x 1 plus 0.6 x 2 plus some slack variable x 3 this is slack variable in order to make it equal to sign. The next is your 3 x 1 minus x 2 plus another new variable x 4 the slack variable is equal to 25 then third equation x 1 plus 6 x 2 plus x 5 another slack variable introduced in third equation to make equality sign. So, our problem is minimize this one and then subject to the constraint of this, so there is a standard LP problem is now formulated. So, you have to solve this one, we know how to solve this one by what is called algebraic approach.

So, first you identify which are the basic variables are there again then you can see once know the basic variables of this one than there are three equations are there three equations. You have a five variables are there, so how many what is called that a basic solution is exist that one can find out immediately. So, our n is equal to how many variables are there five variables are there, how many equations are there, n is the number of three, so number of basic number of basic solution is equal to n c m. So, it is a factorial 5 divided by factorial 3 and this is a factorial 2, so that will be 10, so 10 feasible basic solution are their number of basic solution.

Now, you see the way you want to solve by an algebraic approach, so you have to three basic variables are there and to war what is called non basic variables. So, you selected three basic variable and find out the new basic of the solution, once you find out the basics of the solution find out the objective function value and once to find out. Then you check what is called our x, you check whether you need further to change non basic variable to basic variable or not by computing R D transpose. If the elements of R D transpose D contains negative terms negative values, and then take the largest negative component of r D and corresponding non basic variable will entered as a basic variables.

Then, you from the basic variables which variable will basic variable will entered as a non basic variable that you tested and repeat the process until unless you get that R D transpose. All elements are also positive on set is positive, it indicates father movement is not required to be achieved the minimum value of the functions. So, let us see this one graphically see what is that will make you more clear.

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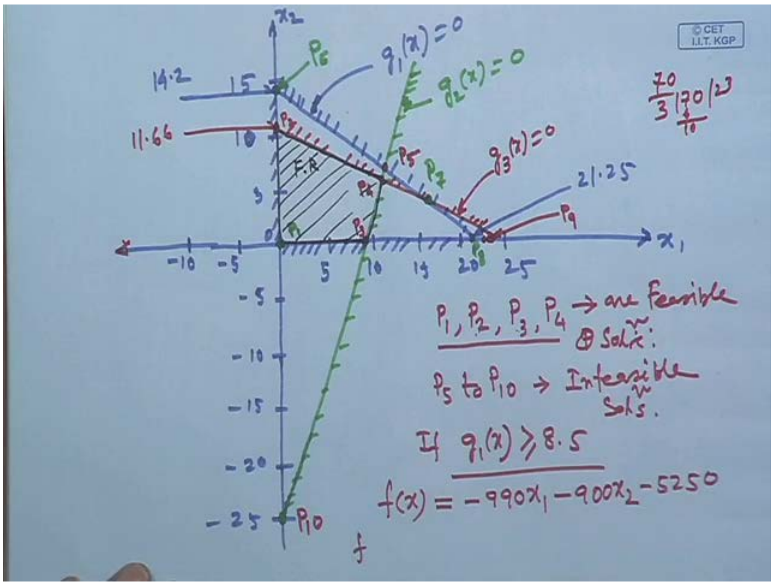

So, let us call this is x 1 and this is x 2, now I am plotting these three equation. So, let us call first I am plotting this one, so before plotting g one before that I am putting 5, 10, 15, 20 and 25. Similarly, 5, 5, and 10 this is 15 and this is minus 5, minus 10, minus 15, minus 20 and minus 25 and let us call minus 10, so let us call this equation I am plotting. So, that is equal to 8.5 when x 1 is 0 then x 2 value will be 8.5 divided by 0.6 that will be near 14.2, so 14.2 will be here 14.2 when x 2 is 0 when x 2 is 0. So, just you plot it with equal to sign when x 2 is equal to 0 that will come near about 8.5 divided by 0.4 that will come near about 21.25.

So, this your equation and since it is an in equality condition equality condition this is this one is g 1 of x is equal to 0, since g 1 x is less than 0 then it indicates this is a this region all regions below this shaded to one this condition is satisfied. So, we also know the x 1 or x 2 is greater than 0 that means this quadrant this whole quadrant x 2 and also x 1 is greater than 0. So, it is only in the first quadrant of this one, so this is our 0, so this our g 1, so see this g 2.

Similarly, you plot you plot g 2, so if you plot g 2 when x 1 is 0 x 2 is minus 25, so it starts from minus 25 and then when x 2 you see when x 2 is 0 then it is a eight point something, so eight point something means here, so 8.3. So, let us call eight is here, so this is the straight line and this line is g 2 of x is equal to g 2 on the line any point on the line g 2 is 0 any point on the line this blue line is g 2 x is 0. So, our since it is less than this our region is that one, now you plot it what is called g 3, g 3 one when x 1 is 0, so x 2 will be 70 by 6 that near about 11 point something, so this one is 11.6, 11.66.

Now, when x 2 is 0 than this is 70 divided by 3, when x 2 is 0 70 divided by 3, so I am just plotting this one with equal to sign. So, this will be near about 20, so this will be a, so 70 by it will be 70 by 3, 27 by 3 is near about 23.3. So, this will be 23.3 somewhere here or somewhere here so on the line this is g 3 of x is equal to g on the line and since it is less than this one that region is that one. So, clearly found this one, our feasible region is that one only any point on this one any point on this shaded region is set is satisfied all constraints, but outside this it does not violate the any one of the constraint out of the three constraint.

So, our think this is the feasible region this, now according the problems stated here ever in the problem stated here you can see that this is the war function and where to minimize this function minimize this function. So, we know if you add some constant in a objective function. Then you minimize this function the what is called the optimum point optimum point will not change anything, but function value change it, but optimum value of these variable decision variable will not change.

So, let us see our objective function or let us see whatever the feasible points are their all basic build solutions are there were other basic solution are there, so we have a these points. Let us call this point is P 1 this point than we consider this is P 2, this point is P 2 then we consider this point is with P 3. Then we consider this point is P 4 or and this point is the your P 5 again and this point is P 6 and one then this cross of this one is let us call P 7. Then this cross is P 8, this cross is P 8 this cross is P 8 and where it cross of that point is P 9 and this point is P 10.

So, this the basic solutions are there again so out of these basic solution you can easily realize from this figure that P 1, P 2, P 3, P 4 are the feasible solution and remaining that the P 5 to retain the right to P 5 to retain are the infeasible solution solutions. Now, let us say P 5, it does not satisfy it satisfies only in the g 1 conditions, but it does not satisfy the other condition g 2, g 3. It does not satisfy that one it satisfies only g 1 and x 1 is greater than 0 x 2 is also greater than 0, but it does not satisfy g 1.

Similarly, you can see other points also let us call P 6, P 6 also is satisfied the only g 1 conditions, but it does not satisfy other conditions when other condition. So, we may have in this particular problem that are what is called ten basic solution are there out of ten only four, only these four P 1, P 2, P 3 are the feasible solution and remaining points verdicts of this one is P 5 to P 6 to P 7 to P 9 and P 10 are the in feasible solution.

Now, see if you change the some of the constant, let us call if I just put if a g 1 of x is greater than equal to 8.5 that is our same thing just only we have changed the iniquity constant instead of less than able greater then these than our region of for g 1 will be above this one. So, we do not have at all any solution of this problem because it violates all the points whatever we got the basic feasible basic solution all points is violates means is we do not have any solution. If we change what is called the constraint with this type these now what is called our or minimum value of this function.

So, our objective function as we mentioned it that our object function is affects if we had our problem is f x minimize the f x subject to the condition and what point what verdicts is this function will minimize same point. If we change the function by constant by adding a constant with this function, it will also minimize the new objective function at the same point that is we have already discussed.

If you add to the objective function some constant term or if you add with, if you multiplied by these with its constant factor, so our optimal point will not change it. So, let us call our objective function was this one that is given is minus 990 x 1 plus minus 900 x 2 minus 5,250. Now, I am changing our new objective function is new objective function this is what we are telling.

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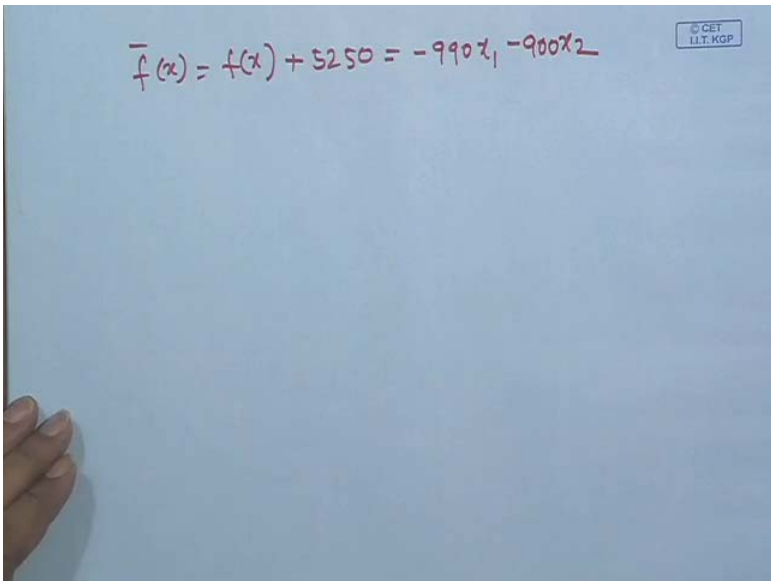

Our new objective function is a bar of x which is nothing but a f of x plus 5,250 that constant term we take in this one so that is a minus 990 x 1 minus 900 x 2.

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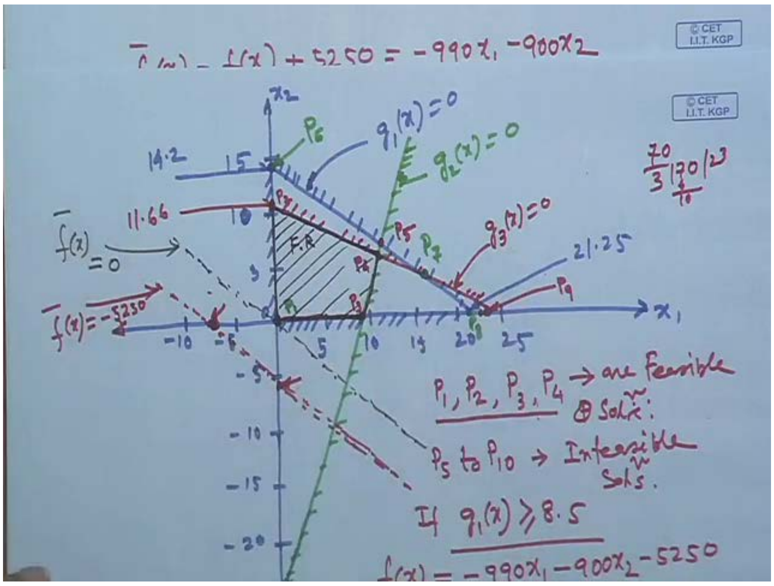

So, graphically you see, so when x 1 is 0 x 2 is 0 the function value is 0. So, if you plot if you plot a line through 0 origin this this is a bar of x is equal to 0 any point on this line is new objective function value is 0 means this one. If you move this new objective function up rather with this one of f bar is equal to 0 up and down the objective value function will increase and decrease. If you go up, the objective value function is increased, if you go down the objective value function f bar will decrease.

Now, let us say if you go up slowly, you are going up parallel to the dotted line parallel to the dotted line. Then function value will go one increasing, but our problem is to minimize the function, so that will be the minimize value of the function bar 0. So, you can go below, supposed you go parallel below this one then let us call another and showing it parallel to this one, so the function value is the case of f bar value is also decreased. So, which in turn f also decrease away, but we have to see whether it is a feasible solution or not.

So, any point below this line in any line parallel to this one if you go downwards then there that will not give you any feasible solution because our feasible region is that one our feasible region is that one. So, let us call this correspondence to this corresponding to f bar of x this corresponding bar of x corresponding f bar of x minus 5,250 and corresponding value.

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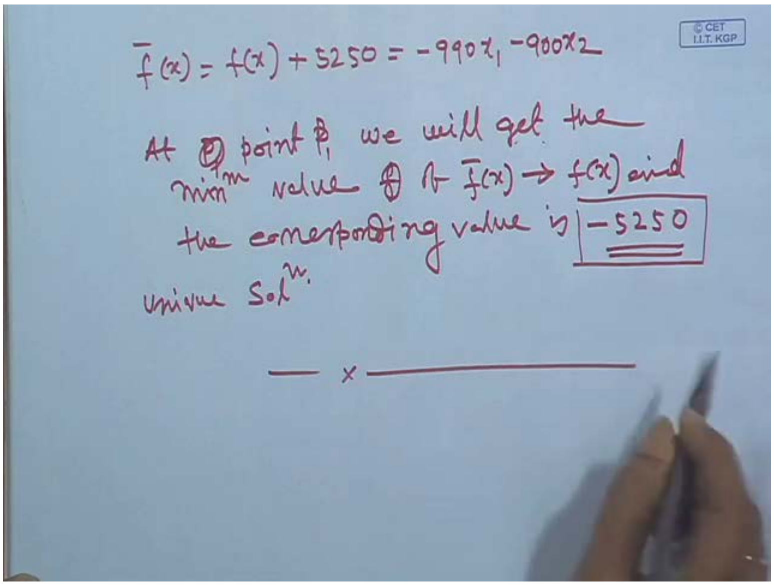

One can easily find out this corresponding the line corresponding to this one, but this value is increasing, sorry decreasing. Any point on this line or any where this is not the feasible solution to the solution though function value is decreased the solution is not acceptable. So, our for this particular problem from graphical interpretation is that the point P 1 is the optimum value of the function at that value that means P 1 value is x 1 is 0 x 2 is 0 again and x 3 x 4 and x 5 value.

We will get correspondingly that I am not just graphically I have to represent what is this. So, I just write at P 1 point, we will get the minimum value of f bar of x which implied f of x and the corresponding value. The corresponding value is your minus 5,250 see this one f bar what a considered f bar just f bar we have considered the value of f bar here see the value of f bar is this is this is your 0, so f bar function.

So, if you just recollect recall that you will get unique solution if the objective function and the constant have the similar slopes unique solution. If the objective function and the constant of similar slopes and there is an infinite slope the objective function slope and if it is same as one of the constraint, one of the slope of the constraint equation or constraint equation. Then you will get the infinite number of solution that means same is parallel if the objective function is parallel with the constraint of the subject to the one of the constraint.

Then, we have an infinite solution, so if you recall all these thing first if you have given the problem convert into standard and the LP problems. Then once a convert into the assembly, but the standard LP problem then find out the vote it is see its basic variable and non basic variables and you find out the objective function value at the initial verdicts. Once you find out the objective function value then you compute the R D transpose which will give the indication whether if you change one of the non basic variable to a basic variable and basic variable to a non basic variable weather function value will reduce further or not that one.

If it shows that the R D transpose the elements of our R D transpose having been more than two negative numbers then consider the largest negative component corresponding to the non basic variable that the basic non basic variable will treated as an basic variable for further iterations. One of the basic variable you have to convert into a transfer enter one of the basic variable will enter as a non basic variable by following further tests, what we have mentioned it. So, this is a summary of the problem, how LP problem is solved by algebraic approach, how it can be solved to this. We will stop here today.