Optimal Control

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Lecture - 11
Post Optimality Analysis, Convex Function and its Properties

So, last class we have seen that how to solve a unconstraint optimization problem using KKT conditions. So, and what is the necessary and sufficient condition also, we have seen and we have worked out some problems in these.

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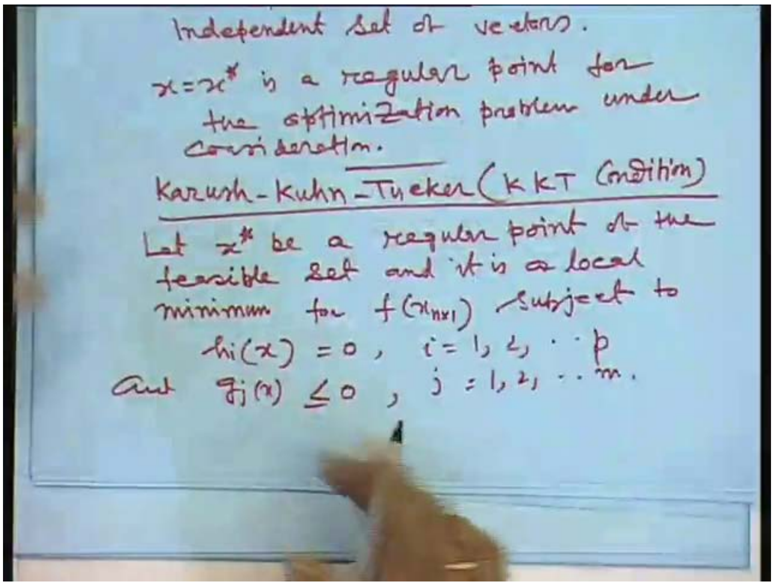

So, quickly if I tell you that what is the, what is the basic problem of this one. We have a function, this function, we have to minimize this function subject to equality constant and inequality constant. So, our problem is to convert this constant optimization problem, into a unconstrained optimization problem by choosing a leaupona function, not sorry, a Lagrangian functions, agree?

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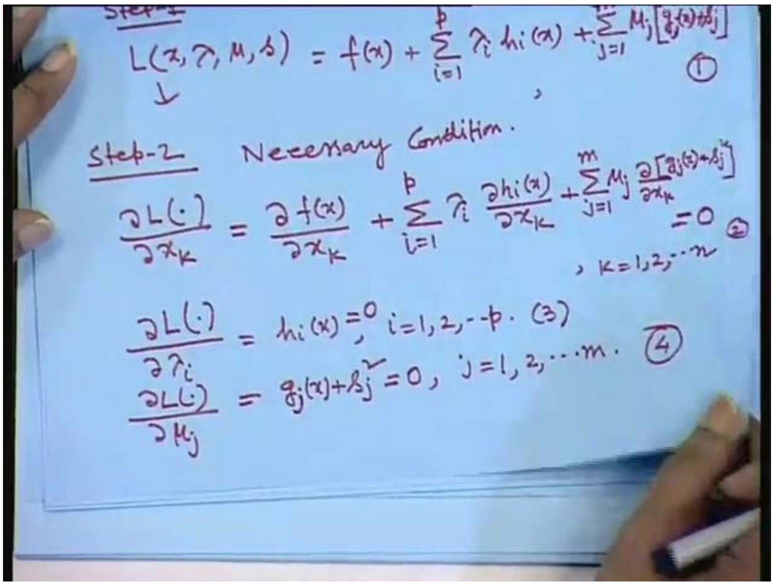

And this is the Lagrangian function, we have considered where, this is the objective function and this lambda i is associate with the, all equality constant and mu i you have to consider, associate with the inequality constant. We made the inequality constant by adding a, what is called a variable s j squared to make it equal to 0, equality constant. The necessary condition means, first that is partial derivative of L with respect to x k lambda k mu k, you have to assign to 0.

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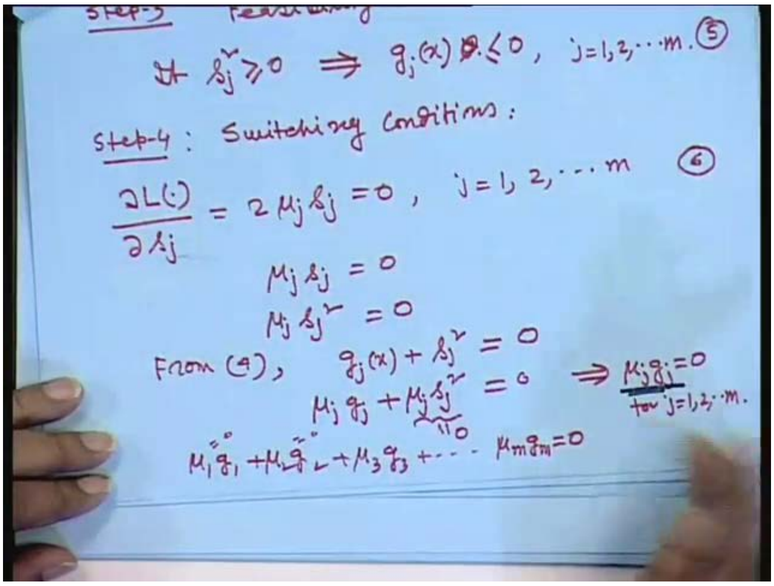

Then we finally, we found out the, this is the expression when you differentiate that Lagrangian function with respect to mu and differential Lagrangian function, with respect to s variables. Then these two constraints, which will be assigned to 0 will ultimately, it will down, boils down to a single what is called expression conditions. So and it, it is, it can be seen that this Lagrangian multipliers mu 1, mu 2 this which is associate with the inequality constant, this g 1 into mu 1, g 2 into mu 2 and g m into mu m, this are the orthogonal, if you form in a vector form and this in a row vector form, they are, orthogonal condition is satisfied.

So, and the when the, when you will get the optimum point of this objective function at that point, at that condition if g 1 or g i is equal to 0, it indicates that it is a active constants is satisfied. When active constant is satisfied, mu i corresponding to mu i value will be greater than equal to 0, that is we have seen it. And also we have seen that, what is the necessary condition, necessary sufficient condition to check whether the function is minimum or maximum this.

So, that hessian matrix of the Lagrangian function you have to check it, if this hessian matrix of Lagrangian function is positive definite then this, that function is a minimum value, that with a minimum value of the function. If it is an negative definite, that is what is called hessian matrix of Lagrangian function is negative definite matrix, that function value half of x will be a maximum, that is we have seen. Now, will see this, what is called the sensitivity analysis, in other words you can say post-optimality analysis.

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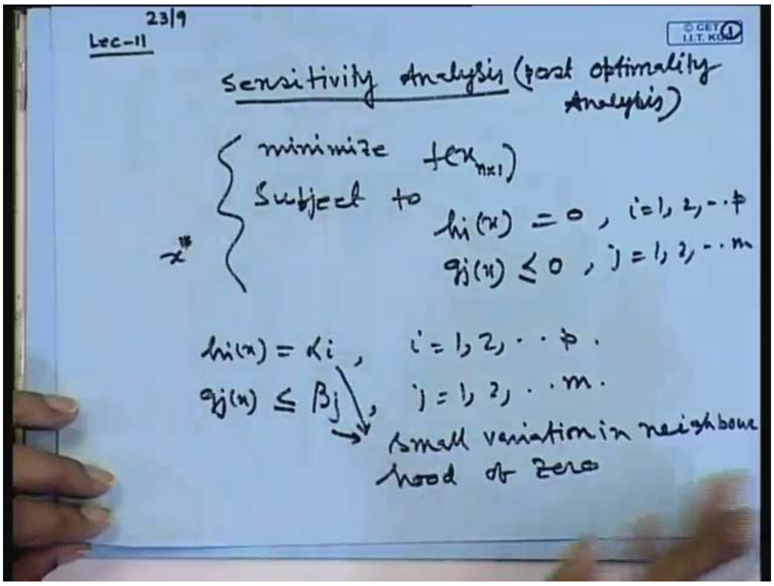

So, now we will see the sensitivity analysis that means post-optimality analysis, post optimality analysis. So, let us call what is the problem is given, minimize a function f of x subject to constants, subject to condition that h i of x is equal to 0, i is varied to that means our problem is minimize f of x and x dimension is n cross 1, subject to, subject to h i of x is equal to 0 and i is equal to, we have p number of equality constants. And g j of x is less than equal to 0 and j is equal to 1 to m constant, that is our basic problem. Now this indicates, we want to study the variation of, in the optimum cost due to the variation in parameters of the original problems.

Means, our original problem is h i x is equal to 0, if in place of 0, if it is equal to f, let us call we consider it as alpha in place of h i x, if this is, in state of this equity constant if it is alpha I, again i is equal to 1 to dot dot p and g j of x in state of less than equality 0, if it is a beta j where j is equal to 1 to m. Naturally this solution will change, previously the solution of this one, if you consider that our solution we obtain the optimum value at this point x is equal to x star. Now, if this in place of 0 this of alpha i equinity constant in place of less than equal to g j less than equal 0, if it is a beta j where this alpha j and beta j are very small part of positive quantity. That means, this are the small variation in neighbourhood of 0, this is.

Then, what is the optimal solution of this problem and how these function value will change from the, this perturbations of parameters so this we have to study this one. So, our problem is now, this is our original problem now that problem. If the parameter alpha is in place 0, if it is alpha which is a positive very small quantity, positive quantity in place of less than equality 0 it is beta j which is positive quantity, but very less than, very small quantity. Then what is the effect of the optimal point on the minimization of these problems so that we want to variation of the minimum optimum value of the function, due to the variation of alpha i beta j, that we are going to study it.

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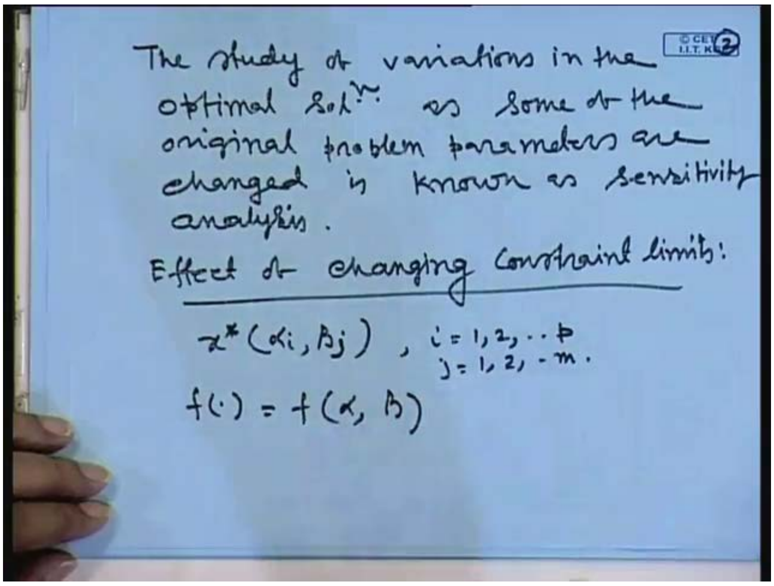

So, in other words the, our main aim is to study, the study of variations in the optimal solution as some of, some of the original, some of the original problem parameters are changed is known as sensitivity analysis, sensitivity analysis. So, that is we are going to study so first we will see the effect of changing constraint limit, constraint limits. So, this is our constant limits, this want this, if we change it with is a positive quantity and small then what is the changing optimal solution of these problems.

So, naturally this x star, let us call x star at the optimum solution which will obtain due to the parameter variation of alpha i and beta i. This new x star is a function of alpha i and beta i agree, then i varies from or you can say that i varies from 1 to alpha i beta j, 1 to dot dot p and j varies from 1 dot dot m. So, this, the solution of, new solution of x star is a function of alpha i and beta I, when alpha is 0 alpha i is all are 0, beta i j we will get our original problem solution. That means, mini, minimization of original problem solution will get it.

So, naturally the function value also, the function value of this one is also function of alpha beta and where alpha means, all elements of alpha 1 alpha 2 dot dot alpha p and beta is all elements of beta 1 beta 2 dot dot beta m. So, this, the function value of f is a, now function of alpha and beta agree. So, if alpha is 0, beta is 0 the vector will get the original value of these, optimum minimum value of the original function at x is equal to x star. Since the alpha is changed to, the 0 is changed to alpha i in the equality constant and inequality constant 0 is now changed to beta I, the solution of this one also change it. So, one can but we do not know how to, how this f is related with alpha and beta x, explicit relationship is not known to us, agree.

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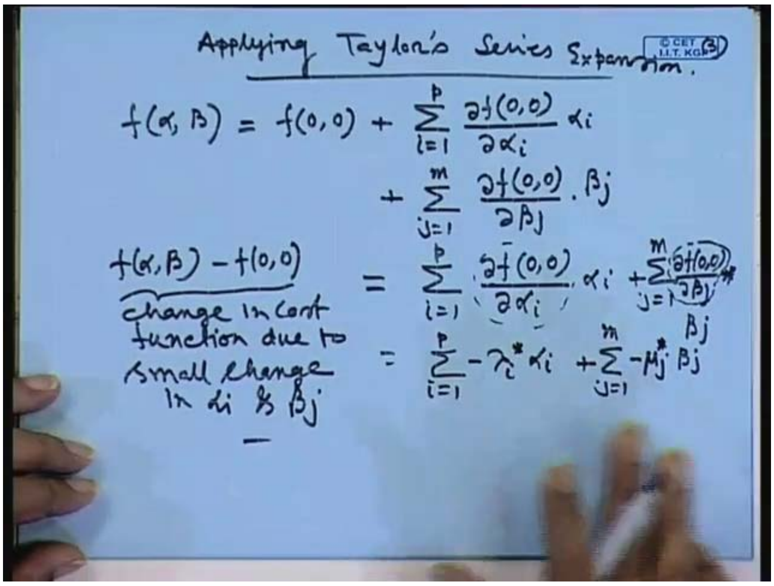

So, we can now write by using the Taylors series expansion in this one, I can write it, now we and write it now like this way applying Taylor series expansion, series expansion, agree. One can write it, now alpha because f is the function of the alpha and beta and alpha and beta are the partavation in right hand side of the equality constants alpha and right hand side of the inequality constant is beta. So, this we can write an alpha is changed from the nominal value is 0, that nominal value in the sense, the original problem in the alpha use 0, in original problem iniquity constant beta is use 0. So, you are doing the Taylor series expansion around 0.

So, 0 0 then we have a summation of, if u see our problem is that we have a p equality constants, summation i is equal to 1 to p. Then del f around 0, we are Taylor series expansion we are doing with respect to alpha i, into alpha i plus, but plus j is equal to 1 to m partial differentiation with respect to around 0, this one with respect to beta j into beta j. So, what is the changing value of the cost function, is alpha beta minus f of 0 is 0, this f of 0 indicates that where, what is a value of the cost function at when there is no perturbation in the equality constant, on the right hand side. That means, equality constraint in the right hand side is said to 0, inequality constraint in right hand side is 0, that is the function value and we correspondingly that, we got it some x star.

So, when there is a partavation is there, that function value of the, optimal value of the function or the minimum value the function, is a function of f is a function of alpha and beta. So, this indicate the changing cost function due to, due to small change in, small change in alpha i and beta j. This is equal to you can write it, summation of i is equal 1 to p delta f 0 0, delta of alpha i into this plus summation of j is equal to 1 to m, delta f 0 0 delta beta j into beta j, that one.

So, this we can write it, if you see this one because this function, this is the, what is this function you can write it, that one, this is nothing but this quantity is nothing but a, i is equal to 1 to p minus lambda i star into alpha i, plus summation of this quantity is mu j. I will explain it, what are it is coming mu j, that is coming mu j minus mu j star into beta j. Let us see how it is coming, that one.

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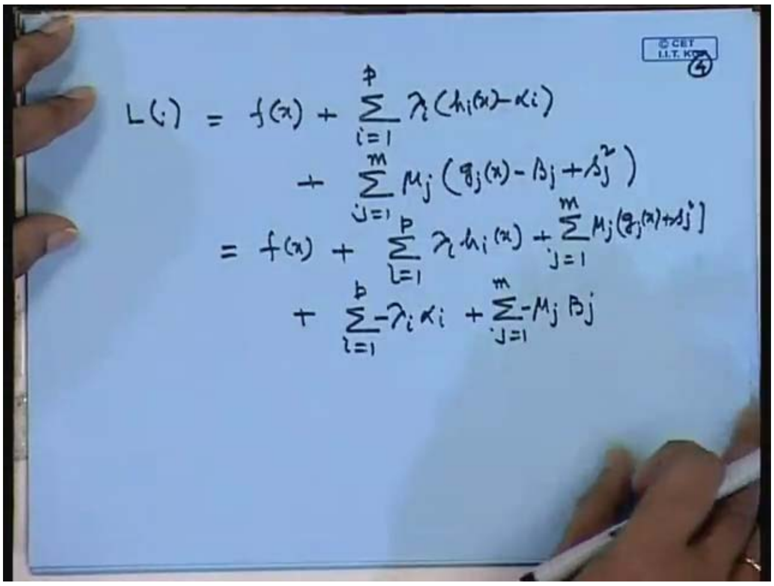

So, we can write the, our Lagrangian function you see, L of this is equal to our f of x plus summation of i is equal to 1 to p. Now our, if you see this one, that is our, this one so what is our equality constraints is part up in alpha i, you take it in left side. So, h i x minus alpha i, I can write it. So, summation of lambda i h i of x minus alpha i, this agree plus summation of j is equal to 1 to m. Now see this one, this is less than equal to this 1 so what we can write it, this I can take it in left side that means g j of x minus beta j is less than 0. So, we have to add something in order to make it equality constant. So, I can write it, it is nothing but a Lagrangian multiplier mu j g j minus, if u call beta j plus s j square, agree.

So, it is nothing but you can see f of x, I can write it this way is equal to 1 to p lambda i h i of x plus mu i is equal to j is equal to 1 to m mu j g j of x plus s j square. This is, when the, is no partavation on the right hand side of the equality constant and inequality constant. This is plus some other, when there is a partavation is there some additional term we are getting here, summation of i is equal to 1 to p lambda i, minus that is, minus alpha i. So, then again plus summation of this j is equal to 1 to m mu j minus beta j, agree. When there is no perturbation is there, we found out the objective function, optimum value of the function of this one.

Now if you differentiate this with respect to lambda, sorry, alpha then you will get it lambda i. So, that is why we have written it here differentiation of, around the origin means, when the perturbation is not there around the origin, this quantity is minus lambda i star, agree. Similarly, if you differentiate these things change in this one, this will be a mu j star minus. So, this now you look at this one, this indicates when there is a partavation inequality constant, partavation in the right hand side, which beta j is a positive quantity in the beginning of considered it is a positive quantity.

This indicates that we are relaxed the, what is called the constant that means, we are, we are, we are when beta is greater than 0, it indicates that we are giving the most, what is called star space. So, there is a possibility of getting further what is called reduction in cost function value. If the space design space or the star space is more compared to earlier stage then there will be a possibility of getting reduction in cost function value.

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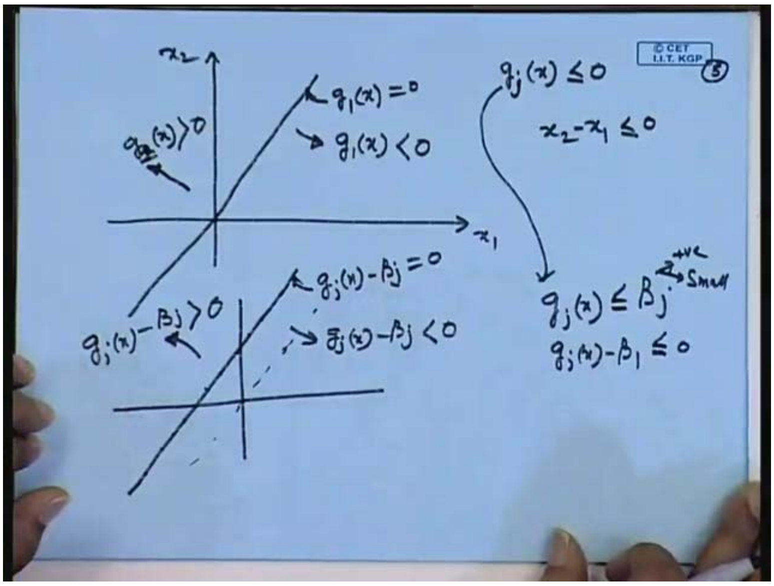

So, just like if you see this one, suppose you have a, just see this one, according to our original problem, if you consider g j g j of x is less than or equal to 0, our original problem we are telling. So, let us call g j is equal to 0, I am this is something like g j, let us call I am showing it this is x 1, you can think of it as if there are two variables are there this is, this equation is g 1 of j g 1 of x is equal to 0, agree. So, you can think of it as if we have a g 1 of x is something like this, x 2 minus x 1 is less than equal to 0. So, this is something like this, this is straight line.

Now, what is this portion? This portion is g 1 of x is less than equal to 0 and this portion is the whole half space, this space and this space is g 2 or g 1, this is g 1 of x is greater than equal to 0. And any point on the line is g 1 is 0 now, if u part up this one, if you part up this one g j of x is equal to beta j, that is what I am telling. Then what is this situation is there? Then this line, if you say this line is now, is becoming here. So, what is this one just see from this equation, if you take this g x is equal to beta, agree then this one will be g j of x minus beta j is equal to 0. I can write it this one like this way, our condition this I am not equal to, this condition I am now part up with beta j.

So, this I can write it now beta j is less than equal to this one so this, if you consider this type of equation, this on the line, any point on the line g j of x minus beta j is 0. Now this is shown that g j of x minus beta j is less than equal to 0, this portion and this portion is showing g j of x minus beta j is greater than equal to 0. Now you see previously, when there is no partavation of the beta, but beta e is positive quantity, this is positive then our star space was geta 1 equal to geta j, i only one constraint with geta 1 of is less than equal to 0. That means, it indicates the whole space of this space is the, our constraint on the line and below this line.

Now, when you put this part up with a inspiration 0, it is a part up with a beta j which is a small quantity is a small variations, agree. Now you see, our design space or the star space is increased previously, previously it was from here this space, now it is become from here to the whole that right up, right portion of the straight line, below the straight line. So, our star space is increased so we may expect that, that function value, cost per that is a cost function or the objective function value may reduce further, from the previous one, agree.

So, there is a possibility is there or at most, at most there will be no change in function value from the previous situation that, that is we can come to conclusion. So, if you relax the, what is called constraint that design space or the, what is called the star space of the optimization problem is the relaxed or increased, which in turn we can say there, there is the possibility of getting reduction, that cost function value is reduction compared to the earlier stage.

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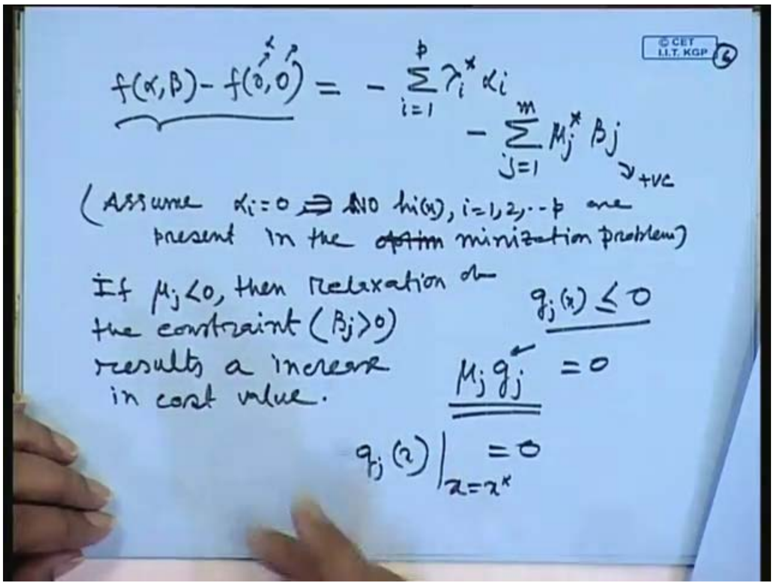

So, keeping all this in mind, we see this one that what is the, we got this our expression, if you see f of alpha beta minus f of 0 of 0, this indicates f of 0 comma 0, this alpha is equal to 0, beta is 0. From there we got the cost function some below, when there is no partavation, when there is a partavation that got function value is change. Because, optimum value of value is changed, optimal value of the optimum point is changed, corresponding function value is changed, which is a function of alpha and beta. So, this value you have seen just now, it is a minus summation of i is equal to 1 to p lambda i star alpha i minus summation of j is equal to 1 to m mu j star into beta j, agree.

Now, you consider from this, we assume, let us call we assume for the time being there is no equality constant present in the optimation problems that means, there is no partavations yet, there is no bigger h of j h of i is not present in the optimation problems. So, this part will not be there let us concentrate with this one, which is the inequality constant is there, which is a less than equal to 0, that is part up with a beta j. Now you see this one assume, assume that alpha i is equal to 0. Means, this implies that h, there is no inequality, no inequality constant present in the optimism problem, no inequality constant, inequality constant for i is equal to 1, 2 dot dot p are present in the optimization problems or minimization problems or in the minimization problems.

In that situation you see, change in function value due to the change in that, what is call beta j is what minus of this one and we have considered, if you recollect this one, will consider this is a small quantity and positive. If beta to a new value, that is a Lagrangian multiplayer value, if it is a, what is called a negative quantity, if it is a negative quantity you see negative and negative, this becomes a positive. So, function value it indicates the function value is increased.

So, this contradicts our assumption what we made it that, if we relax the constraints, that is what if we relax the constants that means, which in turn we have increased is the star space, there is a possibility of reduction in cost function. But here it shows the cost function is increased, when beta is positive and mu value of j Lagrangian leaving a multiplayer value is coming negative, if you consider negative then the function value is increased. So, it contradicts our, that relaxed conditions, this one. So, it cannot be happened.

So, mu value must be what is called a positive quantity, agree. So, mu value must be positive. Then what and what context we are telling this one, mu value must be positive that, if you see this one, if you see one that is what our, our constant equation of I will write it, the constant equation that mu j of x is less than or equal to 0, this constant equation. When this active constants are satisfied at the optimum point, when active constants are satisfied at the optimum point. Means, this means that from the switching condition mu j g j condition is equal to 0, when this is, when this is 0 g j of 0 at the optimum point mu j value is positive, nonnegative number, agree. That is we have shown it now here from, from this one, that should be nonnegative number mu j and.

So, you are writing this one, if mu j is less than 0 then then the relaxation of the constant for beta j greater than 0 results, visible results a increase in cost value. So, this is minus, if it is less than this, this is minus plus was a increasing there. So this is contradicts, this contradicts our assumption that if you relax this one. So, this contradicts our, that assumption, when this constants are relaxed, agree. So, this cannot be happened.

So, mu value mu j value must be positive. So, if you recollect, when we are doing the switching conditions, this condition when solving the KKT, using the KKT necessary condition, we have assume that, we have consider when the active constants are satisfied. That means, when mu j of x, x is equal to x star, if it is 0 that is x term, x, what is called active constants are satisfied.

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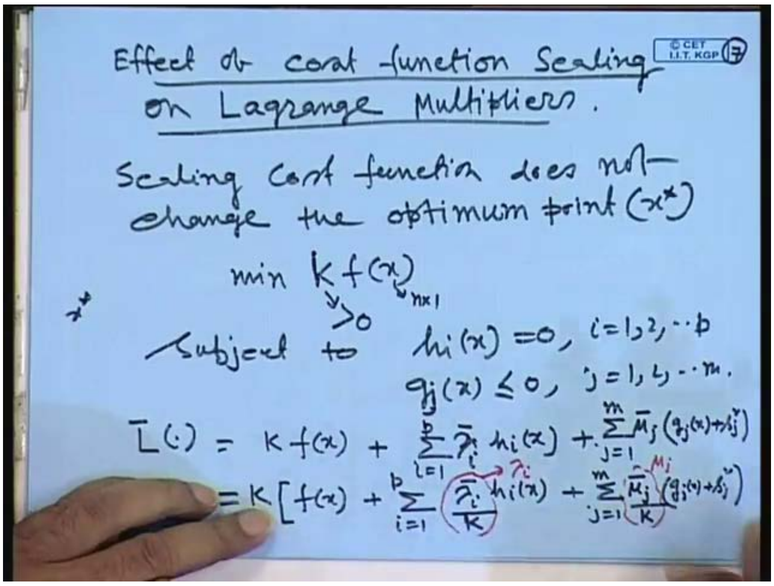

Then mu j value, mu j value will be nonnegative number and this proves here, that should be nonnegative number mu j, it could be negative, agree. So, this is our conditions we have obtained. Now see this one, what is the, if you multiplied by the cost, cost function if you multiplied by the cost function by a positive scalar quantity, scaling the cost function by a scale, what is positive quantity. Then what is affect in the optimal conditions, not conditions, what is the pro-optimal point will change or not.

So, next our study is effect of cost function scaling on, effect of cost function scaling on LaGrange multipliers. So, that is, that means if you scale the objective function or cost function, what is this effect on the Lagrangian multiplayer, that we have to study it. So, let us call scaling cost function does not change the optimum point that means x star, that if the possible point is x star, x star do not without scaling, agree.

If you get the optimum value of the function at the optimum point this, with scaling also you will get the optimum value of the function, you will get different, but at optimum point will remain unchanged, but. So, Lagrangian multiplayer value will change it. Let us see what is this suppose, we have a problem minimize f of x k, k is a greater than 0, positive quantity, agree.

This and this x dimension is a m cross 1, will minimise this one subject to h i of x is equal to 0 and i is equal to 1, 2 dot dot p and g j of x g j of x is less than equal to 0 and j is equal to 1, 2 dot dot m. So, this is our thing so our problem, previous problem was like this way, minimize f of x subject to this constant, agree. Whatever the optimum point you got it, let us call x star now, I have changed the objective function which is multiplied by a constant, positive constant value. Now question is, thus the optimum point will change? Answer is no, first.

Second question is that, what is this effect on the Lagrangian multipliers lambda i and mu j, what is the effect? And definitely the cost function value will change it. So, let us say we, corresponding to this objective function on Lagrangian function L bar, you have considered, is our cost function. Now, our new cost function is k into x plus summation of i is equal to 1 to p, agree. Then lambda, that corresponding to this new object function let us call that lambda i, we have consider that is bar, corresponding to Lagrangian multiplier bar into h i of x plus summation of j is equal to 1 to m and corresponding the Lagrangian multiplier for this one.

Because, cost function is changed by multiplication vector k, positive k that equal to mu j bar and this you have to convert into equity constant, that g j of x plus s j square. So, what is it k, i take it common, k is a scalar quantity, I can take it common f of x plus summation of i is equal to 1 to p, same thing lambda i bar and lambda i bar. And lambda i bar corresponding to the new objective function, Lagrangian multiplier associate to the equality constant. And mu j bar is the Lagrangian multiplayer associate with the inequality constant, corresponding to the new objective function k, into this. So, this is h i of x plus summation of j is equal to 1 to m mu j and that is, I take it common. So, that is divided by k and mu j bar divided by k into g j into g j x plus s j square, this.

Now I will consider, now you see if I consider this is because this is divided by constant quantity. So, whatever the sign will come in turn in lambda i bar, the sign will not change by this scaling. Similarly, mu j, what mu j value i know greater than is equal to 0, nonnegative number is and k is the positive and this will not change, will not affect anything about its sign. So, let us call this I is denoted by this one, I is denoted by lambda i, this I denoted by mu i.

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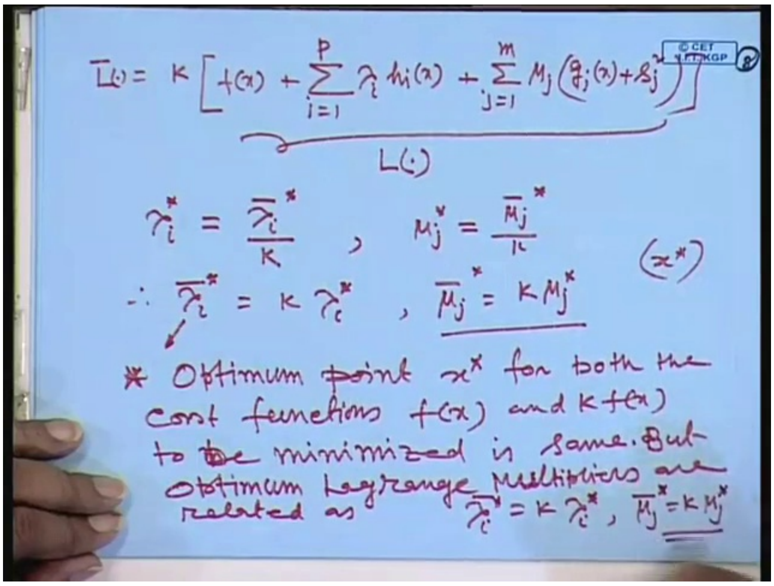

Now, you see it is something like this, instead of Lagrangian function, necessary condition if you see, I am just, it is, this I can write it. Now, this one I can write it, is equal to L bar dot, is equal to k into f of x plus summation, i is equal to 1 to p lambda i, agree h i of x plus, sorry, this is lambda I, yes then j is equal to 1 to m and mu j g j of x plus s j square, this. Now see this one, I have given lambda of this and now it is equivalent to, if you see this one, this equivalent to our original problems, Lagrangian function as if original problem in minimize to f of x subject to this constraint, this corresponding Lagrangian function is this one, agree.

So, minimization of this one is same as the minimization of that one. So, naturally what is called call our, optimum point will not change it, this is you can see from this one. Now, what is the Lagrangian of function for the old, for our old optimization problem our Lagrangian multiplayer is lambda i. And lambda i what is this we got it, lambda for new system is this one, we got it, let us call this is optimum value, this is also you got it optimum value of that one, agree. Similarly, mu j is equal to mu j star is equal to mu j bar star divided by k. Therefore, lambda i bar star is equal to k into lambda i star similarly, mu j bar is equal to k into mu j star.

So, what is our conclusion? Our conclusion is, if our original problem is minimize a function f of x subject to this constant, what is the optimum point will get it, optimum point at which will get the minimum value of the function, agree. Suppose, x star optimum point, now the function is killed by a, that a cost function is killed k into f of x and subject to the same equality and inequality constant then our optimum point will not change it, agree. And our Lagrangian multipliers of the new system, new optimation problem when you multiplied by k, this is multiplied by, k will be multiplied by original problem Lagrangian multiplier, which is associate with the equality constant.

Similarly, the scaled function Lagrangian multiplayer for optimation of scaled function minimisation of scaled function, objective function, this Lagrangian multiplayer will be multiplied by k into that original system Lagrangian multiplayer mu j, which is associated with the inequality constant. So, we can make a conclusion like this way now ultimately, the optimum point x star for both, for both the cost functions f of x, that minimization this and k f of x, both cost function only this, this to be minimized, to be minimized. Optimum point is ours, the optimum point is same, that is what and or you can say, but the optimum Lagrangian, optimum LaGrange multipliers are related by, multipliers are related as.

So, is equal to k into lambda i start that means, scaled, the scale of their cost function objective function, Lagrangian multiplayer is equal to k into the, what is called the original object function Lagrangian multiplayer. This expression another is mu j bar star is equal to k into mu j star, but all the minimum value the function is changed, that we have to multiply by k into f of x what is and at that point optimal point. So, this next is our, what is called scaling a constant.

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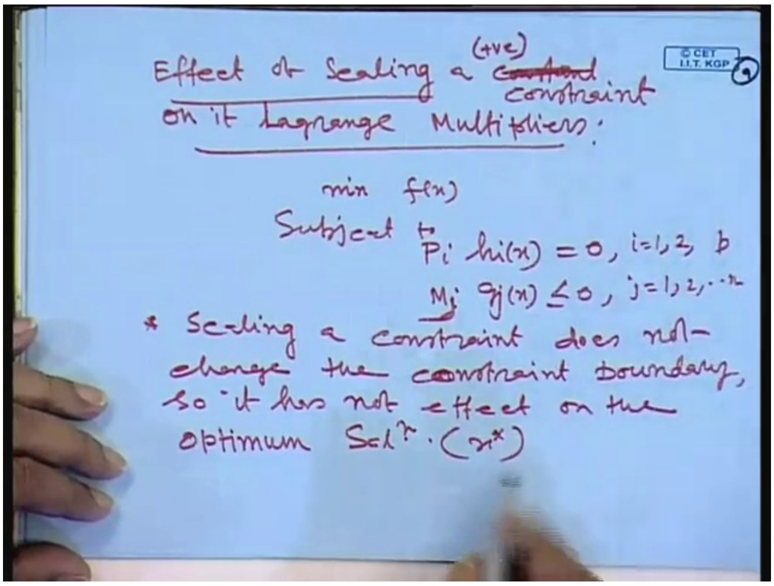

Next is effect of scaling a constant, constraint, effect of scaling a constraint we have a constraint equality constraint and inequality constraint, agree. So, effect of scaling that equality constraint or a inequality constraint, we scaled by a positive number, scaling with a positive quantity, scaling it with a positive quantity, agree, constraint on its LaGrange multipliers. So, this we will study it means, if you recollect this one, that our problem is what minimize, once again minimize f of x subject to h i of x is equal to 0 i is equal to 1, 2, p and g j of x less than equal to 0 j is equal to 1, 2 dot dot m. Now we are doing the scaling, this constraint, equality constraint is scaled by a quantity.

Let us call, multiplied by capital P i, P i is, capital P i is the scaling factor which is greater than 0 means, positive quantity. This is also multiplied by M i that is, this is multiplied by P i, this is multiplied by M i, M j. So, if you multiplied by a positive quantity, the constraints are not change at all. So, this I multiplied by M j so whatever the constants are there, previous problem was there, original problems multiplied by constant mind it, I am telling it is multiplied by positive quantity. The constant will not change this one means, that whatever the whatever, we have the feasible region is there, agree. So, feasible region of this problem will remain unchanged.

So, our, that will see it later, scaling a constraint does not change the constraint boundary, does not change the constraint boundary, if it does not change the constraint boundary naturally it has is no effect on the optimal solution. So, naturally so it has because objective function is changed only we have multiplied by each equality constant or inequality constant by a scalar quantity, which is positive greater than 0. So, the constant boundary will not change, in other words that our design space or star space remained unchanged due to the, even if you multiply by the constant, positive constant quantity. So, it does not so it has not effect, effect on the optimal solution that means, x star.

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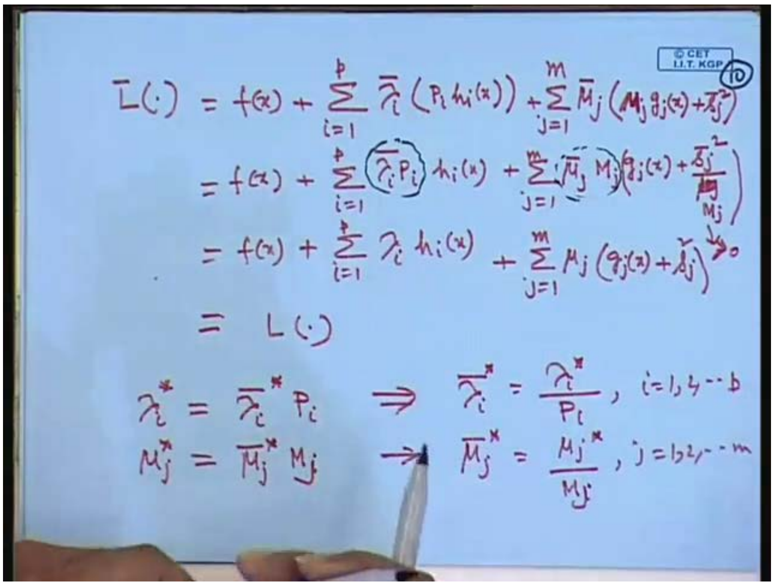

So, let us see this one how it is not so as you recall this one, this is our now constants multiplied by this. Now, what is our Lagrangian this, this one? If you see the Lagrangian equation of our new operation problems is f of x plus summation of i is equal to 1 to p and this constant is changed, agree. When there is the only h of i g j of i, we consider corresponding Lagrangian multiplayer is mu, lambda j associated with s j and mu j associate with g j. Now I am considering new Lagrangian multiplayer lambda bar, our equity constant is p i h i of x, this part is over. Next, summation of j is equal to 1 to m mu j bar and what is our, that equation, mu j, that mu M j not mu, this is M j g j of x plus s j bar mu variables are considered this.

Because, this is our constant now inequality constant, we have to add some variables, positive quantities in order to make it equal to 0 means, equality constant, this so this. So, let us see this one, what you can write it, this, then summation of i is equal to 1 p lambda i bar p i h i of x plus summation of j is equal to 1 to m mu j bar m, M j g j of x or if I take it M j, this M j common, if I take it M j common, this plus s j bar square mu j, sorry, this is not mu j, M j, agree. Now m is a positive quantity, this is positive quantity. So, s square is a positive quantity divided by positive quantity is same here also, this is positive quantity, agree.

So, this I have mentioned it here that you have multiplied it by positive quantity then constant boundary will not change it. So, this you can write it this is greater than 0, this is greater than 0, that one in a positive quantity. Now you see, f of x is equal to, this I am now writing this, since this is associate with only h j. This I can write it summation of i is equal to 1 to p lambda i h i of x plus summation of j is equal to 1 to m, mu this I am writing, this I am writing as a mu j g j of x plus this quantity is another positive quantity mu j square, agree. Now you see, this is nothing but our, this event the Lagrangian function is same as our original problem Lagrangian function, without multiplying the constant with this one.

So, I can write it now this nothing but a, you can defined is nothing but a value. So, the optimum value of this function, agree an optimum point, agree optimum point will not change it. Because, function will, by this function will remain only the minimum, what is called the Lagrangian multiplier are changed. So, you can write it that lambda i is equal to star optimum point, is equal to lambda i bar star P i, which equal to implies that lambda i bar star is real to lambda i star divide by P i and i varies from 1, 2 dot dot p. Similarly, we can write it similarly, can write it mu j star is equal to mu j bar star M i, M j which is equal to mu j star is equal to mu j star is equal to mu j M i M j star, this is M j is equal to 1, 2 dot dot m.

So, if you multiply the constant by its positive square quantity, both the constant equity on the optimum point does not change it, that if you see Lagrangian function remains same. Next we can conclusion that, we have a Lagrangian multiplayer in both cases are scaled by that P i and M i this one, but does it the cost value will change. Since, the optimum value is not changed and the optimum function value, which one these, cost function objective function is same. So, the cost value will remain same. So, this we have seen the, what is call effect of that, what is call optimality testing, what is called post.

So, next we just consider that, if we just consider, see this, our first slide of this one, we have just, the sensitivity analysis, post optimality analysis we have done it. That means, if there is some changing parameters, inequality constraint all these things, we have studied its effect. And from that we have drawn conclusion that mu j, the Lagrangian multiplier associate with the, our objectives, iniquity constraint that quantity must be greater than 0, greater than equal to 0, nonnegative number. When we are dealing with the optimization minimisation of function, that is reaction under what is called active constant that mu j value will be nonnegative.

And this and we have seen that, the if objective functionaries multiplied by a cost function, some scalar quantitative, positive scalar is the quantity then its optimum point does not change it, agree. Only its Lagrangian multiplayer functions will change it by scalar, this one. And next is, if you multiplied the constraints, equality constraint on inequality constraint by, positive quantity the constraint boundary does not change it, agree?

And which in turn the optimum point x star, we have shown it optimum x star will not change and this implies that, the optimum value the cost function will not change it. Only the, what is will change it that one, we have seen the Lagrangian multiplayer, the lambda i star means, new optimization problem lambda, multiplied vector is lambda i star by P i divided by this. So, next class we will just discuss the convex operation problems, how to formulate definition all these things.

Thank you.