Applied Linear Algebra for Signal Processing, Data Analytics and Machine Learning
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Lecture No. 08
Applications of Matrices: Electric Circuits, Traffic Flows

Hello, welcome to another module in this massive open online course. So, let us continue our discussion on the application of matrices and linear algebra, and in this module let us start looking at another very important and interesting area of application and that is related to circuits.

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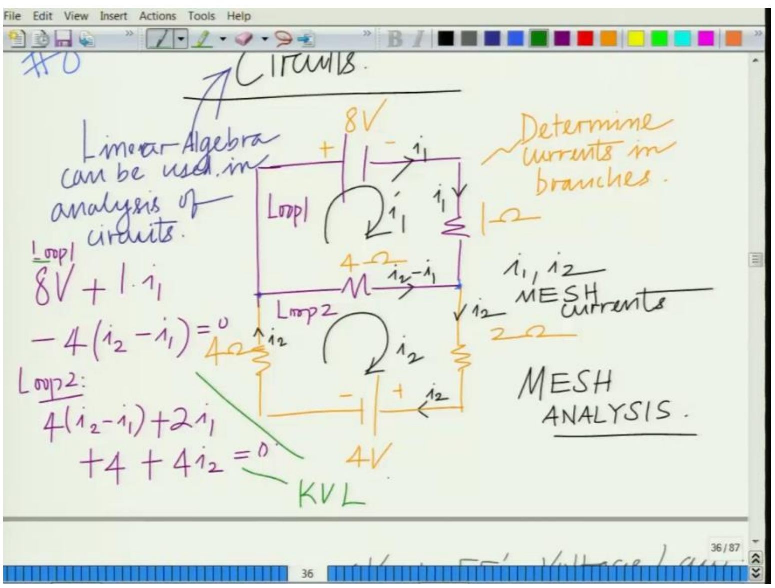

So, circuit analysis is one very important area where these principles of linear algebra can be applied. Let us consider a simple circuit that is shown in screenshot.

So, we want to analyze this circuit to find or to determine the currents in the different branches. So, in this circuit we want to determine. So, once again linear algebra can be used for the analysis of circuits. So, what we were saying is the fact that this linear algebra and principles of linear algebra can be used in the analysis of circuits. So, you want to determine the currents in these branches. So, we use a technique that is known as mesh analysis.

So, what is this mesh analysis, in each mesh we have a separate current. So, i_1 is the current in mesh 1 and i_2 is the current in mesh 2. So, naturally the branch which is both in mesh 1 and 2. You can see there is a current from mesh 2 to the right and current from mesh 1 to the left. So, this will be $i_2 - i_1$. So, this is the mesh analysis i_1 and i_2 these are the mesh currents.

So, we are going to use the principles of mesh analysis to determine these currents. What we have is we have these two independents meshes. So, we have the current i_1 in mesh 1, i_2 in mesh 2 and edges that overlap between these measures you have to determine the currents appropriately like what we have determined in this circuit.

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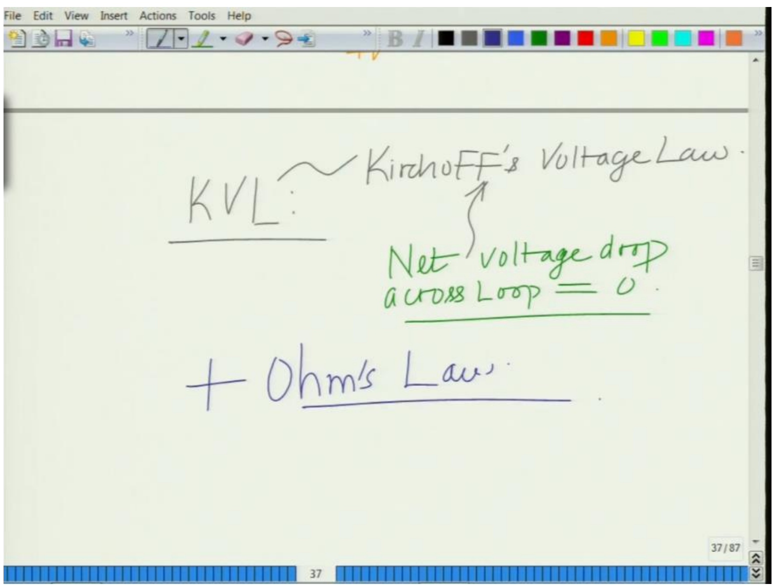

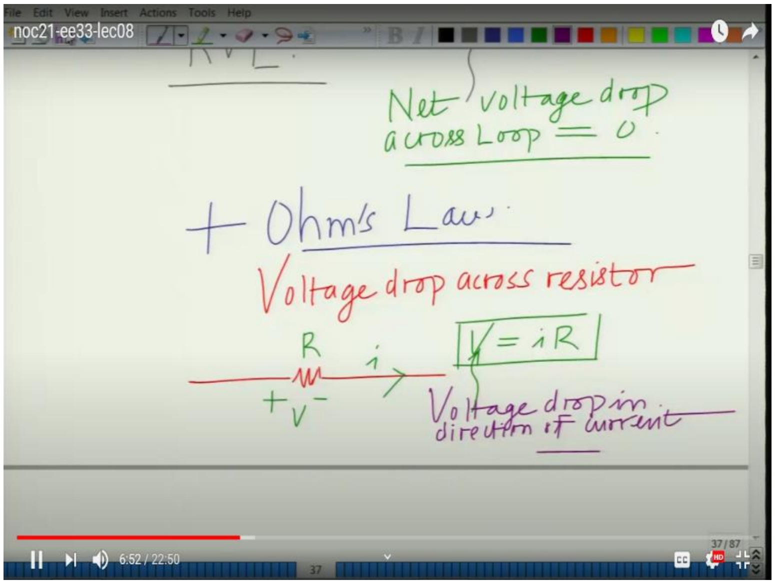

Now, the other principles that we are going to use is, we are going to use KVL. I think most of the people in electrical engineering, I mean even basic engineering, because this is taught as a core subject. So, most of the students who are in engineering and also for that matter of science should be able to appreciate this, this KVL this is Kirchhoff's Voltage Law which basically states that the net voltage drop across a loop is 0.

And plus, we are going to use another law that is the Ohm's law which of course, most of you should be again familiar with it, which is the voltage drop across resistor, which is a linear element, that is, let us take a look at this resistance plus or minus and then you have this resistance R you have the resistance R and then you have the current i.

So, you have $V = iR$. So, voltage drop so, the way to read this is, the voltage drop in the direction of the current, the voltage drop across this resistance in the direction of the current, that is, we are going in the direction of the current across a resistance of value R the voltage drop across that resistance is given as $V = iR$ in the direction of the current i.

So, now, let us go back to our circuit and let us start the analysis of our circuit. Let us start travelling clockwise in this, if you travel clockwise, let me just write these equations here, so that you will be able to follow this. If you go to clockwise, if you call this loop one. And if you call this loop two, clockwise going in loop one you start with the battery elements so, there is

$$8 + i_1 - 4(i_2 - i_1) = 0 \Rightarrow 5i_1 - 4i_2 = -8.$$

So, this is basically your loop one. And if you look at loop two, once again following the same procedure

$$4(i_2 - i_1) + 2i_2 + 4 + 4i_2 = 0 \Rightarrow -4i_1 + 10i_2 = -4.$$

So, basically both of these are obtained using KVL. So, both of these follows essentially from KVL which states that the Kirchhoff's Voltage Law which states that the net voltage drop across a loop is basically 0. So now let us simplify these equations. Let me rewrite these equations for both the loops.

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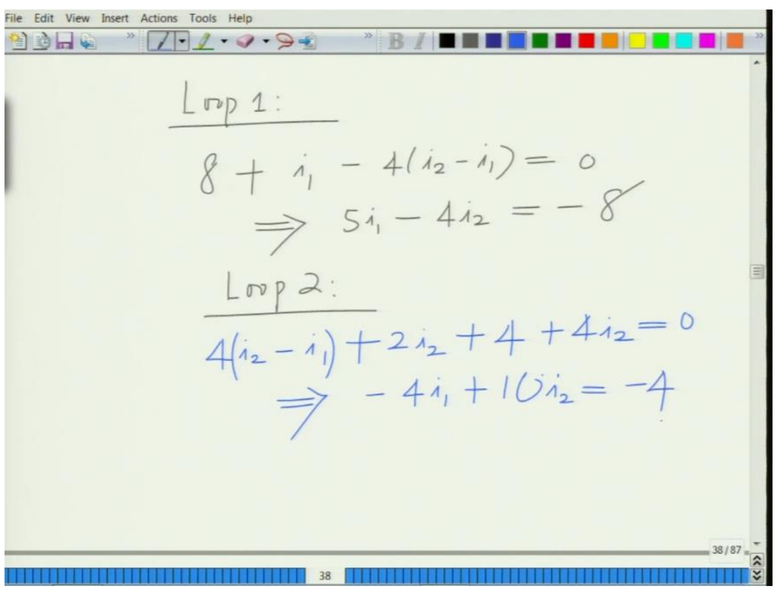

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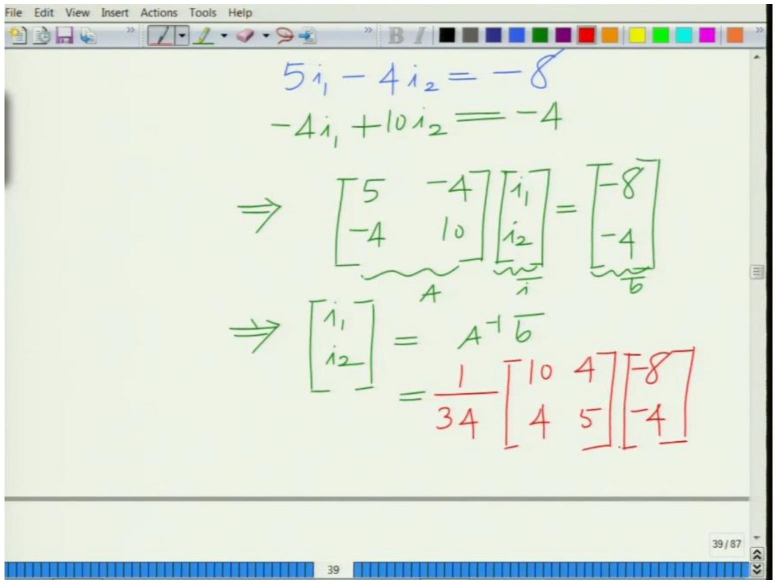

So, put these equations together one obtains

$$5i_1 - 4i_2 = -8$$

$$-4i_1 + 10i_2 = -4,$$

you put these things as a matrix, you will have the matrix by the system of linear equations.

$$\begin{bmatrix} 5 & -4 \\ -4 & 10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -8 \\ -4 \end{bmatrix}$$

So, this is all the form, the matrix **A** times the current vector $\bar{\boldsymbol{\imath}}$ bar is equal to $\bar{\mathbf{b}}$, i.e., $\mathbf{A}\bar{\boldsymbol{\imath}} = \bar{\mathbf{b}}$

So, we have

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} -8 \\ -4 \end{bmatrix}.$$

You know how to compute the inverse of a 2D matrix. So, this is

$$\mathbf{A}^{-1} = \frac{1}{34} \begin{bmatrix} 10 & 4 \\ 4 & 5 \end{bmatrix}.$$

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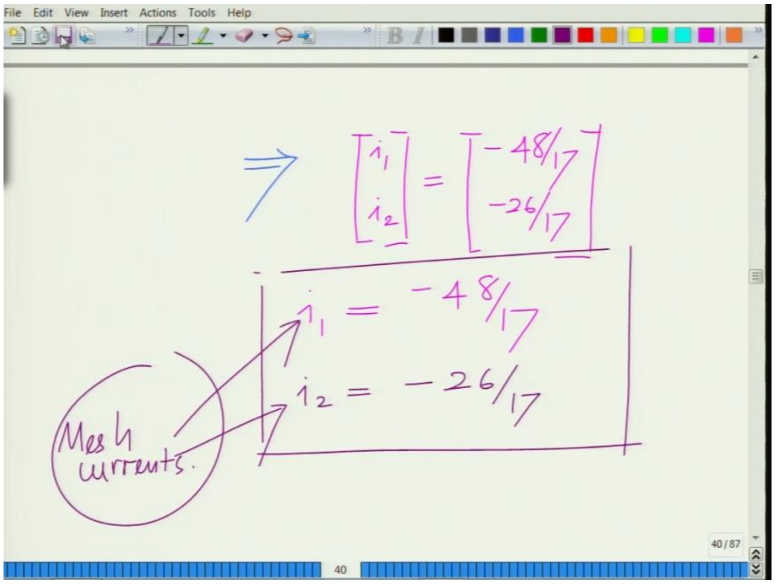

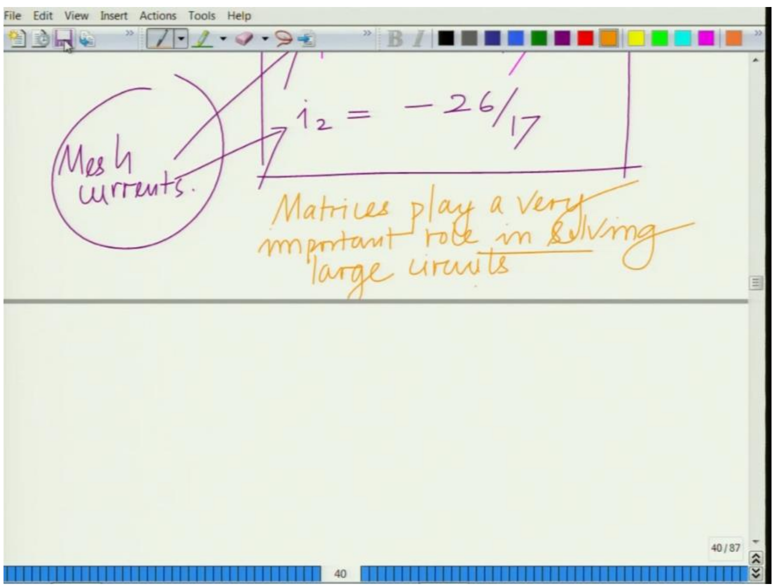

And if I simplify this therefore, you will have the currents

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 10 & 4 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} -8 \\ -4 \end{bmatrix} = \begin{bmatrix} -48/17 \\ -26/17 \end{bmatrix}$$

So, these are the mesh currents so, basically what you have seen is linear algebra can be used to solve circuits. Especially, it is very helpful to solve very large circuits and in fact, this is one kind of analysis you can also do an equivalent analysis to find the node voltages.

In fact, that will then use Kirchhoff’s Current Law which states that the net current that is entering any node or net current that is leaving any node is basically 0. So, this that is known as the KCL, that is the Kirchhoff’s Current Law. So, therefore, what this shows is matrices playing a very important role. So, the idea here is to show through this simple example.

Matrices play a very important role in solving large circuits so, that is the point. So, linear algebra and matrices in general have a number of applications in the analysis and solution of circuits. Let us look at another interesting example that is to solve traffic flows, analyze traffic patterns for instance in cities, across roads and this can be used to public transport and so on and so forth.

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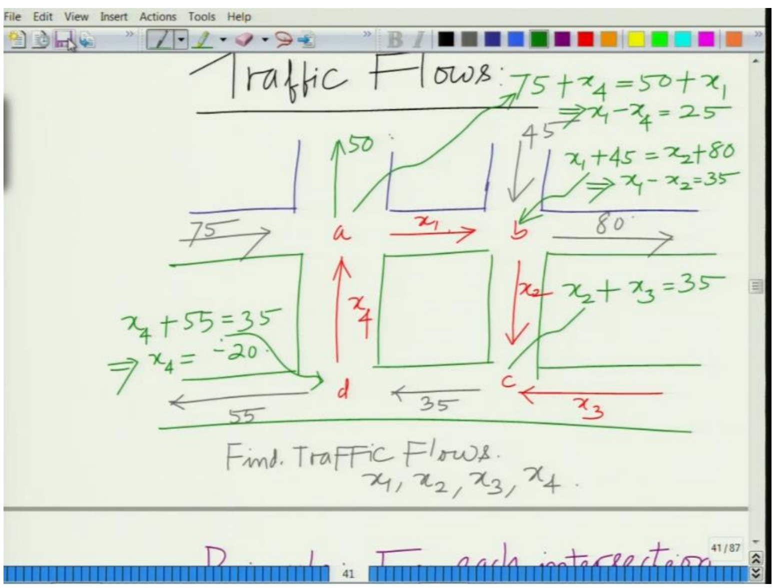

So, another interesting application of this is to basically analyze traffic flows, how do you apply linear algebra to solve traffic flow? Once again, let us take a simple example. Let us take an example of a grid of roads. So, you have a city which is in blocks and then you have the grid of roads. So, let us look at the traffic. So, for instance you have a flow of 75 vehicles, you have a flow here of 50, you have a flow here of 45, you have a flow here of 55 and now you have these unknown flows, so, this is the flow of 80.

Now, let us denote the unknown flows, for instance this is the first floor x_1, x_2, x_3 and this is the unknown floor x_4 and these are the intersections a, b, c, d these are the intersections. Now, we need to find the traffic flows. So, we need to find the traffic flows. Find the unknown traffic flows x_1, x_2, x_3, x_4 and once again we can use the same principle. Similar to Kirchhoff's Current Law. Same principle that is the traffic entering any intersection has to be equal to the traffic leaving that intersection otherwise there is going to be a progressive buildup of traffic in that intersection which is unsustainable. So, that is the principle. So this can be solved using the principles.

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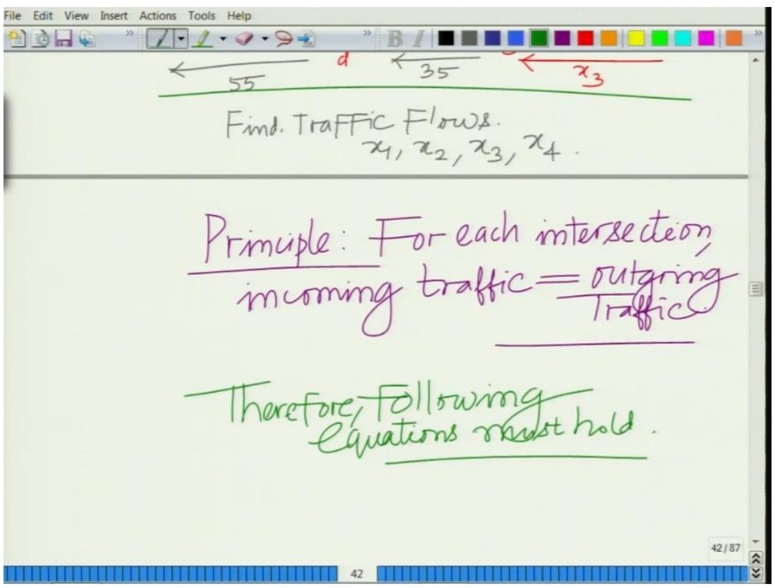

So, what is the simple principle to analyze the traffic flow? What is the principle? For each intersection, incoming traffic has to be equal to outgoing therefore the following equations must hold. Therefore, the following equations must hold let us just write it over here probably because you will find it simple because we have this figure over here. So, if you look at intersection a we must have

$$75 + x_4 = 50 + x_1$$

This implies that

$$x_4 - x_1 = -25$$

for the intersection b we have

$$x_1 + 45 = x_2 + 80 \Rightarrow x_1 - x_2 = 35.$$

for the intersection c we have

$$x_2 + x_3 = 35.$$

For this intersection d, of course, this is simple this intersection is simply

$$x_4 + 55 = 35 \Rightarrow x_4 = -20.$$

So, using this principle at each of the four intersections, we have been able to find the system of equations now, all we have to do is, we have to write the system of equations in the form of a matrix and then you can use the solution of this system of linear equations to solve this traffic flow problem.

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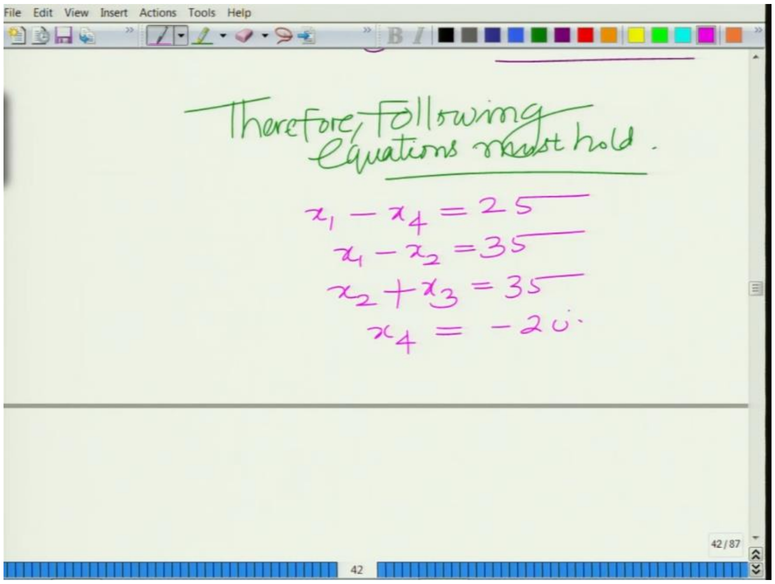

So, essentially what we are going to now write this as we are going to now write this as knowing the four equations you have

$$x_1 - x_4 = 25$$

$$x_1 - x_2 = 35$$

$$x_2 + x_3 = 35.$$

$$x_4 = -20.$$

So, now we are going to write this in the form of a system of equations.

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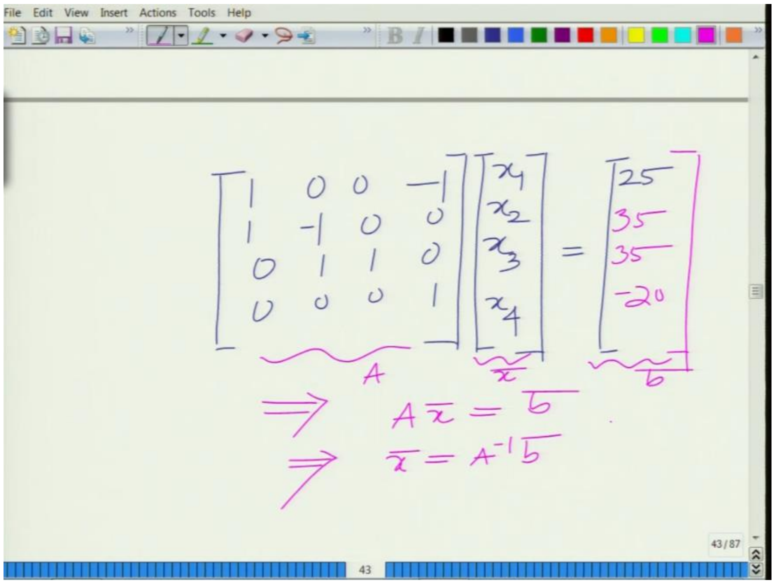

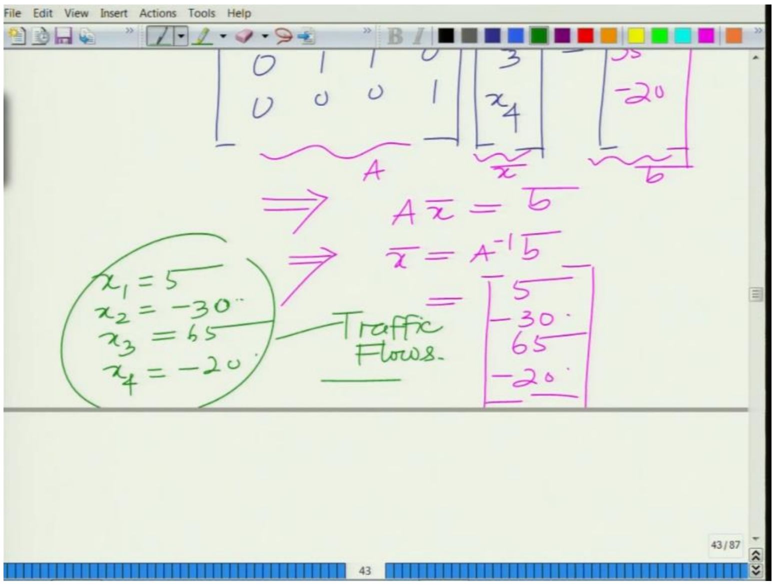

You write the matrix, what is the matrix that is

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 25 \\ 35 \\ 35 \\ -20 \end{bmatrix}$$

What does this imply? This implies of course, now, you have another system which is of the form $\mathbf{A}\bar{\mathbf{x}} = \bar{\mathbf{b}}$. So, this implies that $\bar{\mathbf{x}} = \mathbf{A}^{-1}\bar{\mathbf{b}}$, and if you solve this system of equations, you will find that the solution is basically

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5 \\ -30 \\ 65 \\ -20 \end{bmatrix}$$

So, these are basically your traffic flows, so, you can check and verify these things. So, that is interesting. So, essentially what you have seen in this module is basically seen how powerful linear algebra can be with diverse applications. We have already seen it employed in wireless technology.

We also now seen that it can be employed in traffic flow analysis and can also be employed in circuit analysis and we will see more such applications and we will continue to see more such applications as we move through this course. So, let us stop here and let us continue in the subsequent module. Thank you very much.