Applied Linear Algebra for Signal Processing, Data Analytics and Machine Learning

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Lecture 30

SVD application in MIMO wireless technology: Spatial-multiplexing and high data rates

Hello. Welcome to another module in this massive open online course. So we are talking about the singular value decomposition, we have looked at it and we have noted, it is a very important concept for matrix manipulation, a very important tool rather for handling matrices and manipulating matrices. Let us look at applications, some of the interesting applications of this SVD concept and one of the most interesting applications is in the context of, once again, MIMO wireless communication.

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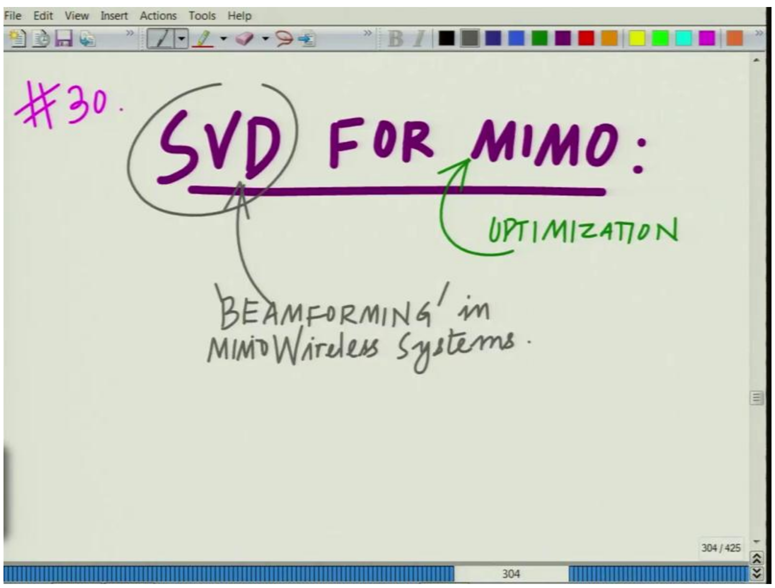

So let us look at SVD specifically for MIMO systems and how SVD helps in the optimization. We are going to eventually look at a very important part of MIMO optimization. In fact, SVD is used for Beam Forming, the idea is that SVD, this can be employed for what we call as Beam Forming, as we have looked at in several times, that is, Beam Forming in wireless systems, especially MIMO wireless systems.

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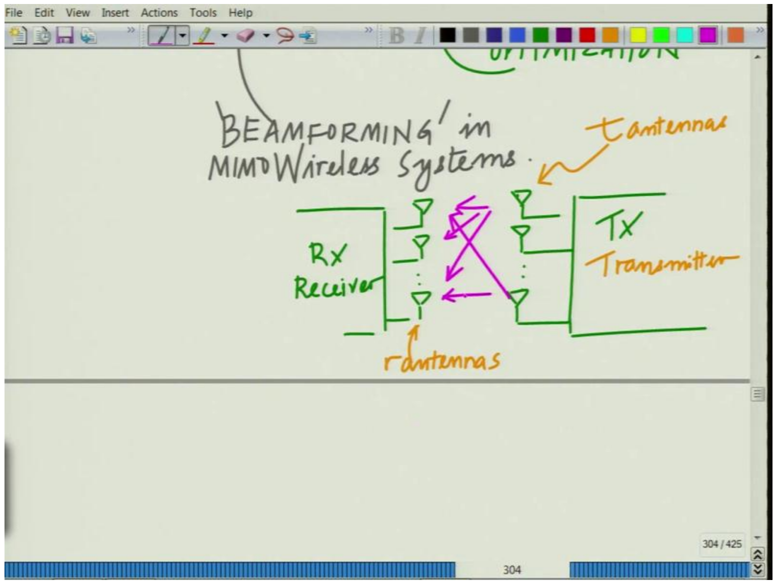

Let us again look at a typical MIMO system, which is basically, has multiple antennas at the receiver and multiple antennas at the transmitter. So you have the receiver, you have the transmitter, you have r antennas at the receiver, t antennas at the transmitter and then you have the channel coefficients where we know that we have the channel matrix. H denotes the channel matrix.

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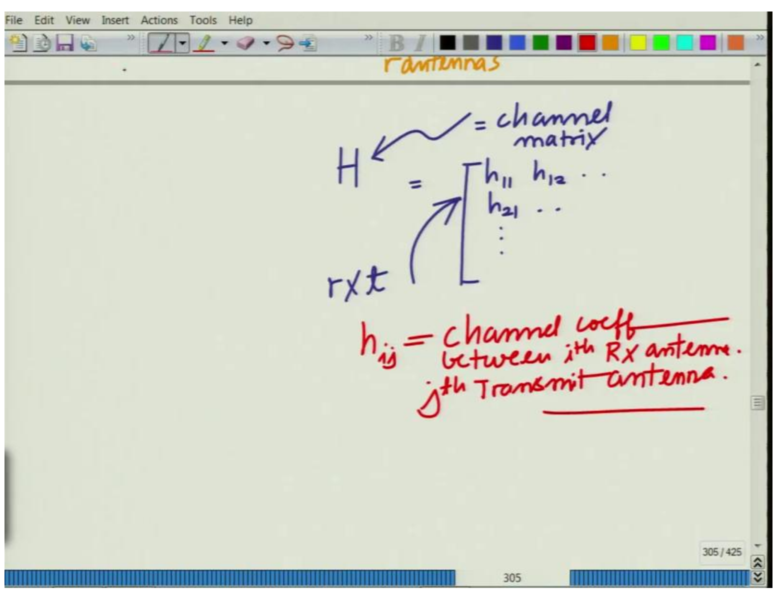

This is the channel matrix which is of the form, h11 h12 h21, so on. And this is an r cross t channel matrix, r cross t channel matrix which contains the channel coefficients. Hij is the channel coefficient between the ith receive antenna and the jth transmit antenna. So if you look at Hij, Hij is the channel coefficient between the ith receive antenna and j th transmit antenna.

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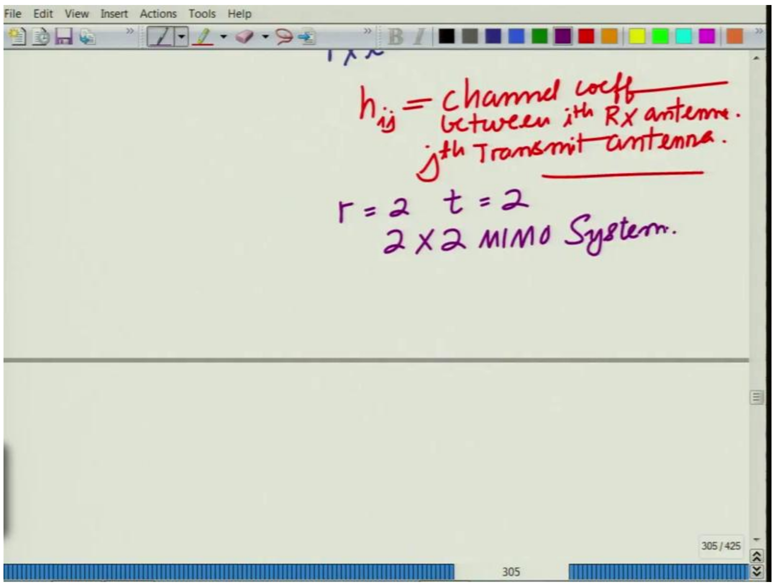

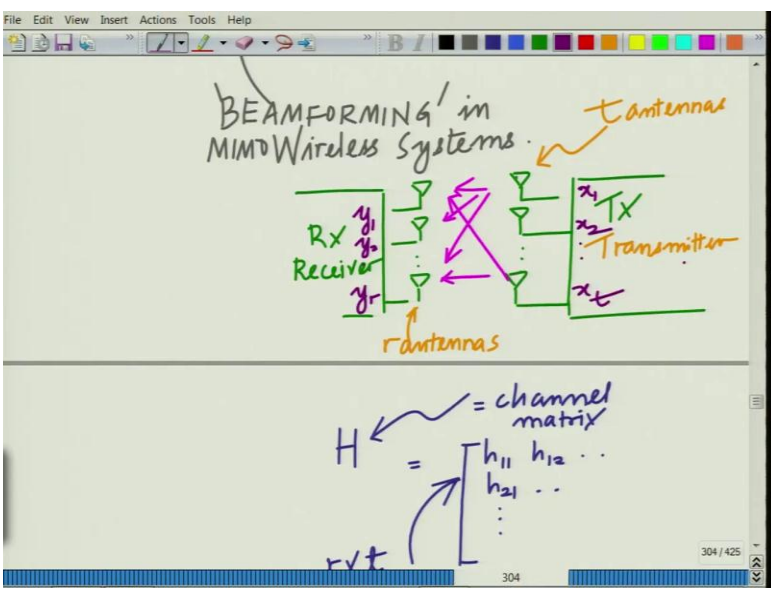

Now, let us look at a typical MIMO system. For instance, let us consider r equal to 2, t equal to 2, this is termed a 2 cross 2 MIMO system. Now, in this 2 cross 2 MMIMO system, now, we have the outputs across the receive antenna, let us call these as y1, y2, y r and we have the inputs which are the transmit symbols, x1, x2, x t.

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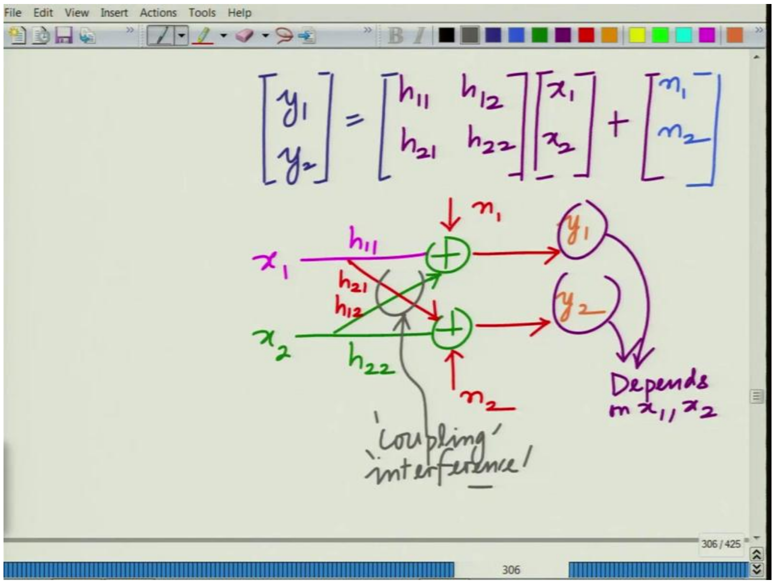

Now, the 2 cross 2 MIMO system model, you have two outputs, y1 comma y2. You can write these as, well, the channel coefficients, h11, h12, h21, h22 times x1 x2 plus the noise which is n1, n2. Now, let us draw a schematic diagram for this. So let us say, you have the symbol, that is your x1. And then you are essentially multiplying this by the channel coefficient, h11 and then you have the symbol x2, which you are multiplying by the channel coefficient h22.

In addition, x2, you multiply by h12, this is h12, and x1 by h21 and this is your noise n1, this is your noise n2 and then you have the outputs which are your y1 and y2. And therefore if you look at this, you can see clearly that y1 depends both, on x1 and x2. Similarly, y2 depends both, on x1 and x2. So there is a sort of coupling between these two systems, so there is interference, is what I mean by that.

So you look at this, y1 depends on x1, x2 and so on both. y1, y2 depends on x1, x2, so there is coupling. So if you look at these arrows, this part, this denotes what we call as the coupling. So these systems are coupled or, basically, there is interference between x1 and x2. x1 and x2 interfere with each other, you can also talk of this as some kind of cross talk between these two channels.

And naturally, if we have to determine x1 and x2 at the receiver, one has to resolve this thing. One has to remove the interference. Now, how to go from this coupled system, can we go from this coupled system to a decouple, how to obtain a decoupled system.

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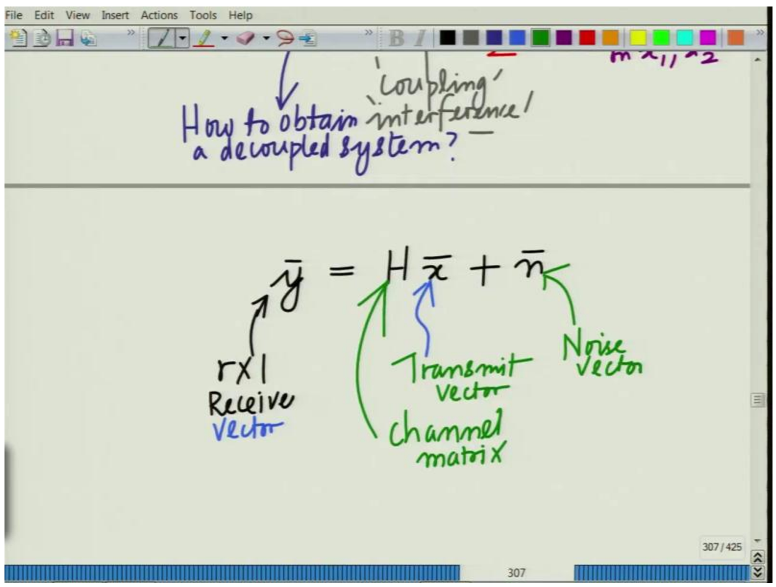

The question that we want to ask here is, how to obtain a decoupled system? And that is where we use the SVD. Now, let us look at the channel. Now, this is our model y bar equals H x bar plus n bar. This is our receive vector, r dimensional, containing the symbols. So this is our receive vector, this is our transmit vector, and this is our channel matrix, and this is the noise vector. Now, what we do over here is first, perform, so this is our channel matrix, perform the SVD of the channel matrix. So this is what we start with.

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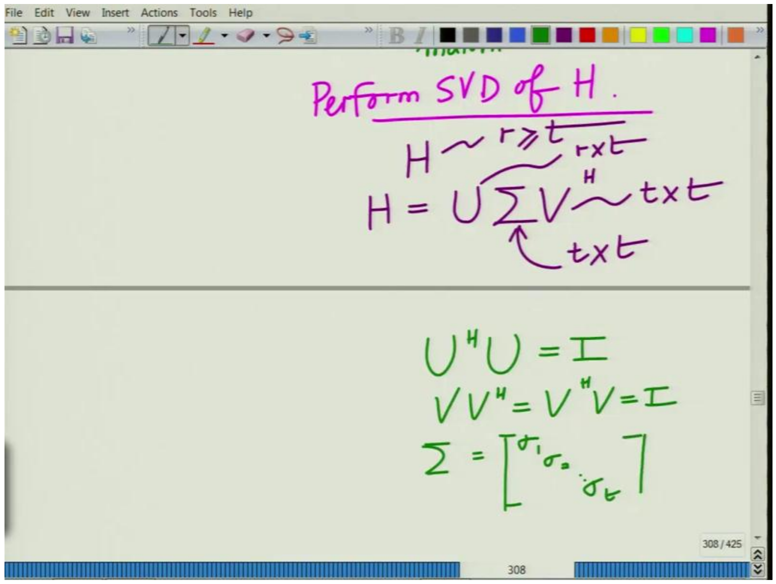

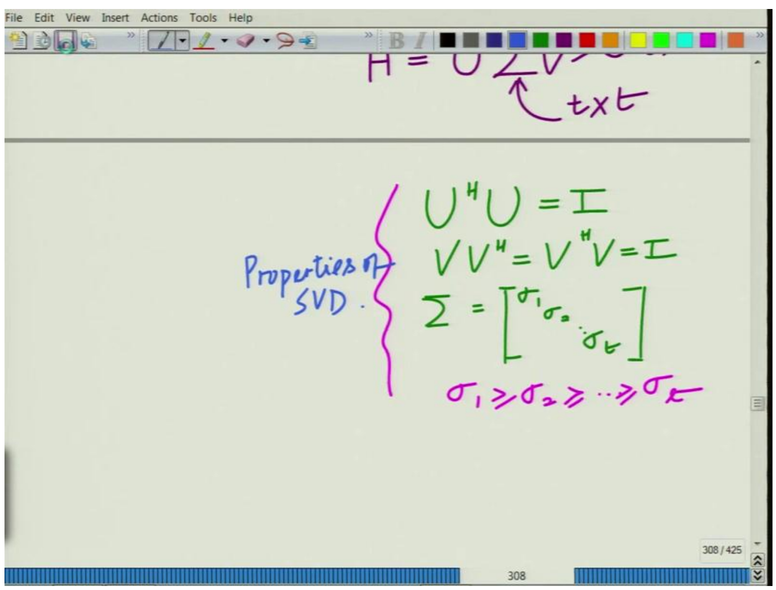

So we perform, we start by performing, in this journey of obtaining a decoupled system, we start by performing the SVD, singular value decomposition of the channel matrix H. And let us again consider a system with r greater than equal to t, number of receive antennas greater than equal to number of transmit antennas.

We can write the SVD as U Sigma V Hermitian, where U is, as we have seen, this is r cross t, this is t cross t and this is a t cross t matrix. And these satisfy the properties, quick recollection, U Hermitian U equals Identity, V V Hermitian equals V Hermitian V equals identity, sigma is a diagonal matrix comprising of the singular values which are non-negative and these are arranged in decreasing order. So this is roughly, if you recall, these are the properties of the singular value decomposition.

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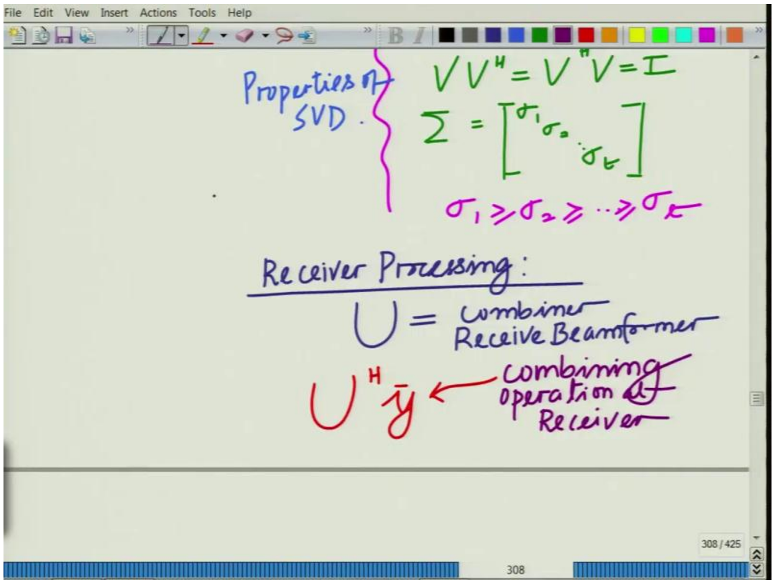

Now, at the receiver, receiver processing in the MIMO system. At the receiver, we perform, so we, U is the combiner, we call this as the combiner matrix or the receive Beam Former, which implies that at the receiver we perform U Hermitian y bar. So this is basically, the combining operation at the receiver.

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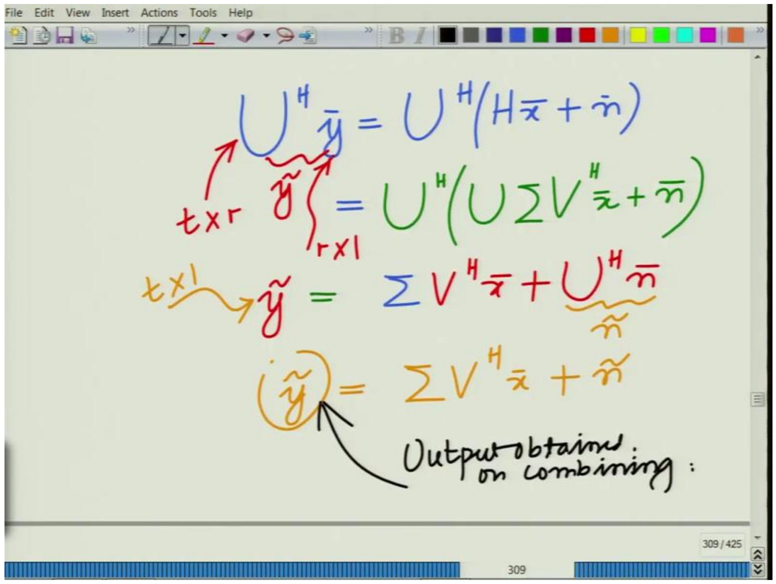

Therefore, now, once you use this in the MIMO model, you have U Hermitian y bar equals U Hermitian H x bar plus n bar. Now, substitute for H equals U Sigma V Hermitian, so this is U Hermitian U Sigma V hermitan x bar plus n bar. Now, we know, U Hermitian U is Identity, U is a semi-unitary matrix, that is, a matrix of left singular vectors. Using that property, now, this U Hermitian U is Identity, this reduces to simply, very interestingly, this reduces to Sigma V Hermitian x bar plus U Hermitian n bar, let us call this is a y tilde.

Now, U Hermitian is t cross r, y bar is r cross 1 which implies y tilde, this is going to be your t cross 1 vector, so y tilde equals and U Hermitian n bar, let us call this as n tilde, so we have y tilde equals Sigma V Hermitian x bar plus n tilde. So this is the model. So this is after, this is the y tilde, this is the combiner, combined vector. That is, output obtained after receive combining.

This is the output obtained after the combining operation at the receiver. Now, at the transmitter what we do is something interesting. At the transmitter, even prior to transmission, we also pre-process. Now, combining is the post-processing. In fact, in the MIMO system we also do pre-processing, that is at the transmitter.

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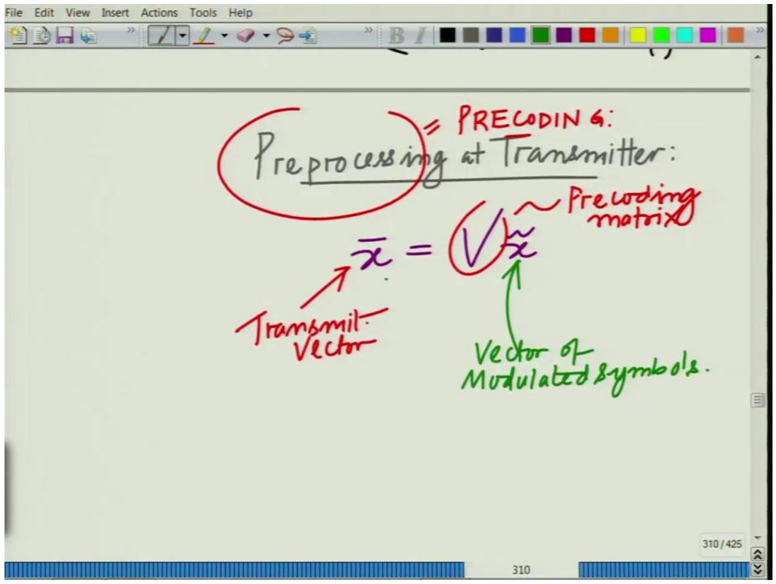

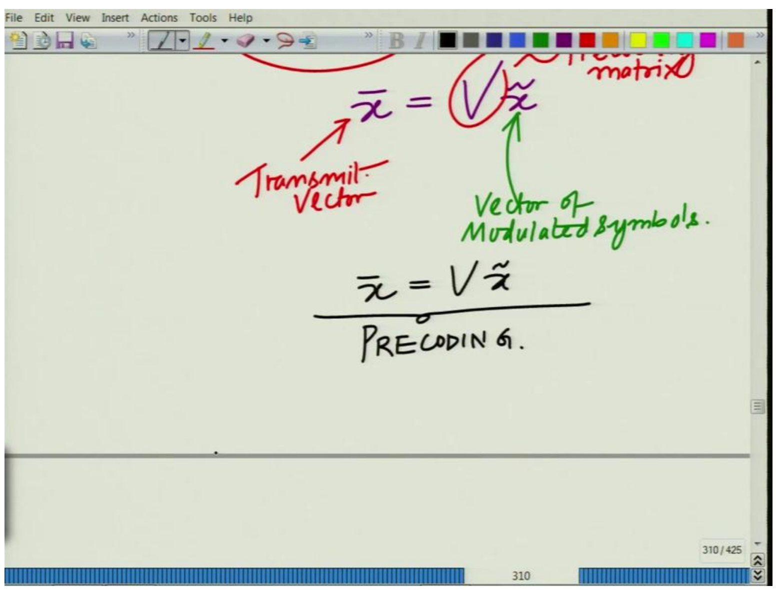

So what is the pre-processing? The processing, the pre-processing at the transmitter. So what happens here is this is your transmit vector x bar, this is generated as x bar equals V times x tilde. Now, this pre-processing, this is also known as a pre-coding. This is an important operation in MIMO. This is also known as condition or pre-conditioning or basically pre-coding. And this is basically, V is basically, your pre-coding matrix. So you have x bar that is a vector of transmit symbols x1, x2. x t and you have the vector of, modulator, the actual modulated information symbols, that is x1 tilde, x2 tilde, x t tilde.

These can be for instance receiver BPSK, QPSK and so on. And we pre-code them, that is, multiplied by the pre-coding matrix V. The pre-coding matrix V acts on this modulated symbols to produce the transmit symbol vectors, x bar. So this is a pre-coding matrix, then think of x bar as the transmit vector. And this is your vector of modulated symbols. And this is your pre-coding operation. x bar equals, so this the vector of modulated symbols and this x bar equals V x tilde. You can think of this as pre-coding. So this is pre-coding.

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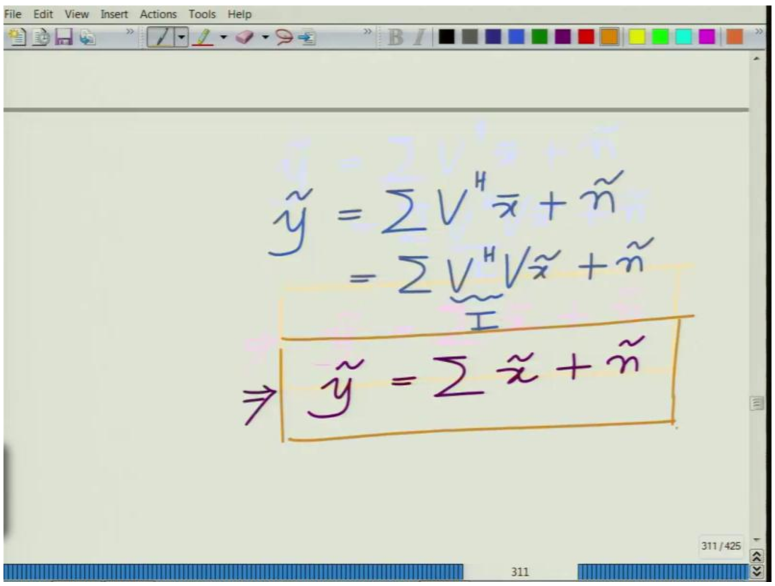

Now, you substitute this in our model that we have over here, that is, you have y tilde equals Sigma V Hermitian x bar plus, n tilde. You substitute the model, the pre-coding model, that is, V Hermitian V times x tilde plus n tilde and V Hermitian V is Identity, so this gives us, this implies, we are left with the model, this reduces to y tilde equals to y tilde equals to Sigma times x tilde plus n tilde. And this is very interesting. Why is this interesting?

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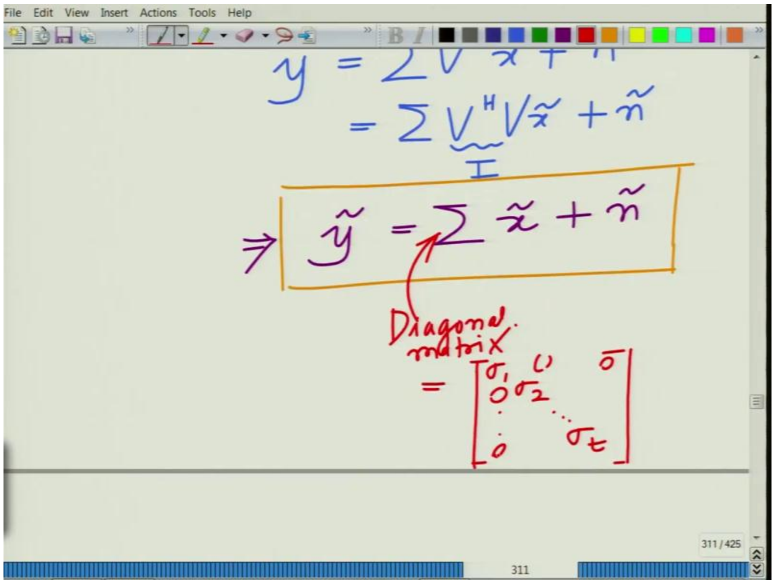

This is interesting because this matrix Sigma, this is a diagonal matrix which comprises of the entries Sigma1, Sigma2, this is a diagonal matrix.

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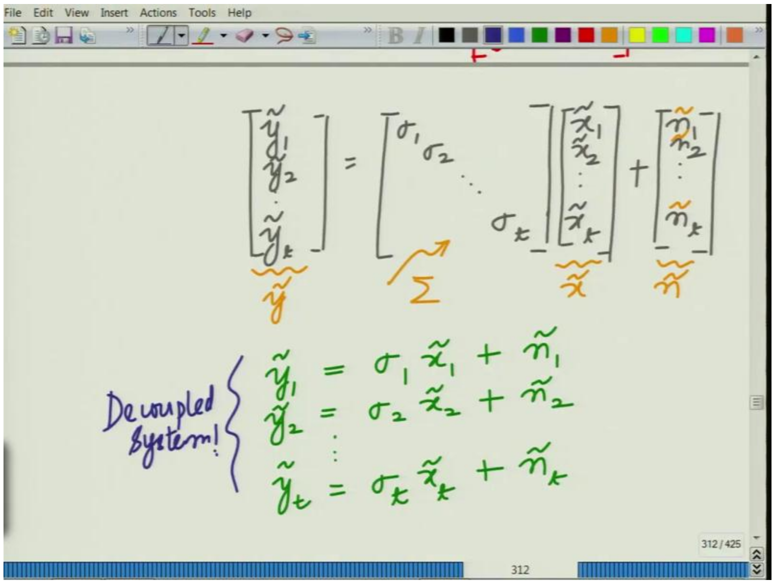

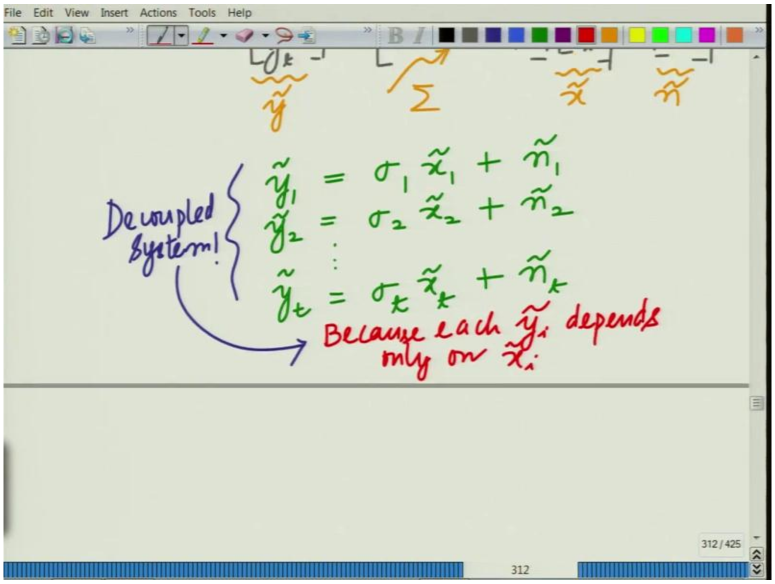

In fact, if you expand this model, you can write this as follows. That is, you have y1 tilde, y2 tilde, so on, y t tilde equals Sigma1, Sigma2, Sigma t times x1, x2, x t plus n1, n2, n t. So this is your MIMO system model. And interestingly if you write, I'm sorry, this is x1 tilde, x2 tilde, x3 tilde. And now, if you write these things and these are n1 tilde, n2 tilde, n t tilde.

And now, if write these things (explicitly), so this is your matrix y tilde, this is the diagonal matrix Sigma, this is your x tilde and this is your n tilde. And if you write these equations explicitly, then what you obtain is that y1 tilde equals Sigma1 x1 tilde plus n1 tilde. y2 tilde equals Sigma2 x2 tilde plus n2 tilde, so on, and you will get y t tilde equals Sigma t x t tilde plus n t tilde and therefore you can see, this is essentially a decoupled system.

Why is this a decoupled system? Because you can see each y i depends only, each y i tilde depends only on x i tilde. that is, y1 tilde depends on x1 tilde, y2 tilde depends on x2 tilde, so on and so forth, y t tilde depends only on x t tilde. So this is a decoupled system because each y tilde depends only on x i tilde.

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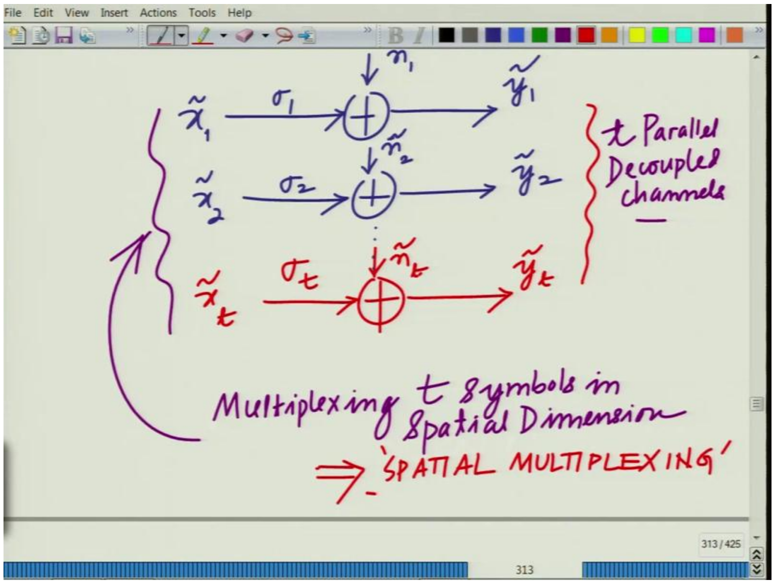

And if you represent this using a diagram, you will find that this is essentially, you have, x1 tilde, multiply this by Sigma1, add the noise, you have n1 tilde, add n1 tilde, you get y1 tilde, x2 tilde, you add the noise, n2 tilde, multiply by Sigma2 x2 tilde, this gives y2 tilde, so on and so forth. Finally, you have n t tilde, you have Sigma t and you have x t tilde and this gives y t tilde. And therefore, what you can see is, essentially you have, essentially this reduces to a set of t parallel decoupled channels. This reduces to a set of t parallel decoupled channels.

And therefore, now, you can see is that the MIMO system, if you think of a MIMO system, the MIMO system, r cross t MIMO system, r greater than equal to t, essentially can be decoupled into a set of t independent parallel channels in which, in each channel you are transmitting the symbol x i tilde. It is acted upon by the gain Sigma i and then of course the noise n i tilde and then you get the output, y i tilde.

And you have t such channels. So it is a collection or a bundling together or t such channels. And you are multiplexing these symbols, they are multiplexing these symbols, x1 tilde, x2 tilde, x3 tilde, same time, same frequency in the spatial dimension. Spatial dimension, by that we mean the antennas and therefore this is known as Spatial Multiplexing.

So this property of the MIMO system essentially, where you are multiplexing these multiple symbols, so you are multiplexing t symbols in the spatial dimension and this is essentially what is termed as Spatial Multiplexing. So you are multiplexing, so what the MIMO system is allowing you to do is to the same time, same frequency, it is allowing you to increase the capacity by a factor of t, that is essentially a minimum of r comma t.

So the capacity increases by a factor of t or the rate increases by a factor of t because you are able to multiplex these t symbols spatially, in the using, in leveraging the spatial dimension which is not, by the way, exploited when you have single transmit and single input antenna. So this is something which is unique to MIMO system. So MIMO system results in a t fold increase in a capacity.

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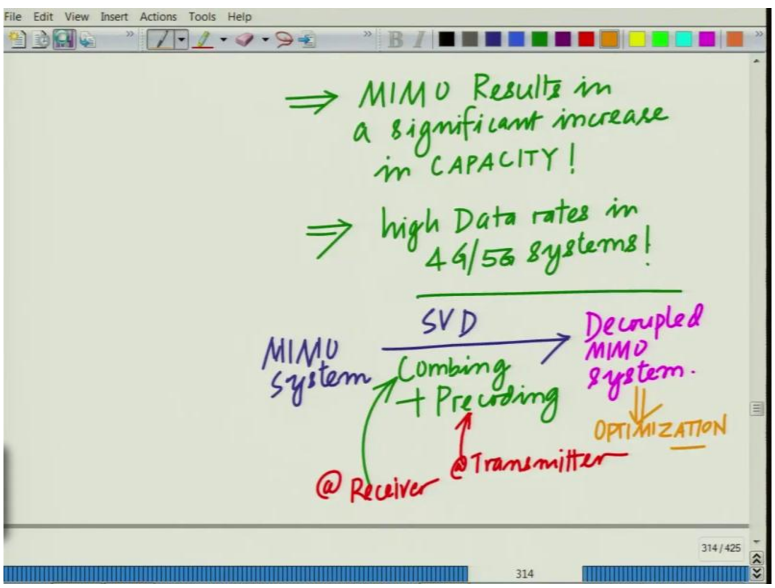

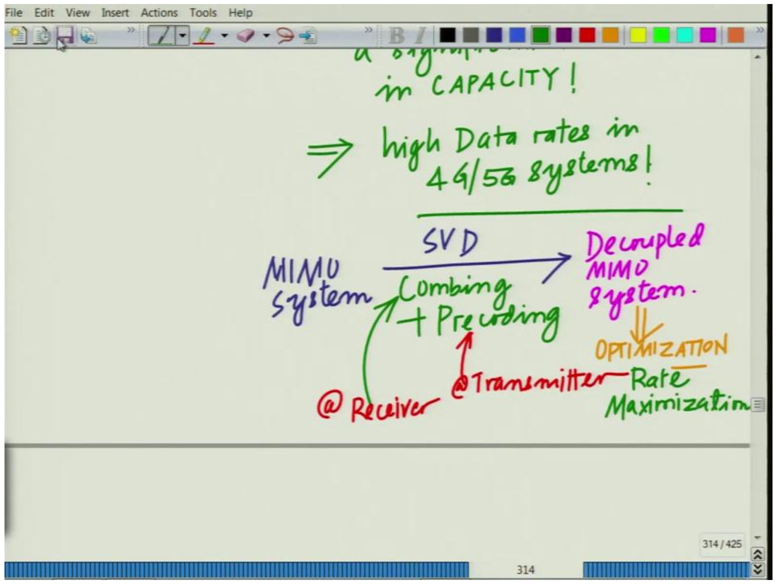

So this implies, because of Spatial Multiplexing, this implies MIMO, and this is what leads to high data rates in 4G, 5G systems. That is, you have, of course, you have a large bandwidth but in addition, you can also, one can also exploit the spatial dimension to transmit or multiplex multiple symbols for the same time and frequency resource.

And therefore, essentially, what this singular value decomposition is doing, is essentially, the singular value decomposition is giving you a technique to convert, so what does this singular, SVD do? SVD is giving you a technique to convert this coupled MIMO system with the interference between the symbols, that is, your x1, x2, x3, and these symbols are interfering as we have seen earlier. So SVD is a very interesting and effective tool to take this coupled MIMO system with interference and go to a decoupled MIMO system via pre-coding at the transmitter and combining at the receiver.

So what we have, essentially, is, you have the MIMO system. We have performed the SVD which is basically your combining plus pre-coding. The combining is at the transmitter, the pre-coding, so the combining is being performed at the transmitter, I am sorry, combining is being performed at the receiver and the pre-coding is being performed at the transmitter. And the SVD gives you, what it gives you is, essentially, it gives you the decoupled. And this can now be optimized as we shall see in the next module.

How can this decoupled MIMO system makes it easier for optimization, processing and so on. So one can use this decoupled MIMO system for optimization. What do we mean by optimization? We can use it for rate maximization, to maximize the transmit rate. I can use this for rate maximization, which is what we will look at in the subsequent module. So, as we have seen SVD has significant practical implications.

One of the most important practical implications, we can say, is to take this MIMO system, convert it from a coupled MIMO system to a decoupled MIMO system and then use it for further optimization and then that in turn, because of the Spatial Multiplexing property of MIMO, where you can transmit several symbols in parallel over the same time, frequency resource, that leads to a significant high, significantly high throughput in 4G and 5G wireless systems. So let us stop here and continue in the subsequent modules. Thank you very much.